# Set-based Formulations for the State Task Network Scheduling Problem

Supplementary material

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## Appendix A. Case study 1

Figure A1 shows the STN considered for this case study.

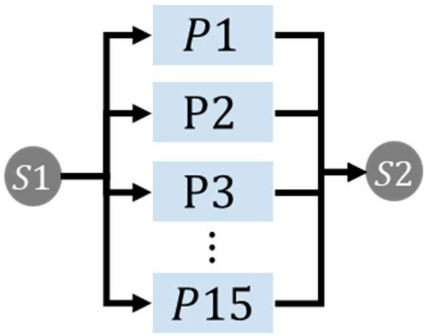


**Figure A1.** STN diagram, where rectangles are tasks and circles are states.

Eqs. (A.1)-(A.3) show the sets that define the connectivity of the STN shown in figure A1.

$$IJ^{\times} = \left\{ \begin{array}{c} (P1,U1),(P2,U1),(P4,U1),(P7,U1),(P9,U1),(P13,U1), \\ (P1,U2),(P2,U2),(P3,U2),(P4,U2),(P5,U2),(P6,U2),(P7,U2),(P8,U2), \\ (P9,U2),(P11,U2),(P13,U2),(P14,U2),(P15,U2), \\ (P1,U3),(P2,U3),(P3,U3),(P4,U3),(P5,U3),(P6,U3),(P7,U3),(P8,U3), \\ (P9,U3),(P10,U3),(P11,U3),(P12,U3),(P13,U3),(P14,U3),(P15,U3) \end{array} \right\} \quad \text{(A.1)}$$

$$IK^{+} = \{(P1,S2),(P2,S2),\dots,(P15,S2)\} \quad \text{(A.2)}$$

$$IK^{-} = \{(P1,S1),(P2,S1),\dots,(P15,S1)\} \quad \text{(A.3)}$$

Tables A1-A4 specify the parameters used for the case study.

**Table A1.** Fraction of materials consumed and produced by different tasks.

| Task-state pair $(i,k)$ | Consumed $(\rho^{-}_{i,k})$ | Produced $(\rho^{+}_{i,k})$ |
|---|---|---|
| $(P1,S1)$ | 1 | 0 |
| $(P2,S1)$ | 1 | 0 |
| ⋮ | ⋮ | ⋮ |
| $(P15,S1)$ | 1 | 0 |
| | | |
| $(P1,S2)$ | 0 | 1 |
| $(P2,S2)$ | 0 | 1 |
| ⋮ | ⋮ | ⋮ |
| $(P15,S2)$ | 0 | 1 |

**Table A2.** Tasks-unit pairs and their corresponding processing times and capacities.

| Task-unit pair $((i,j) \in IJ^{\times})$ | Processing time $(\tau_{i,j})$ | Min. capacity $(\beta_{i,j}^{min})$ | Max. capacity $(\beta_{i,j}^{max})$ |
|---|---|---|---|
| $(P1,U1),(P1,U2),(P1,U3)$ | 3 | 10 | 10 |
| $(P2,U1),(P2,U2),(P2,U3)$ | 2 | 15 | 15 |
| $(P3,U2),(P3,U3)$ | 4 | 20 | 20 |
| $(P4,U1),(P4,U2),(P4,U3)$ | 3 | 12 | 12 |
| $(P5,U2),(P5,U3)$ | 2 | 18 | 18 |
| $(P6,U2),(P6,U3)$ | 5 | 25 | 25 |
| $(P7,U1),(P7,U2),(P7,U3)$ | 3 | 14 | 14 |
| $(P8,U2),(P8,U3)$ | 4 | 22 | 22 |
| $(P9,U1),(P9,U2),(P9,U3)$ | 2 | 9 | 9 |
| $(P10,U3)$ | 5 | 29 | 29 |
| $(P11,U2),(P11,U3)$ | 3 | 16 | 16 |
| $(P12,U3)$ | 4 | 27 | 27 |
| $(P13,U1),(P13,U2),(P13,U3)$ | 3 | 13 | 13 |
| $(P14,U2),(P14,U3)$ | 3 | 21 | 21 |
| $(P15,U2),(P15,U3)$ | 2 | 19 | 19 |

**Table A3.** State storage capacities and initial levels.

| State $(k \in K)$ | Min. capacity $(\gamma_k^{min})$ | Max. capacity $(\gamma_k^{max})$ | Initial storage $(\gamma_k^{init})$ |
|---|---|---|---|
| $S1$ | 0 | 270 | 270 |
| $S2$ | 0 | 270 | 0 |

**Table A4.** Objective function profit parameters.

| Task ($i \in I$) | Profit coefficients ($v_i$) |
|---|---|
| $P1$ | 45 |
| $P2$ | 10 |
| $P3$ | 39 |
| $P4$ | 67 |
| $P5$ | 91 |
| $P6$ | 10 |
| $P7$ | 24 |
| $P8$ | 17 |
| $P9$ | 75 |
| $P10$ | 61 |
| $P11$ | 34 |
| $P12$ | 43 |
| $P13$ | 44 |
| $P14$ | 26 |
| $P15$ | 37 |

Table A5 defines the instances and parameters used to define the sensitivity analysis, whose results are shown in the main manuscript.

**Table A5.** Different scheduling horizons used for sensitivity analysis.

| Instance ID | Scheduling horizon ($n_T$) |
|---|---|
| 1 | 5 |
| 2 | 6 |
| 3 | 7 |
| 4 | 8 |
| 5 | 9 |
| 6 | 10 |
| 7 | 11 |
| 8 | 12 |
| 9 | 13 |
| 10 | 14 |
| 11 | 15 |
| 12 | 16 |

## Appendix B. Case study 2

Figure B1 shows the STN considered for this case study.

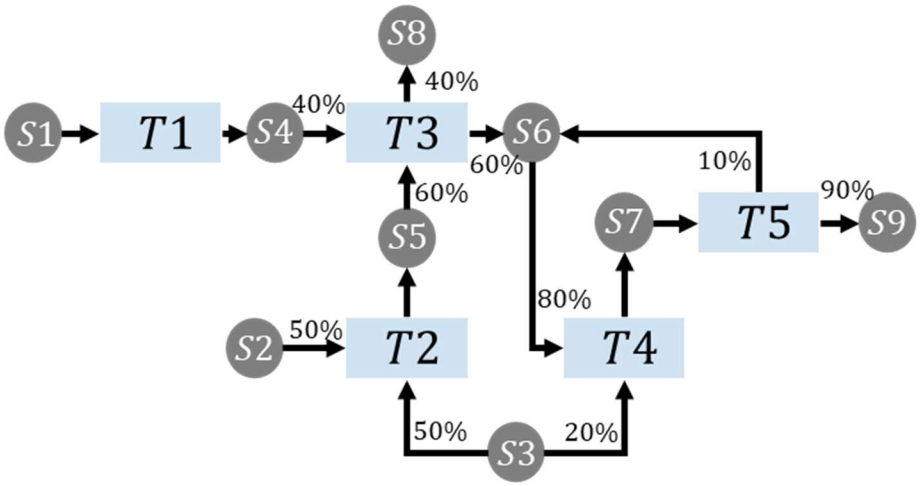


**Figure B1.** STN diagram, where rectangles are tasks and circles are states.

Eqs. (B.1)-(B.3) show the sets that define the connectivity of the STN shown in figure B1.

$$IJ^{\times} = \{(T1, U1), (T2, U2), (T2, U3), (T3, U2), (T3, U3), (T4, U2), (T4, U3), (T5, U4)\} \quad \text{(B.1)}$$

$$IK^{+} = \{(T1, S4), (T2, S5), (T3, S6), (T3, S8), (T4, S7), (T5, S6), (T5, S9)\} \quad \text{(B.2)}$$

$$IK^{-} = \{(T1, S1), (T2, S3), (T2, S2), (T3, S4), (T3, S5), (T4, S6), (T4, S3), (T5, S7)\} \quad \text{(B.3)}$$

Tables B1-B5 specify the parameters used for the case study.

**Table B1.** Fraction of materials consumed and produced by different tasks.

| Task-state pair $(i,k)$ | Consumed $(\rho_{i,k}^{-})$ | Produced $(\rho_{i,k}^{+})$ |
|---|---|---|
| $(T1,S4)$ | 0 | 1 |
| $(T2,S5)$ | 0 | 1 |
| $(T3,S6)$ | 0 | 0.6 |
| $(T3,S8)$ | 0 | 0.4 |
| $(T4,S7)$ | 0 | 1 |
| $(T5,S6)$ | 0 | 0.1 |
| $(T5,S9)$ | 0 | 0.9 |
| $(T1,S1)$ | 1 | 0 |
| $(T2,S3)$ | 0.5 | 0 |
| $(T2,S2)$ | 0.5 | 0 |
| $(T3,S4)$ | 0.4 | 0 |
| $(T3,S5)$ | 0.6 | 0 |
| $(T4,S6)$ | 0.8 | 0 |
| $(T4,S3)$ | 0.2 | 0 |
| $(T5,S7)$ | 1 | 0 |

**Table B2.** Tasks-unit pairs and their corresponding processing times and capacities.

| Task-unit pair $((i,j) \in IJ^{\times})$ | Processing time in units of time $(\tau_{i,j}^{p})$ | Min. capacity $(\beta_{i,j}^{min})$ | Max. capacity $(\beta_{i,j}^{max})$ |
|---|---|---|---|
| $(T1,U1)$ | 0.5 | 10 | 100 |
| $(T2,U2)$ | 0.5 | 10 | 50 |
| $(T2,U3)$ | 1.5 | 10 | 80 |
| $(T3,U2)$ | 1.0 | 10 | 50 |
| $(T3,U3)$ | 2.5 | 10 | 80 |
| $(T4,U2)$ | 1.0 | 10 | 50 |
| $(T4,U3)$ | 5.0 | 10 | 80 |
| $(T5,U4)$ | 1.5 | 10 | 200 |

**Table B3.** State storage capacities and initial levels.

| State ($k \in K$) | Min. capacity ($\gamma_k^{min}$) | Max. capacity ($\gamma_k^{max}$) | Initial storage ($\gamma_k^{init}$) |
|---|---|---|---|
| $S1$ | 0 | 4000 | 4000 |
| $S2$ | 0 | 4000 | 4000 |
| $S3$ | 0 | 4000 | 4000 |
| $S4$ | 0 | 4000 | 0 |
| $S5$ | 0 | 150 | 0 |
| $S6$ | 0 | 500 | 0 |
| $S7$ | 0 | 1000 | 0 |
| $S8$ | 0 | 4000 | 0 |
| $S9$ | 0 | 4000 | 0 |

**Table B4.** Objective function cost coefficients.

| Task-unit pair ($(i,j) \in IJ^{\times}$) | Cost coefficients ($\alpha_{i,j}$) |
|---|---|
| $(T1, U1)$ | 10 |
| $(T2, U2)$ | 15 |
| $(T2, U3)$ | 30 |
| $(T3, U2)$ | 5 |
| $(T3, U3)$ | 25 |
| $(T4, U2)$ | 5 |
| $(T4, U3)$ | 20 |
| $(T5, U4)$ | 20 |

**Table B5.** Objective function revenue coefficients.

| State ($k \in K$) | Revenue coefficients ($\omega_k$) |
|---|---|
| $S1$ | 0 |
| $S2$ | 0 |
| $S3$ | 0 |
| $S4$ | 0 |
| $S5$ | 0 |
| $S6$ | 0 |
| $S7$ | 0 |
| $S8$ | 3 |
| $S9$ | 4 |

Tables B6 and B7 define the instances and parameters used to define the sensitivity analysis, whose results are shown in the main manuscript.

**Table B6.** Sensitivity analysis parameters varying discretization quality.

| Instance ID | Scheduling horizon ($n_T$) | Time step in units of time ($\delta$) |
|---|---|---|
| 1 | 1 | 120 |
| 2 | 2 | 60 |
| 3 | 3 | 40 |
| 4 | 4 | 30 |
| 5 | 5 | 24 |
| 6 | 6 | 20 |
| 7 | 7 | 17 |
| 8 | 8 | 15 |
| 9 | 9 | 13 |
| 10 | 10 | 12 |
| 11 | 12 | 10 |
| 12 | 13 | 9 |
| 13 | 15 | 8 |
| 14 | 17 | 7 |
| 15 | 20 | 6 |
| 16 | 24 | 5 |
| 17 | 30 | 4 |
| 18 | 40 | 3 |
| 19 | 60 | 2 |
| 20 | 120 | 1 |

Based on table B6, the processing time used for each instance can be calculated as shown in Eq. (B.4).

$$\tau_{i,j} = \mathbf{ceil}\left(\frac{\tau_{i,j}^{p}}{\delta}\right) \tag{B.4}$$

**Table B7.** Fixed batching parameters used for the different instances in the sensitivity analysis.

| Instance | Batching parameter ($\psi_{i,j}$) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ID | $(T4, U2)$ | $(T4, U3)$ | $(T3, U2)$ | $(T3, U3)$ | $(T2, U2)$ | $(T1, U1)$ | $(T2, U3)$ | $(T5, U4)$ |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 3 | 0 | 0 | 2 | 2 | 1 | 1 | 1 | 0 |
| 4 | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 1 |
| 5 | 2 | 1 | 1 | 2 | 1 | 1 | 1 | 1 |
| 6 | 1 | 2 | 3 | 2 | 2 | 2 | 1 | 2 |
| 7 | 2 | 2 | 4 | 2 | 1 | 2 | 2 | 2 |
| 8 | 1 | 3 | 4 | 3 | 2 | 2 | 2 | 2 |
| 9 | 2 | 3 | 3 | 4 | 4 | 2 | 1 | 2 |
| 10 | 3 | 3 | 5 | 3 | 1 | 2 | 3 | 2 |
| 11 | 3 | 4 | 5 | 5 | 3 | 3 | 3 | 3 |
| 12 | 1 | 6 | 9 | 3 | 3 | 3 | 3 | 3 |
| 13 | 6 | 4 | 7 | 5 | 1 | 3 | 5 | 4 |
| 14 | 3 | 7 | 10 | 5 | 4 | 4 | 4 | 4 |
| 15 | 7 | 6 | 7 | 9 | 6 | 5 | 4 | 5 |
| 16 | 9 | 7 | 12 | 8 | 2 | 5 | 8 | 6 |
| 17 | 15 | 5 | 8 | 13 | 6 | 6 | 7 | 6 |
| 18 | 26 | 4 | 9 | 20 | 5 | 9 | 12 | 8 |
| 19 | 26 | 6 | 24 | 14 | 9 | 10 | 12 | 9 |
| 20 | 50 | 5 | 50 | 16 | 20 | 16 | 16 | 15 |