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A Deepsets-Guided Framework for Learning Job Priorities In Single-Machine Scheduling

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ABSTRACT

Scheduling is a fundamental decision-making problem in chemical engineering as well as numerous other sectors, arising in manufacturing, energy systems, and supply chains. Many scheduling problems are NP-hard [1], meaning that even small problems are computationally hard to solve deterministically. As a result, existing exact and heuristic methods face trade-offs between scalability, solution quality, and generalizability. This work addresses these limitations through a hybrid machine learning–optimization framework for the single-machine total tardiness scheduling problem (SMTTP). We introduce the DeepSets-Guided Scheduling Framework (DGSF), a hybrid methodology that integrates data-driven priority learning with structured optimization for single-machine scheduling.

First, we propose a geometric instance classification rule that characterizes scheduling instances through aggregate structural parameters, enabling models trained on small instances to generalize to larger instances within the same structural class.

Second, we develop a modified DeepSets [2] machine learning (ML) architecture that processes variable-sized sets of jobs and produces job-aligned priority scores. The model combines job-level feature transformations with an attention-based aggregation mechanism to incorporate instance-level context, allowing priority estimation to depend jointly on job and instance characteristics. Input features are constructed to retain interpretability and include normalized processing, release and due times, slack-based measures, and features derived from classical heuristics.

Third, we introduce a two-stage post-processing step. A fast local pairwise-swap heuristic improves the predicted sequence, which is then used to warm-start a neighborhood-restricted continuous-time mixed-integer programming formulation. By explicitly limiting the search space around the learned solution, this formulation achieves high-quality refinement while controlling combinatorial complexity.

Computational experiments indicate that the learned priority structure aligns with classical one-shot dispatching heuristics while improving solution quality. On instances with up to 120 jobs, DGSF consistently outperforms these heuristics, achieving optimality gaps of 3–6% compared to 26–31% for the best-performing one-shot heuristic. Furthermore, DGSF maintains high-quality solutions across different instance structures, time discretizations, and product catalogues.

REFERENCES

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