Mathematical Model Only

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1 APPENDIX A

1.1 sets

Table 1: Index sets used in the model formulation.

Index / Set	Description
s	Set of suppliers.
i	Set of manufacturing plants.
j	Set of distribution centers.
k	Set of customers.
v	Set of transportation modes.
r	Set of raw materials.
p	Set of products.
e	Set of energy sources.
m	Set of processing equipment or machines.
a	Set of replacement equipment parts.
t_s	Scheduling time scale for operational decisions.
t_n	Network time scale for design decisions.
τ	Lead time.
ω	Set of scenarios.

1.2 Variables

Table 2: Decision Variables Used in the Model Formulation

Symbol	Description
$X^{ship}_{rsivt_nt_s}$	Quantity of raw material r shipped from supplier s to manufacturing plant i using transportation mode v during network time scale t_n and scheduling time scale t_s.
$X^{ship}_{pijvt_nt_s}$	Quantity of product p shipped from manufacturing plant i to distribution center j using transportation mode v during network time scale t_n and scheduling time scale t_s.

Symbol	Description
$X^{ship}_{pjkvt_nt_s}$	Quantity of product p shipped from distribution center j to customer k using transportation mode v during network time scale t_n and scheduling time scale t_s.
Cap_{sivt_n}	Capacity of purchased transportation mode v used to transport material from supplier s to manufacturing plant i at network time scale t_n.
Cap_{ijvt_n}	Capacity of purchased transportation mode v used to transport product from manufacturing plant i to distribution center j at network time scale t_n.
Cap_{jkvt_n}	Capacity of purchased transportation mode v used to transport product from distribution center j to customer k at network time scale t_n.
Cap_{mit_n}	Capacity of processing equipment or machine m installed at manufacturing plant i at network time scale t_n.
$Cap^{store}_{rit_n}$	storage capacity for raw material r installed at i manufacturing plant at network time scale t_n.
$Cap^{store}_{pit_n}$	storage capacity for product p installed at i manufacturing plant at network time scale t_n.
$Cap^{DC}_{jt_n}$	Capacity of distribution center j installed at network time scale t_n.
$X^{use}_{rimt_nt_s}$	Quantity of raw material r used at manufacturing plant i on machine m during network time scale t_n and scheduling time scale t_s.
$X^{prod}_{pimt_nt_s}$	Quantity of product p manufactured at manufacturing plant i using machine m during network time scale t_n and scheduling time scale t_s.
$X^{target}_{pit_nt_s}$	Planned production target of product p at manufacturing plant i during network time scale t_n and scheduling time scale t_s.
$Inv_{rit_nt_s}$	Inventory level of raw material r at manufacturing plant i during network time scale t_n and scheduling time scale t_s.
$Inv_{pit_nt_s}$	Inventory level of product p at manufacturing plant i during network time scale t_n and scheduling time scale t_s.
$Inv_{pjt_nt_s}$	Inventory level of product p at distribution center j during network time scale t_n and scheduling time scale t_s.
$Dem^{fill}_{t_nt_s}$	Demand fulfillment rate at customer k during network time scale t_n and scheduling time scale t_s.
$E^{ship}_{rsivt_nt_s}$	CO_2 emissions generated from transporting raw material r from supplier s to manufacturing plant i using transportation mode v during network time scale t_n and scheduling time scale t_s.
$E^{ship}_{pijvt_nt_s}$	CO_2 emissions generated from transporting product p from manufacturing plant i to distribution center j using transportation mode v during network time scale t_n and scheduling time scale t_s.
$E^{ship}_{pjkvt_nt_s}$	CO_2 emissions generated from transporting product p from distribution center j to customer k using transportation mode v during network time scale t_n and scheduling time scale t_s.

Symbol	Description
$E^{prod}_{pimt_nt_s}$	CO_2 emissions generated from manufacturing product p at plant i using machine m during network time scale t_n and scheduling time scale t_s.
$E^{SCN}_{t_nt_s}$	Total CO_2 emissions of the supply chain network during network time scale t_n and scheduling time scale t_s.
$C^{capex}_{sivt_n}$	CAPEX cost of transportation mode v used between supplier s and manufacturing plant i at network time scale t_n.
$C^{capex}_{ijvt_n}$	CAPEX cost of transportation mode v used between manufacturing plant i and distribution center j at network time scale t_n.
$C^{capex}_{jkvt_n}$	CAPEX cost of transportation mode v used between distribution center j and customer k at network time scale t_n.
$C^{capex}_{mit_n}$	CAPEX cost of purchasing machine m at manufacturing plant i at network time scale t_n.
$C^{capex}_{jt_n}$	CAPEX cost of to build a new distribution center j including storage capacity at network time scale t_n.
$C^{capex,store}_{rit_n}$	CAPEX cost of to build a storage capacity for raw material r a plant i at network time scale t_n.
$C^{capex,store}_{pit_n}$	CAPEX cost of to build a storage capacity for product p a plant i at network time scale t_n.
$C^{capex_total_Network}_{t_n}$	total Capital investment cost associated with machine purchase, transport mode contracting for producing and moving product p throughout the entire supply chain network during network time scale t_n and scheduling time scale t_s.
$C^{opex}_{rsivt_nt_s}$	Operating cost associated with contracting a vehicle v to transport raw material r from supplier s to manufacturing plant i during network time scale t_n and scheduling time scale t_s.
$C^{opex}_{pijvt_nt_s}$	Operating cost associated with contracting a vehicle v to transport product p from manufacturing plant i to distribution center$j$ during network time scale t_n and scheduling time scale t_s.
$C^{opex}_{pjkvt_nt_s}$	Operating cost associated with contracting a vehicle v to transport product p from distribution i to customer k during network time scale t_n and scheduling time scale t_s.
$C^{opex_prod}_{pit_nt_s}$	Operating cost associated with manufacturing product p at plant i during network time scale t_n and scheduling time scale t_s.
$C^{buy_raw}_{rit_nt_s}$	Cost of purchasing raw material r for manufacturing plant i during network time scale t_n and scheduling time scale t_s.
$C^{opex}_{pjt_nt_s}$	Operating cost associated with processing product p at distribution center j during network time scale t_n and scheduling time scale t_s.
$C^{opex_total_Network}_{t_s}$	total Operating cost associated with processing product , shipping product p throughout the entire supply chain network during network time scale t_n and scheduling time scale t_s.
C^{SCN}	Overall total cost of the supply chain network.
y_{sivt_n}	Binary variable equal to 1 if transportation mode v is selected between supplier s and plant i at network time scale t_n; 0 otherwise.

Symbol	Description
y_{ijvt_n}	Binary variable equal to 1 if transportation mode v is selected between plant i and distribution center j at network time scale t_n; 0 otherwise.
y_{jkvt_n}	Binary variable equal to 1 if transportation mode v is selected between distribution center j and customer k at network time scale t_n; 0 otherwise.
y_{mit_n}	Binary variable equal to 1 if machine m is selected for installation at manufacturing plant i at network time scale t_n; 0 otherwise.
$y^{store}_{rit_n}$	Binary variable equal to 1 if storage capacity for raw r is selected for installation at manufacturing plant i at network time scale t_n; 0 otherwise.
$y^{store}_{pit_n}$	Binary variable equal to 1 if storage capacity for product p is selected for installation at manufacturing plant i at network time scale t_n; 0 otherwise.
y_{jt_n}	Binary variable equal to 1 if distribution center j is selected for installation at DC j at network time scale t_n; 0 otherwise.
$Cap^{eng}_{eit_n}$	Installed capacity of energy source e at manufacturing plant i during network time scale t_n.
$W^{gen}_{eit_nt_s}$	Energy generated from source e at manufacturing plant i during network time scale t_n and scheduling time scale t_s.
$W^{total}_{et_nt_s}$	Total energy generated from all energy sources during network time scale t_n and scheduling time scale t_s.
$C^{capex}_{et_n}$	CAPEX cost of energy capacity installed for production at manufacturing plant i at network time scale t_n.
$C^{capex_maint}_{et_n}$	CAPEX fix cost of maintenance of energy generated manufacturing plant i at network time scale t_n.
$C^{capex_sum}_{et_n}$	CAPEX total capex cost for energy generate at network time scale t_n.
$C^{opex_grid}_{eit_n}$	operational cost of for purchasing electricity from the grid for production at manufacturing plant i and operating the Distribution center j at network time scale t_n and scheduling time scale t_s.
$C^{opex_maint}_{et_n}$	operational cost of maintenance of equipment for energy generated for production at manufacturing plant i at network time scale t_n and scheduling time scale t_s.
$C^{opex_sum}_{et_nt_s}$	total operational cost for generating and purchasing energy at network time scale t_n and scheduling time scale t_s.
$C^{SCN}_{et_n}$	overall cost CAPEX and OPEX for energy used across the manufacturing plant i at network time scale t_n and scheduling time scale t_s.
$E^{gen}_{eit_nt_s}$	CO_2 emissions generated from energy source e at manufacturing plant i during network time scale t_n and scheduling time scale t_s.
$E^{gen_SCN}_{et_nt_s}$	CO_2 total emissions generated from energy source e at manufacturing plant i during network time scale t_n and scheduling time scale t_s.

Symbol	Description
$Arriv_{rsivt_nt_s}$	Quantity of raw material r arriving at manufacturing plant i from supplier s using transportation mode v during network time scale t_n and scheduling time scale t_s.
$Arriv_{pijvt_nt_s}$	Quantity of product p arriving at distribution center j from manufacturing plant i using transportation mode v during network time scale t_n and scheduling time scale t_s.
$Arriv_{pjkvt_nt_s}$	Quantity of product p arriving at customer k from distribution center j using transportation mode v during network time scale t_n and scheduling time scale t_s.
$Arriv^{dem}_{pjkvt_nt_s}$	Quantity of product p arriving at customer k from distribution center j based on today's demand at time t_s using transportation mode v during network time scale t_n and scheduling time scale t_s.
$Arriv^{back}_{pjkvt_nt_s}$	Quantity of product p arriving at customer k from distribution center j based on backlog at time t_s using transportation mode v during network time scale t_n and scheduling time scale t_s.
$Backlog_{pkt_nt_s}$	Quantity of unmet demand for product p at customer k carried over to future periods during network time scale t_n and scheduling time scale t_s.
$unmetdemand_{pkt_nt_s}$	unmet demand for product p at customer k , New unmet demand at scheduling period t_s (today's shortfall only)
$C^{backlog}_{t_nt_s}$	Total backlog penalty cost for unmet demand carried overtime during network time scale t_n and scheduling time scale t_s.
$Backlog^{prod}_{pit_nt_s}$	Production backlog only of product p at manufacturing plant i during network time scale t_n and scheduling time scale t_s.
$\mathbb{E}[C^{opex}]$	Expected operating cost of the supply chain network across all scenarios.
$\mathbb{E}[C^{SCN}]$	Expected total supply chain network cost across all scenarios.

1.3 Parameters

Table 3: Parameters Used in the Model Formulation

Symbol	Description
α_{mr}	Conversion factor of raw material r using processing equipment or machine m to produce product p.
τ^{raw}_{si}	Lead time of raw material from supplier s to manufacturing plant i in days.
τ^{prod}_{ij}	Lead time of product from manufacturing plant i to distribution center j in days.
τ^{prod}_{jk}	Lead time of product from distribution center j to customer site k in days.
$I^{0,raw}_{ri}$	Initial inventory of raw material r at manufacturing plant i in kg/day.
$I^{0,prod}_{pi}$	Initial inventory of product p at manufacturing plant i in kg/day.

Symbol	Description
$I_{pit_nt_s}^{target}$	Target ending inventory of product p at manufacturing plant i during network time scale t_n and scheduling time scale t_s in kg/day.
$I_{pj}^{0,DC}$	Initial inventory of product p at distribution center j in kg/day.
η_{pi}^{eng}	Energy needed per kg of product p at manufacturing plant i in KW-day/kg.
ρ_{pi}^{eng}	Demand fill rate for energy required by product p at manufacturing plant i.
κ_e^{capex}	CAPEX cost for installing energy source type e in \$/KW.
κ_e^{maint}	CAPEX Fixed maintenance cost per energy source type e in \$/KW-year.
π_{ei}^{eng}	operational cost, Price per energy source e used at manufacturing plant i in \$/KW-day.
$\kappa_e^{opex_maint}$	operational cost for maintenance per energy source type e in \$/KW-year.
ϕ_e^{cap}	Capacity factor for each energy source e.
$\overline{W}_{peim}$	Maximum energy limit for product p, per energy source e, at manufacturing plant i, and for machine m in KW-day.
ϵ_{ei}^{eng}	Emission factor for energy source type e at manufacturing plant i in kg CO_2-eq/kWh.
ϵ_v^{trans}	Emission factor of transportation mode v in kg CO_2 per kg-unit-mile transported.
ϵ_{pm}^{prod}	Emission factor associated with manufacturing product p using machine m in kg CO_2 per kg of product produced.
d_{si}	Distance from supplier node s to manufacturing plant i in miles.
d_{ij}	Distance from manufacturing plant i to distribution center j in miles.
d_{jk}	Distance from distribution center j to customer node k in miles.
ρ_{pk}^{dem}	Demand fulfillment rate requirement for product p at customer node k.
$Dem_{pkt_nt_s}$	Customer demand of product p at customer node k during network time scale t_n and scheduling time scale t_s in kg.
$\kappa_{mi}^{capex_m}$	Annualized CAPEX cost for purchasing and installing processing machine m at manufacturing plant i.
$\kappa_{ri}^{capex_store}$	Annualized CAPEX cost for installing a storage capacity for raw material r at manufacturing plant i.
$\kappa_{pi}^{capex_store}$	Annualized CAPEX cost for installing a storage capacity for product p at manufacturing plant i.
$\kappa_j^{capex_DC}$	Annualized CAPEX cost for constructing and installing distribution center j.
$\kappa_v^{capex_car}$	Annualized CAPEX cost for purchasing transportation mode v.
c_i^{opex}	Operating cost of manufacturing plant i, including manufacturing, facility management, storage, and utilities in \$/kg.
c_j^{opex}	Operating cost of distribution center j, including storage and utility costs in \$/kg.

Symbol	Description
c_v^{trans}	Operating transportation cost associated with transportation mode v, including fuel and driver cost in \$/kg transported.
$c_r^{buy_raw}$	Purchasing price of raw material r in \$/kg.
$\overline{Cap}_s^{big_M}$	Maximum supply capacity of supplier node s.
$\underline{Cap}_m^{min}$	Minimum processing capacity of machine m at manufacturing plant i in kg.
$\overline{Cap}_m^{max}$	Maximum processing capacity of machine m at manufacturing plant i in kg.
$\overline{Cap}_j^{max}$	Maximum storage and handling capacity of distribution center j.
$\underline{Cap}_j^{min}$	Minimum operating capacity requirement of distribution center j.
$\underline{Cap}_v^{min}$	Minimum transportation capacity of vehicle or transportation mode v in kg of product.
$\overline{Cap}_v^{max}$	Maximum transportation capacity of vehicle or transportation mode v in kg of product.
$\gamma_{imt_nt_s}$	Availability factor of machine m at manufacturing plant i during network time scale t_n and scheduling time scale t_s.
c_k^{unmet}	fix backlog penalty cost associated with one unit of unmet demand backlog at customer node k network time scale t_n and scheduling time scale t_s.
$c_k^{backlog}$	Penalty cost associated with carried over one unit of unmet demand backlog at customer node k.

2 Objective Functions

The proposed supply chain optimization model considers two objective functions. The first objective minimizes the total supply chain network cost, while the second objective minimizes the total supply chain network emissions.

2.0.1 Objective 1: Total Supply Chain Cost Minimization

The first objective function minimizes the overall supply chain network cost, including total operating expenditures and capital expenditures across the full planning horizon.

$$\min Z_1 = C^{SCN} \tag{1}$$

where the total supply chain cost is defined as:

$$C^{SCN} = \sum_{t_n \in T_N} \sum_{t_s \in T_S} C_{t_n t_s}^{opex} + \sum_{t_n \in T_N} C_{t_n}^{capex} \tag{2}$$

2.0.2 Objective 2: Total Supply Chain Emission Minimization

The second objective function minimizes the overall supply chain network emissions, including transportation-related emissions, production-related emissions, and energy-related emissions across the full planning horizon. these are representative of emissions scope 1 scope 2 and scope 3.

$$\min Z_2 = E^{SCN} \tag{3}$$

where the total supply chain emission is defined as:

$$E^{SCN} = E^{trans,total} + \sum_{t_n \in T_N} \sum_{t_s \in T_S} E^{prod}_{t_n t_s} + \sum_{t_n \in T_N} \sum_{t_s \in T_S} E^{genSCN}_{t_n t_s} \tag{4}$$

3 CONSTRAINTS

3.1 supply Constraints

The raw material usage constraint ensures that the quantity of raw material consumed at each manufacturing plant and machine is proportional to the quantity of product produced through the corresponding conversion factor.

$$X^{use}_{rimt_nt_s} = \sum_{p \in P} \alpha_{mr} X^{prod}_{pimt_nt_s} \qquad \forall r \in R,\ i \in I,\ m \in M,\ t_n \in T_N,\ t_s \in T_S \tag{5}$$

3.2 Lead time Constraints

This constraint ensures that raw material shipped from supplier s to manufacturing plant i arrives after the corresponding raw-material transportation lead time. It's important to note that, lead time parameter where assumed to be zero in the deterministic and stochastic.

$$Arriv_{rsivt_nt_s} = \begin{cases} X^{ship}_{rsivt_nt_s}, & \text{if } \tau^{raw}_{si} = 0 \\ 0, & \text{if } t_s < \tau^{raw}_{si} \\ X^{ship}_{rsiv,t_n,t_s-\tau^{raw}_{si}}, & \text{if } t_s \geq \tau^{raw}_{si} \end{cases} \qquad \forall r \in R,\ s \in S,\ i \in I,\ v \in V,\ t_n \in T_N,\ t_s \in T_S \tag{6}$$

This constraint ensures that product shipped from manufacturing plant i to distribution center j arrives after the corresponding plant-to-distribution-center lead time.

$$Arriv_{pijvt_nt_s} = \begin{cases} 0, & \text{if } t_s - \tau^{prod}_{ij} \notin T_S \\ X^{ship}_{pijv,t_n,t_s-\tau^{prod}_{ij}}, & \text{if } t_s - \tau^{prod}_{ij} \in T_S \end{cases} \qquad \forall p \in P,\ i \in I,\ j \in J,\ v \in V,\ t_n \in T_N,\ t_s \in T_S \tag{7}$$

This constraint ensures that product shipped from distribution center j to customer k arrives after the corresponding distribution-center-to-customer lead time.

$$Arriv_{pjkvt_nt_s} = \begin{cases} X^{ship}_{pjkvt_nt_s}, & \text{if } \tau^{prod}_{jk} = 0 \\ 0, & \text{if } t_s < \tau^{prod}_{jk} \\ X^{ship}_{pjkv,t_n,t_s-\tau^{prod}_{jk}}, & \text{if } t_s \geq \tau^{prod}_{jk} \end{cases} \qquad \forall p \in P,\ j \in J,\ k \in K,\ v \in V,\ t_n \in T_N,\ t_s \in T_S \tag{8}$$

3.3 Inventory Constraints

The inventory balance constraints track material and product flow of inventories at manufacturing plants and distribution centers over both the network time scale t_n and the scheduling time scale t_s.

The raw material inventory balance ensures that the inventory of raw material r at manufacturing plant i equals the previous raw material inventory plus raw material arrivals from suppliers, minus the raw material consumed by all machines.

$$Inv_{rit_nt_s} = Inv^{prev}_{rit_nt_s} + \sum_{s\in S}\sum_{v\in V} Arriv_{rsivt_nt_s} - \sum_{m\in M} X^{use}_{rimt_nt_s} \qquad \forall r \in R,\ i \in I,\ t_n \in T_N,\ t_s \in T_S \tag{9}$$

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The inventory level of each raw material stored at a manufacturing plant cannot exceed the installed raw material storage capacity of that facility.

$$Inv_{rit_nt_s} \le Cap^{store}_{rit_n}, \qquad \forall r \in R,\ i \in I,\ t_n \in T_n,\ t_s \in T_s \tag{10}$$

The product inventory balance at the manufacturing plant ensures that finished product inventory equals previous inventory plus current production, minus product shipped from the plant to distribution centers.

$$Inv_{pit_nt_s} = Inv^{prev}_{pit_nt_s} + \sum_{m\in M} X^{prod}_{pimt_nt_s} - \sum_{j\in J}\sum_{v\in V} X^{ship}_{pijvt_nt_s} \qquad \forall p \in P,\ i \in I,\ t_n \in T_N,\ t_s \in T_S \tag{11}$$

Similarly, the inventory level of finished products stored at a manufacturing plant cannot exceed the installed product storage capacity.

$$Inv_{pit_nt_s} \le Cap^{store}_{pit_n}, \qquad \forall p \in P,\ i \in I,\ t_n \in T_n,\ t_s \in T_s \tag{12}$$

The product inventory balance at the distribution center ensures that product inventory equals previous distribution center inventory plus arrivals from manufacturing plants, minus product shipments to customers.

$$Inv_{pjt_nt_s} = Inv^{prev}_{pjt_nt_s} + \sum_{i\in I}\sum_{v\in V} Arriv_{pijvt_nt_s} - \sum_{k\in K}\sum_{v\in V} X^{ship}_{pjkvt_nt_s} \qquad \forall p \in P,\ j \in J,\ t_n \in T_N,\ t_s \in T_S \tag{13}$$

The previous inventory terms define the inventory carried into the current scheduling period. At the first network and scheduling period, the previous inventory is equal to the initial inventory. At the first scheduling period of a later network period, the previous inventory is the ending inventory from the last scheduling period of the previous network period. Otherwise, the previous inventory is the inventory from the immediately preceding scheduling period.

$$Inv^{prev}_{rit_nt_s} = \begin{cases} I^{0,raw}_{ri}, & \text{if } t_n = t_n^1 \text{ and } t_s = t_s^1 \\ Inv_{ri,t_n-1,t_s^{last}}, & \text{if } t_s = t_s^1 \text{ and } t_n > t_n^1 \\ Inv_{ri,t_n,t_s-1}, & \text{otherwise} \end{cases} \tag{14}$$

$$Inv^{prev}_{pit_nt_s} = \begin{cases} I^{0,prod}_{pi}, & \text{if } t_n = t_n^1 \text{ and } t_s = t_s^1 \\ Inv_{pi,t_n-1,t_s^{last}}, & \text{if } t_s = t_s^1 \text{ and } t_n > t_n^1 \\ Inv_{pi,t_n,t_s-1}, & \text{otherwise} \end{cases} \tag{15}$$

$$Inv^{prev}_{pjt_nt_s} = \begin{cases} I^{0,DC}_{pj}, & \text{if } t_n = t_n^1 \text{ and } t_s = t_s^1 \\ Inv_{pj,t_n-1,t_s^{last}}, & \text{if } t_s = t_s^1 \text{ and } t_n > t_n^1 \\ Inv_{pj,t_n,t_s-1}, & \text{otherwise} \end{cases} \tag{16}$$

where t_s^{last} represents the last scheduling time period within the network horizon, t_n^1 represents the first network time period, and t_s^1 represents the first scheduling time period within the network horizon.

3.4 Energy Constraints

The energy constraints represent energy generation, energy demand satisfaction, energy-related cost, and energy-related emissions across manufacturing plants,at a particular network time periods, and scheduling time periods.

The energy generation capacity constraint ensures that the energy generated from each energy source cannot exceed the installed energy capacity multiplied by the capacity factor.

$$W^{gen}_{eit_nt_s} \leq \phi^{cap}_e Cap^{eng}_{eit_n} \qquad \forall e \in E,\ i \in I,\ t_n \in T_N,\ t_s \in T_S \tag{17}$$

The energy demand satisfaction constraint ensures that the total energy generated at each manufacturing plant is equal to the energy required to support production at that plant.

$$\sum_{e\in E} W^{gen}_{eit_nt_s} = \sum_{p\in P}\sum_{m\in M} X^{prod}_{pimt_nt_s}\rho^{eng}_{pi}\eta^{eng}_{pi} \qquad \forall i \in I,\ t_n \in T_N,\ t_s \in T_S \tag{18}$$

The total energy generation constraint calculates the total amount of energy generated from all energy sources and plants during each time period.

$$W^{total}_{t_nt_s} = \sum_{e\in E}\sum_{i\in I} W^{gen}_{eit_nt_s} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{19}$$

The grid energy limit constraint ensures that the amount of grid electricity used at each manufacturing plant does not exceed the maximum grid energy allowed for production.

$$W^{gen}_{Grid,it_nt_s} \leq \sum_{p\in P}\sum_{m\in M} W_{p,Grid,i,m} X^{prod}_{pimt_nt_s} \qquad \forall i \in I,\ t_n \in T_N,\ t_s \in T_S \tag{20}$$

The energy installation CAPEX cost is calculated as the installed capacity of each energy source multiplied by the corresponding installation cost.

$$C^{capex}_{et_n} = \sum_{e\in E}\sum_{i\in I} Cap^{eng}_{eit_n}\kappa^{capex}_e \qquad \forall t_n \in T_N \tag{21}$$

The energy CAPEX maintenance cost is calculated as the total energy generated from each source multiplied by its CAPEX maintenance cost.

$$C^{capexmaint}_{et_n} = \sum_{e\in E}\sum_{i\in I} Cap^{eng}_{eit_n}\kappa^{maint}_e \qquad \forall t_n \in T_N \tag{22}$$

The total energy CAPEX cost is equal to the energy installation cost plus the CAPEX maintenance cost.

$$C^{capexsum}_{et_n} = C^{capex}_{et_n} + C^{capexmaint}_{et_n} \qquad \forall t_n \in T_N \tag{23}$$

The grid electricity purchase cost is calculated as the grid energy used at each plant multiplied by the grid electricity price.

$$C^{opexgrid}_{t_nt_s} = \sum_{i\in I} W^{gen}_{Grid,it_nt_s}\pi^{eng}_{Grid,i} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{24}$$

The energy OPEX maintenance cost is calculated as the energy generated from each energy source multiplied by the corresponding OPEX maintenance cost.

$$C^{opexmaint}_{t_nt_s} = \sum_{e\in E}\sum_{i\in I} W^{gen}_{eit_nt_s}\kappa^{opexmaint}_e \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{25}$$

The total energy OPEX cost at each time period is equal to the grid electricity purchase cost plus the energy OPEX maintenance cost.

$$C^{opexsum}_{et_nt_s} = C^{opexgrid}_{t_nt_s} + C^{opexmaint}_{t_nt_s} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{26}$$

The total energy OPEX cost over the full time horizon is calculated by summing grid electricity costs and OPEX maintenance costs over all network and scheduling time periods.

$$C_{energy}^{opex,total} = \sum_{t_n \in T_N} \sum_{t_s \in T_S} C_{t_n t_s}^{opexgrid} + \sum_{t_n \in T_N} \sum_{t_s \in T_S} C_{t_n t_s}^{opexmaint} \tag{27}$$

The overall total energy cost includes grid electricity cost, OPEX maintenance cost, energy installation CAPEX, and CAPEX maintenance cost over the full time horizon.

$$C_{energy}^{SCN} = \sum_{t_n \in T_N} \sum_{t_s \in T_S} C_{t_n t_s}^{opexgrid} + \sum_{t_n \in T_N} \sum_{t_s \in T_S} C_{t_n t_s}^{opexmaint} + \sum_{t_n \in T_N} C_{et_n}^{capex} + \sum_{t_n \in T_N} C_{et_n}^{capexmaint} \tag{28}$$

The energy emission constraint calculates the CO_2 emissions generated from each energy source at each plant.

$$E_{eit_n t_s}^{gen} = W_{eit_n t_s}^{gen} \epsilon_{ei}^{eng} \qquad \forall e \in E,\ i \in I,\ t_n \in T_N,\ t_s \in T_S \tag{29}$$

The total energy emissions at each time period equal the sum of energy-related emissions from all energy sources across all plants.

$$E_{t_n t_s}^{genSCN} = \sum_{e \in E} \sum_{i \in I} E_{eit_n t_s}^{gen} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{30}$$

The overall total energy emissions are calculated by summing energy-related emissions over all network and scheduling time periods.

$$E^{gen,total} = \sum_{t_n \in T_N} \sum_{t_s \in T_S} E_{t_n t_s}^{genSCN} \tag{31}$$

3.5 Emission Constraints

The emission constraints account for transportation emissions, production emissions, energy-related emissions, and total supply chain network emissions across both the network time scale t_n and scheduling time scale t_s.

The raw material transportation emission constraint calculates the total emissions generated from transporting raw material from suppliers to manufacturing plants.

$$E_{rsi,t_n t_s}^{ship} = \sum_{r \in R} \sum_{s \in S} \sum_{i \in I} \sum_{v \in V} \epsilon_{siv}^{trans} * d_{si} * X_{rsivt_n t_s}^{ship} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{32}$$

The plant-to-distribution-center transportation emission constraint calculates the total emissions generated from transporting product from manufacturing plants to distribution centers.

$$E_{pij,t_n t_s}^{ship} = \sum_{p \in P} \sum_{i \in I} \sum_{j \in J} \sum_{v \in V} \epsilon_{ijv}^{trans} * d_{ij} * X_{pijvt_n t_s}^{ship} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{33}$$

The distribution-center-to-customer transportation emission constraint calculates the total emissions generated from transporting product from distribution centers to customers.

$$E_{pjk,t_n t_s}^{ship} = \sum_{p \in P} \sum_{j \in J} \sum_{k \in K} \sum_{v \in V} \epsilon_{jkv}^{trans} * d_{jk} * X_{pjkvt_n t_s}^{ship} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{34}$$

The total transportation emission over the full planning horizon is the sum of raw material transportation emissions, plant-to-distribution-center transportation emissions, and distribution-center-to-customer transportation emissions.

$$E^{trans,total} = \sum_{t_n \in T_N} \sum_{t_s \in T_S} E_{rs,t_n t_s}^{ship} + \sum_{t_n \in T_N} \sum_{t_s \in T_S} E_{ij,t_n t_s}^{ship} + \sum_{t_n \in T_N} \sum_{t_s \in T_S} E_{jk,t_n t_s}^{ship} \tag{35}$$

The production emission constraint calculates the emissions generated from manufacturing products at each plant using each processing machine.

$$E^{prod}_{pimt_nt_s} = \sum_{p\in P}\sum_{i\in I}\sum_{m\in M} \epsilon^{prod}_{pm} X^{prod}_{pimt_nt_s} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{36}$$

The total production emission over the full planning horizon is the sum of production emissions over all network and scheduling time periods.

$$E^{prod,total} = \sum_{t_n\in T_N}\sum_{t_s\in T_S} E^{prod}_{t_nt_s} \tag{37}$$

The total supply chain emission at each time period is the sum of transportation emissions, production emissions, and energy-related emissions.

$$E^{SCN}_{t_nt_s} = E^{ship}_{rs,t_nt_s} + E^{ship}_{ij,t_nt_s} + E^{ship}_{jk,t_nt_s} + E^{prod}_{t_nt_s} + E^{genSCN}_{t_nt_s} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{38}$$

The overall total supply chain emission is calculated by summing transportation emissions, production emissions, and energy-related emissions over the full planning horizon (scope 3, 1 and 2 respectively).

$$E^{SCN} = E^{trans,total} + \sum_{t_n\in T_N}\sum_{t_s\in T_S} E^{prod}_{t_nt_s} + \sum_{t_n\in T_N}\sum_{t_s\in T_S} E^{genSCN}_{t_nt_s} \tag{39}$$

3.6 Emission Epsilon Constraint

The total supply chain emissions are restricted by an emission cap that is determined using the ε-constraint approach:

$$E^{cap} = E^{\min} + \varepsilon\left(E^{\max} - E^{\min}\right) \tag{40}$$

The total supply chain emissions must satisfy:

$$E^{SCN} \leq E^{cap} + \tau \tag{41}$$

where:

- E^{SCN} is the total supply chain network emission,
- E^{cap} is the allowable emission limit,
- ε is the emission trade-off parameter,
- τ is a small numerical tolerance used to avoid solver precision issues. τ was set to 10e-4

3.7 Demand Constraints

The demand fulfillment constraint ensures that the total quantity of product delivered to each customer over the each scheduling horizon satisfies the required fraction of total customer demand.

For each product p and customer k, the total amount of product arriving at the customer from all distribution centers and transportation modes must be greater than or equal to the required demand level.

$$\sum_{j\in J}\sum_{v\in V}\sum_{t_n\in T_N}\sum_{t_s\in T_S} Arriv_{pjkvt_nt_s} \geq \sum_{t_n\in T_N}\sum_{t_s\in T_S} Dem_{pkt_nt_s}\rho^{dem}_{pk} \qquad \forall p \in P,\ k \in K \tag{42}$$

3.8 Cost Constraints

The cost constraints account for transportation CAPEX, machine CAPEX, distribution center CAPEX, transportation OPEX, production OPEX, raw material purchasing cost, storage cost, energy cost, backlog cost, and the overall total supply chain network cost.

The transportation CAPEX cost is calculated for each transportation layer based on the selected transportation mode and its annualized purchase cost.

$$C^{capex}_{siv,t_n} = \sum_{s\in S}\sum_{i\in I}\sum_{v\in V} \kappa^{capexcar}_{v} y_{vsit_n} \qquad \forall t_n \in T_N \tag{43}$$

$$C^{capex}_{ijv,t_n} = \sum_{i\in I}\sum_{j\in J}\sum_{v\in V} \kappa^{capexcar}_{v} y_{vijt_n} \qquad \forall t_n \in T_N \tag{44}$$

$$C^{capex}_{jkv,t_n} = \sum_{j\in J}\sum_{k\in K}\sum_{v\in V} \kappa^{capexcar}_{v} y_{vjkt_n} \qquad \forall t_n \in T_N \tag{45}$$

$$C^{capex,car}_{t_n} = C^{capex}_{siv,t_n} + C^{capex}_{ijv,t_n} + C^{capex}_{jkv,t_n} \qquad \forall t_n \in T_N \tag{46}$$

3.8.1 Machine CAPEX Cost

The machine CAPEX cost is calculated as the annualized cost of each installed processing machine.

$$C^{capex,machine}_{t_n} = \sum_{i\in I}\sum_{m\in M} \kappa^{capexm}_{mi} y_{mit_n} \qquad \forall t_n \in T_N \tag{47}$$

3.8.2 CAPEX for Storage Capacity

Similarly, the CAPEX cost associated with the raw material and product storage installation can be calculated as below:

$$C^{capex,store}_{rt_n} = \sum_{i\in I} \kappa^{capex\_store}_{ri} y^{store}_{rit_n}, \qquad \forall r \in R,\ t_n \in T_n \tag{48}$$

$$C^{capex,store}_{pt_n} = \sum_{i\in I} \kappa^{capex\_store}_{pi} y^{store}_{pit_n}, \qquad \forall p \in P,\ t_n \in T_n \tag{49}$$

3.8.3 Distribution Center CAPEX Cost

The distribution center CAPEX cost is calculated as the annualized construction cost of each selected distribution center. it is important to note that the distribution center installation cost include storage capacity cost as well.

$$C^{capex,DC}_{t_n} = \sum_{j\in J} \kappa^{capexDC}_{j} y_{jt_n} \qquad \forall t_n \in T_N \tag{50}$$

3.8.4 Transportation OPEX Cost

The transportation OPEX cost is calculated by multiplying shipment quantities by their corresponding route-level transportation cost.

$$C^{opex}_{rsiv,t_nt_s} = \sum_{r\in R}\sum_{s\in S}\sum_{i\in I}\sum_{v\in V} X^{ship}_{rsivt_nt_s} c^{trans}_{siv} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{51}$$

$$C^{opex}_{pijv,t_nt_s} = \sum_{p\in P}\sum_{i\in I}\sum_{j\in J}\sum_{v\in V} X^{ship}_{pijvt_nt_s} c^{trans}_{ijv} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{52}$$

$$C^{opex}_{pjkv,t_nt_s} = \sum_{p\in P}\sum_{j\in J}\sum_{k\in K}\sum_{v\in V} X^{ship}_{pjkvt_nt_s} c^{trans}_{jkv} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{53}$$

3.8.5 Production OPEX Cost

The production operating cost is calculated by multiplying the production quantity at each plant by the plant operating cost.

$$C^{opex,prod}_{pit_nt_s} = \sum_{p\in P}\sum_{i\in I}\sum_{m\in M} c^{opex}_i X^{prod}_{pimt_nt_s} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{54}$$

3.8.6 Raw Material Purchasing Cost

The raw material purchasing cost is calculated by multiplying the amount of raw material shipped by the purchasing price of each raw material.

$$C^{buyraw}_{rit_nt_s} = \sum_{r\in R}\sum_{s\in S}\sum_{i\in I}\sum_{v\in V} X^{ship}_{rsivt_nt_s} c^{buy}_r \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{55}$$

3.8.7 Distribution Center Storage Cost

The distribution center storage cost is calculated by multiplying inventory stored at each distribution center by the distribution center operating cost.

$$C^{opex}_{pjt_nt_s} = \sum_{p\in P}\sum_{j\in J} c^{opex}_j Inv_{pjt_nt_s} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{56}$$

3.8.8 Total OPEX Cost

The total operating cost at each time period includes transportation cost, production cost, raw material purchasing cost, distribution center storage cost, energy operating cost, and backlog penalty cost.

$$C^{opex}_{t_nt_s} = C^{opex}_{rsiv,t_nt_s} + C^{opex}_{pijv,t_nt_s} + C^{opex}_{pjkv,t_nt_s} + C^{opex,prod}_{pit_nt_s} + C^{buyraw}_{rit_nt_s} + C^{opex}_{pjt_nt_s} + C^{opexsum}_{et_nt_s} + C^{backlog}_{t_nt_s} \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{57}$$

3.8.9 Total CAPEX Cost

The total capital cost at each network time period includes transportation CAPEX, machine CAPEX, distribution center CAPEX, and energy CAPEX.

$$C^{capex}_{t_n} = C^{capex}_{siv,t_n} + C^{capex}_{ijv,t_n} + C^{capex}_{jkv,t_n} + C^{capex,machine}_{t_n} + C^{capex,store}_{rt_n} + C^{capex,store}_{pt_n} + C^{capex,DC}_{t_n} + C^{capexsum}_{et_n} \qquad \forall t_n \in T_N \tag{58}$$

3.8.10 Overall Total Supply Chain Cost

The overall total supply chain network cost is calculated as the sum of all operating costs and capital costs over the full planning horizon.

$$C^{SCN} = \sum_{t_n\in T_N}\sum_{t_s\in T_S} C^{opex}_{t_nt_s} + \sum_{t_n\in T_N} C^{capex}_{t_n} \tag{59}$$

3.9 Capacity Constraints

3.9.1 Machine Capacity Constraints

The installed machine capacity should be greater than the total quantity of product manufacture at plant i. This install capacity is bounded by the minimum and maximum allowable capacity

$$\sum_{p\in P} X^{prod}_{pimt_nt_s} \le Cap_{mit_n}\gamma_{imt_nt_s} \qquad \forall i\in I,\ m\in M,\ t_n\in T_N,\ t_s\in T_S \tag{60}$$

$$Cap^{min}_m y_{mit_n} \le Cap_{mit_n} \le Cap^{max}_m y_{mit_n} \qquad \forall i\in I,\ m\in M,\ t_n\in T_N \tag{61}$$

3.9.2 Storage Capacity Installation Constraints

Similarly, the installed storage capacity for raw materials is bounded by the minimum and maximum allowable storage capacities only when the corresponding storage installation binary variable is activated.

$$Cap^{store,min}_{it_n} y^{store}_{rit_n} \le Cap^{store}_{rit_n} \le Cap^{store,max}_{it_n} y^{store}_{rit_n}, \qquad \forall r\in R,\ i\in I,\ t_n\in T_n \tag{62}$$

Similarly, the installed storage capacity for finished products is bounded by the minimum and maximum allowable storage capacities when the corresponding product storage installation binary variable is activated.

$$Cap^{store,min}_{it_n} y^{store}_{pit_n} \le Cap^{store}_{pit_n} \le Cap^{store,max}_{it_n} y^{store}_{pit_n}, \qquad \forall p\in P,\ i\in I,\ t_n\in T_n \tag{63}$$

3.9.3 Distribution Center Capacity Constraints

The installed DC capacity should be greater than the total quantity of product shipped at distribution center j. This install capacity is bounded by the minimum and maximum allowable capacity

$$Cap^{min}_j y_{jt_n} \le Cap^{DC}_{jt_n} \le Cap^{max}_j y_{jt_n} \qquad \forall j\in J,\ t_n\in T_N \tag{64}$$

$$\sum_{p\in P}\sum_{k\in K}\sum_{v\in V} X^{ship}_{pjkvt_nt_s} \le Cap^{DC}_{jt_n} \qquad \forall j\in J,\ t_n\in T_N,\ t_s\in T_S \tag{65}$$

3.9.4 Distribution Center storage Capacity Constraint

The total quantity of products shipped from distribution center j during scheduling period t_s cannot exceed the available distribution center capacity after accounting for the inventory currently stored at the facility.

$$\sum_{p\in P}\sum_{k\in K}\sum_{v\in V} Ship_{pjkvt_nt_s} \le Cap^{DC}_{jt_n} - \sum_{p\in P} Inv_{pjt_nt_s} \qquad \forall j\in J,\ t_n\in T_n,\ t_s\in T_s \tag{66}$$

3.9.5 Supplier Capacity Constraint

$$\sum_{r\in R}\sum_{i\in I}\sum_{v\in V} X^{ship}_{rsivt_nt_s} \le Cap^{bigM}_s \qquad \forall s\in S,\ t_n\in T_N,\ t_s\in T_S \tag{67}$$

3.9.6 Supplier-to-Plant Vehicle Capacity Constraints

$$\sum_{r\in R} X^{ship}_{rsivt_nt_s} \le Cap_{sivt_n} \qquad \forall s\in S,\ i\in I,\ v\in V,\ t_n\in T_N,\ t_s\in T_S \tag{68}$$

$$Cap^{min}_v y_{sivt_n} \le Cap_{vsit_n} \le Cap^{max}_v y_{sivt_n} \qquad \forall s\in S,\ i\in I,\ v\in V,\ t_n\in T_N \tag{69}$$

3.9.7 Plant-to-Distribution Vehicle Capacity Constraints

$$\sum_{p\in P} X^{ship}_{pijvt_nt_s} \le Cap_{ijvt_n} \qquad \forall i\in I,\ j\in J,\ v\in V,\ t_n\in T_N,\ t_s\in T_S \tag{70}$$

$$Cap^{min}_v y_{ijvt_n} \le Cap_{vijt_n} \le Cap^{max}_v y_{ijvt_n} \qquad \forall i\in I,\ j\in J,\ v\in V,\ t_n\in T_N \tag{71}$$

3.9.8 Distribution-to-Customer Vehicle Capacity Constraints

$$\sum_{p \in P} X^{ship}_{pjkvt_nt_s} \leq Cap_{jkvt_n} \qquad \forall j \in J,\ k \in K,\ v \in V,\ t_n \in T_N,\ t_s \in T_S \tag{72}$$

$$Cap^{min}_v y_{jkvt_n} \leq Cap_{vjkt_n} \leq Cap^{max}_v y_{jkvt_n} \qquad \forall j \in J,\ k \in K,\ v \in V,\ t_n \in T_N \tag{73}$$

3.10 Backlog Constraints

3.10.1 Customer Arrival Decomposition

Equation (74) decomposes the total quantity arriving at customer k into two components: the portion used to satisfy current-period demand and the portion used to reduce previously accumulated backlog.

$$Arriv_{pjkvt_nt_s} = Arriv^{dem}_{pjkvt_nt_s} + Arriv^{back}_{pjkvt_nt_s} \qquad \forall p \in P,\ j \in J,\ k \in K,\ v \in V,\ t_n \in T_N,\ t_s \in T_S \tag{74}$$

3.10.2 New Unmet Demand Constraint

$$UnmetDemand_{pkt_nt_s} = Dem_{pkt_nt_s}\rho^{dem}_{pk} - \sum_{j \in J}\sum_{v \in V} Arriv^{dem}_{pjkvt_nt_s} \qquad \forall p \in P,\ k \in K,\ t_n \in T_N,\ t_s \in T_S \tag{75}$$

$$\sum_{j \in J}\sum_{v \in V} Arriv^{dem}_{pjkvt_nt_s} \leq Dem_{pkt_nt_s}\rho^{dem}_{pk} \qquad \forall p \in P,\ k \in K,\ t_n \in T_N,\ t_s \in T_S \tag{76}$$

Equation (75) defines new unmet demand as the portion of current-period demand not satisfied by arrivals allocated to current demand. Constraint (76) prevents the model from assigning more current-demand arrivals than the demand required in that period.

3.10.3 Customer Backlog Balance

$$Backlog_{pkt_nt_s} = Backlog^{prev}_{pkt_nt_s} + UnmetDemand^{prev}_{pkt_nt_s} - \sum_{j \in J}\sum_{v \in V} Arriv^{back}_{pjkvt_nt_s} \qquad \forall p \in P,\ k \in K,\ t_n \in T_N,\ t_s \in T_S \tag{77}$$

$$Backlog^{prev}_{pkt_nt_s} = \begin{cases} 0, & \text{if } t_n = t_n^1 \text{ and } t_s = t_s^1 \\ Backlog_{pk,t_n-1,t_s^{last}}, & \text{if } t_s = t_s^1 \text{ and } t_n > t_n^1 \\ Backlog_{pk,t_n,t_s-1}, & \text{otherwise} \end{cases} \tag{78}$$

$$UnmetDemand^{prev}_{pkt_nt_s} = \begin{cases} 0, & \text{if } t_n = t_n^1 \text{ and } t_s = t_s^1 \\ UnmetDemand_{pk,t_n-1,t_s^{last}}, & \text{if } t_s = t_s^1 \text{ and } t_n > t_n^1 \\ UnmetDemand_{pk,t_n,t_s-1}, & \text{otherwise} \end{cases} \tag{79}$$

where t_s^{last} represents the last scheduling time period within a network period, t_n^1 represents the first network time period, and t_s^1 represents the first scheduling time period.

3.10.4 Backlog Penalty Cost

The backlog cost at each time period includes two parts: the penalty for new unmet demand and the penalty for accumulated backlog carried over from previous periods

$$C^{backlog}_{t_nt_s} = \sum_{p \in P}\sum_{k \in K}\left(c^{unmet}_k UnmetDemand_{pkt_nt_s} + c^{backlog}_k Backlog_{pkt_nt_s}\right) \qquad \forall t_n \in T_N,\ t_s \in T_S \tag{80}$$

4 2-Stages Stochastic Model

4.1 New Objective Function

Since the model considers two uncertain parameters, machine availability and demand, the stochastic scenario set is defined as the Cartesian product of the availability scenario set and the demand scenario set:

$$\Omega = \Omega^A \times \Omega^D \tag{81}$$

where Ω^A is the set of machine availability scenarios and Ω^D is the set of demand scenarios. Therefore, each joint scenario $\omega \in \Omega$ represents one realization of both machine availability and customer demand.

The probability of each joint scenario is defined as:

$$\pi_\omega = \pi_{\omega^A} \pi_{\omega^D}, \qquad \forall \omega = (\omega^A, \omega^D) \in \Omega \tag{82}$$

The two-stage stochastic objective minimizes the sum of deterministic first-stage investment cost and the expected second-stage operating cost:

$$\min\ Z = \underbrace{\sum_{t_n \in T_N} C^{\text{capex}}_{t_n}}_{\text{first-stage investment cost}} + \underbrace{\sum_{\omega \in \Omega} \pi_\omega \left(\sum_{t_n \in T_N} \sum_{t_s \in T_S} C^{\text{opex}}_{t_n, t_s, \omega} \right)}_{\text{expected second-stage operating cost}} \tag{83}$$

4.2 Joint Uncertainty Scenario Representation

The stochastic model considers two independent sources of uncertainty: machine availability and customer demand. Let $a \in A$ denote the machine availability scenario and $d \in D$ denote the demand scenario. The combined scenario is defined as:

$$w = (a, d), \qquad w \in W = A \times D \tag{84}$$

where W is the set of joint uncertainty scenarios.

The probability of each joint scenario is given by:

$$\pi_w = \pi_a \pi_d, \qquad \forall w = (a, d) \in W \tag{85}$$

where π_a is the probability of machine availability scenario a and π_d is the probability of demand scenario d.

4.3 Demand Uncertainty

Scenario-dependent demand is defined as:

$$D^w_{pkt_nt_s} = \theta_d D_{pkt_nt_s}, \qquad \forall p \in P,\ k \in K,\ t_n \in T_N,\ t_s \in T_S,\ w = (a, d) \in W \tag{86}$$

where $D_{pkt_nt_s}$ is the baseline demand and θ_d is the demand multiplier under demand scenario d.

Table 4: Demand uncertainty scenarios.

Demand Scenario	Demand Multiplier θ_d	Probability π_d	Description
D_1	1.05	1/3	5% demand increase
D_2	1.00	1/3	Baseline demand
D_3	1.10	1/3	10% demand increase

The minimum demand fulfillment requirement must hold under each joint scenario:

$$\sum_{j \in J} \sum_{v \in V} \sum_{t_n \in T_N} \sum_{t_s \in T_S} Arriv^w_{pjkvt_nt_s} \geq \sum_{t_n \in T_N} \sum_{t_s \in T_S} \alpha_{pk} D^w_{pkt_nt_s}, \qquad \forall p \in P,\ k \in K,\ w \in W \tag{87}$$

4.4 Machine Availability Uncertainty

Machine availability under each joint scenario is defined as:

$$\gamma^{w}_{imt_nt_s} = \gamma_a, \qquad \forall i \in I,\ m \in M,\ t_n \in T_N,\ t_s \in T_S,\ w = (a, d) \in W \tag{88}$$

where γ_a is the machine availability factor associated with availability scenario a.

The available production capacity is adjusted by the machine availability factor:

$$\sum_{p \in P} Q^{w}_{pimt_nt_s} \leq Cap_{imt_n} \gamma^{w}_{imt_nt_s}, \qquad \forall i \in I,\ m \in M,\ t_n \in T_N,\ t_s \in T_S,\ w \in W \tag{89}$$

The Process availability is modeled using an adapted PID proposed by Kusolsongtawee et al. [14] See simplified PID diagram below.

OREDA database [17] data were used to run the Monte Carlos simmulation can be see in the table below.

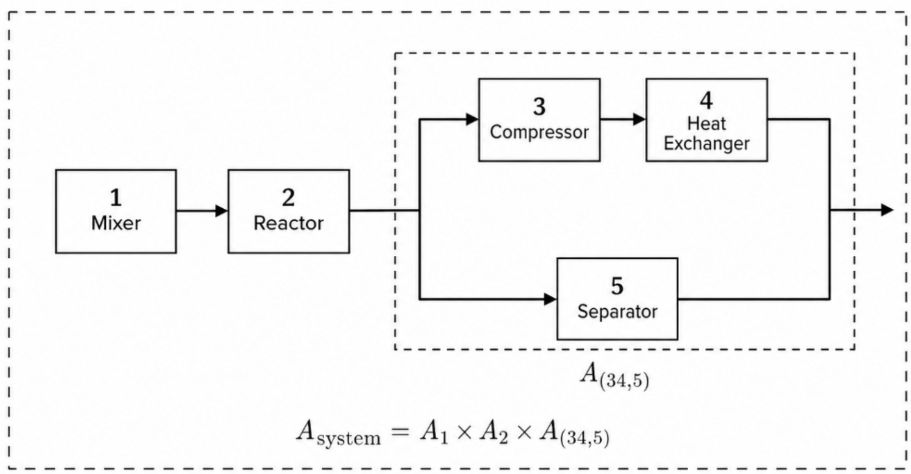

Figure 1: System availability block diagram for the LLDPE production process.

Table 5: Reliability data for the major processing equipment from OREDA DATA BASE.

Equipment	MTBF Low [h]	MTBF Mean [h]	MTBF High [h]	MTTR Low [h]	MTTR Mode [h]	MTTR High [h]
Mixer	6,310	3,916	2,696	2	15.3	66
Reactor	12,450	4,906	2,333	58	125.8	217
Compressor	5,000,000	5,626	1,363	0.3	15.9	410
Heat Exchanger	119,904	19,818	8,193	1	21.1	279
Separator 1	144,509	7,299	2,439	1	13.4	52
Separator 2	4,545,455	12,620	3,222	1	9.6	47

At the process level, The system reliability is represent with the availability metric to account for the failure rate on the an equipment in the production process. The Availability metric if formally defined as **[citation]**:

Availability Metric : *Probability of failure calculation*

$$A_i = \prod \frac{MTBF_{n,j}}{MTBF_{n,j} + MTTR_{n,j}} \tag{90}$$

Here we assumed the process is made of a mixer, a reactor, a compressor, an heat exchanger and of one or two separator install at the first stage decision. When the system decide on the number of separator to install anmoung the two, the system availability is calculate as below:

$$A_{\text{system}} = A_1 \times A_2 \times A_{(34,5)} \tag{91}$$

$$A_{34} = A_3 \times A_4 \tag{92}$$

$$A_{(34,5)} = 1 - (1 - A_{34})(1 - A_5) \tag{93}$$

$$A_{\text{system}} = A_1 A_2 \left[1 - (1 - A_3 A_4)(1 - A_5)\right] \tag{94}$$

Where,

$$\begin{aligned} 1 &= \text{Mixer}, \\ 2 &= \text{Reactor}, \\ 3 &= \text{Compressor}, \\ 4 &= \text{Heat Exchanger}, \\ 5 &= \text{Separator}. \end{aligned}$$

Following the above, discrete probability were generated for the system availability as below:

Table 6: Machine availability uncertainty scenarios.

Availability Scenario	Availability Factor γ_a	Probability π_a	Description
A_1	0.30	0.0044	Severe disruption
A_2	0.40	0.0434	Below-average availability
A_3	0.60	0.1698	Base-case availability
A_4	0.80	0.4476	Above-average availability
A_5	1.00	0.3346	Full availability

5 APPENDIX

5.1 Appendix A : Deterministic Case Study Parameters

Table 7: Customer demand quantities and penalty fees during the first year.

Customer Location	Quantity LLDPE [kg/day]	Backlog Penalty Fee [$/kg]	Unmet Demand Penalty Fee [$/kg]
Ohio	600	500	500
Philadelphia	1200	800	1000
Georgia	1400	500	500

The below table show all input for the energy generation for LLDPE grade 1 produce. (see table 4)

Table 8: Raw Material Conversion and cost.

Raw Material	Conversion factor	Price [$ /kg]
Ethylene	0.911088	0.07
Nitrogene	0.006699	0.01
Hydrogene	0.00003915	1.50
Cyclohexane	0.00002175	0.90

Table 9: Raw Material Conversion factor and purchasing cost

Table 10: Manufacturing plant raw material and product storage capacity.

Location	**Min Capacity [kg]**	**Max Capacity [kg]**	**Capex over 30 yrs [$]**
Texas Plant Raw Storage	50	70,000	1,050,000
Louisiana Plant Raw Storage	20	60,000	900,000
Texas Plant Product Storage	50	43,750	300,000
Louisiana Plant Product Storage	20	37,500	257,143

Table 11: Maximum capacity and economic characteristics of processing technologies at each manufacturing plant.

Technology	**Min. Capacity [kg]**	**Max. Capacity [kg]**	**Conversion Factor [%]**	**CAPEX over 30 yrs [$]**
Machine 1	300	350,000	97	4,200,000
Machine 2	100	300,000	90	5,600,000

Table 12: Maintenance Cost at Each plant.

Plant Location	Maintenacne Cost[$ /kg]
Texas	0.12
Louisiana	0.07

Table 13: Vehicle characteristics including capacity limits, purchase restrictions, emission factors, operating expenditures (OPEX), and capital expenditures (CAPEX).

Vehicle Type	**Min. Capacity [kg]**	**Max. Capacity [kg]**	**Purchase Limit**	**Emission Factor [kg CO_2/ kg-mile]**	**OPEX [$/mile]**	**CAPEX over 10 Years [$]**
Heavy Duty Truck	15,000	36,000	5	2.05×10^{-4}	0.110	120,000
Mid-size Truck	3,000	12,000	3	2.05×10^{-4}	0.118	70,000
Light Truck	1,500	4,000	5	2.20×10^{-5}	0.125	40,000

Table 14: Distribution center storage capacities and economic parameters.

Location	**Min. Capacity [kg]**	**Max. Capacity [kg]**	**CAPEX over 30 yrs [$]**	**OPEX [$/kg]**
Texas	500	1,095,000	525,600	0.02
Philadelphia	100	730,000	420,480	0.13

Table 15: Energy demand Intensity per product.

Plant Location	Quantity LLDPE [KW- day]
Texas	1.01
Louisiana	1.01

Table 16: Energy capacity Factor and emission.

Energy Source	Capacity factor	Emission factor [kg C02 eq/ KWH]
Wind Turbine	0.3246	0.00957
PV Solar	0.2165	0.0324
Grid	1	0.4192498424
Gas Turbine	0.90	0.353

Table 17: Capacity factor for energy conversion and emission factor

Table 18: Energy source capital and maintenance costs.

Energy Source	**CAPEX [$/kW]**	**Maintenance CAPEX [$/kW-year]**	**Maintenance OPEX [$/kWh]**
Wind Turbine	7.6371	24.5	0
PV Solar	1.1835	11.0	0
Grid	0	0	0
Gas Turbine	2.2729	10.0	0.0354

Table 19: Grid electricity prices and maximum grid purchase limits at each manufacturing plant.

Plant Location	**Grid Price [$/kWh]**	**Grid Purchase Limit [kWh/day]**
Texas	0.0637	0.151
Louisiana	0.0541	0.151