

Real-time dynamic optimisation for sustainable biogas production through anaerobic co-digestion with hybrid models

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ABSTRACT

Renewable energy and energy efficiency are increasingly recognised as crucial for creating new economic opportunities and mitigating environmental impacts. Anaerobic digestion (AD) transforms organic materials into a clean, renewable energy source. Co-digestion of various organic wastes and energy crops addresses the disadvantages of single-substrate digestion, increasing production flexibility yet adding process complexity and sensitivity. This study employs a two-pronged approach to optimise biogas production while considering global warming potential: a nonlinear programming (NLP) model for dynamic system economic optimisation with a model predictive control (MPC) strategy for precise temperature regulation within the digester. The NLP model integrates a combined heat and power (CHP) system to leverage dynamic electricity, heat, and gas prices, accounting for physical and economic parameters such as biomethane potential, chemical oxygen demand, and substrate density. A cardinal temperature and pH model ensures accurate depiction of substrate degradation and gas production rates under varying conditions. The MPC scheme, formulated as a system of differential-algebraic equations, offers fine-grained temperature control, capturing real-world complexities like heating/cooling delays, ambient conditions, and multiple feed components with different optimal digestion temperatures. Results demonstrate that this integrated model optimises the interaction between electricity production, biogas generation, and CHP operation for real-time multi-objective optimisation of profit, global warming potential and temperature control. A case study validates the model's capability for guiding decision-making in biogas facilities, emphasising strategic feedstock management and precise temperature control. Overall, this integrated approach advances the modelling and control of anaerobic co-digestion systems, enhancing both efficiency and profitability in biogas production.

Keywords: Pyomo, Food & Agricultural Processes, Optimization, Process Control, Biofuels

INTRODUCTION

In addition to reducing carbon emissions, renewable energy sources lessen dependency on fossil fuels[1]. Anaerobic digestion (AD) is a well-established and reliable method to convert organic feedstocks into clean energy. However, single-substrate digestion often limits operational flexibility and productivity [2]. Co-digestion—processing multiple substrates, such as organic waste

and energy crops, simultaneously can significantly enhance biogas yields by leveraging the complementary characteristics of diverse feedstocks [3]. Despite these advantages, co-digestion introduces greater process complexity, increasing sensitivity to variables such as substrate composition, retention times, temperature control, and microbial balance [4]. These interdependent factors influence microbial growth, digestion kinetics, and, ultimately, system efficiency.

Optimising co-digestion processes therefore requires innovative solutions that integrate both the physical and economic dimensions of AD systems. A range of control strategies has been developed for AD; however, few of these strategies are deployed in industry due to the variability in waste characteristics and operating conditions [5]. Classical PID controllers, fuzzy logic expert systems, and neural networks are among the most common approaches, targeting controlled variables such as methane flow rate, pH, and chemical oxygen demand (COD). Moreover, several researchers have explored using model predictive control (MPC) in AD[6], motivated by its capability to handle multivariable processes with constraints. Despite this progress, predictive control in AD remains challenging due to the nonlinear, and the uncertain nature of the biochemical processes, which involve numerous microbial populations. These complexities are amplified in co-digestion systems because of the diverse range of substrates being processed simultaneously[7].

Recognising temperature as a crucial operational factor, particularly for mesophilic AD, this study applies MPC to regulate digester temperature, while simultaneously considering optimal feeding and gas processing strategy. Temperature exerts a direct influence on microbial activity and reaction rates, making its precise control pivotal for stable and efficient biogas production. While earlier work has addressed MPC for single-substrate AD, to the best of our knowledge, this is the first application of predictive control in an agricultural co-digestion context. By addressing both the inherent process complexity and the added challenges introduced by multiple substrates, MPC offers a promising avenue to improve the robustness and performance of AD systems under real-world operating conditions. In parallel with the challenges of process control, fluctuating energy prices underscore the need for strategic operational decisions in AD. Currently, studies often focus on isolated variables—such as temperature regulation—without capturing the interplay between feedstock availability, and market conditions. Without predictive insights into market trends, operators risk missing opportunities to maximise profitability and reduce operational risks [8].

Accordingly, this study proposes an integrated framework that combines dynamic optimisation, precise temperature management, and advanced price forecasting to enhance co-digestion-based AD systems. First, a nonlinear programming (NLP) model is formulated to optimise resource allocation and operational decisions, considering factors such as feedstock mix, digester capacity, and cost constraints. Second, a model predictive control (MPC) scheme ensures tight temperature management, accounting for real-world dynamics and uncertainties. Third, the Facebook Prophet model is used to forecast energy prices over time,

enabling informed, data-driven decisions about biogas injection and energy generation. This model was selected due to its proven effectiveness in tackling complex market-related time-series forecasting challenges [9]. By synchronising these components, the framework aims to maximise production flexibility, profitability, and sustainability, addressing the growing demand for efficient, low environmental impact renewable energy solutions.

METHODOLOGY

This study presents an integrated framework that combines dynamic real-time process optimisation, MPC, and energy price forecasting to enhance both the sustainability and economic viability of biogas production in AD systems. Figure 1 outlines the principal components of this approach: feedstock management, AD, energy generation, and product utilisation. Feedstock (e.g., maize and manure) is collected from harvesting sites and farms, where it can either be stored for later use or fed directly into the digester. The produced biogas follows two possible pathways: (1) injection into the gas grid following CO₂ removal, or (2) utilisation as a fuel source for CHP units. The residual digestate is employed as an organic fertilizer, further closing the loop on waste management.

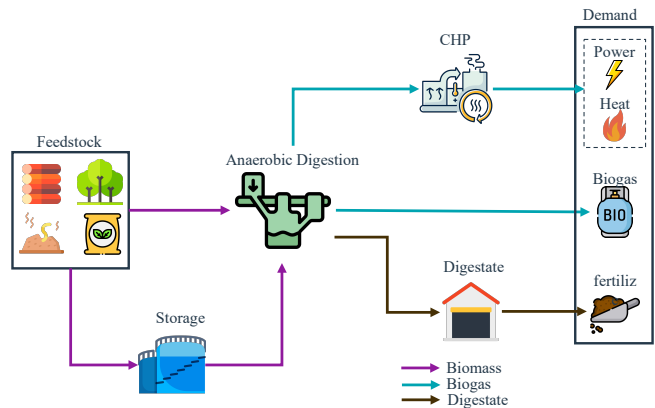


Figure 1. Superstructure of proposed model

To optimise resource utilisation and operational efficiency, an NLP model is formulated with a multi-objective function:

$$\text{Max}[Z] = \omega_1 \text{Profit} - \omega_2 \text{GWP} - \omega_3 \text{Penalty}_{\text{MPC}}, \sum \omega_i = 1 \quad (1)$$

Here, *Profit* presents total profit of the AD process over the prediction horizon including profit of selling biogas and electricity and production costs; *GWP* is the overall global warming potential (GWP) of the process that is calculated by summation of the greenhouse gas emissions from cultivation, transport, CHP operation and, biogas leakage [10]; and *Penalty_{MPC}* penalises temperature deviations from the optimal range. The weighting

factors ω_i allow for customising the balance between maximising biogas output, minimising costs, and maintaining stable operating conditions. Physical and economic constraints, such as biomethane potential, chemical oxygen demand, and substrate density, ensure robust substrate utilisation and system performance.

A key innovation of this work is the incorporation of an advanced MPC scheme to regulate the digester temperature. By predicting future disturbances over a defined time horizon, the MPC dynamically adjusts heating ($Q_{heating}$) and cooling ($Q_{cooling}$) inputs to maintain optimal microbial activity. The thermal balance is captured by the following equation, which includes heating, direct thermal losses, cooling, and feed-related heat terms without delay for Q_{loss} and Q_{feed} :

$$M_{digester} \cdot \frac{C_p dT_{digester}}{dt} = Q_{heating}^{delayed} - Q_{loss} - Q_{cooling}^{delayed} + Q_{feed}, \quad (2)$$

where $M_{digester}$ and C_p are mass of the digester contents and specific heat capacity, respectively. To account for realistic response times, the control inputs for heating and cooling are represented by delayed states. These delays are modeled dynamically using first-order delay equations $\frac{dQ^{delayed}}{dt} = \frac{Q - Q^{delayed}}{\tau_{delay}}$. The parameter τ_{delay} represents the characteristic time constant for the delay associated with each process. Physically, it accounts for the lag in system responses due to the time required for heating elements, cooling mechanisms, or fluid dynamics to affect the digester temperature. By incorporating τ_{delay} , the model ensures that control actions better reflect real-world system dynamics, improving the accuracy and robustness of the MPC.

The digester mass balance is expressed as: $\frac{dM}{dt} = f_{in} - f_{out}$, subject to a capacity constraint $M \leq \rho V_{dg}$. Here f_{in} , f_{out} , ρ , and V_{dg} are the flows of input and output, density of feedstock and volume of digester, respectively. The inflow rate is determined by summing mass contributions of substrate and water content, whereas the outflow depends on a smooth step function governed by hydraulic retention time. This combination of material and energy balance constraints enables the MPC to maintain the digester temperature near an ideal setpoint, thus preserving optimal microbial activity and stabilising biogas output.

To accurately model microbial activity and biogas production over time, the study incorporates a cardinal temperature model (CTM) model [11]. This involves defining constraints for the specific growth rate for substrate j at time t ($\mu_{j,t}^T$) and biogas production:

$$\mu_{j,t}^T = \frac{\mu_j^{opt} (T_t - T_{max,j})(T_t - T_{min,j})^2}{(T_{opt,j} - T_{min,j})[(T_{opt,j} - T_{min,j})(T_t - T_{opt,j}) - (T_t - T_{opt,j})(T_{opt,j} + T_{min,j} - 2T_t)]} \quad (3)$$

where μ_j^{opt} is the optimal growth rate, T_t is the digester

temperature, $T_{min,j}$, $T_{max,j}$, and $T_{opt,j}$ are the minimum, maximum, and optimal temperatures for each substrate, and μ_j^{opt} is the optimal growth rate. By using $\mu_{j,t}^T$, the superimposed first order model [12] was used to predict the maximum biomethane production in time (P_t):

$$BP_t = \left(\sum_{j,\tau} \frac{AV_{t,j}}{\sum_j AV_{t,j}} \right) BMP_j e^{1 - \mu_{j,t}^T (t - \tau)}, \quad (4)$$

where $AV_{t,j}$ is the available substrate for species j at time t , and BMP_j is the biomethane potential of species j . Here, τ is the time when substrate j is added, and $(t - \tau)$ is the elapsed degradation time since its introduction into the digester. In addition to temperature constraints, other factors such as the carbon-to-nitrogen (C/N) ratio and total solids (TS) content are integrated into the model as constraints to maintain feasible operation within industrially relevant bands. These parameters also affect microbial activity and are thus accounted for in calculating overall biogas production, ensuring a more comprehensive and accurate representation of the AD process.

Future energy price forecasting is integral to optimising operational decisions, particularly concerning the timing of biogas injection into the grid versus CHP utilisation for electricity production. In this study, the Prophet model is used to predict electricity prices based on historical data, seasonal factors, and longer-term trends. Formally, Facebook Prophet can be represented as [13]:

$$y(t) = g(t) + s(t) + h(t) + \epsilon_t, \quad (3)$$

where $g(t)$ captures the piecewise linear or logistic trend, $s(t)$ represents seasonality, $h(t)$ accounts for holiday or special event effects, and ϵ_t is an error term. To refine forecasting accuracy, Bayesian optimisation is conducted over key hyperparameters—such as the changepoint prior scale, seasonality prior scale, and changepoint range—using a rolling-window cross-validation approach. The root mean square error (RMSE) is used to evaluate each parameter set; if no improvement is observed after multiple iterations, the search terminates early.

Once the optimal hyperparameters are identified, the final Prophet model is retrained on a rolling basis to produce accurate forecasts for the upcoming months. These forecasts guide strategic decisions, enabling operators to schedule biogas injection or CHP production during periods of favourable energy prices. By aligning operational activities with real-time market signals, the proposed methodology maximises revenues, promotes energy self-sufficiency, and supports environmental sustainability within biogas-based systems.

Case Study

To demonstrate the performance of the model, we selected as case study a farm-scale AD plant with a digester capacity of 7,000 m³, designed to process energy crop maize and manures. The feeding rate is constrained

by a solid retention time (SRT) of 90 days, with the maximum total solids content of the digester feed limited to 35 %. All specified capacities and feed information are adapted from real site data provided by industrial partners with some changes to feed costs. The estimated parameters in equation (3) is presented in Table 1 [14], [15]:

Table 2. Cardinal temperature model parameters

	T_{min} (C)	T_{opt} (C)	T_{max} (C)	μ^{opt} (1/d)
Maize	-1.27	37.26	45.12	2.63
Manure	3.60	41.10	45.06	1.29

In an hourly timestep with a nine-day prediction horizon, electricity and gas price data were obtained from the Office for National Statistics of the United Kingdom [16], [17]. The results in Figure 2 illustrate that the Prophet model captures key price trends for both electricity and gas, while uncertainty bands (shaded in blue) reflect forecast confidence. The forecasted prices (blue lines) track the observed values (black dots) reasonably well, indicating satisfactory predictive performance. Using a shorter horizon helps reduce computational complexity while still providing actionable price estimates for operational decision-making.

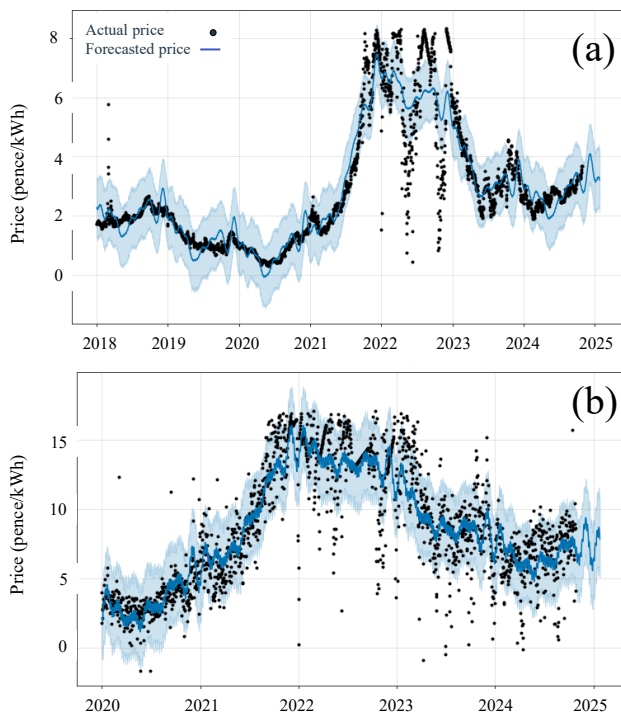


Figure 2. Historical data and forecasting performance for (a) gas price and (b) electricity prices.

RESULTS AND DISCUSSION

The solutions of the case study were computed by implementing the proposed model in Pyomo —

comprising 6,941 constraints (1,735 nonlinear) and 7,148 variables — and using the IPOPT solver, which effectively handles the non-linearities present in both the digester dynamics and the market-driven optimisation. The model was solved in 700 CPUs, demonstrating efficient computational performance given the problem's scale and complexity.

As depicted in Figure 3, the digester temperature tracks the ideal temperature setpoint with a small offset. Initially, the system demands a rapid increase in heating power (up to a maximum of 9000 W) to bring the digester from ambient conditions to near-optimal temperature, while significant heat losses occur due to the temperature gradient between the digester contents and the external environment. Given the higher cost of cooling, the maximum cooling power is limited to 900 W. Notably, an ambient temperature difference of about 20 °C was used to test the model's robustness under a challenging scenario. Once the system stabilises, the MPC strategy finely tunes both heating ($Q_{heating}^{delayed}$) and cooling ($Q_{cooling}^{delayed}$), ensuring close setpoint tracking. These observations affirm the efficacy of the delayed control action in sustaining microbial activity and preventing temperature overshoot. However, it is seen that the model does not always prioritise temperature control due to its weighting in the objective function being relatively low, leading to trade-offs in how the system balances temperature regulation against other operational goals.

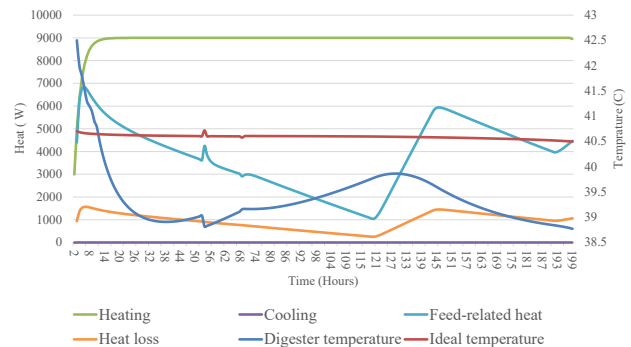


Figure 3. Performance of the proposed MPC.

As Figure 4 shows, electricity generation (blue bars) is presented alongside biogas sold to the grid (red line). The fluctuations in biogas sales are a result of real-time optimisation of feed rates and substrate compositions, driven by both technical constraints and market opportunities. During periods of higher electricity prices, the optimised solution diverts a greater proportion of biogas to the CHP unit, thereby maximising on-site power output for export. Conversely, when injecting gas into the grid proves more profitable, surplus biogas is channelled away from the CHP, underscoring the value of coupling accurate price forecasting with flexible production pathways.

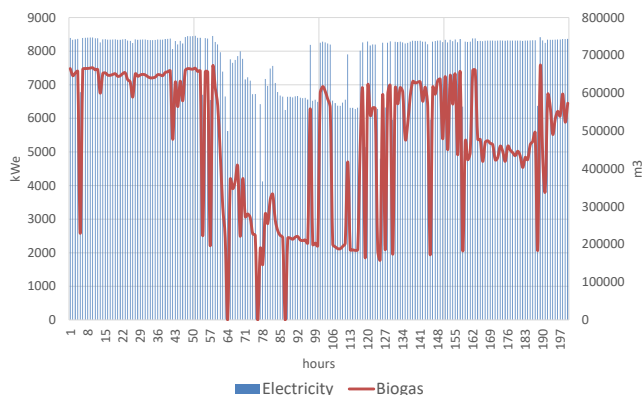


Figure 4. Optimal energy distribution by the process.

Figure 5 displays the co-digestion ratio shifts over time, with maize shown in blue and manure in red. By dynamically balancing these two substrates, the system exploits maize's higher biomethane potential while benefiting from manure's stabilizing influence on nutrient levels. Sudden changes in the substrate ratio frequently correlate with fluctuations in biogas yield, highlighting the intricate relationship between feedstock composition, microbial performance, and shifts in up- and down-stream pricing and feed availability. Overall, these results confirm that adaptively managing feed composition, temperature control, and energy allocation in response to real-time conditions can significantly enhance biogas production, ensure process stability, and optimise economic returns.

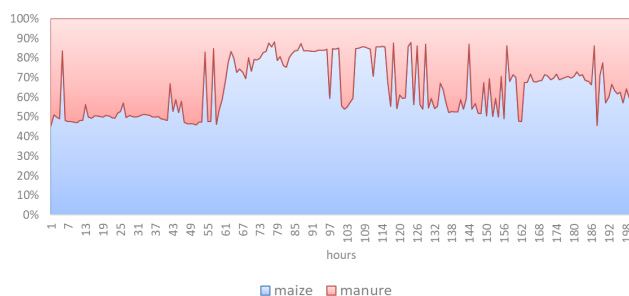


Figure 5. substrate co-digestion ratios in the process

Figure 6 shows four distinct contributions to GWP: substrate cultivation, transportation, CHP operation, and biogas leakage. Leakage, assumed to be 2% of biogas production[18], emerges as the predominant driver of GWP, highlighting the importance of robust sealing and monitoring to minimise fugitive methane emissions. The other three GWP components remain relatively modest, indicating that carefully managed process heating and CHP usage do not substantially elevate the plant's carbon footprint. Notably, periods with reduced gas leakage coincide with improved gas-tight integrity or targeted operational adjustments.

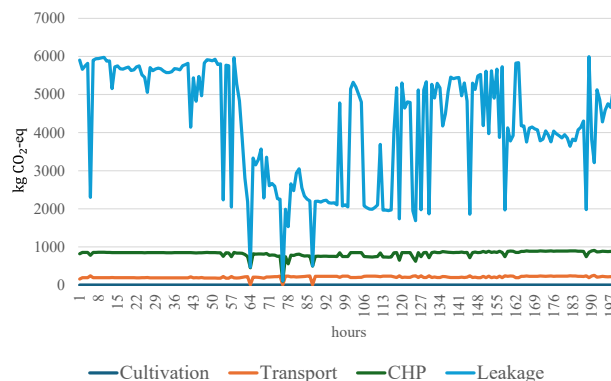


Figure 6. Global warming potential related to the process.

CONCLUSION

This study demonstrates a comprehensive and synergistic approach to optimising anaerobic co-digestion systems by integrating an NLP model, temperature control via MPC, and advanced energy price forecasting. By leveraging the CHP unit and dynamic co-digestion strategies, the framework effectively adapts to varying substrate compositions, operational constraints, and fluctuating market conditions. The proposed delayed control scheme ensures tight temperature regulation, sustaining microbial activity near optimal levels and minimising the risk of thermal shocks. Additionally, the energy price forecasting component, based on the Prophet model, allows for strategic decisions on whether to allocate biogas to onsite electricity generation or injection into the grid, thereby maximising revenue streams.

Results indicate that co-digestion of maize and manure significantly enhances biogas yields while maintaining nutrient balance but also underscores the critical need to minimise methane leakage—the principal contributor to global warming potential. By identifying and mitigating fugitive emissions, operators can substantially reduce the environmental footprint of biogas production. Despite these advances, there remain avenues for improvement. Future work will focus on refining the biogas production model to capture higher fidelity microbial and kinetic responses with more accurate substrate interactions. In addition, efforts to decrease computational time—such as implementing advanced solution techniques or simplifying the problem structure—would make this framework more practical for larger-scale or real-time applications. Finally, exploring methods to remove certain nonlinear parameters could strengthen the model's scalability and facilitate adoption in a broader range of biogas facilities. By pursuing these improvements, the integrated approach stands to further enhance operational efficiency, profitability, and environmental sustainability in anaerobic co-digestion systems.

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