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Kernel-based estimation of wind farm power probability density considering wind speed and wake effects due to wind direction

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ABSTRACT

This study compares the probability density function (PDF) of the power generated by a wind farm obtained analytically with the PDF considering the wake effect between wind turbines, a phenomenon that reduces the power generation capacity of wind farms. Instead of considering the wake effect in the analytical method, which is complex and difficult to solve, it has been proposed to use kernel estimators to obtain the PDF. To calculate it, a wind farm power output data set has been used. This data set was generated using historical wind speed and direction data and the Katic multiple wake model. Discrepancies between the analytical PDF and PDF fitted with the kernel estimators, can lead to an overstatement of the annual available energy by 4 an 9 %, depending on the complexity of the wind farm layout. These inconsistencies can have significant implications for production planning, wind farm design, and integration of wind power into the grid. Therefore, this analysis underscores the necessity of incorporating the wake effect in wind farm modelling, to guarantee more precise projections of generation and energy availability.

Keywords: Wake effect, wind farm power distribution, kernel estimators

INTRODUCTION

When planning wind farm projects, it is crucial to quantify and assess the wind resource of the candidate site. This assessment is typically conducted using the wind energy density [1], also called wind power density [2], which requires the probability distribution of wind speed $f_v(v)$ and the power curve of a wind turbine $P_{WT}(v)$ to calculate it. Furthermore, based on $f_v(v)$, $P_{WT}(v)$ and using the change-of-variable technique, it is possible to obtain the probability density function of a wind turbine power output, $f_{PWT}(P_{WT})$, which provides additional insight into the energy that can be produced [3].

However, to optimize the management of a wind farm, it is necessary to know the probability density function of the power generated by the entire wind farm, $f_{PWF}(P_{WF})$. This function enables the estimation of the variability and availability of the power generated by the wind farm, facilitating production planning over a given period. It also ensures the integration of wind energy into the electricity grid by allowing the determination of the probability of achieving a specific power output. This probability-based approach supports optimal decision making [4]. One way to obtain $f_{PWF}(P_{WF})$ is through the probability distribution of wind speed $f_v(v)$, and the wind farm's power curve, $P_{WF}(v)$, as described before for the single turbines. This requires a simple analytical expression of the wind farm power curve $P_{WF}(v)$, which allows the change-of-variable technique to be used. For example, the power produced by a wind farm can be approximated as the power output of a single wind turbine multiplied by the number of turbines, $P_{WF}(v)=n_{turb}\cdot P_{WT}(v)$.

However, this approach has the disadvantage of not considering a significant source of power loss such as the wake effect (turbines shading each other) due to wind direction. A first alternative to consider the wake effect is to use a wind probability density function and a wind farm power curve that depend on wind speed (v) and direction (θ): $f_{V,\Theta}(v,\theta)$, $P_{WF}(v,\theta)$. Nevertheless, deriving these expressions and applying the change-of-variable technique to bivariable distribution functions is highly complex.

For this reason, some authors have tried to keep the

$P_{WF}(v)=n_{turb}\cdot P_{WT}(v)$ approximation and add a wake coefficient term [5], simplifying a wake model such as Jensen's [6], but this approximation is only valid for wind farms with *n*x*m* rectangular geometry.

This paper proposes generating a sample of wind farm power output data based on historical wind speed and direction data from a given location. The Katic wake model [7] is used to calculate the effective incident velocity for each wind turbine. Subsequently, the power generated by each turbine, $P_{WT}(v)$, is determined, and the total wind farm power is obtained by summing the individual contributions. Finally, the wind farm power probability density function, $f_{PWF}(P_{WF})$, is estimated using kernel method. A similar application of kernel estimators to model the probability density function of a group of wind farms is described in [8]; however, their approach also neglects the wake effect.

The paper is structured as follows. Section 2 explains the two alternatives proposed to obtain $f_{PWF}(P_{WF})$ and the wake model used; section 3 gives a brief description of the data used; section 4 shows the results; and section 5 ends with some brief conclusions.

MODELLING

This section begins with an explanation of the wake effect in wind farms and how it has been modelled in this work. Next, subsection *Theoretical wind farm power PDF* explains how the probability density function of the wind farm power is obtained theoretically. Finally, subsection *Wind farm power PDF considering wake effect* presents our proposal, based on kernel estimators, to obtain the probability density function of the wind farm power considering the wake effect.

Wake effect

The wake effect is a phenomenon whereby wind turbines located geographically behind others in relation to the wind front will see their capacity to produce electrical energy diminished due to the loss of energy and increased turbulence that the wind undergoes after passing through a turbine.

Jensen's model is the simplest model to describe this phenomenon [6]. It is based on the assumption that there are no deterministic or periodic vortices immediately behind the turbine, but that the air behaves as a turbulent fluid (the particles move disorderly and the particle trajectories meet to form aperiodic vortices). This model also considers that within the wake, the velocity is constant rather than following a Gaussian or other distribution. As shown in Figure 1, the wake behind the turbine starts at the diameter of the turbine and opens linearly with distance, forming a cone.

In the case of wind farms where there may be more wakes that may also partially affect a turbine, the Jensen's multiple wake model or Katic's model [7] is applied.

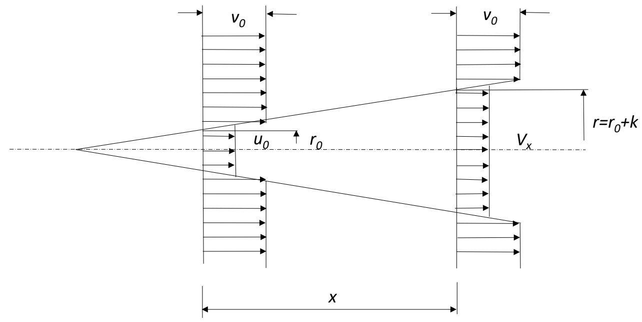

Figure 1. Wake generated by the turbine on the left at distance x downstream.

There are many ways to incorporate in a turbine *j* the wake effect of all other turbines *i* upstream. One possibility is to assume that the kinetic energy deficit in turbine *j*, attributable to a mixture of wakes with respect to the undisturbed wind v_0, is equal to the sum of the kinetic energy deficits in turbine *j* due to each individual wake of turbine *i* with respect to the undisturbed wind v_0, see equation (1).

$$\left(v_0 - v_j\right)^2 = \sum_{i=1}^{n}\left(v_0 - v_{ij}\right)^2 \qquad (1)$$

Considering equation (1), together with the Katic's multiple wake model, equation (2) is derived.

$$v_j = v_0\left[1 - \sqrt{\sum_{i=1}^{n}\left(1-\sqrt{1-C_T}\right)^2\left(\frac{r_i}{r_i(x_{ij})}\right)^4\left(\frac{A_{sh,i}}{A_j}\right)^2}\right] = v_0\left[1 - \sqrt{\sum_{i=1}^{n}\delta_{ij}^2\left(\frac{A_{sh,i}}{A_j}\right)^2}\right] \qquad (2)$$

Where subscript *i* refers to the wind turbines generating a wake on a given wind turbine *j*, v_0 is the undisturbed velocity affecting the wind farm (at a position sufficiently far from all turbines), C_T is the thrust coefficient of the wind turbine, $A_{sh,i}$ is the overlapping wake area of wind turbine *i* on wind turbine *j*, and A_j is the swept area of the blades of wind turbine *j*. Finally, $r_i(x_{ij})$ is the radius of the wake of wind turbine *i* (wake generator) at the height of wind turbine *j* (affected by the wake), which can be calculated using equation (3) in terms of the radius of the wind turbine generating the wake, r_i, and the wake decay coefficient (4).

$$r_i\left(x_{ij}\right) = r_i + k\cdot x_{ij} \qquad (3)$$

The wake decay coefficient is a function of the height of the wind turbine hub, *h*, and the roughness of the terrain, z_0.

$$k = \frac{0.5}{ln\left(\frac{h}{z_0}\right)} \qquad (4)$$

Considering the different cones generated by the turbines, some of which do not necessarily fully affect the downstream wind turbines, the overlap area $A_{sh,I}$ can be calculated according to three situations:

- If the swept area A_j is outside of the wake area, or wind turbine i is stopped, then $A_{sh,i} = 0$.
- If the swept area A_j is completely inside the wake area, and wind turbine i is running, then $A_{sh,i} = A_j$.
- If the swept area A_j is partially inside the wake area, and wind turbine i is running, then $A_{sh,i}$ is calculated using equation (5).

$$A_{sh,i} = [r_i(x_{ij})]^2 cos^{-1}\left(\frac{L_{ij}}{r_i(x_{ij})}\right) + r_j^2 \cos^{-1}\left(\frac{d_{ij} - L_{ij}}{r_j}\right)$$

$$-L_{ij}\sqrt{r(x_{ij})^2 - L_{ij}^2} - (d_{ij} - L_{ij})\sqrt{r_j^2 - (d_{ij} - L_{ij})^2} \quad (5)$$

Being L_{ij} described in equation (6).

$$L_{ij} = \frac{r_i^2(x_{ij}) - r_j^2 + d_{ij}^2}{2\, d_{ij}} \quad (6)$$

Analytical PDF of wind farm power

As mentioned in the introduction, the PDF of the wind farm power $f_{PWF}(P_{WF})$ can be obtained theoretically from the PDF of the wind speed $f_v(v)$ and from the power curve of a wind farm $P_{WF}(v)$, using the change-of-variable technique described in [3].

The power curve of a wind farm, if all wind turbines in the wind farm are equal, can be expressed in megawatts, according to equation (7) and the number of turbines *n_turb*.

$$P_{WF}(v) = \frac{n\_turb}{10^6} \cdot P_{WT}(v) \quad (7)$$

The power curve of a wind turbine, $P_{WT}(v)$, establishes the relationship between the wind speed and the power generated by a wind turbine. It can be expressed theoretically as a piecewise function (8), whose parts are defined by v_{in}, which is the lower wind speed at which power can be extracted, v_r is the wind speed at which the nominal power of the wind farm (P_{RWT}) can be obtained, and v_{off} is the wind speed at which the wind turbine must be stopped in order to prevent damage. Between v_{in} and v_r the power extracted by the wind turbine is a function of the air density ρ, the radius R of the area swept by the wind turbine blades, and the power coefficient Cp. This coefficient, in turn, is a function of the turbine rotation speed and the pitch angle. The value of this coefficient depends on the turbine model, and its maximum value is set by the Betz limit at 0.593 [9]. In this study, a fixed value has been calculated based on the rated speed of the wind turbine, the rated power of the turbine, and the mean air density at the location of the wind farms.

$$P_{WT}(v) = \begin{cases} 0 & 0 \le v < v_{in} \\ \frac{1}{2} \cdot \pi \cdot R^2 \cdot \rho \cdot v^3 \cdot Cp & v_{in} \le v < v_r \\ P_{RWT} & v_r \le v < v_{off} \\ 0 & v \ge v_{off} \end{cases} \quad (8)$$

The probability distribution function (PDF) of the wind speed is commonly modelled with a Weibull distribution [1], as it is shown in equation (9).

$$f_v(v) = \begin{cases} \frac{k}{C}\left(\frac{v}{C}\right)^{k-1} \cdot e^{-\left(\frac{v}{C}\right)^k} & v \ge 0 \\ 0 & v < 0 \end{cases} \quad (9)$$

Where C and k are the scale and the shape parameters, respectively.

Once $f_v(v)$ and $P_{WF}(v)$ are defined, it is possible to obtain the PDF of the wind farm power $f_{PWF}(P_{WF})$, which is described in the equation (10).

$$f_{PWF}(P_{WF}) = \begin{cases} \left(1 + e^{-\left(v_{off}/c\right)^k} - e^{-\left(v_{in}/c\right)^k}\right) \cdot \delta(P_{WF}) & P_{WF} = 0 \\ 0 & 0 < P_{WF} < P_{INWF} \\ \frac{k'}{C'} \cdot \left(\frac{P_{WF}}{C'}\right)^{k'-1} \cdot e^{-\left(P_{WF}/C'\right)^{k'-1}} & P_{INWF} \le P_{WF} < P_{RWF} \\ \left(e^{-\left(v_r/c\right)^k} - e^{-\left(v_{off}/c\right)^k}\right) \cdot \delta(P_{WF} - P_{RWF}) & P_{WF} = P_{RWF} \\ 0 & P_{WF} > P_{RWF} \end{cases} \quad (10)$$

Where P_{WF} is the wind farm power, P_{INWF} is the wind farm power to v_{in}, P_{RWF} is the wind farm rated power, δ is Dirac's delta, and K'' and C'' are parameters that depend on the previous scale and shape parameters, $k'' = k/3$ and $C'' = (C^3 \cdot \pi \cdot R^2 \cdot Cp \cdot n\_turb)/(2 \cdot 10^6)$.

Finally, the cumulative distribution function (CDF) of the wind power $F_{PWF}(P_{WF})$ is obtained by integrating the PDF of the wind farm (10) between zero and infinity, resulting in (11).

$$F_{PWF}(P_{WF}) = \begin{cases} \left(1 + e^{-\left(v_{off}/c\right)^k} - e^{-\left(v_{in}/c\right)^k}\right) & 0 < P_{WF} < P_{INWF} \\ 1 - e^{-\left(P_{WF}/C'\right)^{k'-1}} & P_{INWF} \le P_{WF} < P_{RWF} \\ 1 - e^{-\left(P_{WF}/C'\right)^{k'-1}} + e^{-\left(v_r/c\right)^k} - e^{-\left(v_{off}/c\right)^k} & P_{WF} = P_{RWF} \\ 1 & P_{WF} > P_{RWF} \end{cases} \quad (11)$$

PDF of wind farm power considering wake effect

The PDF of the wind farm power obtained previously does not consider the wake effect in any way. In this paper it is proposed to use kernel estimators to estimate the PDF of the wind farm power including the wake effect $f_{PWF\_WE}(P_{WF\_WE})$. For this purpose, historical wind speed and wind direction data are used to calculate the incident wind speed at each wind turbine (v_j) of a wind farm using a valid wake model. In this study, Katic's model is used, but another model could also be applied while the methodology and conclusions of this paper remain valid. Then, the power generated by each turbine is calculated using $P_{WT}(v)$. Finally, the power of the wind farm is obtained as the sum of the power generated by each turbine (12).

$$P_{WF\_WE}(v) = \sum_{j=1}^{n\_turb} P_{WT}(v_j) \quad (12)$$

Once the wind farm power data set has been generated, kernel estimators, whose basic equation is shown in equation (13), are used to fit the function $f_{PWF\_WE}(P_{WF\_WE})$. However, since the power of a wind farm is constrained between 0 and P_{RWF}, and the conventional kernel estimator does not consider this constraint, it is necessary to modify it. In this case, the reflection method [10] has been employed, as it is the approach implemented in the kdensity() function of the Statistics and Machine Learning Toolbox [11] from Matlab™ [12], and that will be used to perform the fitting.

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right) \quad (13)$$

Finally, $f_{PWF\_WE}(P_{WF\_WE})$ and $F_{PWF\_WE}(P_{WF\_WE})$ are described in equation (14) and (15) respectively.

$$f_{PWF\_WE}(P_{WF\_WE}) = \frac{1}{nh}\sum_{i=1}^{n}\left[K\left(\frac{x-2L+x_i}{h}\right) + K\left(\frac{x-x_i}{h}\right) + K\left(\frac{x-2U+x_i}{h}\right)\right] \quad (14)$$

$$F_{PWF\_WE}(P_{WF\_WE}) = \frac{1}{n}\sum_{i=1}^{n}\left[G\left(\frac{x-2L+x_i}{h}\right) + G\left(\frac{x-x_i}{h}\right) + G\left(\frac{x-2U+x_i}{h}\right)\right] - \frac{1}{n}\sum_{i=1}^{n}\left[G\left(\frac{-L+x_i}{h}\right) + G\left(\frac{L-x_i}{h}\right) + G\left(\frac{x-2U+x_i}{h}\right)\right] \quad (15)$$

Where n is the number of power samples, L = 0, U = P_{RWF}, $K(\cdot)$ is the kernel estimator, $G(\cdot)$ is the cumulative of $K(\cdot)$, and x = P_{WF_WE}.

Data

The comparison of the two methods will be carried out using simulated data from the New European Atlas [13], [14]. On the NEWA website [15] it is possible to access, free of charge, the time series of wind speed and direction at a given site at different heights (50, 75, 100, 150, 200, 250 and 500 metres), from 2005 to 2018 at half-hourly intervals, a total of 245424 data, of which 195 are missing.

The study is carried out with all data from 2005 to 2018 for two different sites, corresponding to the Rabinaldo wind farm (WF1), located in the province of Burgos (latitude 42° 35' 28.6", longitude -3° 42' 19.7"), and Las Panaderas (WF2) in the province of Valladolid (latitude 41° 47' 21.6", longitude -5° 1' 36.3"). Starting from the velocity time series at the heights available in NEWA, Hellman's law [16] is used to generate the velocity series at the heights of the wind turbine hubs (78 and 114 metres respectively).

However, for the wind direction data series, the NEWA height closest to the hub height (75 and 100 metres respectively) is chosen, since there is no simple analytical expression that relates the variability of the wind direction with height. Information on the wind farms comes also from various official sources. Figure 2 shows the resulting wind roses for each wind farm at wind turbine hub height.

In order to calculate the wake effect, it is also necessary to know the spatial distribution of the wind turbines in each wind farm, which are shown in Figure 3.

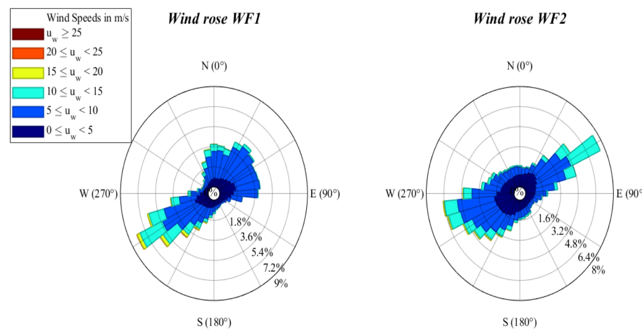

Figure 2. Wind roses of wind farms.

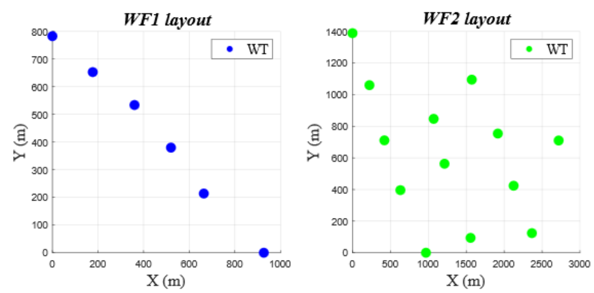

Figure 3. Wind farms layouts.

RESULTS AND DISCUSSION

In this section, the probability density and cumulative distribution functions of the wind farms without taking into account the wake effect, $f_{PWF}(P_{WF})$ and $F_{PWF}(P_{WF})$, are compared with those that consider the wake effect, $f_{PWF\_WE}(P_{WF\_WE})$ and $F_{PWF\_WE}(P_{WF\_WE})$. For this comparison, a Gaussian kernel and the plug-in method were used as the optimal bandwidth selection method. The results can be seen in the Figure 4.

The first relevant result is that the bandwidth required to adequately represent $f_{PWF\_WE}(P_{WF\_WE})$ and $F_{PWF\_WE}(P_{WF\_WE})$ has very small values, being 2.8209E-4 for WF1 and 8.2103E-5 for WF2. This level of detail is crucial for capturing specific features, such as the zone of $f_{PWF}(P_{WF})$ = 0 between $P_{WF} > 0$ and $P_{WF} < P_{INWF}$, described in the theoretical approach. However, for the estimation with kernels, this $f_{PWF\_WE}(P_{WF\_WE})$ = 0 zone is smaller, because with the wake effect, P_{INWF}, is reduced. This can be clearly seen in Figure 5, an amplified view of the two PDFs

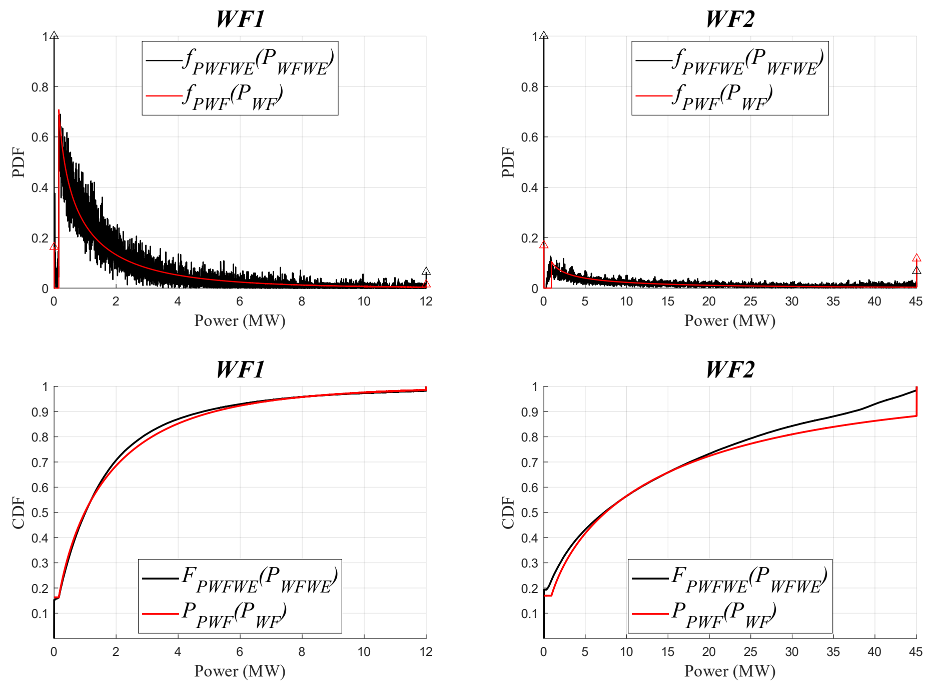

Figure 4. Probability density functions and cumulative distribution functions obtained analyticaly without wake effect $f_{PWF}(P_{WF})$ and $F_{PWF}(P_{WF})$ vs $f_{PWF\_WE}(P_{WF\_WE})$ and $F_{PWF\_WE}(P_{WF\_WE})$ with wake effect obtained using kernel estimators both for WF1 and WF2.

of WF1, which shows that for $f_{PWF}(P_{WF})$, P_{INWF} is equal to 0.18 MW but for $f_{PWF\_WE}(P_{WF\_WE})$, P_{INWF} is only 0.02 MW.

Table 1. Annual energy (AE) estimation using PDF without including wake effect and including it.

	AE without wake effect	AE with wake effect	Overestimation
WF1	16,935 MWh	16,250 MWh	4 %
WF2	123,061 MWh	111,813 MWh	9.1 %

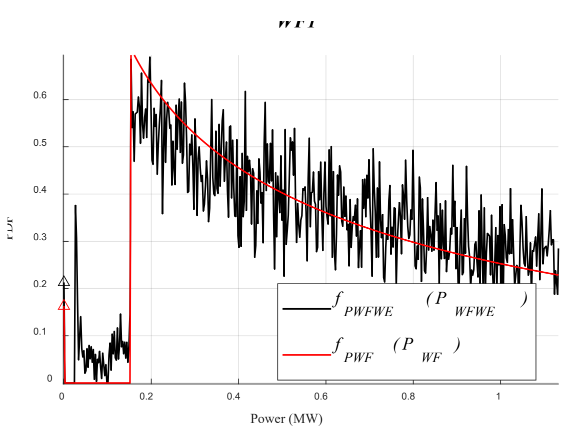

Figure 5. Zoom view of the PDFs of WF1.

Then, the annual energy generated by the wind farm (AE), which is simply the expected value of the probability distributions multiplied by the number of hours of the year, 8760, is calculated using $f_{PWF}(P_{WF})$ or $f_{PWF\_WE}(P_{WF\_WE})$ for WF1 and WF2. It can be seen in Table 1 that for WF1, which has a fairly simple layout and is properly oriented with respect to the prevailing wind directions (northeast-southwest) (Figure 2 and 3), the annual energy without considering the wake effect is 16,935 MWh, while taking the wake effect into account reduces it to 16,250 MWh, overestimating the producible energy by the wind farm by only 4%. The difference is small because along the predominant wind direction there is no wake effect, due to the specific layout of the wind farm. However, for WF2, which has a more complex layout with several wind turbines behind others even towards the prevailing wind direction (northeast-southwest) (Figure 2 and 3), the annual energy is 123,061 MWh without considering the wake effect, and 111,813 MWh with it, overestimating the wind farm's producible power by 9.1%.

There are also discrepancies for both wind farms between $F_{PWF}(P_{WF})$ and $F_{PWF\_WE}(P_{WF\_WE})$. This implies incorrectly estimating the probability of occurrence of the different power values or power intervals, which can have a negative effect on long-term production decisions. For example, for WF2, the wind farm most affected by the wake effect, the probability of P_{WF} being less than 4 MW is 0.38 for $f_{PWF}(P_{WF})$ and 0.4 for $f_{PWF\_WE}(P_{WF\_WE})$. Although this difference may seem small, it can have a large impact on energy production planification in the long run, as its effect is multiplied by the time interval of the decision period. These results highlight the importance of including the wake effect in the analysis because inadequate modelling can underestimate the available power density.

CONCLUSIONS

In this study, it has been proposed to model the PDF of the power generated by a wind farm taking into account the wake effect, $f_{PWF\_WE}(P_{WF\_WE})$, and has been compared with the analytical modelling (without wake effect) of the PDF of the power of a wind farm $f_{PWF}(P_{WF})$. To model $f_{PWF\_WE}(P_{WF\_WE})$ it has been proposed to use a historical wind speed and direction data set and the Katic multiple wake model to generate a data set of wind farm power output, and then use kernel estimators to model the PDF.

A comparison of the two distributions shows that ignoring this effect can result in an overestimation of the available energy of up to 9.1% for wind farms with more complex layouts. The differences seen in PDF modelling also have an effect on the CDF, again proving how a competitive layout can have large differences in cumulative contributions in the low and high-power ranges. These discrepancies can have important implications for production planning and wind farm grid integration, highlighting the importance of incorporating the wake effect into wind farm modelling to ensure more accurate projections of generation and energy availability.

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