

Comparison of optimization methods for studying the energy mix of infrastructures. Application to an infrastructure in Oise, France

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ABSTRACT

In the last decades, the growing awareness of climate change and the high political sensitivity of critical resources such as energy have emphasized a need for local, renewable and optimized energy mixes at various scales. Several studies have therefore aimed to optimize renewable energy technologies and plant locations to develop more renewable and efficient Energy Mixes. Following this trend, this paper applies and compares Goal Programming, Branch-and-Cut and NSGA-II to a multi-objective combinatorial optimization problem focused on the energy mix of Oise, France. Results show more optimality for Goal Programming and Branch-and-Cut, accompanied by a high sensitivity to constraints, while NSGA-II provides more technological diversity in the computed solutions.

Keywords: Energy Systems, Optimization, Genetic Algorithm, Stochastic Optimization, Goal Programming, Branch-and-Cut, Energy Mix

1. INTRODUCTION

The global rise of climate change awareness and its urgency has prompted governmental and intergovernmental organisations to update their policies in several areas. Environmental considerations have become paramount in the design of energy mixes, and renewable energy sources are now widely preferred to fossil fuels. The latter have also become highly sensitive to geopolitical issues and conflicts, making energy a critical resource more than ever. These environmental and economic incentives have led economic regions at different scales to improve their energy mixes and achieve energy independence [1]. The development of optimized, renewable and local energy mixes is therefore strongly supported by both environmental and economic situations globally [2].

The optimization of renewable energy technologies, plant locations and energy mixes has been the objective of several studies in the last decades, aiming to develop more renewable and efficient energy mixes [3]. Most of

these optimization problems are solved using Mixed-Integer Linear Programming (MILP), Mixed-Integer Non-Linear Programming (MINLP) or heuristic algorithms [3].

This study aims to compare three optimization methods in their application to the optimization of the energy mix of Oise, France, focused on the Paris-Beauvais Airport, from environmental and economic perspectives.

The focus is put on annual production potential at a regional scale and therefore does not consider decomposition optimization methods, better suited to problems involving temporal variation or multiple time periods. For this study, Goal Programming, Branch-and-Cut and Non-dominated Sorting Genetic Algorithm II (NSGA-II) were selected for their wide use and coverage of problem formulations. The first two can easily handle MILP and MINLP problems but require the problem to be expressed as a single objective function. On the other hand, NSGA-II can handle both objectives separately [4,5].

After census of energy resources already present in the target area, the available energy mix undergoes a

carbon footprint evaluation that serves as the environmental component of the problem [6]. The economic component is based on the Levelized Cost of Energy (LCOE), an aggregation of operating, maintenance and installation costs. These two components constitute the objectives of the problem, either treated separately or weighted in a single objective function depending on the optimization method. The three selected methods are applied to the problem, and the results provided by each are gathered and assessed based on criteria such as optimality of solutions, sensitivity to constraints and parameters, execution time and diversity of solutions. The evaluated methods are then compared against these criteria, to identify their strengths and weaknesses of each method regarding the studied problem.

The aim of the study is to assess not only optimality but also diversity of solutions and to identify acceptable trade-offs between the two. This could lead to the development of a hybrid method ideally fitted to this energy mix optimization problem.

2. METHODS

2.1 Choice of optimization methods

The methods compared in this study are Goal Programming, Branch-and-Cut and NSGA-II. They approach problems very differently but are all recognized and widely used for multi-objective combinatorial optimization problems. Each of these methods has its own strengths and weaknesses, but all three are suitable for the energy mix optimization problem studied and described later in this paper [7,8].

2.1.1 Goal Programming

Goal Programming is known for being ideal in multi-criteria decision-making problems, especially with conflicting objectives. Computationally, it is the simplest method of the three, essentially defining an ideal solution from given objectives and calculating deviations between possible solutions and the ideal one. In this way, it can handle multiple conflicting objectives, allowing the decision-makers to prioritize one objective over others, addressing trade-offs explicitly. In the case of energy-related problems, it has been applied to several energy-planning issues, especially to handle the conflict between financial costs and greenhouse gases emissions [7,9]. Its simplicity and its ease in dealing with conflicting objectives make Goal Programming fitted to the energy mix optimization problem later described in this paper.

2.1.2 Branch-and-Cut

The Branch-and-Cut optimization method combines Branch-and-Bound and Cutting-Planes, two algorithmic approaches, to solve mixed-integer linear programming

(MILP) problems. Branch-and-Cut explores the space of feasible solutions by creating branches of decision variables, then refines the solution space with cutting planes to remove identified non-optimal solutions. While it can find guaranteed global optima for MILP problems, its computation times can rise quickly depending on problem size. However, Branch-and-Cut is well suited to problems involving discrete variables, which is often the case with Energy-related scenarios [10]. Despite it being a combinatorial optimization problem, the studied optimization problem is rather small and can be optimized using the Branch-and-Cut method.

2.1.3 Non-dominated Sorting Genetic Algorithm (NSGA-II)

Unlike previous methods, NSGA-II uses a stochastic approach to solve more complex, non-linear problems. It is widely used for multi-objective optimization problems, including energy mix optimization problems. It generates successive populations of candidate solutions before ranking them according to Pareto dominance. Although it is computationally intensive, it provides flexibility in handling non-linear, non-convex problems, making it a widely used method in a variety of fields, including energy problems for energy development and greenhouse gases (GHG) emission reduction [8,11]. In the case of the studied energy mix optimization problem, the key asset it provides is the Pareto-front solutions which, while not guaranteeing global optimality, allows to test trade-offs without having to implement weights. A comparison of optimality between NSGA-II solutions and deterministic algorithms will allow to assess the acceptability of solutions regarding optimality, while still exploring technological diversity in the computed energy mix optimizations.

2.2 Studied optimization problem

The case study for this work is a combinatorial optimization problem for the energy mix of Oise, France. The problem consists in the selection of energy sources from a given dataset [12–15], presenting as a list of potential installations. The optimization algorithms then combine multiple of these installations to build an energy mix. To do so, the objective function is based on two criteria, an environmental criterion and a economic criterion. The first is derived from the energy mix evaluation method proposed in [6]. It consists of three emission components, expressed in tons of expressed in tons of CO₂eq per year. The first component is a product of energy production and an energy source emission factor, the second represents emissions due to urbanization, and the third translate the emissions depending on the distance between the energy source and the consuming infrastructure.

The economic criterion is based on the Levelized Cost Of Energy (LCOE) data from [16]. It also depends on

the annual production and is expressed in euros per year. The problem is therefore expressed in Equations (1), (2) and (3).

$$\min f_{env}(P) = \sum_{i=1}^n (P_i * EES_i + CFU_i + CFD_i) \quad (1)$$

$$\min f_{eco}(P) = \sum_{i=1}^n (P_i * LCOE_i) \quad (2)$$

$$\sum_{i=1}^n (P_i) \geq PMin \quad (3)$$

With EES the GHG emission factor assigned to production, CFU the emissions of urbanization and CFD the emissions assigned to distance [6,12,13]. PMin is the target annual production to reach and varies between 0 and the total annual production for the dataset.

2.3 Python Implementation

The optimization of the problem with Goal Programming, Branch-and-Cut and NSGA-II was implemented using the Python programming language, chosen for its versatility and wide number of libraries. Moreover, Python offers accessibility and is ideal for open-source applications.

2.3.1 Goal Programming

The solving of the optimization problem by Goal Programming was implemented using PuLP, an open-source Python library. To guide the algorithm in decisions between the conflicting objectives, weights α and β were introduced, summing to 1 and weighting the environmental and economic criteria respectively. Both components were then added up in a single objective function and therefore needed to be comparable. The LCOE value, expressed in €/y, was converted to tCO_{2eq}/y using the Social Cost of Carbon (SCC) as the average of calculations in the literature from 2018 to 2022 [17]. The final objective function for Goal Programming application is expressed in Equation (4).

$$\min f_{env}(P) = \alpha \sum_{i=1}^n (P_i * EES_i + CFU_i + CFD_i) + \beta \sum_{i=1}^n (P_i * LCOE_i * SCC) \quad (4)$$

Using the PuLP library, a Linear Programming problem was specified. The decision variable for each item was defined as a binary value: each item is either included or not included in the final solution. Deviations were defined from zero for objectives to minimize, and from PMin for the target production constraint.

2.3.2 Branch-and-Cut

The solving of the optimization problem with Branch-and-Cut was also implemented using PuLP. The weights implemented in the Goal Programming method were retained to allow comparison between the two approaches. Similarly, both objectives were combined into a single objective function, and the LCOE value was also converted to tCO_{2eq}/y using the SCC. The PuLP library-implementation also uses the *LpProblem* function and a binary decision variable. The cutting plane methods used for this optimization were Gomory cuts, Knapsack cuts, suitable for combinatorial problems, and Probing cuts, which are suited to problems with binary variables.

2.3.3 Non-dominated Sorting Genetic Algorithm (NSGA-II)

The solving of the optimization problem with NSGA-II was implemented using the DEAP Python library. Unlike previous methods, both objectives can be evaluated together without having to sum them or implement weights. The algorithm was initially run with a population of 100 at generation 0, a crossover probability of 75% and a mutation probability of 20%, for a duration of 300 generations. To ensure that the production constraint is met, an infinite penalty is applied to both minimization objectives for all individuals that do not reach the production objective.

3. RESULTS

3.1 Goal Programming

Goal Programming was applied to the problem with a weight of 0.65 for the α parameter and 0.35 for β , while the minimum energy production (PMin) was set at 1422 GWh/y, representing 60% of the maximum possible production. Results showed a significant dominance of wind energy, with 53 out of the 66 selected installations being wind turbines, accounting for approximately 81% of the total production (**Table 1**).

Table 1: Summed results for the Goal Programming optimization, with PMin=1422, α =0.65, β =0.35. P_Tot is the total production, in GWh/y. T_Env is the environmental component of the objective function and T_Eco the economic component of the objective function, both in tCO_{2eq}/y.

Energy	Count	P_Tot	T_Env	T_Eco
Wind	53	1153.2	25112	20373
Solar	2	0.1	5	4
Biomethane	11	268.7	6290	8508
Total	66	1422.0	31408	28885

Further computations were performed with weight values varying between 0.05 and 0.95, and target production varying between 40% and 60% of total production. PMin is the most influential parameter on results,

and the algorithm is less sensitive to weighting when PMin is lower, resulting in fewer optimal solutions and more convergence (**Figure 1**).

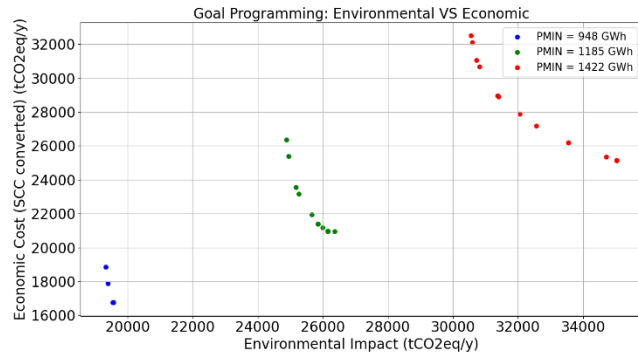


Figure 1: Compared results of the optimization with the Goal Programming approach, with varying PMin, α and β

3.2 Branch-and-Cut

The Branch-and-Cut method was applied to the optimization problem, with the same values of PMin, α , and β , and with the same variations as with Goal Programming. The results obtained were identical: the total values were the same, as the exact same elements were selected by both approaches. The trends observed based on the variables therefore remain consistent for Branch-and-Cut. The only difference observed between the methods is a variation in execution time, with Branch-and-Cut being 8% faster than Goal Programming.

3.3 NSGA-II

Unlike previous methods, no weights were applied in the NSGA-II optimization. As a stochastic approach, the algorithm was run multiple times with the same PMin value and could inherently explore different trade-offs. Different iterations with identical parameters therefore resulted in slightly different Pareto-fronts. The computed results differ significantly from those produced by Goal Programming and Branch-and-Cut (**Figure 2**).

NSGA-II computed results are globally less optimal for both criteria, and tend to exceed the production constraint more than necessary, often by more than 1 GWh/y. They also tend to return higher values for the environmental objective and favour minimizing the economic component of the problem. Computation times for the NSGA-II optimizations were about 15 times faster than Goal Programming or Branch-and-Cut and rarely lasting more than a dozen seconds. The initial population size was therefore increased to 2000 and 5000, which showed a positive impact and partially resolves the optimality problem, with results much closer to Goal Programming and Branch-and-Cut, although still less optimal. However, computation times were raised significantly (circa 500 times longer for a population of 5000).

The population size increase mitigated the excess production in solutions, although solutions provided still exceed the threshold production constraint. On the other hand, NSGA-II solutions appear to provide more diversity in the technologies used to reach that threshold (**Table**

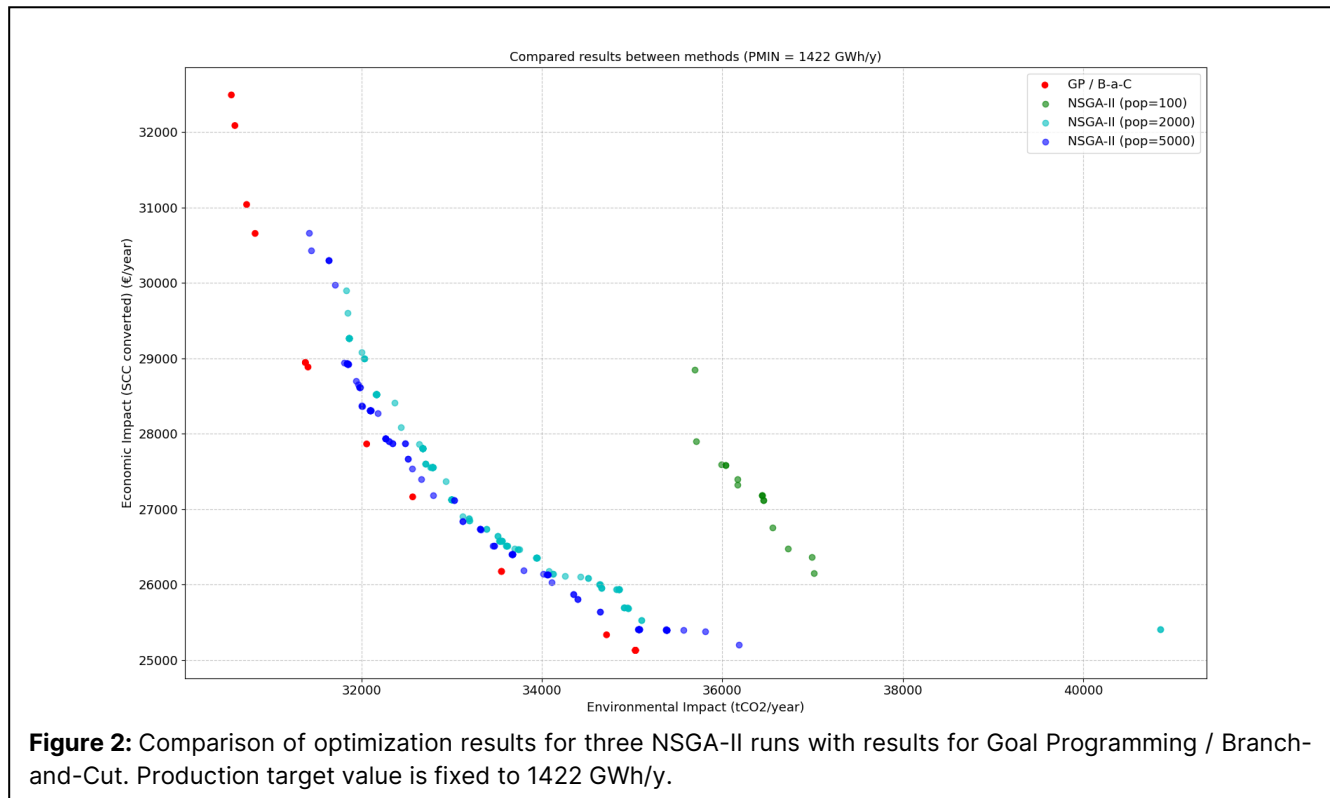


Figure 2: Comparison of optimization results for three NSGA-II runs with results for Goal Programming / Branch-and-Cut. Production target value is fixed to 1422 GWh/y.

2), using more solar plants than Goal Programming or Branch-and-Cut methods. Upscaling tests were performed, and results suggest that the optimality difference between NSGA-II and deterministic methods increases with the number of selectable items, as well as the computation times, while maintaining technological diversity.

Table 2: Mean results for NSGA-II optimization computation with PMin=1422. P_Tot is the reached production, in GWh/y.

Energy	Count	P_Tot
Wind	54.79	1231.59
Solar	111.22	10.58
Biomethane	8.88	181.03
Bioenergy	0.32	0.53
Total	175.22	1423.74

4. DISCUSSION

The Goal Programming results show an over-representation of wind energy installations compared to the original dataset. This can be explained by the evaluation methods, which both favor wind power installation. Moreover, wind turbines are individually some of the most productive installations, so the reward for incorporating them in the mix is high. This bias could be controlled and limited by adding a third objective, possibly regarding the flexibility or controllability of the production, or social acceptance of the project.

The high similarity between Goal Programming and Branch-and-Cut results confirms the optimality of found solutions. These two computational approaches, both deterministic, are therefore computationally closer to each other than to NSGA-II, thus explaining the observed disparities with the latter's results. Though they provide optimality in this study, further work may introduce additional objectives and constraints that may be too complex for deterministic approaches and require an evolution of the method. In addition, the algorithms require weights to explore the solution space, which induces a high dependency between the results and the α and β parameters, especially with high PMin values. This could mean that a small number of items in the dataset are significantly better than others in regard to both criteria and would therefore be systematically preferred in cases with low PMin, resulting in a weight dependence increasing with target production. On the other hand, NSGA-II can explore different regions of the solution space and find trade-offs without these weights.

Concern is raised regarding NSGA-II, since the pareto fronts produced were not as optimal as the results of previous methods. The efficiency of the method on this problem can be doubted, although some explanation

can also be provided by the strong penalties implemented in the computation. The algorithm is so strongly encouraged to reach the production constraint, that it seems reluctant to slightly reduce production to find more optimal solutions in terms of the environmental and economic objectives. Although increasing the population size partially solved this issue, upscaling the problem could prove difficult, with a larger dataset implying even longer computation times. However, the technological diversity provided by NSGA-II is significantly better than that of deterministic approaches, which supports interest in pursuing the research of trade-offs between optimality and technological diversity, as long as computational times remain acceptable. To pursue this goal, a hybrid approach could be considered, incorporating portions of the deterministic approaches to reduce the solution space and thus guide the NSGA-II towards more optimized results.

5. CONCLUSION

The comparative analysis of Goal Programming, Branch-and-Cut, and NSGA-II for a combinatorial energy mix optimization problem in Oise, France, shows nuanced insights. Goal Programming and Branch-and-Cut produced remarkably similar results, demonstrating good solution optimality and computational efficiency with a strong dependence to the input dataset, translating to a distinct preference for wind energy installations. This underlines an interesting development opportunity for introducing additional objectives like production flexibility or social acceptance to create more fitted optimization approaches.

NSGA-II provided less optimal solutions than deterministic methods across both environmental and economic criteria, which correlates with existing literature. However, the optimality difference reduces with the population size used for NSGA-II computation, with an increase in computational time. The method's performance was constrained by strict production penalties, which may have limited its ability to find optimal solutions and explore more nuanced trade-offs. However, its overperformance regarding the production threshold criteria and the greater technological diversity it provides in its solutions suggest that an acceptable trade-off between optimality, computation time and technological diversity can be found.

Future research should focus on refining penalty mechanisms, implementing additional objectives and constraints to progress towards better suited optimization algorithms, both in terms of computational methodology and parameter definition. Opportunities for hybrid methods should not be ignored and will be considered in future works.

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