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Integration of MILP and Discrete-Event Simulation for Flowshop Scheduling Using Benders Decomposition

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ABSTRACT

Real-world flowshop problems which are very common in the chemical industry are often difficult to solve in a reasonable time with allocation, sequencing, and lot-sizing decisions. Although great progress has been made in the last 20 years regarding MILP model formulations and solution algorithms, realistically-sized flowshop problems with resource and buffer constraints are still difficult to solve. On the other hand, discrete-event simulation (DES) allows for very detailed modelling of process plants, but lacking of optimization capabilities. Simulation Optimization (SO) combines the high-detail DES with mathematical optimization. We show that is possible to integrate MILP and DES using Benders decomposition. We explain the Benders-DES (BDES) approach with a small motivation example with makespan minimization objective and apply it to a real-world case study of a formulation plant with seven formulation and filling lines with sequencing, allocation, and lot-sizing decisions. We show that the BDES approach performs comparably to the original monolithic-sequential MILP-DES approach. However, the key advantage of this method is the ability to leverage MILP optimization power without requiring a detailed model description.

Keywords: Planning & Scheduling, Optimization, Algorithms, Process Operations, Batch Process, Benders Decomposition

INTRODUCTION

General Motivation

In the chemical process industry, optimization-based scheduling is a critical advantage in today's fast-paced and interconnected world. However, the complexity when optimizing chemical processes is high: A chemical process that requires personnel and processing equipment, consumes raw materials and utilities, and is linked to a complex supply chain, is naturally subject to many constraints and objectives [1]. Discrete Event Simulation (DES) models can describe complex real-world processes to a great level of detail, have relatively short computation times, and allow to include uncertain parameters. However, since DES models have limited optimization capabilities, the solutions of DES models may be far away from the optimum. While MILP models enable global optimization, they quickly grow to intractable size, when trying to include all relevant constraints. In addition, MILP models can be difficult to set up, validate, and maintain for real-world applications.

Simulation Optimization

Simulation Optimization (SO) combines the high-detail DES with mathematical optimization. SO has repeatedly shown success in solving process design and operations problems [2]. The interested reader is referred to the review [3] on DES-based optimization methods for industrial engineering problems.

However, SO approaches struggle with complex real-world models that contain many discrete and continuous decision variables and constraints. For example, in [2] the same hybrid flowshop as in this paper is solved with a genetic algorithm (GA). The authors use a detailed DES model of the plant and include lot-sizing decisions. The GA requires 200 generations with a population size of 20, which leads to several hours of solution time on large computer resources.

In [4], a simulation-optimization approach for adaptive manufacturing capacity planning in small and medium-sized enterprises is developed. An artificial neural

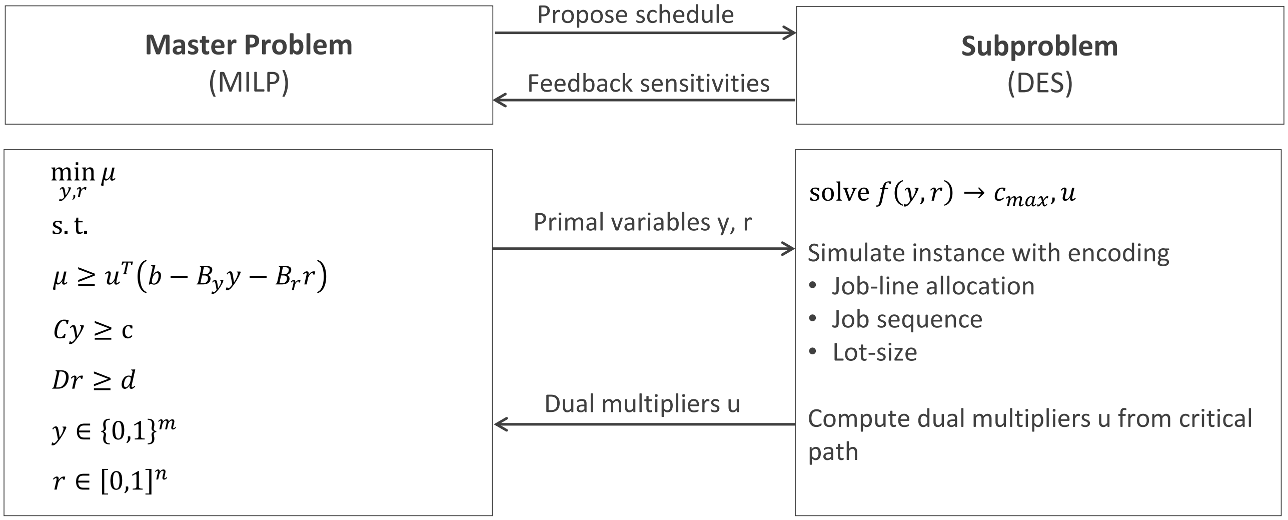

Figure 1: Benders-DES algorithm for hybrid flowshops. y represents the discrete allocation and sequence decision variables; r the continuous lot-sizing variables as split factors, which indicate what percentage of the job is covered by the lot. The objective is to minimize the makespan c_{max}.

network (ANN) is trained on production data of an industrial case study of pastry production and a GA is used to optimize the ANN. While this is a practical solution approach for complex problems it is concluded that the GA can be a limiting factor when exact solutions with guaranteed optimality are required.

In [5], a Digital Twin-driven dynamic scheduling approach is presented featuring a MILP model and a 3D shop floor simulation model to address a hybrid flow shop problem in perfume manufacturing. While the approach is effective and practical, it requires a “human-in-the-loop” to assess the results of the simulation model. Similarly to [6], there is no automatic feedback from the simulation to the optimization model which is a disadvantage compared to the aforementioned SO approaches.

Contribution of this Work

This work presents a novel SO approach that integrates DES and Benders Decomposition. Sequencing, allocation, and lot-sizing are included in the Benders master problem, while important real-world constraints such as sequence-dependent changeover times, limited personnel resources, and limited intermediate buffer capacities are considered in the Benders subproblem. It is shown that the BDES can be applied to a real-world hybrid flowshop as presented in [6] and that good schedules in a reasonable amount of time can be obtained.

METHODOLGY

Benders Decomposition Framework

The Benders decomposition framework is shown in Figure 1. This master MILP problem proposes a new schedule (job sequence) as well as job-line allocation and lot-sizing decisions if required. Based on this information, the DES subproblem calculates the resulting schedule and returns an upper bound of the makespan and sensitivities (dual multipliers) using the concept of critical path mapping, which is explained in the next section. The dual multipliers are used to set up the Benders cuts. For every DES schedule, one Benders cut is added to the MILP master problem leading a new lower bound for the makespan. If the lower bound reaches the upper bound or the upper bound is not improving any more, the algorithm will stop. Further details can be found in [7].

Motivating Example

The BDES approach is explained with a small flowshop problem with no intermediate storage, processing only 3 jobs (see Figure 2). The objective is to find the job sequence which minimizes the makespan. For the sake of simplicity, allocation and lot-sizing decisions play no role here. It can be shown by enumeration that *Schedule 2* of Figure 2 is the optimal one with a makespan of 6 h.

For the above mentioned flowshop, a MILP problem (1) – (6) is set up. The model formulation is based on precedence variables. The precedence variable y_{ij} is one, if a job $i \in I = \{1,2,3\}$ is processed before job $j \in I = \{1,2,3\}$. s_{il} is the start time variable of a job i on stage $l \in L = \{1,2\}$. p_{il} are the processing time parameters which can be read from Figure 2. T is the Big-M constant. A valid (but weak) estimate for this value can be the summation of all processing times.

The objective (1) is to minimize the makespan c_{max}. Constraint (2) is the makespan constraint. Constraint (3) ensures that a job on a stage only starts after the processing of the previous stage concluded. The constraint

(4) ensures that only one job is processed at a given time on a given stage. (5) is a logical constraint.

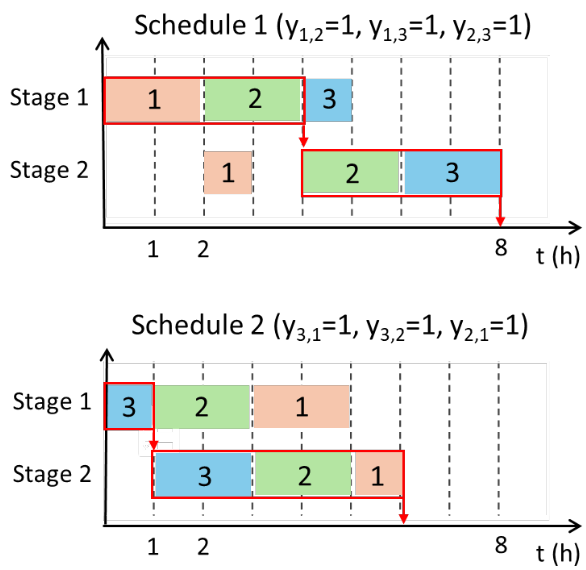

Figure 2. Small flowshop problem with 3 jobs and no intermediate storage (numbers in the Gantt bars indicate the job number, not the processing time). The critical paths are are indicated by the red boxes.

$$\min c_{max} \quad (1)$$

$$\text{s.t.} \quad c_{max} \geq s_{i2} + p_{i2} \; \forall \, i \in I \quad (2)$$

$$s_{il} + p_{il} - s_{il+1} \leq 0 \; \forall \, i \in I, l \in L \backslash \{2\} \quad (3)$$

$$s_{jl} + p_{jl} - T y_{ij} - s_{il} \leq 0 \; \forall \, l \in L, i, j \in I : i \neq j \quad (4)$$

$$y_{ij} + y_{ji} = 1 \quad \forall \, i, j \in I : i \neq j \quad (5)$$

When fixing the precedence variables y_{ij}, the MILP reduces to a LP. Defining y_{ij} as complicated variables, the LP becomes the Benders subproblem in the classical Benders decomposition approach [8]. In the BDES approach, the subproblem corresponds to one DES simulation.

The LP solution returns the information of the binding constraints. For example, for *Schedule 1* in Figure 2, the following sequencing constraints are active:

$$s_{32} + p_{32} - c_{max} \leq 0 \quad (6)$$

$$s_{21} + p_{21} - s_{22} \leq 0 \quad (7)$$

$$s_{11} + p_{11} - T y_{21} - s_{21} \leq 0 \quad (8)$$

$$s_{22} + p_{22} - T y_{32} - s_{32} \leq 0 \quad (9)$$

All active constraints are associated with a non-zero dual multiplier u which is -1 in case of makespan minimization, and the Benders cut can then be set up accordingly:

$$\mu \geq -u \cdot (p_{32} + p_{21} + p_{11} - T y_{21} + p_{22} - T y_{32}) \quad (10)$$

This dual information can also be obtained from the critical path of the Gantt diagram of the DES simulation (see Figure 2). The makespan is obtained by summing over all lot processing times on the critical path. For *Schedule 1* ($y_{21} = 0$ and $y_{32} = 0$), the makespan can be derived from the Benders cut (11): $p_{11} + p_{21} + p_{22} + p_{32} = 8$. For every DES simulation, a new Benders cut is generated and added to the MILP, as shown in Figure 1.

CASE STUDY

As a case study, a hybrid flowshop problem of a agrochemical formulation plant as presented in [6] is considered. The process is a two-stage formulation and filling process that takes place in a multi-product batch plant, as shown in Figure 3. In the formulation stage, raw materials are milled, mixed, and brought to reaction. In the filling stage, the intermediate products are filled into final containers of different sizes. The plant has seven parallel production lines (machines) per stage, each with different run rates per product and sequence-dependent changeover times. There are seven optional buffer tanks to decouple formulation and filling between the two stages. In addition, shift personnel is limited and organized into two different schedules. The process is exposed to 50 raw materials on the input side and a highly fragmented demand for 83 finished products on the output side. The goal is to process various products (jobs) in the shortest possible makespan, which results in a complex flowshop optimization problem.

PROBLEM FORMULATION

MILP Core Problem

The core problem of the presented case study is a hybrid flowshop MILP problem. The model formulation is based on assignment and precedence variables as suggested in [8]. Binary assignment variables a_{biml} indicate whether lot $b \in \{1, \ldots, B\}$ of job $i \in \{1, \ldots, N\}$ is processed on machine $m \in \{1, \ldots, M\}$ in process stage $l \in \{1, \ldots, L\}$. Binary indicator variables $h_{bb'ijml}$ indicate whether lot $b \in \{1, \ldots, B\}$ of job $i \in \{1, \ldots, N\}$ and lot $b' \in \{1, \ldots, B\}$ of job $j \in \{1, \ldots, N\}$ are both processed on the same machine $m \in \{1, \ldots, M\}$ in process stage $l \in \{1, \ldots, L\}$. Binary precedence variables $y_{bb'ijml}$ indicate whether lot b of job i is processed before lot b' of job j on machine m in process stage l. Continuous lot-sizing variables $r_{bi} \in [0,1]$ represent the split factors, which indicate what percentage of job i is covered by lot b. Start time variables s_{biml}, completion time variables c_{biml}, and processing time parameters t_{iml} define the resulting schedule. T is the Big-M constant.

The objective (11) is to minimize the makespan c_{max}.

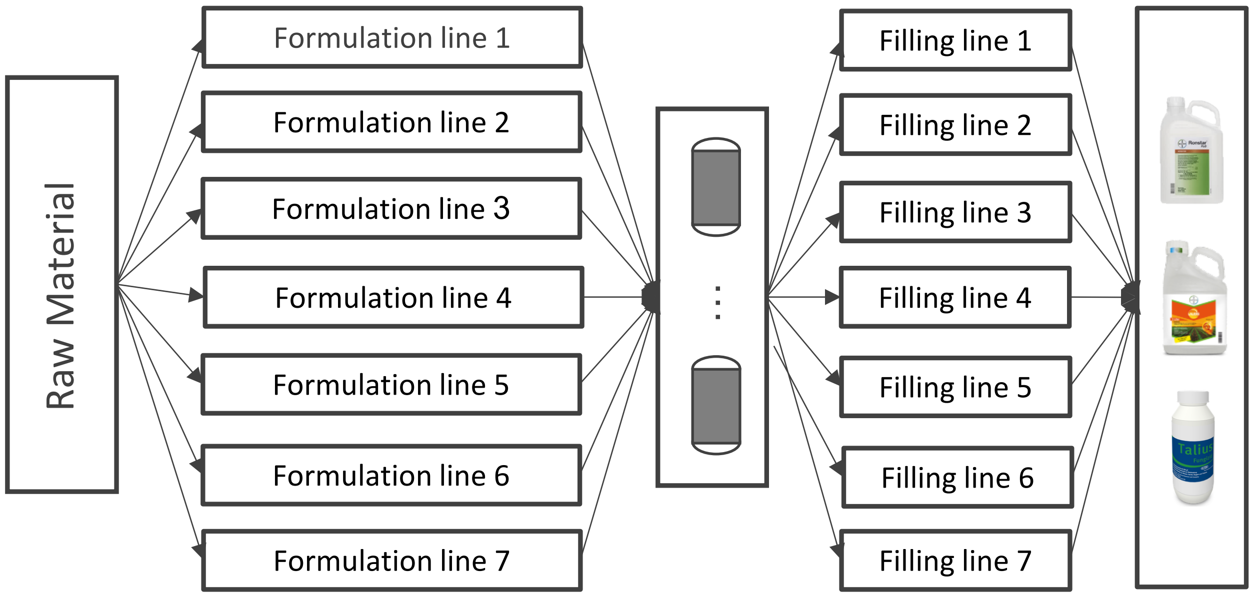

Figure 3: Equipment layout of the agrochemical formation plant treated in the case study. The case study is a two-stage flexible flowshop problem with seven formulation lines (stage 1) and seven filling lines (stage 2). The intermediated (formulated) product can be stored in seven buffer tanks before being processed in the filling lines.

(12) is the makespan constraint. Constraint (13) sets the start and completion times according to the processing times, (14) enforces the lot sequence on each machine, (15) ensures valid start times of lots when switching from one stage to the next, (16) forces each lot to be processed on exactly one machine in each stage, and (17) defines for each machine the precedence relation between lots which are assigned to that machine. Constraints (18) to (20) build the binary indicator variables $h_{bb'ijml}$ and (21) enforces valid lot sizes.

$$\min c_{max} \tag{11}$$

$$\text{s.t.} \quad c_{max} \geq c_{bimL} \; \forall \, b \in B, i \in N, m \in M \tag{12}$$

$$s_{biml} + r_{bi} t_{iml} \leq c_{biml} + T(1 - a_{biml}) \; \forall \, b \in B, i \in N, m \in M, l \in L \tag{13}$$

$$c_{biml} + r_{b'j} t_{jml} \leq c_{b'jml} + T\left(1 - y_{bb'ijml}\right) \; \forall \, b, b' \in B, i, j \in N: i \neq j, m \in M, l \in L \tag{14}$$

$$T(a_{biml} - 1) + s_{biml} + r_{bi} t_{iml} \leq s_{binl+1} + T(1 - a_{binl+1}) \; \forall \, b \in B, i \in N, m, n \in M, l \in L \setminus \{Lmax\} \tag{15}$$

$$\textstyle\sum_{m=1}^{M} a_{biml} = 1 \; \forall \, b \in B, i \in N, l \in L \setminus \{Lmax\} \tag{16}$$

$$y_{bb'ijml} + y_{b'bjiml} = h_{bb'ijml} \; \forall \, b, b' \in B, i, j \in N: i \neq j, m \in M, l \in L \tag{17}$$

$$h_{bb'ijml} \leq a_{biml} \; \forall \, b, b' \in B, i, j \in N: i \neq j, m \in M, l \in L \tag{18}$$

$$h_{bb'ijml} \leq a_{b'jml} \; \forall \, b, b' \in B, i, j \in N: i \neq j, m \in M, l \in L \tag{19}$$

$$h_{bb'ijml} \geq a_{biml} + a_{b'jml} - 1 \; \forall \, b, b' \in B, i, j \in N: i \neq j, m \in M, l \in L \tag{20}$$

$$\textstyle\sum_{b=1}^{B} r_{bi} = 1 \quad \forall \, i \in N \tag{21}$$

Benders Master Problem

From the MILP core problem, the Benders master problem (22) to (29) is derived. The Benders cuts in (23) are based on the dual variables u_{biml} which assume a value of -1 if lot b of job i on machine m in stage l is on the critical path, and 0 otherwise. Constraints (24) to (27) ensure valid lot-machine assignments and lot sequencing. Constraint (29) enforce valid lot sizes.

$$\min \mu \tag{22}$$

$$s.t. \quad \mu \geq \textstyle\sum_{b=1}^{B} \sum_{b'=1}^{B} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{m=1}^{M} \sum_{l=1}^{L} -u_{biml} \left(r_{bi} t_{iml} - T\left(1 - y_{bb'ijml}\right)\right) \tag{23}$$

$$\textstyle\sum_{m=1}^{M} a_{biml} = 1 \; \forall \, b \in B, i \in N, l \in L \tag{24}$$

$$y_{bb'ijml} + y_{b'bjiml} = h_{bb'ijml} \; \forall \, b, b' \in B, i, j \in N: i \neq j, m \in M, l \in L \tag{25}$$

$$h_{bb'ijml} \leq a_{biml} \; \forall \, b, b' \in B, i, j \in N: i \neq j, m \in M, l \in L \tag{26}$$

$$h_{bb'ijml} \leq a_{b'jml} \; \forall \, b, b' \in B, i, j \in N: i \neq j, m \in M, l \in L \tag{27}$$

$$h_{bb'ijml} \geq a_{biml} + a_{b'jml} - 1 \; \forall \, b, b' \in B, i, j \in N: i \neq j, m \in M, l \in L \tag{28}$$

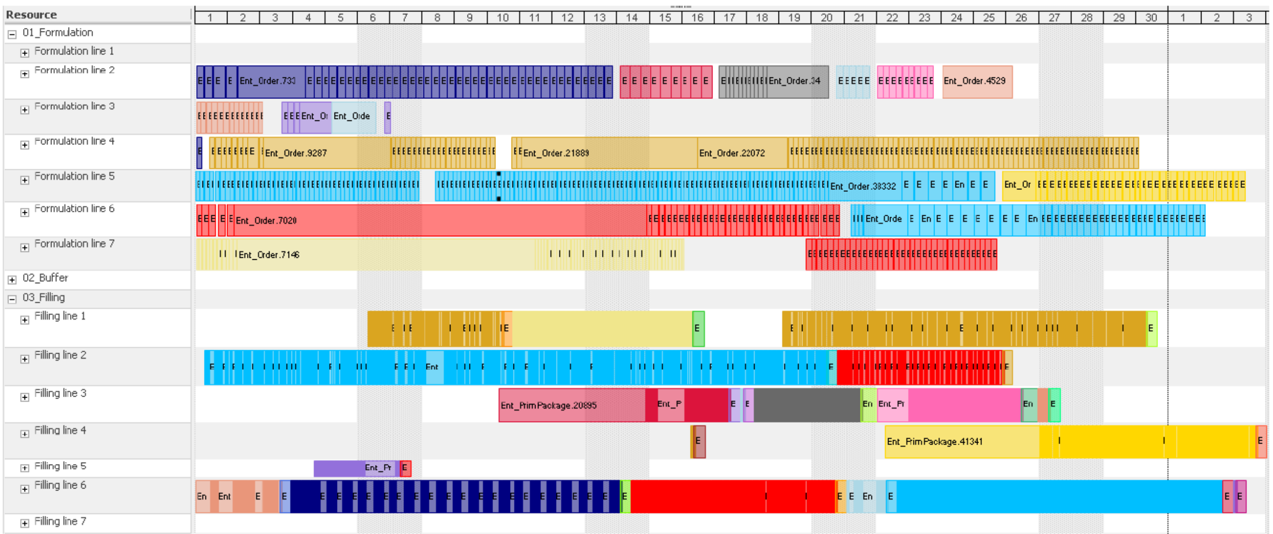

Figure 4: Gantt chart of best solution after solving one-month production dataset with BDES for 360 seconds.

$$\sum_{b=1}^{B} r_{bi} = 1 \quad \forall\, i \in N \tag{29}$$

Note that the cuts (23) only include the processing times as contributors to the critical path because they were derived from the core flow shop problem. If these cuts are applied to the detailed DES model of the case study, they remain valid inequalities as discussed in [7], but omit certain features of the problem such as secondary resources and changeover times. In this case, the cuts represent valid underestimators to the DES makespan with a gap $\delta \geq c_{max} - \mu^* \geq 0$.

Benders Subproblem

For the Benders subproblem, a DES model of the case study flow shop is assumed. In contrast to the MILP, the DES does not suffer from the curse of dimensionality, and computation times do not increase exponentially with problem size. This makes it possible to accurately model, simulate, and analyze complex systems. The DES model is considered as a black box function $f: y_{bb'ijml}, a_{biml}, r_{bi} \rightarrow s_{biml}, c_{biml}, c_{max}$ that computes a solution for the non-complicating variables and dual multipliers, given an input solution of the complicating variables. In the case study, the input contains the lot sequences, lot-line assignments, and lot sizes. The output includes the starting and completion times of all lots. In addition, the dual multipliers are calculated from the critical path.

RESULTS

BDES is applied to the one-month production dataset 2 of the case study in [6, Table 4]. As DES Python-based SimPy 4.1.1 is used, and for MILP gurobipy 11.0.1 with Gurobi 9.5.2. Similar to the monolithic-sequential MILP-DES approach of [6], BDES can compute an optimized one-month production schedule for the formulation and filling plant. This schedule is found after approximately one minute of solution time, which is faster than the monolithic MILP model, which required approximately ten minutes for the first optimization step to schedule the production within the month without backlog. However, a shortcut DES model was used, and the makespan in the same full-space DES model as used in [6] is 33 days (see Figure 4). This result is comparable to the manually generated DES schedule and implies that some backlog quantities occur.

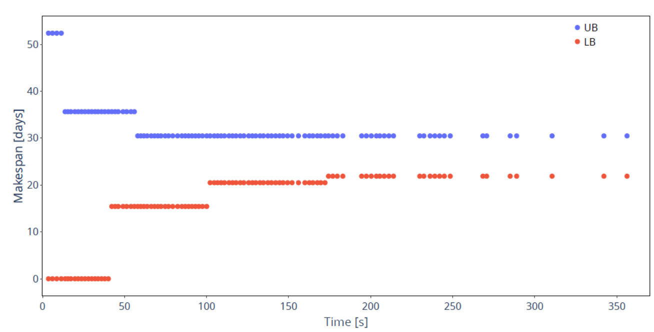

Figure 5. Makespan bounds progression when solving a one-month production dataset with BDES for 360 seconds.

Figure 5 shows that the lower bounds on the makespan increase after a solution with a makespan of 30.5 days has already been found. This information can be helpful in practice to terminate the search process prematurely, as it can be assured that no better solutions than with a makespan of 20 days can be found. Meaningful lower bounds can be computed for the one-month case study dataset because eligibility constraints are added to the master problem. These constraints rule out many lot-line assignments, reducing the search space a priori.

Lot-sizing is an important degree of freedom that enables efficient scheduling with minimal makespan. However, BDES only exploits splitting for some products. For example, the first products on formulation lines 5 and 6 exploited the split factor of two (see Figure 4). Therefore, a split factor of two already supports a high utilization of formulation and filling lines. An unlimited split factor as implemented in [6] may lead to further makespan improvements but also increases the model complexity. In this work, experiments with higher split factors indicated that the complexity can be prohibitive for BDES because the search space for the Benders master problem grows exponentially.

Furthermore, Figure 4 shows that fewer changeovers in the filling stage occur compared to the result in [6]. This is because the objective function in this work is not to minimize changeover costs but the makespan. In [6], many changeovers occurred due to favorable changeover cycles in the filling stage that minimized the overall changeover costs.

The application of BDES to the one-month production dataset shows that the performance of BDES is generally comparable to the manual-DES approach in [6] for real-world scheduling tasks. Furthermore, BDES requires similar solution times as the MILP-DES approach in [6] but yields slightly worse results. However, building the complex, monolithic MILP model and manually reconciling the MILP results with the DES model is not necessary. Instead, a Benders master model of a generic, hybrid flowshop (see Section “Problem Formulation”) is used to guide the search process in the DES subproblem.

CONCLUSIONS

This work presents a novel BDES algorithm to minimize the makespan of hybrid flowshop with allocation, sequencing, and lot-sizing decisions as well as secondary resource constraints. BDES performs similarly to the sequential monolithic MILP-DES approach of [6]. Although BDES yields slightly worse results compared to the approach of [6], the setup of BDES is less complex compared to a monolithic MILP approach.

The main advantage of BDES is that it can apply MILP in the form of simple Benders cuts to highly detailed DES models. It is not necessary to build a complex MILP model, but a MILP Benders master model of a generic hybrid flowshop with precedence variables is used while the real-world constraints being moved to the DES subproblem. This work shows that combining the strengths of rigorous optimization and detailed modeling can be attractive from a practical perspective since it allows for fast model development and implementation.

Future research will focus on the investigation of the generality of the approach, i.e., the influence of different problem instances on the solution quality as well as strengthening the Benders cuts and generation of robust schedules.

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