

# Implementation and assessment of fractional controllers for an extractive distillation system

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## ABSTRACT

This work presents an approach to implement and assess fractional controllers in an extractive distillation system. The experimental dynamic data for an extractive distillation column is used as a case study. A strategy is developed to fit the operation data to fractional-order transfer functions. Then, the fractional controllers are designed in the Simulink environment in Matlab, tuning the controllers through a hybrid optimization approach. First, the approach uses a genetic algorithm to find an initial point, and then the solution is improved through the fmincon algorithm. According to the results of the design of fractional controllers, the sum of the square of errors is below  $2.9 \times 10^{-6}$  for perturbations in heat duty, and  $1.2 \times 10^{-5}$  for perturbations in the reflux ratio. Moreover, after controller tuning, a minimal value for ISE of 1,278.12 is obtained, which is approximately 8% lower than the value obtained for an integer-order controller.

**Keywords:** Fractional calculus, fractional controllers, extractive distillation

## INTRODUCTION

Process intensification is a strategy applied to chemical engineering which is devoted to the development of technologies that enhance the performance of the operations in a chemical process. The philosophy of process intensification can be summarized through its four principles: (i) maximizing the effectiveness of intra and intermolecular events, (ii) giving each molecule the same processing experience, (iii) optimizing the driving forces at every scale, and (iv) maximizing the synergistic effects from partial processes [1]. This is achieved through the implementation of modified equipment and/or multi-tasking equipment, among other approaches. Among the common operations in chemical processes, distillation is a candidate for intensification, since it is a separation process with high energy requirements and, consequently, high operational costs and high environmental impact. Diverse approaches have been proposed to intensify distillation structures, such as thermally coupled distillation schemes, where modified

configurations are used to make a proper use of the heat of the internal streams and the composition profiles to reduce the energy requirements (e.g. [2]); or reactive distillation, where the operations of reaction and distillation are performed in a single unit [3]. Extractive distillation is another example of an intensified distillation operation, where an entrainer is employed to allow the separation of azeotropic mixtures [4]. Further enhancements can be obtained by using advanced entrainers [5] or with the implementation of thermal couplings to the extractive distillation system [6].

Various studies have demonstrated that the dynamic properties of intensified systems can be, for some cases, better than those of conventional configurations (e.g. [7]). In the case of extractive distillation, there are various aspects that may affect the performance of the control structure, such as the kind of solvent employed [8], the solvent flow rate [9], and the pressure [10]. In a study presented by Yi and Chien [11], a modification of the steady-state design is proposed to enhance the performance of the control system. This indicates that the

development of better control structures is still necessary [12]. The use of fractional controllers can be an alternative to achieve this target.

Fractional PID controllers (or  $PI^\lambda D^\mu$  controllers) take advantage of the concepts of fractional calculus, increasing the flexibility and robustness of the controller by allowing fractional orders for the derivative and the integral actions. These characteristics make fractional controllers adequate to be implemented in systems with complex dynamics, which may be difficult to control with classical PID controllers [13]. However, the additional flexibility implies a higher complexity to perform the tuning of the controller, since additional parameters must be included in the analysis. Fractional order controls have been widely implemented to process units like reactors [14,15]. Only few works have reported the implementation of fractional controllers for distillation systems. Mishra et al. [16] assessed the performance of a fractional order fuzzy PID controller for a binary distillation column, reporting a superior performance when compared to a fuzzy PID controller. Haji Haji and Monje [17] used a dynamic version of the metaheuristic bat algorithm to perform the tuning of a fractional-order PID controller in a distillation column. Marques de Almeida et al. [18] reported the implementation of a decentralized fractional-order PI controller on a TITO distillation column, reporting a superior performance of the fractional system in comparison with integer-order controllers and model predictive control structures. In the case of intensified distillation systems, to the authors knowledge, the only antecedent is given by the work of Edet [19], who reported the implementation of a fractional PI controller to a reactive distillation system. However, the implementation of  $PI^\lambda D^\mu$  controllers for intensified distillation systems has not been reported. Thus, in this work a fractional-order model, based on transfer functions, is developed for an extractive distillation system. The model is obtained through experimental data to ensure that the proposed fractional approach properly represents actual operational data. A fractional controller is implemented in the model, developing a tuning strategy based on rigorous optimization. The performance of the fractional controller is compared with that of an integer order controller.

## METHODOLOGY

The experimental dynamic data reported by Kumar et al. [20] for an extractive distillation column has been used as a case study. The column is used to separate the mixture acetone-methanol, employing water as solvent. In this study, changes in the measured variables reported for perturbations in the reflux rate and the steam pressure on the reboiler are used to perform the analysis. Since the experimental data had been reported in graphical form, the tool *WebPlotDigitalizer4.6* has been used

to extract the numerical data.

The experimental data has been fitted to fractional order functions. The general form of the transfer function is given by Equation (1).

$$G(s) = \frac{Ke^{-\theta s}}{As^\alpha + Bs^\beta} \quad (1)$$

Where  $0 < \alpha < 1$  for a pseudo-first order model, and  $1 < \alpha < 2$  for a pseudo-second order model. The numerical fitting has been performed in the *FOMCON* toolbox of Matlab, using the Grünwald-Letnikov definition of the fractional derivative [21]. The parameters of the model are adjusted, aiming to minimize the deviation of the predictions of the model in comparison to the experimental data. The effect of the step size  $h$  over the fitting is also analyzed.

The implementation of the fractional controllers is performed in the Simulink environment of Matlab. The response of the fractional controller,  $u(t)$ , is given by Equation (2).

$$u(t) = K_p e(t) + K_i D^{-\lambda} e(t) + K_d D^\mu e(t) \quad (2)$$

Where  $K_p$ ,  $K_i$  and  $K_d$  are the gains for the proportional, integral and derivative actions, respectively. On the other hand,  $\lambda$  and  $\mu$  are the orders for the fractional integral and derivative operators.

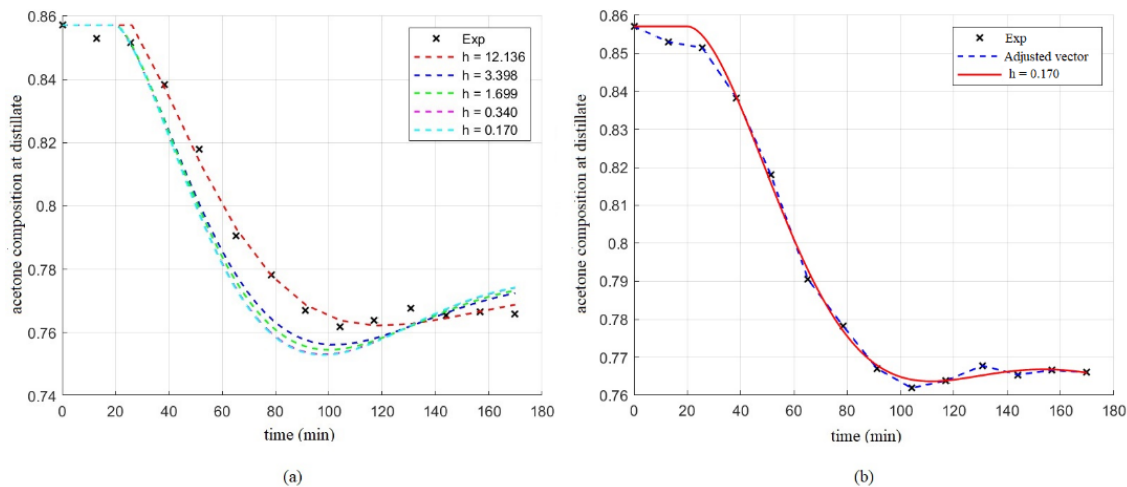
An LV configuration is employed, using the reflux rate and the steam pressure in the reboiler as the manipulated variables. The measured variables are the compositions on the distillate and the bottoms. Decoupling elements are added to the control structure to reduce the interactions among the input and output variables of the control loops. The implementation of the fractional elements is performed through the interaction between Simulink and *FOMCON*. The controllers are tuned using a hybrid optimization approach. First, a genetic algorithm is used to find an initial guess. Then, the solution is refined with the *fmincon* algorithm. The objective in the optimization procedure is to minimize the total value for the integral square error (ISE). A penalization term  $F$  is incorporated to avoid solutions with high overshooting. Thus, the objective function  $f$  is given by Equation (3).

$$f = F \sum_{i=1}^n ISE_i \quad (3)$$

Where  $i$  represents the number of the corresponding product stream. For the distillate,  $i = 1$ . On the other hand, for the bottoms,  $i = 2$ .

## RESULTS

For fitting the experimental data to a fractional order function, a strong dependence of the fitting over the step size  $h$  has been observed. Figure 1.a shows that, based on the experimental data available, the fitting is adequate only for a relatively large step size. As the step size is reduced, the predictions of the model have a higher



**Figure 1:** Effect of the step size in the adjustment of experimental data to fractional order transfer functions, (a) adjustment with the original data, (b) adjustment with the interpolation of data.

deviation from the experimental data. For  $h = 12.136$ , an acceptable fit is obtained, but there are still experimental points out of the trend. To further enhance the prediction, 1000 additional points have been generated by interpolating the experimental data. Using this approach, the prediction produced with low step sizes is considerably enhanced, as shown in Figure 1.b.

Tables 1 and 2 show the parameters obtained for the fractional transfer functions and the deviation for each fitting, where  $\theta$  is given in minutes. It is important to notice that the delay time considers the period before the first experimental response is reported, which causes apparently large values for this parameter.

In general terms, low deviations are obtained through the fitting to fractional order transfer functions. It is also observed that, for all the cases, the values of  $\alpha$  indicate pseudo-second order models. However, the values of that parameter for most cases are far from being integer. A similar trend is observed for the values of  $\beta$ . Thus, the relaxation of the order for the denominator in the transfer function allows flexibility in the data fitting. Although small deviations are obtained, it is important to mention that the error obtained when using integer-order transfer functions is similar to that obtained with the fractional functions. Thus, the studied system does not show a significant fractional response.

Table 3 shows the parameters of the fractional controllers after the optimization of the objective function given by Equation (2). It is observed that the integral and derivative actions are both relevant for the controller of the acetone composition in the distillate, with orders relatively close to unity. However, for the control of methanol composition in the bottoms, the derivative action has a low order, relatively close to zero. The controller in the distillate has a higher proportional gain, showing an

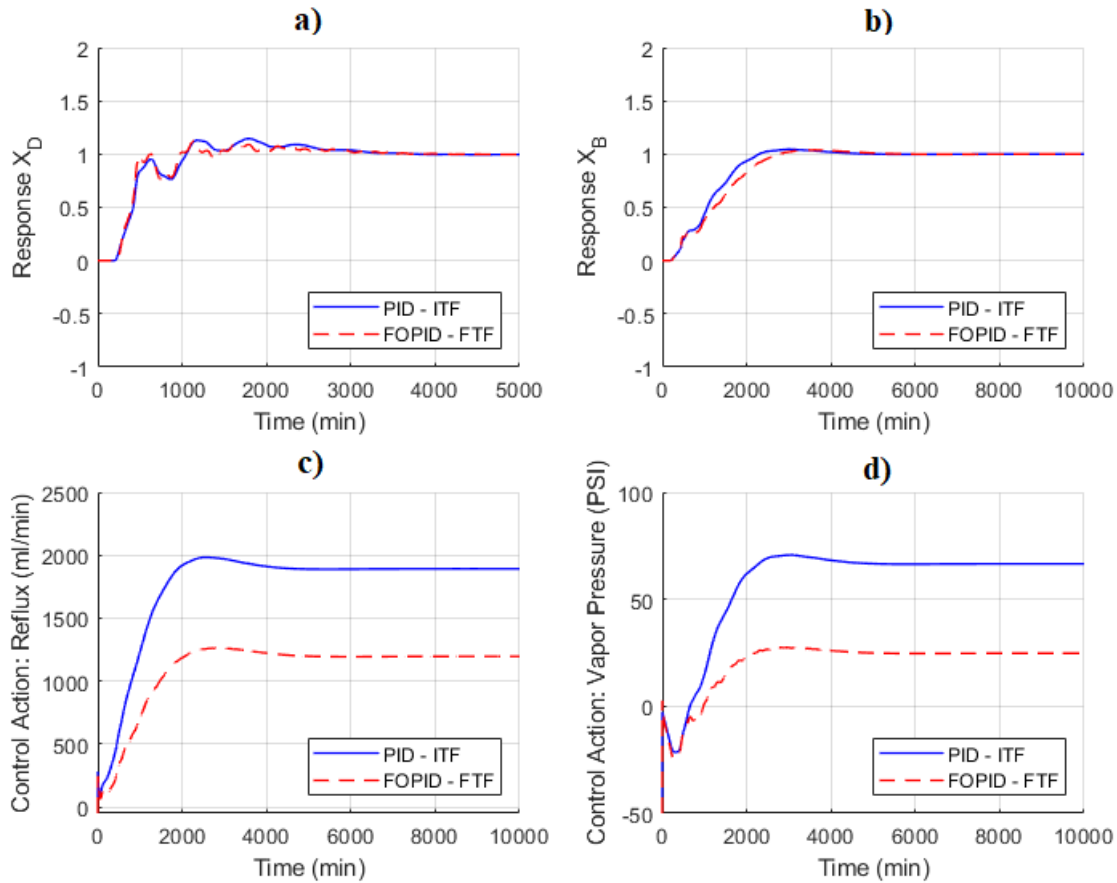
oscillatory response.

**Table 1:** Results for the fitting of experimental data to fractional-order transfer functions when a perturbation in reflux rate occurs.

|           | Acetone composition at distillate | Methanol composition at bottoms |
|-----------|-----------------------------------|---------------------------------|
| K         | 0.00080453                        | 0.00014573                      |
| A         | 267.27                            | 43.855                          |
| B         | 1                                 | 1                               |
| $\theta$  | 415.444                           | 434.55                          |
| $\alpha$  | 1.7388                            | 1.5568                          |
| $\beta$   | 0.11565                           | 0.29164                         |
| Deviation | $3.2015 \times 10^{-5}$           | $1.2120 \times 10^{-5}$         |

**Table 2:** Results for the fitting of experimental data to fractional-order transfer functions when a perturbation in steam pressure occurs.

|           | Acetone composition at distillate | Methanol composition at bottoms |
|-----------|-----------------------------------|---------------------------------|
| K         | -0.0074095                        | -0.00069622                     |
| A         | 121.14                            | 23.576                          |
| B         | 1                                 | 1                               |
| $\theta$  | 206.69                            | 196.69                          |
| $\alpha$  | 1.8119                            | 1.6776                          |
| $\beta$   | 0.2643                            | 0.41674                         |
| Deviation | $5.2494 \times 10^{-5}$           | $2.9317 \times 10^{-6}$         |



**Figure 2:** Responses and control actions for the  $PI^\lambda D^\mu$  controller, compared with a PID controller, (a) response for the composition of acetone in the distillate, (b) response for the composition of methanol in the bottoms, (c) control action over the reflux rate, (d) control action over the steam pressure on the reboiler.

**Table 3:** Optimized parameters for the fractional controllers.

| Parameter | Distillate controller | Bottoms controller |
|-----------|-----------------------|--------------------|
| $K_p$     | -128.0115             | 18.6006            |
| $K_i$     | -3.2562               | 0.2160             |
| $K_d$     | -0.7867               | -9.2389            |
| $\lambda$ | 0.8587                | 0.8229             |
| $\mu$     | 0.7091                | 0.1674             |

With the objective function used in the analysis, an overshooting percentage of 9.622 and 0% for the controllers in the distillate and the bottoms has been obtained, respectively. This is a lower percentage compared to that obtained if an integer-order controller is used, where the optimized parameters lead to overshooting values of 14.593 and 4.502% for the distillate and the bottoms, respectively. The objective function for the fractional controller has a value of 1278.12, which is approximately 8% lower than the value obtained for an

integer-order controller.

Figure 2 presents the responses of the compositions when the fractional-order controller is used. The responses are compared with those obtained with an integer-order PID controller, where an integer-order transfer function is used to represent the dynamic performance of the column. Both cases use the optimized parameters of each controller. The optimized parameters for the integer-order controllers are presented in Table 4. Figure 2 also shows the corresponding control action.

**Table 4:** Optimized parameters for integer-order controllers.

| Parameter | Distillate controller | Bottoms controller |
|-----------|-----------------------|--------------------|
| $K_p$     | -120.1442             | 12.9061            |
| $K_i$     | -1.8031               | 0.1253             |
| $K_d$     | -2.3642               | -6.3010            |

A similar performance is observed for the responses

of the measured variables for both kinds of controllers. The time required to reach a new steady state is similar for both kinds of controllers, with approximately 3,000 minutes needed for the stabilization of the response for both, the acetone and the methanol compositions. However, a higher over-shooting is observed for the integer-order controller, which is consistent with the over-shooting percentages described in the previous lines. The reduction on the over-shoot is highly desirable for the control of a chemical process [22,23]. Additionally, the fractional controllers allow reaching the new steady state with a lower increase in the control variables, as can be observed in Figures 2.c and 2.d. Whereas the integer controller requires increasing the reflux to a maximum value of 2,000 ml/min, the fractional controller only requires increasing the variable to approximately 1,300 ml/min. Similarly, the steam pressure in the reboiler changes to a maximum of approximately 30 psi with the fractional controller, whereas the integer controller requires increasing the manipulated variable to a value of approximately 70 psi. However, it is important to notice that this may be associated with multiplicity, which is a common situation in distillation systems [24]. Thus, the comparison in terms of the control action must be carefully verified.

## CONCLUSIONS

In this work, the use of fractional controllers in an intensified distillation system has been proposed. The studied system is an extractive distillation column. Experimental data has been used to ensure that the model fits actual operating trends. The data is successfully fitted to fractional-order transfer functions, obtaining low deviations after the fitting of parameters. However, to obtain a better fitting, it is necessary to generate additional data points. Although the fitting is adequate, the improvement achieved with fractional control is not significant when compared to that resulting from the use of an integer-order transfer function; such result indicates that the dynamic response may not have significant memory effects. In fact, for the controllers, the dynamic responses are not considerably different between the integer and the fractional-order controllers. On the other hand, the fractional controllers have been successfully designed, showing that the use of such controllers allows reducing the overshooting while obtaining responses with a lower change in the control variables. Moreover, the changes performed by the controller over the control variables to achieve the new steady-state have lower magnitudes than those required when the integer-order controller is used.

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## REFERENCES

1. Ramírez Corona N, Palacios Rosas A, Generalities about process intensification. In: *Process Intensification – Design Methodologies*. Ed: Gómez-Castro FI, Segovia-Hernández JG. De Gruyter (2019).
2. Calzon-McConville CJ, Rosales-Zamora MB, Segovia-Hernández JG, Hernández S, Rico-Ramírez V, Design and optimization of thermally coupled distillation schemes for the separation of multicomponent mixtures. *Ind Eng Chem Res* 45:724-732 (2006) <https://doi.org/10.1021/ie050961s>
3. Malone MF, Doherty MF, Reactive distillation. *Ind Eng Chem Res* 39:3953-3957 (2000) <https://doi.org/10.1021/ie000633m>
4. Aquilon AF, Cargullo DM, Onayan J, Sarno J, Molino VM, Lopez ECR, Recent advances in extractive distillation. *Eng Proc* 56:11 (2023) <https://doi.org/10.3390/ASEC2023-15261>
5. Han J, Lei Z, Dong Y, Dai C, Chen B, Process intensification on the separation of benzene and thiophene by extractive distillation. *AIChE J* 61:4470-4480 (2015) <https://doi.org/10.1002/aic.15009>
6. Pan C, Qiu X, Guo J, Liu Y, Feng B, Li G, Gai H, Song H, Xiao M, Huang T, Zhu Q, Efficient purification of n-butanol by thermally coupled extractive distillation with mixed entrainer. *Sep Purif Technol* 336:126365 (2024) <https://doi.org/10.1016/j.seppur.2024.126365>
7. Segovia-Hernández JG, Hernández S, Rico-Ramírez V, Jiménez A, A comparison of the feedback control behavior between thermally coupled and conventional distillation schemes. *Comput Chem Eng* 28:811-819 (2004) <https://doi.org/10.1016/j.compchemeng.2004.02.019>
8. Luyben WL, Effect of solvent on controllability in extractive distillation. *Ind Eng Chem Res* 47:4425-4439 (2008) <https://doi.org/10.1021/ie701757d>
9. Wang Y, Shisheng L, Bu G, Liu W, Zhang Z, Zhu Z, Effect of solvent flow rates on controllability of extractive distillation for separating binary azeotropic mixture. *Ind Eng Chem Res* 54:12908-12919 (2015) <https://doi.org/10.1021/acs.iecr.5b03666>
10. Zhu Z, Yu M, Zhang W, Liu Y, Cui P, Yang J, Wang

- Y, Gao J, Exploration of the effects of pressure on the controllability of extractive distillation for separating pressure-sensitive azeotropes. *Sep Purif Technol* 227:115681 (2019) <https://doi.org/10.1016/j.seppur.2019.115681>
11. Yi C-C, Chien I-L, Control study to enhance the controllability of heterogeneous extractive distillation: cyclohexane/cyclohexene separation. *Ind Eng Chem Res* 58:3211-3224 (2019) <https://doi.org/10.1021/acs.iecr.9b00341>
  12. Wang C, Wang C, Cui Y, Guang C, Zhang Z, Economics and controllability of conventional and intensified extractive distillation configurations for acetonitrile/ethanol/benzene mixtures. *Ind Eng Chem Res* 57:10551-10563 (2018) <https://doi.org/10.1021/acs.iecr.8b01875>
  13. Patnaik S, Hollkamp JP, Semperlotti F, Applications of variable-order fractional operators: a review. *Proc R Soc A: Math Phys Eng Sci* 476:20190498 (2020) <https://doi.org/10.1098/rspa.2019.0498>
  14. Gupta D, Goyal V, Kumar J, Design of fractional-order NPID controller for the NPK model of advanced nuclear reactor. *Prog Nucl Energy* 150:104319 (2022) <https://doi.org/10.1016/j.pnucene.2022.104319>
  15. Mukherjee D, Lloyds Raja G, Kundu P, Ghosh A, Improved fractional augmented control strategies for continuously stirred tank reactors. *Asian J Control* 25:2165-2182 (2023) <https://doi.org/10.1002/asjc.2887>
  16. Mishra P, Kumar V, Rana KPS, A fractional order fuzzy PID controller for binary distillation column control. *Expert Syst Appl* 42:8533-8549 (2015) <https://doi.org/10.1016/j.eswa.2015.07.008>
  17. Haji Haji V, Monje CA, Fractional-order PID control of a MIMO distillation column process using improved bat algorithm. *Soft Comput* 23:8887-8906 (2019) <https://doi.org/10.1007/s00500-018-3488-z>
  18. Marques de Almeida A, Kaminski Lenzi M, Kaminski Lenzi E, Decentralized fractional-order PI $\lambda$  control applied to a TITO distillation column: a comparative study with PID and MPC. *Cuadernos de Educación y Desarrollo* 16:e6021 (2024) <https://doi.org/10.55905/cuadv16n10-131>
  19. Edet E, On fractional order predictive PI controller design for a MIMO reactive distillation process. *IFAC-PapersOnLine* 53:3644-3649 (2020) <https://doi.org/10.1007/s00500-018-3488-z>
  20. Kumar S, Wright JD, Taylor PA, Modelling and dynamics of an extractive distillation column. *Can J Chem Eng* 62:185-192 (1984) <https://doi.org/10.1002/cjce.5450620608>
  21. Vinagre BM, Monje CA, Calderón AJ, Suárez JI, Fractional PID controllers for industrial application. A brief introduction. *J Vib Control* 13:1419-1429 (2007) <https://doi.org/10.1177/1077546307077498>
  22. Khare YB, Singh Y, PID control of heat exchanger system. *Int J Comput Appl* 8:22-28 (2010) <https://doi.org/10.5120/1213-1742>
  23. Tsavnin A, Efimov S, Zamyatin S, Overshoot elimination for control systems with parametric uncertainty via a PID controller. *Symmetry* 12:1092 (2020) <https://doi.org/10.3390/sym12071092>
  24. Sánchez-Correa CA, Arturo-Calvache JE, Gil-Chaves ID, Rodríguez-Niño G, Multiplicities in ethanol dehydration by extractive distillation processes with ethyleneglycol. *Chem Eng Process* 174:108886 (2022) <https://doi.org/10.3390/sym12071092>

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