

# *Food for thought: Delicious problems for Process System Engineering (PSE) courses*



Daniel R. Lewin,  
Department of Chemical Engineering,  
Technion,  
Haifa, Israel

# ESCAPE | 35

6 → 9 JULY 2025

KU LEUVEN CAMPUS GHENT • BioTeC+



Computer Aided  
Process Engineering



EFCE's Working  
Party





- **Active Learning**
  - Improves classroom outcomes by allowing students to apply knowledge
  - Encourages making mistakes, correcting them, and repeating
- **Flipped Classroom** - An effective way to implement active learning
- **Purpose of the Presentation**
  - Introduce four food-themed problems for numerical methods and process control courses, that engage students in practical, problem-solving activities:
    - Chocolate bar formulation
    - Pizza shop production scheduling
    - Optimal frying time for “fried ice cream”
    - Control of continuous pancake batter production



# *An Example LP Problem: “Willy Wonka”*



Willy Wonka has hired you to formulate "Super-Choc," weighing 100 g. The bar consists of two types of chocolate: African (A), Belgian (B), and sugar. The optimal weight of each of the components needs to be specified to minimize costs, while meeting the following five requirements:



1. Each bar should contain no more than 60 g chocolate
2. The amount of A should not exceed the amount of B by more than 10 g.
3. The bar's *Sweetness* (S) should not exceed 30 units/bar, where each gram of A reduces S by 0.3 unit (relative to sugar), while each gram of B raises S by one unit.
4. The bar's *Mouth Feel* (M) should be at least 40 units. Each gram of A increases M by 2.2 units, while each gram of B raises M by one unit.
5. The bar's *Luster* (L) should be at least 70 units. Each gram of A increases L by one unit, while each gram of B raises L by 3.9 units.

If the cost of B is three times the cost of A, and the cost of sugar can be neglected, compute the optimal composition of a bar of "Super-Choc."



# *Solution of the “Willy Wonka” Problem*



Willy Wonka has hired you to formulate a new chocolate bar called "Super-Choc," weighing 100 grams. The bar consists of two types of chocolate: African (A) and Belgian (B), and sugar. If the cost of Belgian chocolate is three times the cost of African chocolate, and the cost of sugar can be neglected, compute the optimal composition of a bar of "Super-Choc."

Defining  $x_1$  and  $x_2$  as the mass in grams, of African and Belgian chocolate, respectively, the problem as defined can be translated to the following LP:

$$\min_{x_1, x_2} J = x_1 + 3x_2 \quad (\text{B costs 3 times more than A})$$

Subject to:

1. Each bar should contain no more than 60 grams of chocolate

$$x_1 + x_2 \leq 60 \quad (\text{chocolate content of at most 60 g})$$

2. The amount of African chocolate should not exceed the amount of Belgian chocolate by more than 10 grams.

$$x_1 - x_2 \leq 10 \quad (\text{A content must be up to 10 g more than B})$$



3. The bar's *Sweetness* ( $S$ ) should not exceed 30 units per bar, where each gram of African chocolate reduces  $S$  by 0.3 unit (relative to sugar), while each gram of Belgian chocolate raises  $S$  by one unit.

$$-0.3x_1 + x_2 \leq 30 \text{ (S must be at most 30)}$$

4. The bar's *Mouth Feel* ( $M$ ) should be at least 40 units. Each gram of African chocolate increases  $M$  by 2.2 units, while each gram of Belgian chocolate raises  $M$  by one unit.

$$2.2x_1 + x_2 \geq 40 \text{ (M must be at least 40)}$$

5. The bar's *Luster* ( $L$ ) should be at least 70 units. Each gram of African chocolate increases  $L$  by one unit, while each gram of Belgian chocolate raises  $L$  by 3.9 units.

$$x_1 + 3.9x_2 \geq 70 \text{ (L must be at least 70)}$$

$$x_1, x_2 \geq 0$$



Defining  $x_1$  and  $x_2$  as the mass in grams, of African and Belgian chocolate, respectively, the problem as defined can be translated to the following LP:

$$\min_{x_1, x_2} J = x_1 + 3x_2 \text{ (B costs 3 times more than A)}$$

Subject to:

$$x_1 + x_2 \leq 60 \text{ (chocolate content of at most 60 g)}$$

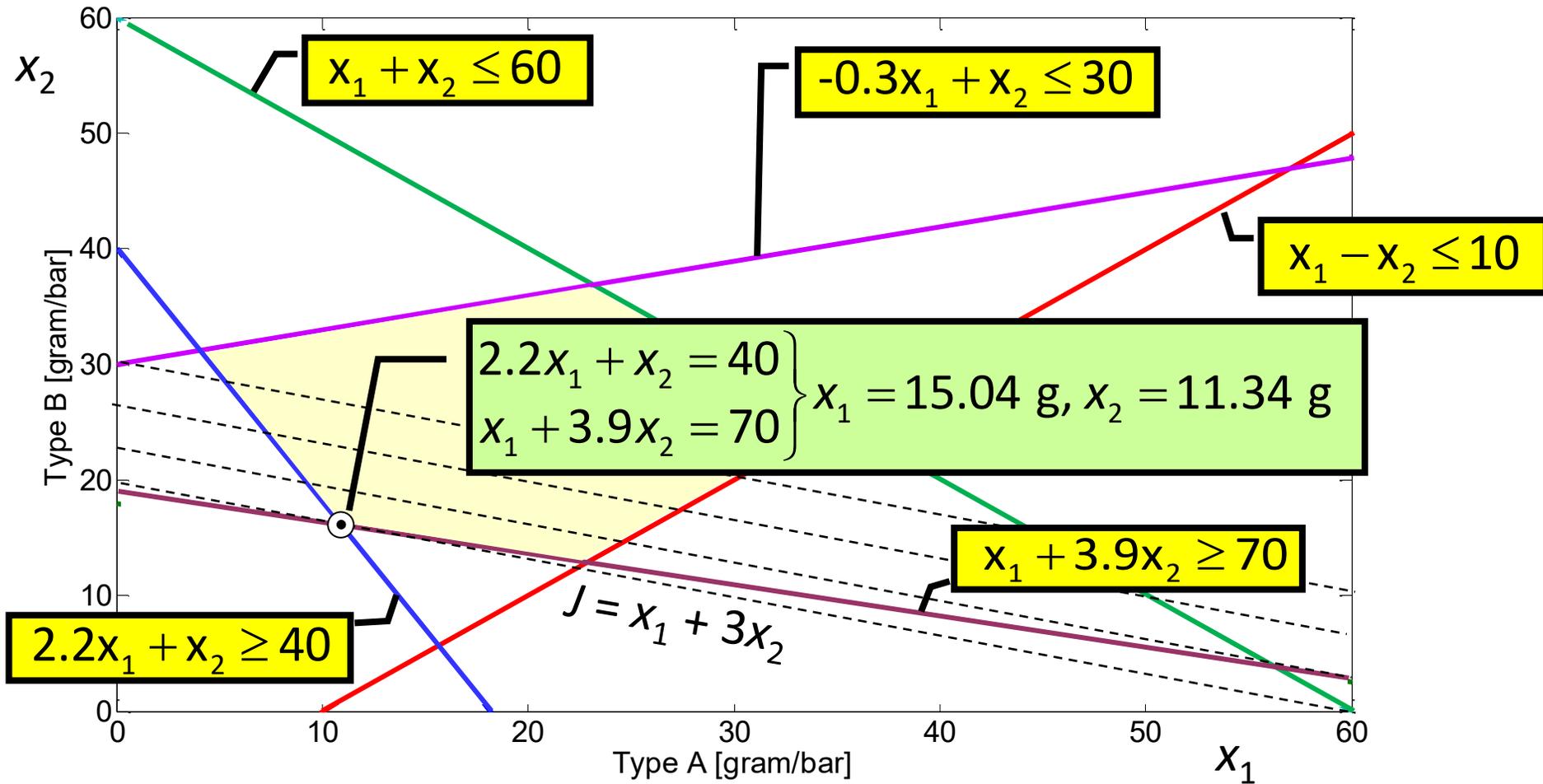
$$x_1 - x_2 \leq 10 \text{ (A content must be up to 10 g more than B)}$$

$$-0.3x_1 + x_2 \leq 30 \text{ (S must be at most 30)}$$

$$2.2x_1 + x_2 \geq 40 \text{ (M must be at least 40)}$$

$$x_1 + 3.9x_2 \geq 70 \text{ (L must be at least 70)}$$

$$x_1, x_2 \geq 0$$





# *Example MILP Problem: “Matrix Pizza”*

# M\_Pizza Problem - Definition

“Matrix Pizza” is a pizza bakery that provides quality pizza pies to the college campus in mid-West USA. The bakery produces three types of pizza pies: Matrix Special, sold for \$10/pie, Matrix Pro, sold for \$7/pie, and Matrix College, sold for \$5/pie. The three types of pizza differ only in the toppings, whose costs are summarized below.

Grade	Price (\$)	Tomatoes	Mozzarella	Parmasan	Olives	Tuna
Matrix Special	\$10	200g	200g	100g	100g	100g
Matrix Pro	\$7	250g	250g	100g	100g	-
Matrix College	\$5	350g	350g	-	-	-
Cost (\$/kg)		2	2	15	20	10

**Staff costs:** \$1/pizza pie

**Equipment and maintenance costs:** \$1,000/oven/week

**Capacity:** 1,000 pizzas/oven/week

Pizza Weekly Demands	Matrix Special	Matrix Pro	Matrix College
No Saturday football match	320	480	600
With Saturday home games	520	580	700



- a) Formulate and solve a MILP whose solution is the optimal quantities of each type of pizza that should be produced each week, including the number of ovens that should be installed in the bakery to maximize profits.

$$\max_{x_1, x_2, x_3, y} J = 3.7x_1 + 1.5x_2 + 2.6x_3 - 1,000y. \quad x_1, x_2, x_3, y - \text{integer.}$$

Subject to:

$$x_1 + x_2 + x_3 \leq 1,000y$$

$$x_i \geq 0, i = 1, 2, 3$$

$$x_1 \leq 320$$

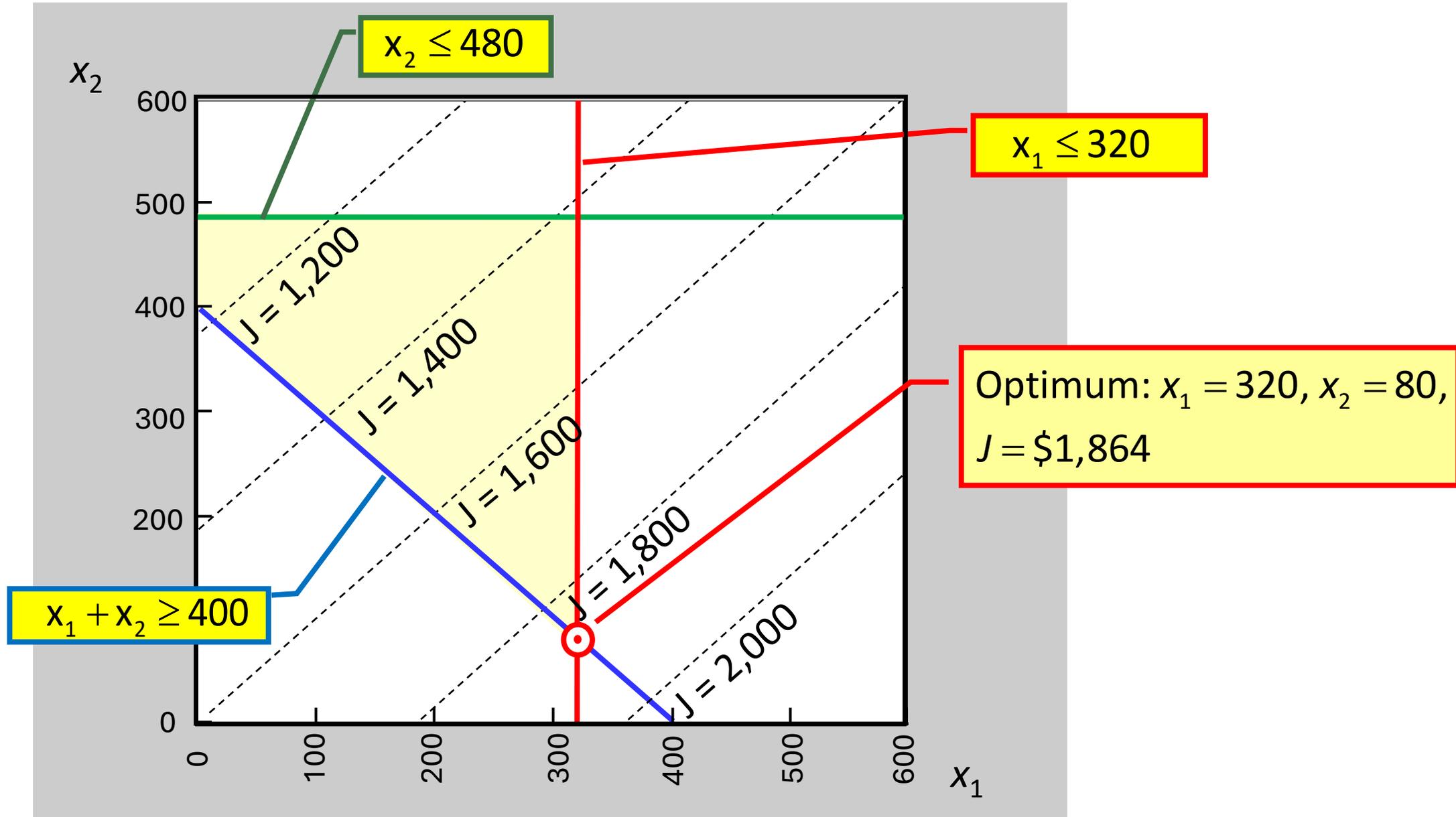
$$x_2 \leq 480$$

$$x_3 \leq 600$$

$$y \geq 1$$



$$y = 1$$





$$y = 2$$

$$\max_{x_1, x_2, x_3, y} J = 3.7x_1 + 1.5x_2 + 2.6x_3 - 2,000. \quad x_1, x_2, x_3, y - \text{integer.}$$

Subject to:

$$x_1 + x_2 + x_3 \leq 2,000$$

$$x_i \geq 0, i = 1, 2, 3$$

$$x_1 = 320$$

$$x_2 = 480$$

$$x_2 = 600$$

$$y \geq 1$$

$$\left. \begin{array}{l} x_1 = 320 \\ x_2 = 480 \\ x_2 = 600 \end{array} \right\} J = \$1,464. \quad 2^{\text{nd}} \text{ oven not justified!}$$



b) The college's football team is planning to hold home games each Saturday night. How will this affect the optimal solution, if the home games change the pizza demands on Saturdays according to the demand table.

$$\max_{x_1, x_2, x_3, y} J = 3.7x_1 + 1.5x_2 + 2.6x_3 - 1,000y. \quad x_1, x_2, x_3, y - \text{integer.}$$

Subject to:

$$x_1 + x_2 + x_3 \leq 1,000y$$

$$x_i \geq 0, i = 1, 2, 3$$

$$x_1 \leq 520$$

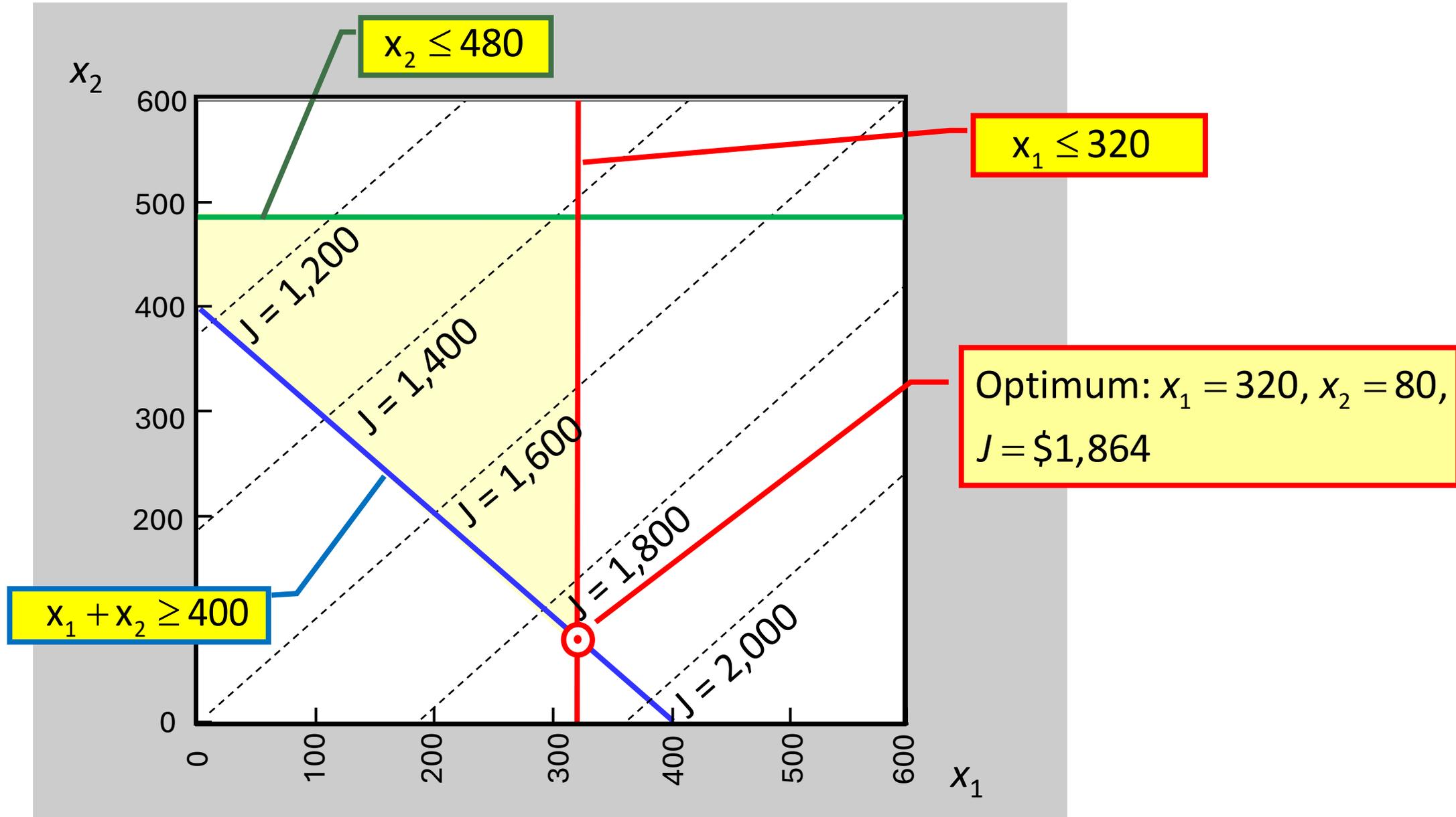
$$x_2 \leq 580$$

$$x_3 \leq 700$$

$$y \geq 1$$



$$y = 1$$



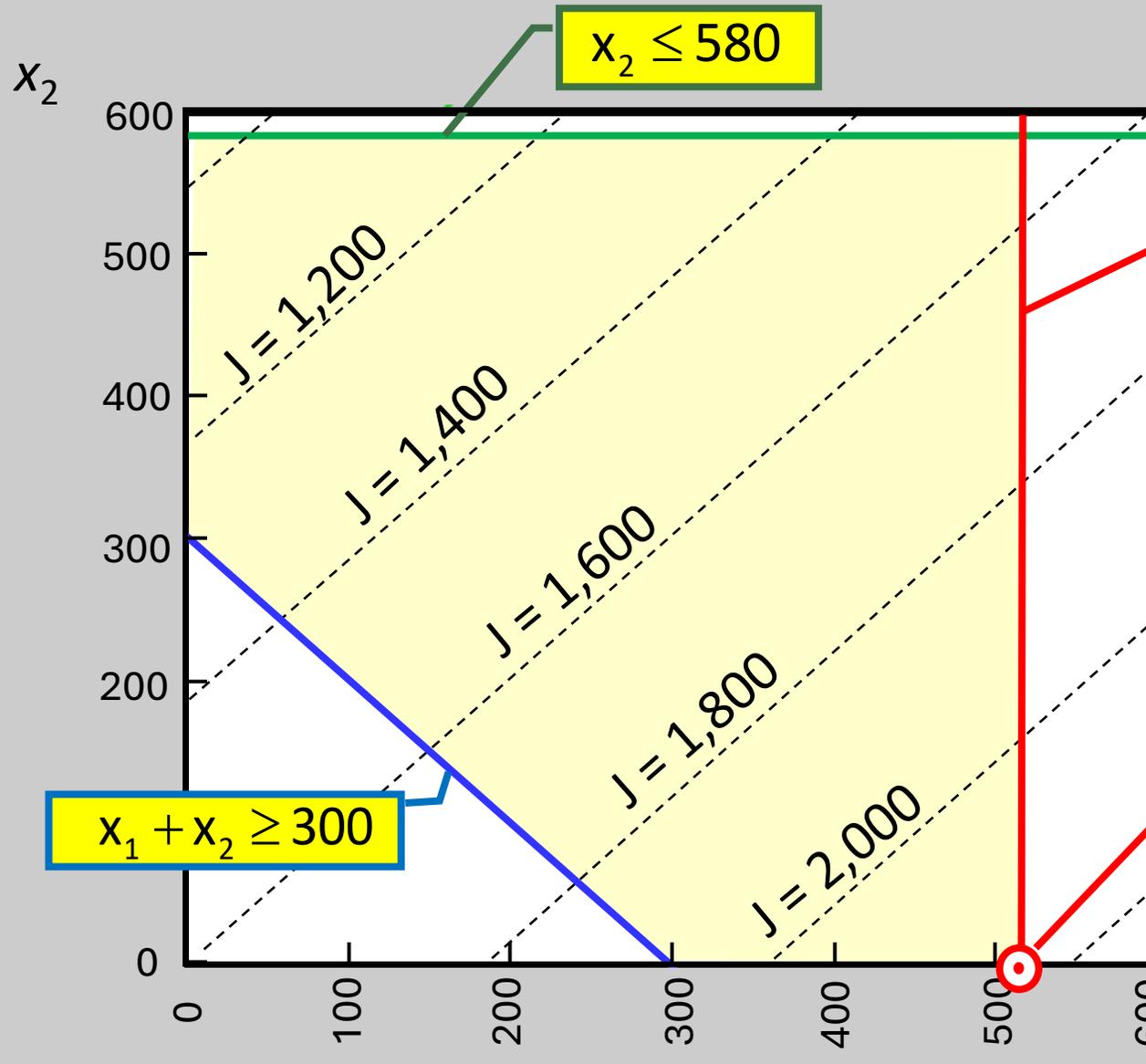
# Matrix Pizza Graphical Solution

ESCAPE | 35

6 → 9 JULY 2025



$$y = 1$$



$$x_2 \leq 580$$

$$x_1 \leq 520$$

$$x_1 + x_2 \geq 300$$

Optimum:  $x_1 = 520$ ,  $x_2 = 0$ ,  
 $J = \$2,172$



$$y = 2$$

$$\max_{x_1, x_2, x_3, y} J = 3.7x_1 + 1.5x_2 + 2.6x_3 - 2,000. \quad x_1, x_2, x_3, y - \text{integer.}$$

Subject to:

$$x_1 + x_2 + x_3 \leq 2,000$$

$$x_i \geq 0, i = 1, 2, 3$$

$$x_1 = 520$$

$$x_2 = 580$$

$$x_3 = 700$$

$$y \geq 1$$

$$J = \$2,614.$$

2 ovens are optimal!



*Example IVPDE Problem: “Fried Ice Cream”*

# “Fried Ice Cream”- Definition

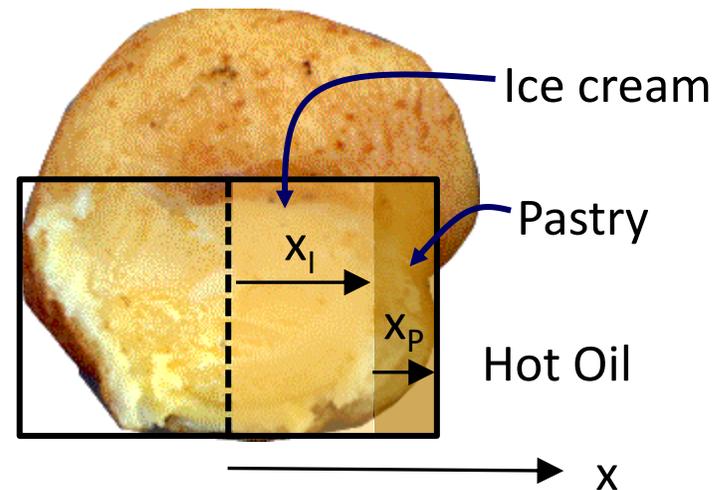


“Fried ice cream” is the house specialty of the “Sea Waves” restaurant.



A block of ice cream is wrapped in pastry dough and deep-frozen. The required frying time needs to be computed.

A mathematical model is set up to compute the temperature of the pastry and ice cream as functions of time and position.

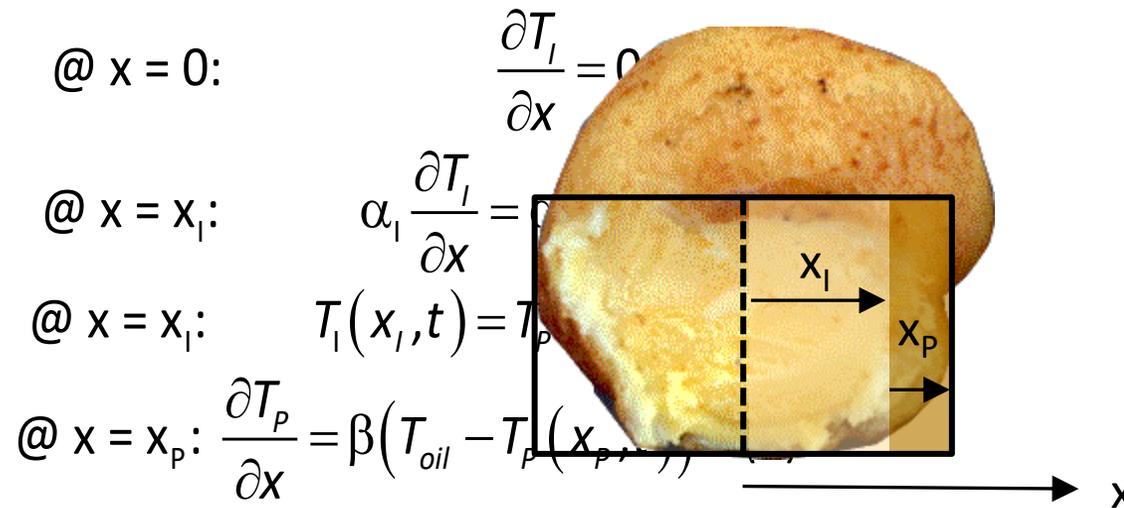




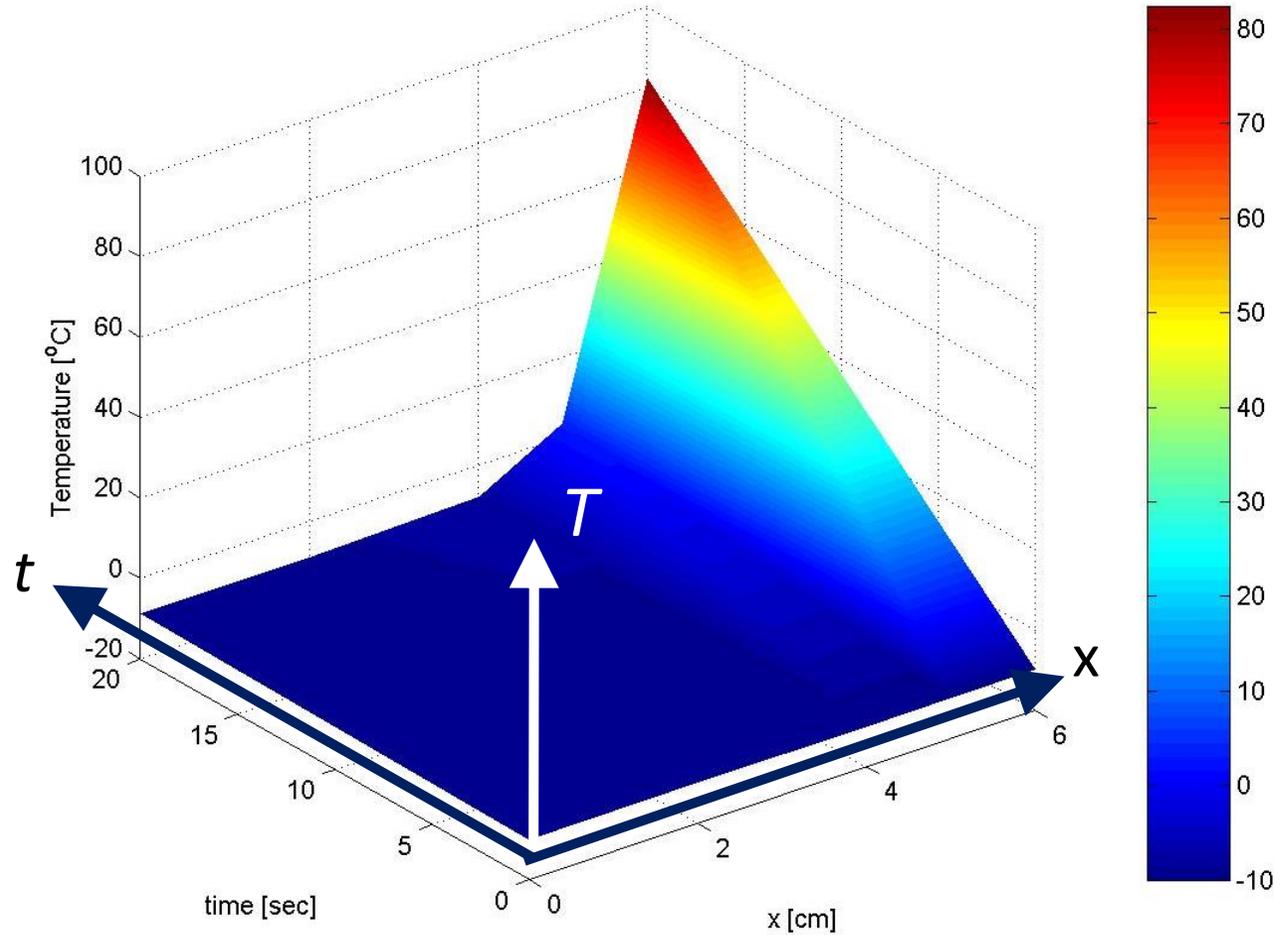
For simplicity, a one-dimensional model is constructed making use of symmetry:

For ice cream:  $\frac{\partial T_i}{\partial t} = \alpha_i \frac{\partial^2 T_i}{\partial x^2}, T_i(x,0) = T^0, 0 \leq x \leq x_i \quad (1)$

For pastry:  $\frac{\partial T_p}{\partial t} = \alpha_p \frac{\partial^2 T_p}{\partial x^2}, T_p(x,0) = T^0, x_i < x \leq x_p \quad (2)$



# “Fried Ice Cream”- Solution



$$(T_{1,j} \quad T_{2,j} \quad T_{3,j} \quad T_{4,j} \quad T_{5,j}) = (-8.857 \quad -8.857 \quad -7.943 \quad 3.483 \quad 82.32)$$



*Example Control Problem:  
“Uncle Kane’s Pancake Batter Machine”*

# *Conclusions*



The four main takeaways are:

- ❑ Active learning strategies, especially in flipped classrooms, enhance student engagement and learning outcomes in process systems engineering courses.
- ❑ The flipped classroom paradigm releases contact time for open-ended problem-solving and practical exercises, requiring banks of appropriate problems that can be used by students for practice.
- ❑ Food-themed problems, with varying difficulty levels:
  - spark students' curiosity
  - motivate them
  - promote their critical thinking, problem-solving, and technical skills.
- ❑ The exercises introduced today have been used over decades, incorporating each of them in one exam and then in classroom practice.