

Hybrid Physics-Informed Neural Networks for Thermal Process Identification and Control

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ABSTRACT

Physics-Informed Neural Networks (PINNs) offer a promising approach for integrating first-principles modeling with data-driven methods, especially in dynamic thermal systems. This study introduces a hybrid PINN framework for a one-dimensional heating rod governed by heat transfer equations. Unlike traditional PINNs that rely on time-dependent automatic differentiation, this approach employs numerical derivatives to bypass gradient saturation and enhance robustness. The proposed model demonstrates accurate extrapolation and generalization with limited training data and is effectively used as a surrogate in a Model Predictive Control (MPC) framework for rod-tip temperature regulation. Additionally, a plan is outlined to apply physics-informed dimensionality reduction and model order reduction to improve computational efficiency and enable real-time application. The findings affirm PINNs' potential as control-oriented reduced models for thermal processes.

Keywords: Physics-Informed Neural Networks, Heat Transfer, Model Predictive Control, Model Order Reduction, Thermal Systems

INTRODUCTION

The importance of modeling and system identification has increased as modern process systems have grown more complex. First-principles modeling requires extensive domain expertise and is often time-consuming and costly to implement, particularly for complex or poorly characterized systems. Data-driven black-box models alleviate some of these challenges by relying solely on available measurements; however, they typically suffer from poor extrapolation performance and limited physical interpretability [1].

Neural networks, while flexible and expressive, are highly sensitive to data quality and initial parameter values, and their training may converge to local minima. To address these limitations, Physics-Informed Neural Networks (PINNs) incorporate governing equations directly into the learning process, thereby reducing data requirements and improving generalization. Since the seminal work of Raissi et al., PINNs have been successfully applied to a wide range of differential-algebraic and partial differential equation (PDE) systems [2].

Some studies predict the thermophysical properties of chemical components by imposing physical constraints on the neural network model structure [3].

PINNs have been successfully employed to enforce heat-transfer physics in complex thermal-flow problems, demonstrating their ability to solve PDEs with limited or no training data in a specific domain [4].

Recent literature demonstrates a growing interest in physics-informed neural networks for thermal systems and control applications, ranging from building energy systems [5] and thermal energy storage [6] to real-time MPC implementations in industrial processes [7], [8]. Despite these advances, integrating PINNs as lightweight, control-oriented surrogate models for dynamic thermal systems, while avoiding time-dependent automatic differentiation and maintaining robustness, remains an open challenge. PINNs are mostly used in time-series modeling for batch processes, since common automatic differentiation approaches require time values as inputs to the surrogate model.

This work addresses this gap by developing a PINN framework for a one-dimensional heating rod system that

leverages numerically evaluated derivatives to enforce physical consistency, achieves reliable extrapolation with sparse data, and enables efficient deployment within a model predictive control scheme.

METHODS

This section discusses the methods, case study, data, and model structure to implement a PINN-MPC controller.

Physics-Informed Neural Network Framework

Physics-informed neural network architectures have been extended toward control-oriented recurrent structures, enabling improved generalization and robustness in predictive control applications [9].

The proposed PINN follows a hybrid modeling strategy in which first-principles heat transfer equations guide the learning process, while a neural network approximates the unknown system dynamics. In contrast to conventional PINN formulations that explicitly treat time as a network input and rely on automatic differentiation, the present framework enforces physical consistency through numerically evaluated derivatives. This strategy improves robustness, mitigates gradient saturation from monotonic time inputs, and enhances extrapolation performance, making the model suitable for control-oriented applications.

Network Representation

The neural network is used to approximate the system input–output relationship and is represented as:

$$\hat{y}(t) = f_{\theta}(u(t), x(t)), \quad (1)$$

where $f_{\theta}(\cdot)$ denotes the neural network parameterized by θ , $u(t)$ represents the heater duty cycle, and $\hat{y}(t)$ corresponds to the predicted rod-tip temperature. The network architecture is kept deliberately compact to support real-time control applications [10].

Physics-Informed Training Objective

Physical knowledge is incorporated into the learning process through a composite loss function of the form

$$L = L_{\theta} + \lambda L_{res}, \quad (2)$$

where L_{θ} enforces agreement with measured data and L_{res} penalizes violations of the discretized heat transfer equations. The weighting parameter λ balances data fidelity and physical consistency. Figure 1 illustrates the PINN framework.

Numerical Derivative Enforcement

Unlike classical soft-constraint PINNs that rely on time-dependent automatic differentiation, this work employs numerical differentiation to evaluate temporal derivatives in the governing equations. This choice avoids

activation function saturation caused by ramp-like time inputs in continuous thermal systems and leads to improved training reliability and prediction for continuous processes.

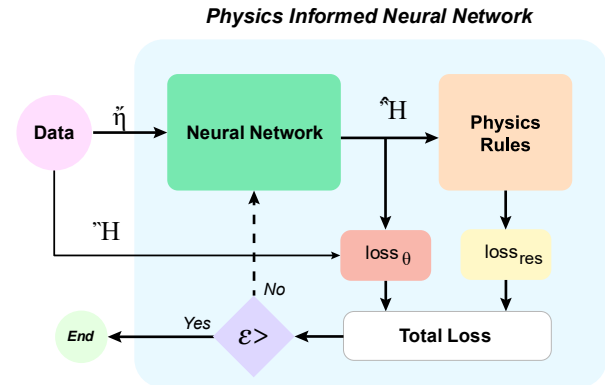


Figure 1. A schematic of the training process in a physics-informed neural network

Heating Rod Case Study

The case study considered in this work is a one-dimensional heating rod, which serves as a benchmark for thermal processes commonly encountered in industrial heating, furnaces, and laboratory-scale thermal systems. Heat transfer along the rod is governed primarily by conduction, with heat input applied through an electrical heater and heat losses occurring due to convection and radiation at the boundaries.

The system dynamics can be described by a one-dimensional transient heat conduction equation with appropriate boundary and initial conditions. These governing equations provide the physical foundation that is embedded in the learning process of the proposed PINN framework. A schematic of the system is shown in Figure (2).

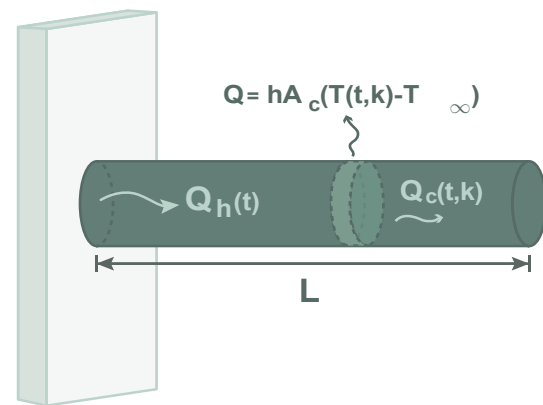


Figure 2. Heating rod schematic.

Data and Training Setup

Training data for the PINN model are obtained from simulated temperature of the heating rod under varying heater duty cycles. The dataset is intentionally limited in size to evaluate the extrapolation capability of the proposed physics-informed approach. Input-output trajectories are split into training and validation sets, ensuring that the validation data include operating conditions outside the training range.

The neural network is trained with the Adam optimizer, and physical constraints are incorporated via a physics-informed loss term. At the end of the training session, the SLSQP optimization method was used to fine-tune the model weights. Hyperparameters are selected to balance model accuracy, robustness, and computational efficiency. A summary of model hyperparameters is presented in Table 1. The neural network model uses the location between zero and the rod length, along with past temperature measurements and heater duty, as the manipulated variable to predict the system output temperature in the next timestep. The input and output data are normalized.

Table 1. Summary of the PINN model structure

#	Layer Type	Unit	Activation Function
1	Dense	3	-
2	Dense	10	tanh
3	Dense	10	tanh
4	Dense	1	linear

The proposed framework aims to identify dominant thermal dynamics while maintaining physical consistency, enabling reduced-order, control-oriented modeling [11]. The learning rate decreases from 10^{-3} - 10^{-5} based on the number of epochs to improve the convergence and create a smooth training loss curve.

Performance Metrics

Model performance is evaluated using standard regression metrics, including mean squared error (3), as well as a qualitative assessment of extrapolation behavior.

$$\text{MSE} = 1/n \sum_{i=1}^n (\hat{y}_i - y_i)^2 \quad (3)$$

An early-stopping criterion of 10 epochs is used to prevent overfitting on the training data. Particular emphasis is placed on the model's ability to produce physically consistent predictions beyond the training domain.

PINN-MPC Controller Design

Control Strategy

Recent studies have integrated physics-informed neural networks into nonlinear model predictive control

frameworks, demonstrating improved tracking performance and computational efficiency [12]. At each sampling instant, a model-predictive control problem is formulated, using the trained PINN as a surrogate for the system dynamics. The objective function penalizes deviations of the predicted rod-tip temperature from the desired setpoint over a finite prediction horizon, while a move-suppression term is included to limit aggressive changes in the heater duty cycle. System evolution over the prediction horizon is governed by the PINN model, and the optimization is solved in a receding-horizon fashion, with only the first control action applied to the process.

This formulation is the minimal formulation for the MPC optimization problem:

$$\min_{u_k} \sum_{k=0}^{N_p} (\hat{y}_k - y_k^{ref})^2 + \rho \sum_{k=0}^{N_c} \Delta u_k^2 \quad (4)$$

subject to:

$$\hat{y}_{k+1} = f_{\text{PINN}}(\hat{y}_k, u_k) \quad (5)$$

where:

N_p : prediction horizon

N_c : control horizon

y^{ref} : temperature setpoint

ρ : move-suppression weight

Constraints and Implementation

Input constraints on heater duty are enforced to reflect physical actuator limits. A limit on the maximum change in heater duty at each controller action is applied. The optimization problem is solved using a receding-horizon approach, and only the first control action is applied at each sampling instant.

RESULTS AND DISCUSSION

The trained PINN demonstrates accurate prediction performance both within and outside the training domain. Figure 3 illustrates the model's ability to extrapolate beyond the training dataset while maintaining physical consistency. The heating rod model was reduced to a lower-order dimension using a PINN, since the surrogate model can predict the temperature at each location on the heating rod by getting the distance from the base as the input.

The proposed PINN model is embedded as a surrogate model within a Model Predictive Control (MPC) framework. The controller manipulates the heater duty to regulate the rod-tip temperature. This concept can be applied to industrial cases, where the cooling profile of the final product affects its quality. Figure 4 shows successful setpoint tracking performance under closed-loop operation. The PINN-MPC controller forecasts the dynamic behavior of the controlled variable (CV) and determines the optimal control action to meet the setpoint with

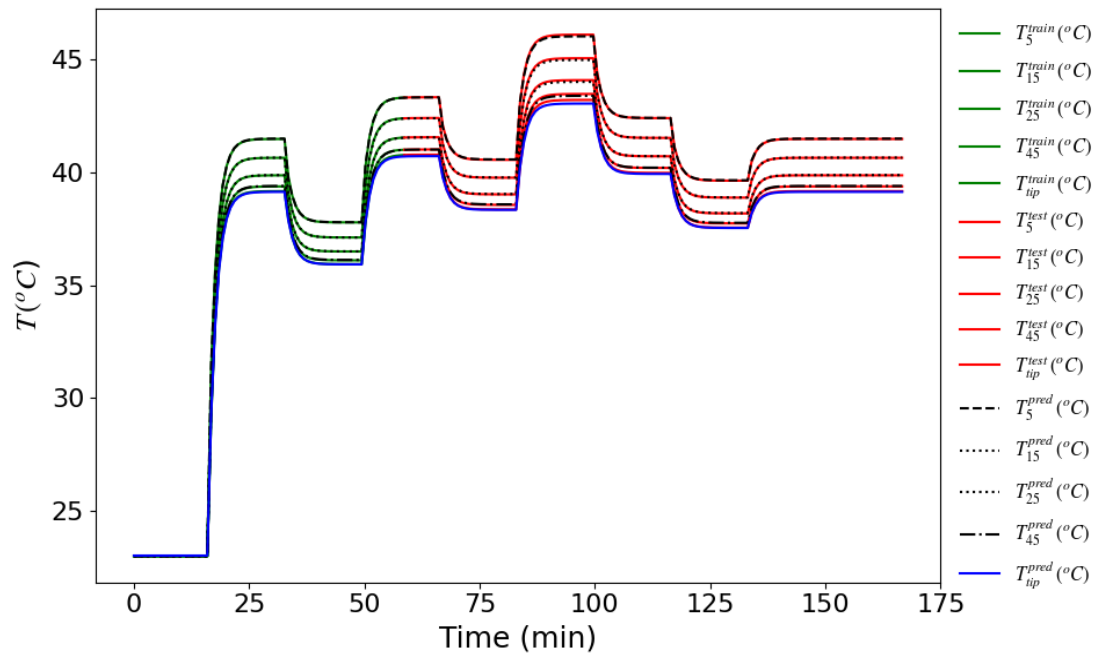


Figure 3. The prediction results from the PINN heating rod model against training and validation data at different locations on the rod

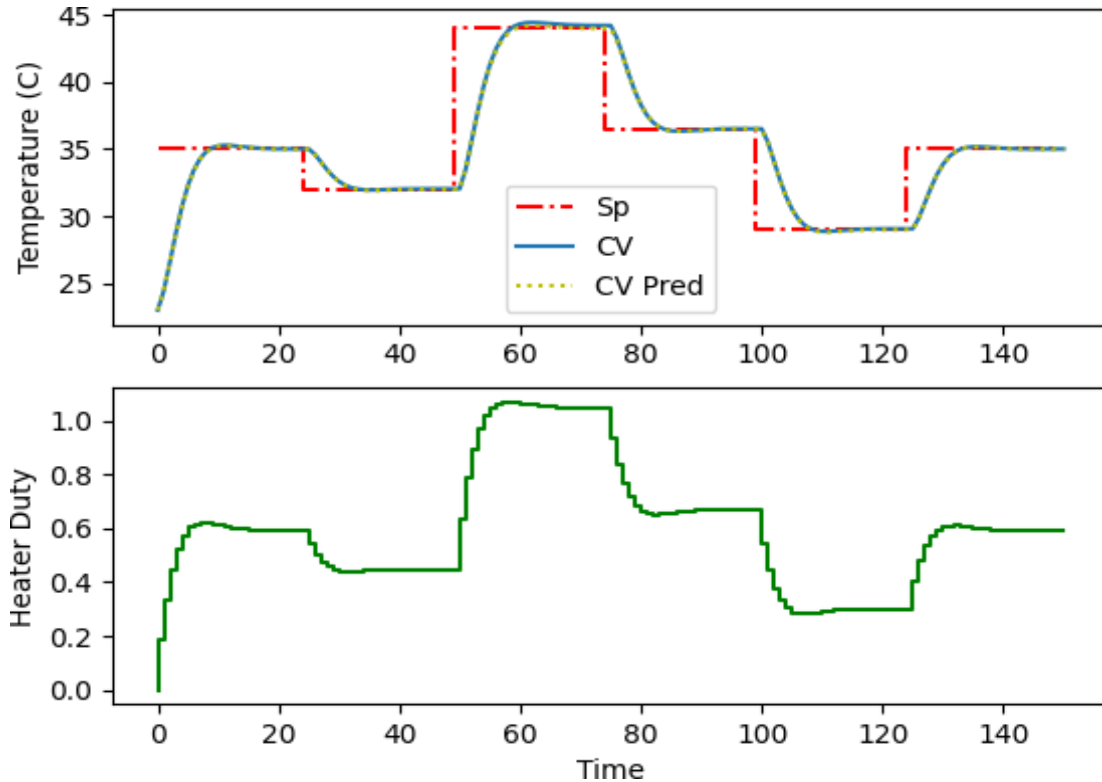


Figure 4. Setpoint tracking performance of PINN-MPC for the heating rod system.

acceptable overshoot. The penalty applied to control action in the MPC objective function avoids the oscillation around the setpoint, as shown in Figure 4.

CONCLUSION

This work presented a hybrid Physics-Informed Neural Network framework for thermal system identification and control. By enforcing physical constraints through numerically evaluated derivatives, the proposed approach achieves reliable prediction and extrapolation

performance using limited training data, while improving robustness compared to time-dependent PINN formulations. The developed PINN model was successfully employed as a control-oriented surrogate within a Model Predictive Control framework, enabling efficient real-time temperature regulation of a one-dimensional heating rod system.

Beyond the considered case study, the proposed framework provides a foundation for integrating physics-informed dimensionality reduction and feature selection techniques. For high-dimensional thermal systems, incorporating model-order reduction methods within the network architecture can significantly reduce computational complexity while preserving dominant dynamics. Future work will focus on extending the approach to multi-dimensional thermal processes, incorporating uncertainty quantification, and exploring advanced neural architectures, such as attention-based and transformer models, to further enhance modeling accuracy and control performance.

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