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A Comparison of Robust Modeling Approaches to Cope with Uncertainty in Independent Terms, considering the Forest Supply Chain Case Study

Frank Piedra-Jimenez[a], Ana Inés Torres[b], and Maria Analia Rodriguez[a]*

[a] Universidad Nacional de Córdoba, CONICET, Instituto de Investigación y Desarrollo en Ingeniería de Procesos y Química Aplicada, Córdoba, Córdoba, Argentina
[b] Carnegie Mellon University, Department of Chemical Engineering, Pittsburgh, Pennsylvania, United States
* Corresponding Author: analia.rodriguez@unc.edu.ar.

ABSTRACT

Uncertainty plays a crucial role in strategic supply chain design. In this study, we explore robust approaches to model uncertainty when the non-deterministic parameters are placed in the independent term, on the right-hand side (RHS) of the constraints. We consider the "disjunctive adjustable column-wise robust optimization" (DACWRO), a disjunctive formulation introduced previously in our group, and compare it with the adjustable column-wise robust optimization (ACWRO) formulation, a specific technique for solving robust optimization problems when the original robust optimization approach may assume too-conservative results. Given that the proposed method is based on the generalized disjunctive programming (GDP) technique, it is a higher lever modelling approach that represents the discrete nature of the decision process. In addition, it provides alternative MILP representations that can be further tested and compared. The analysis assesses the computational performance and reformulation tightness of both approaches in the context of the forest supply chain design problem.

Keywords: Design Under Uncertainty, Modelling, Optimization, Supply Chain, Biomass

INTRODUCTION

The need to consider uncertainty in the decision-making process is widely acknowledged in the PSE community, which distinguishes three main modeling paradigms for optimization under uncertainty, namely robust optimization (RO), stochastic programming (SP), and chance-constrained programming (CCP). The last two paradigms are challenging due to the need for complete distributional knowledge or the involvement of non-linearities [1]. Instead, RO does not require the probabilistic behavior of uncertain parameters and strikes a good balance between solution quality and computational tractability [2].

One widely used method for RO is the static robust optimization one initially proposed by Bertsimas and Sim [3]. They proposed the budgeted uncertainty set, which allows flexible handling of the level of conservatism of robust solutions in terms of probabilistic limits of constraint violations. It defines for each uncertain parameter a deviation bound from its nominal value and a budget parameter to determine the number of uncertain parameters that are allowed to take their worst value in each equation. In the case that there is only one uncertain parameter in the independent term, on the RHS of the equations, this method may adopt a too-conservative perspective, considering the worst-case scenario for each constraint.

To address this concern, an extension of this method is proposed in [4] and applied to the water distribution system design problem. This method is recognized to better handle uncertainty in this context, such as in [5,6]. The method is called “adjustable column-wise robust optimization” (ACWRO) because it defines the number of uncertain realizations that the decision-maker is willing to satisfy. Initially presented as a nonlinear model, it is later reformulated to achieve a linear formulation.

Motivated by the disjunctive nature of the decision process of this case, the present paper proposes an alternative method applying a linear disjunctive formulation, called "disjunctive adjustable column-wise robust optimization" (DACWRO). One of the contributions of this article is that it extends the use of Generalized Disjunctive Programming (GDP) to a new application domain. Additionally, taking advantage of the high-level representation capability of GDP, Big-M and Hull MILP reformulations are derived from the proposed formulation. Here, the proposed method is applied to the forest supply chain design problem and it is compared with the ACWRO approach for a large number of instances showing the tightness of each reformulation and computational performance.

DISJUNCTIVE ADJUSTABLE COLUMN-WISE ROBUST OPTIMIZATION APPROACH

In this section, we develop the DACWRO approach to deal with uncertainty when there is only one uncertain parameter in the constraint and it is on the right-hand side (RHS) of the equation. The novel approach is an alternative robust reformulation of the ACWRO. For details of this approach, the reader is referred to this reference [4]. Let us examine the following linear optimization model:

$$Min\, Z = \sum_j c_j \cdot x_j \tag{1}$$

$$\sum_j A_{i,j} \cdot x_j \geq \tilde{b}_i \quad \forall i \tag{2}$$

$$L \leq x_j \leq U \quad \forall i \tag{3}$$

where $\tilde{b}_i$ are the uncertain parameters. Here, the uncertain realization of each $\tilde{b}_i$ is handled by a box uncertainty set $U_{box} = \{(\boldsymbol{b} | b_i^L \leq \tilde{b}_i \leq b_i^U, \forall i)\}$. This defines the range of each uncertain parameter in the vector of uncertain parameters $\boldsymbol{b}$. That is, b_i^L and b_i^U represent the lower and the upper bounds of each uncertain parameter, respectively. Assuming a symmetric variation of the uncertain parameter, Eqs. (4) and (5) are presented:

$$b_i - \hat{b}_i = b_i^L \quad \forall i \tag{4}$$

$$b_i + \hat{b}_i = b_i^U \quad \forall i \tag{5}$$

where b_i is the nominal value and $\hat{b}_i$ represents the maximal absolute variation that the uncertain parameter is subject to change. In the robust approach, each uncertain parameter $\tilde{b}_i$ is transformed into a decision variable that represents the value of the uncertain parameter the decision maker is willing to satisfy. Given Eqs. (4) and (5), this variable can be expressed by a new variable $\eta_i = (\tilde{b}_i - b_i)/\hat{b}_i$ that takes values in the interval $[-1,1]$. Additionally, a new redundant constraint is introduced, i.e., Eq. (2) is summed for all i and the RHS is rewritten in terms of parameters b_i, $\hat{b}_i$ and variable η_i, in Eq. (6).

$$\sum_i \sum_j A_{i,j} \cdot x_j \geq \sum_i b_i + \sum_i \eta_i \cdot \hat{b}_i \tag{6}$$

Eq. (6) is now rewritten as (7) introducing a protection function, where larger values of $\sum_i \eta_i \cdot \hat{b}_i$ result in higher protection against uncertainty. Thus, Eq. (6) is replaced by Eq. (7) and the variable η_i takes values in the interval $[0,1]$. Note that I is a set of uncertain parameters, S_i is the subset of I that takes their worst-case values. The number of elements of S_i is given by the budget parameter Γ, that indicates the level of conservatism of the system. Conceptually, the protection function is a fundamental goal of robust optimization: finding a solution that not only maximizes the objective function Z but also assures that Z remains optimal with high probability given some error in predicting $\tilde{b}_i$ values [3]. This inner problem is reformulated by Eqs. (8) - (10).

$$\sum_i \sum_j A_{i,j} \cdot x_j \geq \sum_i b_i + \underset{\{S_i | S_i \subseteq I, |S_i| = \Gamma\}}{Max} \{\sum_i \eta_i \cdot \hat{b}_i\} \tag{7}$$

$$Max \sum_i \eta_i \cdot \hat{b}_i \tag{8}$$

$$0 \leq \eta_i \leq 1 \quad \forall i \tag{9}$$

$$\sum_i \eta_i \leq \Gamma \tag{10}$$

Given that problem in Eqs. (8)–(10) is feasible and bounded by Γ, its dual pair is also feasible and bounded with the identical objective value according to the strong duality property. The problem in Eqs. (1)–(3), (7)–(10) is reformulated as Eqs. (1), (3), (11)–(13).

$$\sum_i \sum_j A_{i,j} \cdot x_j \geq \sum_i b_i + \Phi \cdot \Gamma + \sum_i \Omega_i \tag{11}$$

$$\Phi + \Omega_i \geq \hat{b}_i \quad \forall i \tag{12}$$

$$\sum_j A_{i,j} \cdot x_j \geq b_i + \hat{b}_i \quad \forall i \tag{13}$$

Note that Eq. (11) is equivalent to Eq. (7) where Φ and Ω_i are dual variables. Variable Φ represents the common variation for all b_i elements, i.e., Φ remains constant for each uncertain constraint $i \in I$, but the value of Ω_i differs according to the corresponding value of $\hat{b}_i$. Equation (12) determines to the sum of the dual variables corresponding to each equation from the primal model (Eq. (8) – (10)), while Eq. (13) replaces Eq. (2) assuming the Bertsimas and Sim approach [3]. That is, the worst-case scenario is assumed for each constraint subject to this type of uncertainty.

As mentioned, the aim of the proposed approach, as in [4], is to define the degree of uncertainty the decision maker is willing to cover. For this purpose, a disjunctive formulation is introduced to control the degree of conservatism Γ. Equation (13) is replaced by Eqs. (14) – (17). Note that Eqs. (14) and (15) ensure that for all $i \in I$, the uncertain parameter will remain between its nominal value and its worst-case value. The disjunction is introduced in Eq. (16) where Ψ_i is a Boolean variable. In the positive case, the uncertain parameter of constraint $i \in S_i$ must take its worst value, which is determined by the auxiliary variable Λ_i that is equal to the sum of the dual variables (Φ and Ω_i) generated from the robust reformulation. In the negative case ($\neg\Psi_i$), the uncertain parameter remains at its nominal value. Therefore, auxiliary variable Λ_i and the specific variation of element b_i determined by dual variable Ω_i are zero. Finally, the number of uncertain parameters that take their worst-case value is controlled by the Γ parameter, as shown in Eq. (17). Note that the degree of conservatism (Γ) must necessarily be an integer parameter that takes values in the range $[0, |S_i|]$. Finally, DAWRO formulation is given by Eqs. (1), (3), (11)–(12), (14)–(17).

$$\sum_j A_{i,j} \cdot x_j \leq b_i + \hat{b}_i \qquad \forall i \tag{14}$$

$$\sum_j A_{i,j} \cdot x_j \geq b_i + \Lambda_i \qquad \forall i \tag{15}$$

$$\begin{bmatrix} \Psi_i \\ \Lambda_i = \Phi + \Omega_i \\ \psi_i = 1 \end{bmatrix} \vee \begin{bmatrix} \neg\Psi_i \\ \Lambda_i = 0 \\ \Omega_i = 0 \\ \psi_i = 0 \end{bmatrix} \qquad \forall i \tag{16}$$

$$\sum_i \psi_i \geq \Gamma \tag{17}$$

FOREST SUPPLY CHAIN RE-DESIGN AND PLANNING PROBLEM

In this section the case study is briefly described. The problem consists of the optimal design and planning of the forest supply chain in the Argentinean context. There are feedstock nodes that produce pine and eucalyptus logs, forest residues, and sawmill residues; the processing facilities convert biomass into paper pulp, fuels, and intermediates; and the consumer nodes are biodiesel plants or petroleum refineries. There are some fixed facilities and some potential locations for new stand-alone and integrated facilities. The model decides in each time period the location of forest nodes, harvesting decisions, technologies and capacity levels in the industrial locations, product mix, production levels and compounds' flows along all transportations links to meet specific demand levels and minimize present costs. The complete model formulation and case study data is presented in [7]. Let introduce the uncertain constraint from this formulation given by Eq. (18), which means that the amount of product *o* delivered to each demand node *l* in each period *t* must satisfy the demand level given by uncertain parameter $\widetilde{\chi_{o,l,t}}$.

$$\sum_{\substack{m,k \\ m \in M_O}} F3_{o,k,m,l,t} \geq \widetilde{\chi_{o,l,t}} \qquad \forall\,(o,l) \in O_L, t \tag{18}$$

Applying the proposed robust reformulation, the following set of constraints is introduced, as shown in Eqs. (19) – (24).

$$\sum_{\substack{o,k,m,l,t \\ m \in M_O \\ (l,o) \in O_L}} F3_{o,k,m,l,t} \geq \sum_{\substack{o,l,t \\ (l,o) \in O_L}} \left(\chi_{o,l,t} + \Omega_{o,l,t}\right) + \Phi^{de} \cdot \Gamma^{de} \tag{19}$$

$$\Phi^{de} + \Omega_{o,l,t} \geq \hat{\chi}_{o,l,t} \qquad \forall\,(o,l) \in O_L, t \tag{20}$$

$$\sum_{\substack{m,k \\ m \in M_O}} F3_{o,k,m,l,t} \leq \chi_{o,l,t} + \hat{\chi}_{o,l,t} \quad \forall\,(o,l) \in O_L, t \tag{21}$$

$$\sum_{\substack{m,k \\ m \in M_O}} F3_{o,k,m,l,t} \geq \chi_{o,l,t} + \Lambda_{o,l,t} \quad \forall\,(o,l) \in O_L, t \tag{22}$$

$$\begin{bmatrix} \Psi^{de}_{o,l,t} \\ \Lambda_{o,l,t} = \Phi^{de} + \Omega_{o,l,t} \\ \psi^{de}_{o,l,t} = 1 \end{bmatrix} \veebar \begin{bmatrix} \neg\Psi^{de}_{o,l,t} \\ \Lambda_{o,l,t} = 0 \\ \Omega_{o,l,t} = 0 \\ \psi^{de}_{o,l,t} = 0 \end{bmatrix} \quad \forall\,(o,l) \in O_L, t \tag{23}$$

$$\sum_{\substack{o,l,t \\ (l,o,t) \in O_L}} \psi^{de}_{o,l,t} \geq \Gamma^{de} \tag{24}$$

This formulation can be reformulated by applying Big-M and Hull reformulations, which are called DACWRO-BM and DACWRO-H, respectively.

RESULTS AND DISCUSSION

The key features and dimensions of the sample runs are outlined in Table 1. Four supply chain structures are set corresponding to each example. For each example, a set of instances is determined by varying the budget parameter (third column). The number of instances for each example is presented in the fourth column. Model sizes are displayed in the last three columns. The supply chain structure in Example 1 has 10 supply sources, 3 kraft pulp mills, 13 potential biorefinery locations, and 10 demand nodes. In subsequent examples, the number of nodes gradually increased to 25 supply nodes and 23 demand nodes. Different levels of deviation in uncertain demand parameters are also considered.

Figure 1 shows that all approaches present very similar performance in terms of the mean value of the CPU time. As expected, as the size of the examples are increased, the execution time is also greater. In examples 1 to 3, all instances tested present an execution time near the mean value. However, heterogenous results are obtained among instances of example 4, that is, large dispersion is observed in the execution time, showing data dependency when the size of the model is increased. Figure 2 presents the ratio of the relaxed solution in the root node with respect to the final optimal solution for each method. It is noteworthy that the three methods present very tight results and they are homogeneous among the examples and approaches. Note that all mean values of this ratio are the same for each method what may explain the similar performance in terms of CPU time (Fig. 1). DACWRO is as good as the ACWRO in terms of computational performance, but it represents the disjunctive nature of level of uncertainty selection process in a more straightforward way.

Table 1. Features of executed examples and their size

Example	Reformulation	Budget parameter	Number of instances executed	Number of equations	Number of continuous variables	Number of binary variables
1	ACWRO	20 to 70	6	27721	30531	2700
1	DACWRO-BM	20 to 70	6	27721	30531	2700
1	DACWRO-H	20 to 70	6	27791	30671	2700
2	ACWRO	30 to 100	24	29911	48261	2730
2	DACWRO-BM	30 to 100	24	29911	48261	2730
2	DACWRO-H	30 to 100	24	30011	48461	2730
3	ACWRO	10 to 150	30	30261	49311	2780
3	DACWRO-BM	10 to 150	30	30261	49311	2780
3	DAWRO-H	10 to 150	30	30811	50761	2780
4	ACWRO	10 to 200	30	45291	75461	3890
4	DACWRO-BM	10 to 200	30	45291	75461	3890
4	DACWRO-H	10 to 200	30	45591	76061	3890

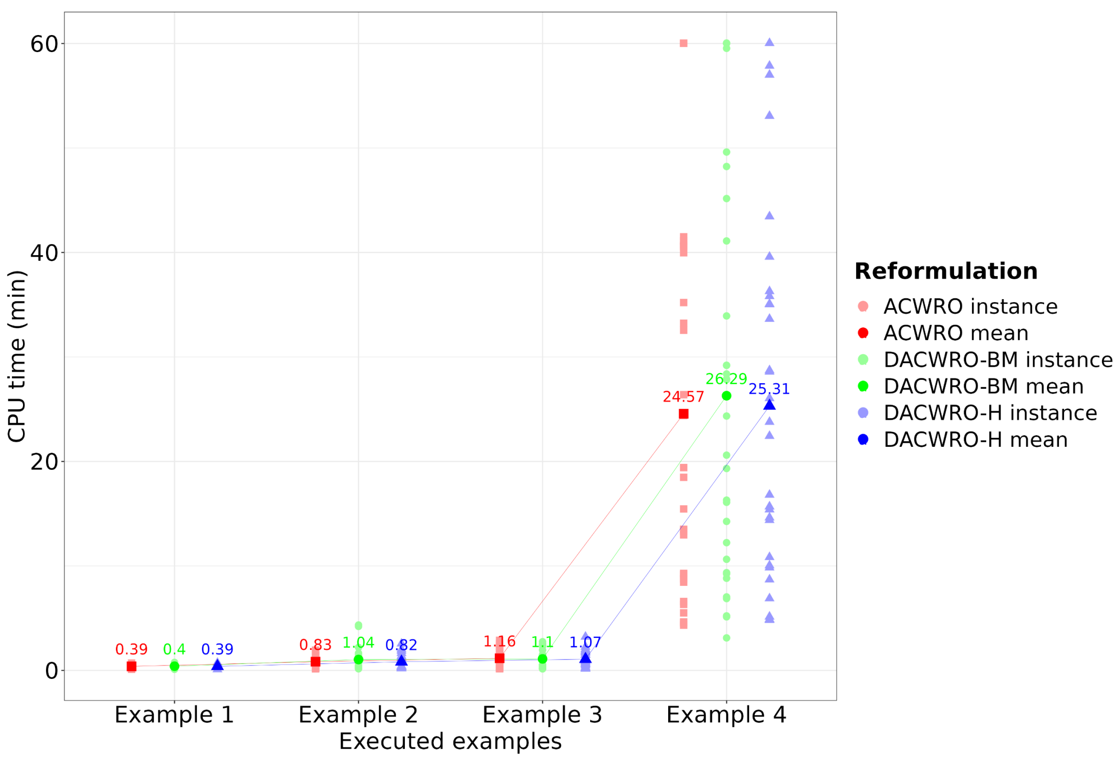

Figure 1. Execution time (min).

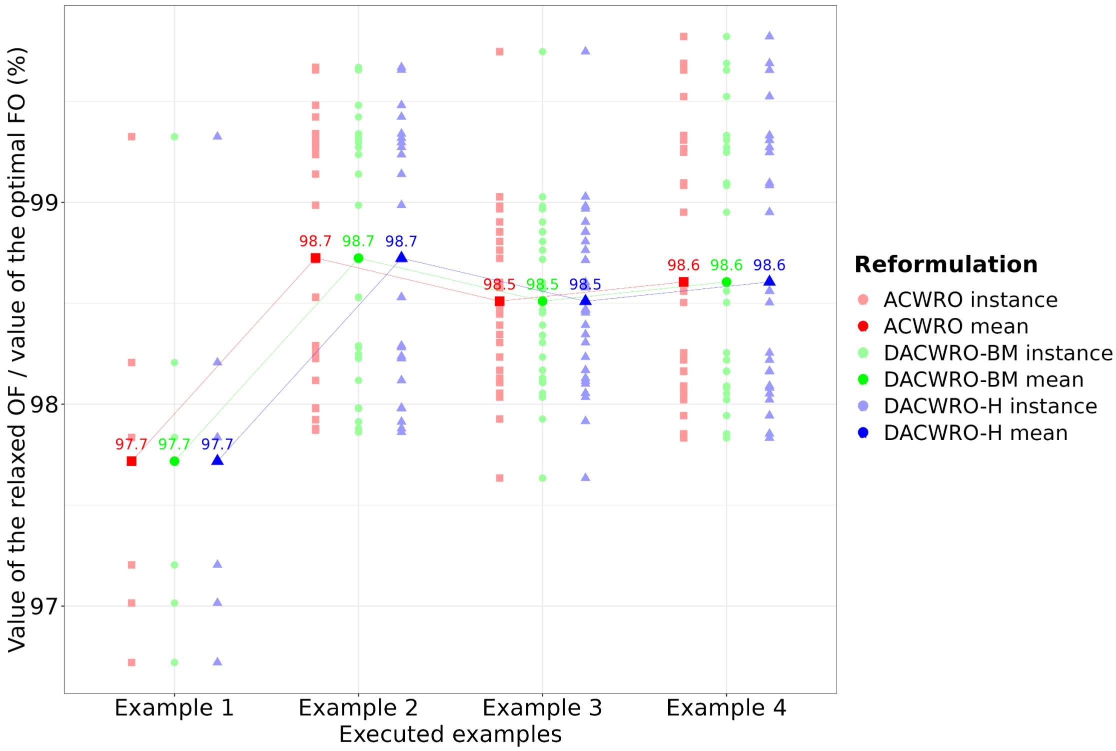

Figure 2. Initial relaxed solution vs. optimal solution.

CONCLUSIONS

In this article a new method, named DACWRO, is proposed to model uncertainty that is compared to ACWRO [4]. These methods present the advantage that the decision maker can select the level of conservatism is willing to assume. These approaches are particularly interesting in the case the uncertain parameters are part of the independent term of the equations. The proposed method is based on a disjunctive formulation that is reformulated applying Big-M and Hull relaxations to obtain the equivalent mixed integer linear models.

To compare the approaches, the formulations were applied to the forest supply chain design problem assuming uncertainty in the demand constraints. A set of 270 instances were executed. From the instances tested, due to a tight reformulation, efficient and homogeneous results are obtained in all cases. The proposed method extends the use of GDP to a novel application domain. Additionally, it capitalizes on the high-level representational power of GDP to develop Big-M and Hull MILP reformulations based on the proposed formulation. Future work includes analyzing larger instances to evaluate the behaviour of the approaches for more challenging cases as well as expanding the scope of the present approach to other types of uncertainties. Also, new problem domains and case studies will be considered to check differences between methods in other contexts.

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