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Mathematical Optimization of Separator Network Design for Sand Management

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ABSTRACT

Sand produced along with well-production fluids accumulates in the surface facilities over time, taking valuable space, while the sand carried with the fluids damages downstream equipment. Thus, sand is separated from the fluid in the sand traps and separators and removed during periodic clean-ups. But at high sand productions, the probability of unscheduled facilities shutdowns increases. Such extreme production conditions can be handled by strategic planning and optimal design of the separator network to enable maximum sand separation at minimal equipment cost while ensuring the accumulation extent is within tolerable limits. This paper develops a mathematical model to optimize the separator network design to maximize sand separation while the sand accumulation extent and total equipment cost are minimal. The optimization model is formulated using multi-objective mixed-integer nonlinear programming (MINLP). The capabilities of the developed model to assist sand management in the separator network are demonstrated with a case study of optimizing the network for two wells producing sand particles of different sizes. A residence time distribution-based model is used to predict sand settling behavior. The developed Pareto Front shows the trade-off between the increase in total sand accumulation rate and total equipment cost for an increase in the fraction of sand settled.

Keywords: Optimization, Planning, Oil and Gas, Separator, Sand

INTRODUCTION

On the surface, sand produced with oil and gas accumulates in the separators, leading to costly unscheduled shutdowns, while the sand carried over damages the downstream equipment [1]. Sand management practices include sand separation from the fluid in the sand traps and separators and removal during periodic clean-ups. Multiple separators are generally necessary to separate the multiphase production containing oil, gas, water, and sand. The extent of sand separation and accumulation in separators depends on the production conditions, separator design, and separator network. Previous studies have developed algorithms for simultaneous optimization of the well production and separation systems for production management and enhancement [2,3]. However, these studies do not account for the sand separation in the separators.

This paper develops a mathematical programming model to optimize separator network design with three objectives: (i) to maximize the overall sand separation, (ii) to minimize sand accumulation in each separator, and (iii) to minimize the total purchased equipment cost. The model is formulated as a multi-objective mixed-integer nonlinear programming model (MINLP) and solved using epsilon-constraint method.

PROBLEM STATEMENT

The main goal is to develop an optimization-based framework that assists the design of the separator network by (i) selecting separators from each layer of the separator network for the separation of produced fluid and sand, (ii) determining the design specifications of the selected separators, and (iii) estimating sand settling rate in the separators. The objective is to maximize sand separation from the produced fluid while minimizing sand accumulation in each separator and the total equipment cost of the separators.

The superstructure in Figure 1 illustrates the separator network. Production fluids ($F_{nf,nw}$) and sand (S_{nw}) from wells (W_{nw}) enter the network. The sets $nf \in NF$ and $nw \in NW$ represent the fluid phases and wells. The layers correspond to different stages of separation, and each layer consists of separators ($SF_{nl,ns}$). The sets $nl \in NL$ and $ns \in NS$ represent the layers and separators in each layer. The well fluids are separated into their constituents in each layer.

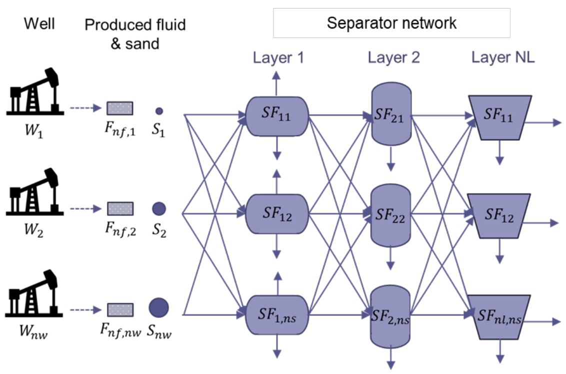

Figure 1. The superstructure of the separator network.

MULTI-OBJECTIVE OPTIMIZATION MODEL FORMULATION

The three objectives of the separator network design optimization model are: i) to maximize the overall sand separation, ii) to minimize the sand accumulation in each separator, and iii) to minimize the total purchased equipment cost of the separators in the network. The overall sand separation is represented as a fraction of sand produced by the wells that are separated from the produced fluid in the separators (FS) (Eq. 1). It is computed using the total sand production rate from the wells and the total sand settling rate in the separators in each layer of the network. The sand accumulation in each separator in the network is minimized by minimizing the summation of the sand accumulation rate in each separator (AC). The sand accumulation rate is determined as the rate at which the sand bed height increases in the vessel (Eq. 2). The total purchased equipment cost of the separators network (TC) is computed from the equipment cost of each separator in the network (Eq. 3).

$$Max\ FS = \left(\sum_{nl1 \in NL1} ms1'_{nl1} + ms2'_{nl2} + \cdots + \sum_{nln \in NLN} msn'_{nln}\right) / \sum_{nw \in NW} Mw_{nw} \quad (1)$$

$$Min\ AC = \sum_{nl1 \in NL1} ac1_{nl1} + \cdots + \sum_{nln \in NLN} acn_{nln} \quad (2)$$

$$Min\ TC = \sum_{nl1 \in NL1} ce1_{nl1} + \cdots + \sum_{nln \in NLN} cen_{nln} \quad (3)$$

The sets $nw \in NW$ and $nl1 \in NL1,\ nl2 \in NL2, \ldots, nln \in NlN$ represent the oil and gas wells and separators in the different layers of the network, respectively. The parameter Mw_{nw} represents the sand production rate from each well nw. The variables $ms1'_{nl1}$, $ms2'_{nl2}$, and msn'_{nln} represent the sand settling rate in each separator in each network layer. The variable FS represents the fraction of produced sand separated (i.e., settled) in the separators network. The variables $ac1_{nl1}$ and acn_{nln} represent the sand accumulation rate in each separator in each network layer and the summation of the accumulation rates is represented by the variable AC. The variables $ce1_{nl1}$ and cen_{nln} represent the purchased equipment cost of the separators in different network layers, and TC represents the total equipment cost of all the separators in the network.

Binary Variables

A set of binary variables, $y1_{nl1}$, ..., yn_{nln}, is defined in Eqs. (4) and (5) to represent the selection of separators in each network layer ($nl1 \in NL1, \ldots, nln \in NlN$). Another binary variable, $l1_{nw,nl1}$, is defined in Eq. (6) to represent the interaction between the wells and the first-layer separators ($nl1 \in NL1$). Similarly, another set of binary variables, $l2_{nf,nl1,nl2}$, ..., $ln_{nf,nln-1,nln}$, is defined in Eqs. (7) and (8) to represent the interaction between the first- and second-layer separators and the separators in the subsequent $NLN-1$ and NLN layers. If fluid phase separation occurs in the first-layer separators, the individual or mixed phases (the phases that are still not separated) enter the second network along with sand. Thus, the interaction variables $l2_{nf,nl1,nl2}$, ..., $ln_{nf,nln-1,nln}$ are written in terms of the set of fluid phases $nf \in NF$. All fluid phases from the NLN layer separators exit the separator network.

$$y1_{nl1} = \begin{cases} 1, \text{if separator } nl1 \text{ in } NL1 \text{ layer is selected} \\ 0, \text{otherwise} \end{cases} \quad (4)$$

$$yn_{nln} = \begin{cases} 1, \text{if sep. } nln \text{ in } NLN \text{ layer is selected} \\ 0, \text{otherwise} \end{cases} \quad (5)$$

$$l1_{nw,nl1} = \begin{cases} 1, \text{if fluid from well } nw \text{ enters} \\ \quad \text{sep. } nl1 \text{ in } NL1 \text{ layer} \\ 0, \text{otherwise} \end{cases} \quad (6)$$

$$l2_{nf,nl1,nl2} = \begin{cases} 1, \text{if phase } nf \text{ from sep. } nl1 \text{ in } NL1 \\ \text{layer enters sep. } nl2 \text{ in } NL2 \text{ layer} \\ 0, \text{otherwise} \end{cases} \quad (7)$$

$$ln_{nf,nln-1,nln} = \begin{cases} 1, \text{if phase } nf \text{ from sep. } nln-1 \\ \quad \text{in } NLN-1 \text{enters} \\ \quad \text{sep. } nln \text{ in } NLN \text{ layer} \\ 0, \text{otherwise} \end{cases} \quad (8)$$

$$\forall\, nf \in NF, nw \in NW, nl1 \in NL1, \ldots, nln \in NlN$$

Parameters

The parameters are defined for the produced fluid, produced sand, separators design, and separators cost models. (i) Produced fluid: The volumetric flow rate of each fluid phase nf ($Q_{nf,nw}$) produced from each well nw are given. Similarly, the physical properties of each fluid phase ($\varepsilon_{nf,nw}$), such as density and viscosity, are given for each well nw. (ii) Produced sand: The sand density

(ρs), sphericity (ψ), and the sand production rate (Mw_{nw}) and mean particle diameter of sand produced (d_{nw}) from each well nw are given. The fraction of produced sand dispersed in each fluid phase ($Fd_{nf,nw}$) is given for each well nw. (iii) Separators design: Lower and upper limits are given for each design specification of the separators in the network, Dl and Du, respectively. (iv) Separators cost models: The cost model parameters, $\alpha 1_{nl1}$, $\alpha 2_{nl2}$, ..., αn_{nln} specific to separators in each network layer are given.

Constraints

If the separator $nl1$ is selected in the first layer of the network, the binary variable $y1_{nl1}$ takes a value of one, and if the separator has input from the well nw, the binary $l1_{nw,nl1}$ takes a value of one. The fraction of the total produced fluid and sand from well nw that enters the separator $nl1$ is represented by the variable $fr1_{nw,nl1}$. Thus, it can take a value of zero to one. If the separator $nl1$ does not have input from the well nw, the variables $l1_{nw,nl1}$ and $fr1_{nw,nl1}$ take a value of zero. This disjunction is given in Eq. (9). Similarly, the interaction between separators in the subsequent layers $NLN-1$ and NLN are represented by the binary variable $ln_{nf,nln-1,nln}$ and the fraction of each fluid phase nf entering the separator in NLN layer from $NLN-1$ layer is represented by the variable $frn_{nf,nln-1,nln}$ (Eq. 10).

$$\begin{bmatrix} \boldsymbol{l1_{nw,nl1} = 1} \\ 0 \le fr1_{nw,nl1} \le 1 \end{bmatrix} \vee \begin{bmatrix} \boldsymbol{l1_{nw,nl1} = 0} \\ fr1_{nw,nl1} = 0 \end{bmatrix} \tag{9}$$

$$\forall\, nw \in NW, nl1 \in NL1$$

$$\begin{bmatrix} \boldsymbol{ln_{nf,nln-1,nln} = 1} \\ 0 \le frn_{nf,nln-1,nln} \le 1 \end{bmatrix} \vee \begin{bmatrix} \boldsymbol{ln_{nf,nln-1,nln} = 0} \\ frn_{nf,nln-1,nln} = 0 \end{bmatrix} \tag{10}$$

$$\forall\, nf \in NF,\ nln-1 \in NLN-1, nlN \in NLN$$

The design specifications, fluid flow rate and properties, and sand properties in the separator nln selected in NLN network layer are given in Eq. (11). The design specifications of the separator (Dn_{nln}) should be within the specified lower and upper bounds (Dl and Du). The total volumetric rate of each fluid phase ($qn_{nf,nln}$) entering the separator is computed from the fraction of each fluid phase entering the separator nln from each separator $nln-1$ in the previous layer $NLN-1$ ($frn_{nf,nln-1,nln}$) and the volumetric rate of each fluid phase in the previous layer separators ($qn_{nf,nln-1}$). Similarly, the rate at which sand of size nw dispersed in phase nf enters the separator nln ($men_{nf,nw,nln}$) is computed using the amount of sand carried by the fluid phase from the previous layer separators ($mcn_{nw,nf,nln-1}$) and the fraction of that fluid phase entering the separator nln, $frn_{nf,nln-1,nln}$.

In the previous description, the set $nw \in NW$ is used to denote the different particle sizes from different wells in the network. The physical properties of each fluid phase are estimated ($\varepsilon n'_{nf,nln}$) based on the properties of fluid from each previous layer separator ($\varepsilon n_{nf,nln-1}$). The design specifications, fluid flow rate and properties, and sand amount variables take a value of zero if the separator is not selected in the network, i.e., when $yn_{nln} = 0$. The design specifications (Dn_{nln}) depend on the total volumetric flow rate of each fluid phase in the separator ($qn_{nf,nln}$) and other separator type-specific factors (σn_{nln}) (Eq. 12).

$$\begin{bmatrix} \boldsymbol{yn_{nln} = 1} \\ Dl \le Dn_{nln} \le Du \\ qn_{nf,nln} = \sum_{nln-1} frn_{nf,nln-1,nln} qn_{nf,nln-1} \\ men_{nf,nw,nln} = frn_{nf,nln-1,nln} mcn_{nw,nf,nln-1} \\ \varepsilon n'_{nf,nln} = f(frn_{nf,nln-1,nln}, \varepsilon n_{nf,nln-1}) \end{bmatrix} \vee \begin{bmatrix} \boldsymbol{yn_{nln} = 0} \\ Dn_{nln} = 0 \\ qn_{nf,nln} = 0 \\ men_{nf,nw,nln} = 0 \\ \varepsilon n'_{nf,nln} = 0 \end{bmatrix} \tag{11}$$

$$Dn_{nln} = f(qn_{nf,nln}, \sigma n_{nln}) \tag{12}$$

$$\forall\, nf \in NF, nw \in NW, nln-1 \in NLN-1, nln \in NLN$$

Sand particles of different sizes (d_{nw}) from different wells are settling in the separator nln. The fraction of sand particles of each size initially dispersed in phase nf that settle to the vessel bottom ($Fsn_{nw,nf,nln}$) depends on the properties of the fluid ($\varepsilon n'_{nf,nln}$) it is dispersed in, separator design specifications (Dn_{nln}), and particle diameter (d_{nw}) (Eq. 13). Similarly, the fraction of each sand size carried by each phase from the separator nln ($Fcn_{nw,nf,nln}$) is estimated. Some amount of sand initially dispersed in the fluid phase $nf = 1$ can move to the phase $nf = 2$, and vice versa. This amount is accounted for by adding the variables $\delta sn_{nw,nf,nl1}$ and $\delta cn_{nw,nf,nl1}$ to the fraction settled and fraction carried, respectively. This variable can take a value in the range $[-1, 1]$ based on the sand dispersion and settling model selected for the separator. The rate of sand settling at the bottom ($msn_{nw,nf,nln}$) and sand being carried ($mcn_{nw,nf,nln}$) for different sand sizes are computed from the amount of each sand size entering the separator ($men_{nf,nw,nln}$). If the separator nln is not selected in the network, all these variables take a value of zero.

$$\begin{bmatrix} \boldsymbol{yn_{nln} = 1} \\ Fsn_{nw,nf,nln} = f\left(\varepsilon n'_{nf,nln}, Dn_{nln}, d_{nw}\right) + \delta sn_{nw,nf,nln} \\ Fcn_{nw,nf,nln} = f\left(\varepsilon n'_{nf,nln}, Dn_{nln}, d_{nw}\right) + \delta cn_{nw,nf,nln} \\ msn_{nw,nf,nln} = men_{nf,nw,nln} Fsn_{nw,nf,nln} \\ mcn_{nw,nf,nln} = men_{nf,nw,nln} Fcn_{nw,nf,nln} \end{bmatrix}$$

$$\vee \begin{bmatrix} \boldsymbol{yn_{nln} = 0} \\ Fsn_{nw,nf,nln} = 0 \\ Fcn_{nw,nf,nln} = 0 \\ msn_{nw,nf,nln} = 0 \\ mcn_{nw,nf,nln} = 0 \end{bmatrix} \quad (13)$$

$$\forall\, nf \in NF, nw \in NW, nln \in NLN$$

The fluid and sand produced from each well should be treated in the separator network. Thus, the total fraction of fluid entering from each well into different separators in the first network layer ($fr1_{nw,nl1}$) is equal to one (Eq. 14). The fluid phases exiting separators in each network layer either enter the separators in the subsequent layer or exit the network based on the interaction variable $frn_{nf,nln-1,nln}$ given in the Eq. (10).

$$\textstyle\sum_{nl1} fr1_{nw,nl1} = 1 \quad \forall\, nw \in NW \quad (14)$$

The total amount of sand settled in each separator nln (msn'_{nln}) is computed as the total amount of each sand size dispersed in each phase that has settled to the separator bottom ($msn_{nw,nf,nln}$) (Eq. 15). The sand accumulation rate (i.e., rate of increase of sand bed height) in each separator is computed based on the amount of sand particles of each size settled in the separator and the void space between the settled particles. The void space is accounted for using void fraction, which depends on the mean particle size, particle size distribution, and the sphericity of the particles (ψ) [7]. The mean particle size and particle size distribution are estimated based on the amount of different particle sizes settled in the separator. Thus, the void fraction of the sand particles settled in the separator (ϕn_{nln}) depends on the particle sizes present (d_{nw}) and the amount of each particle size settled in the separators ($msn_{nw,nf,nln}$) (Eq. 16). The sand accumulation rate in the separator (acn_{nln}) is computed using the void fraction (ϕn_{nln}), the total amount of sand settled in the separator (msn'_{nln}), sand density (ρs), and cross-sectional area of the separator (CSn_{nln}) (Eq. 17).

$$msn'_{nln} = \textstyle\sum_{nw,nf} msn_{nw,nf,nln} \quad \forall\, nln \in NLN \quad (15)$$

$$\phi n_{nln} = f(d_{nw}, msn_{nw,nf,nln}) \quad (16)$$

$$acn_{nln} = msn'_{nln} / (CSn_{nln}\, \rho s (1 - \phi n_{nln})) \quad (17)$$

$$\forall\, nf \in NF, nw \in NW, nln \in NLN$$

The purchased equipment cost of each separator depends on its design specifications. The cost of the separator nln (cen_{nln}) is estimated based on its design specifications (Dn_{nln}) using the separator-specific cost model (αn_{nln}) (Eq. 18).

$$cen_{nln} = f(Dn_{nln}, \alpha n_{nln}) \quad \forall\, nln \in NLN \quad (18)$$

CASE STUDY

The equations (1) to (18) yield a deterministic multi-objective mixed-integer nonlinear programming model (MINLP) that maximizes sand separation, minimizes sand accumulation, and minimizes the total purchased equipment cost of the separator network. The disjunctive constraints in the model are reformulated using Big-M Reformulation [4]. In the developed multi-objective model, a set of decision variables improving one objective's value could negatively affect another objective and vice versa. Thus, a single optimal solution is not possible; rather, a set of optimal solutions is obtained, which forms the Pareto front [5]. This work uses the epsilon-constraint method [6] to obtain the Pareto front for the developed model. The model is solved by selecting one of the objective functions as the only objective and the remaining objective functions as constraints. Then, the constraint bounds are systematically varied to obtain the Pareto front.

The capabilities of the developed model to optimize a separator network design for sand management are demonstrated with a test problem. The well production and the separation of the produced fluid and sand in the separator network are shown in Figure 2. Two wells producing oil, gas, water, and sand are considered. The amount of fluid produced, the fluid's physical properties, and the sand's size differ for the two wells. It is assumed that the production from the two wells is completely mixed before it enters the separator network. The produced fluid now contains two different sizes of sand particles, which are assumed to be evenly dispersed in the liquid phases (oil and water). The produced fluid and sand particles enter the separator network containing two layers of separators, and each layer contains two separators. The separators in the first layer separate the produced fluid into its constituents: oil, water, and gas. A fraction of the sand particles is separated from the produced fluid by settling to the bottom of the separators. The remaining sand particles are carried by oil and water. These liquid phases carrying sand enter the separators in the second layer, where more sand particles are separated from the liquid. It is assumed that liquid phases separated in the first-layer separators cannot enter the same second-layer separator, i.e., the second-layer separators can handle sand/oil or sand/water only. Thus, a two-phase vertical gravity separator is used for separation in the second layer.

The design specifications of the four-phase gravity separator are given in Eqs. (19-21). The vessel diameter (D_S) is selected based on the volumetric flow rate of liquid (oil and water) separated in the vessel (Q_o and Q_w) and the mean residence time of the liquid phases in the vessel (τ). The separator height (H_S) is selected based on the ratio of the height to the diameter for the separator (r). The vessel's capacity should be large enough that the liquid level is within the maximum allowable level ($H_{S,lmax}$) (Eq. 19). A minimum vessel diameter ($Dmin$) is required to slow the gas to a velocity at which the liquid droplets settle and separate from the gas phase [8]. This minimum diameter depends on the volumetric flow rate of the gas (Q_g) and the settling velocity of liquid droplets in the gas phase in the separator (v_l) (Eq. 20). Thus, the vessel diameter should be greater than the minimum diameter (Eq. 21). The two-phase gravity separator has only sand/oil or sand/water phases, and it is also designed using Eqs. (19-21).

$$(Q_o + Q_w)\tau/(\pi/4D_S{}^2H_S) = Hmax/\ H_S \quad (19)$$

$$Dmin = \left[4Q_g/(\pi v_l)\right]^{0.5} \quad (20)$$

$$D_S \geq Dmin \quad (21)$$

Sand settles at the bottom of the gravity separator when its settling time is less than the residence time of the fluid it is dispersed. The residence time of the fluid is expressed using Residence Time Distribution (RTD) models [9]. In our previous work [10], an RTD-based model was developed to compute the fraction of sand settled in a vertical gravity separator. This model is used to estimate the extent of sand separation in each separator in the network.

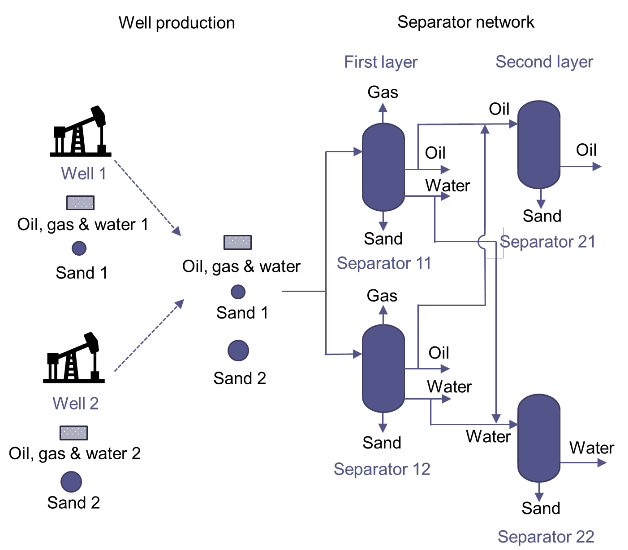

Figure 2. Well production (oil, gas, water, and sand) from two wells enters a separator network with two layers. The separators in the first layer separate the produced fluids. Sand is separated from the produced fluids in both layers.

The model assumptions for the sand settling behavior in a four-phase gravity separator are: (i) As oil and water separate inside the vessel, the sand particles initially dispersed in the oil move to the oil column, while the particles dispersed in the water move to the water column. (ii) Sand particles in the oil column gradually settle with a constant terminal settling velocity, during which a fraction of the sand particles get carried away by the flowing oil. Similarly, in the water column, the sand particles settle down at a constant velocity specific to the water phase, and the water carries a fraction of these particles. (iii) The remaining sand particles in the water column eventually settle down at the bottom of the separator. The remaining particles in the oil column enter the water phase and settle at a constant velocity. The water carries away some particles, and the rest settle at the separator's bottom.

The terminal sand settling velocity in the oil and water phases is estimated using Stokes' law [11]. The settling time of a sand particle in a liquid column is estimated as the time required to settle through the entire liquid column height at terminal velocity. The distribution of residence time ($E(t)$) is expressed by the RTD model for a single CSTR [9] (Eq. 22). The mean residence time is represented as τ.

$$E(t) = \frac{e^{-t/\tau}}{\tau} \quad (22)$$

The fraction of sand particles initially dispersed in the oil phase is Fd_o, and the rest in the water phase (Fd_w). The fraction of sand dispersed in oil fraction with residence time less than sand settling time in oil (ts_o) is carried away by oil ($F_{o,co}$), and it is estimated by Eq. (23). The remaining fraction of sand in oil ($F_{o,ew}$) settles through the oil column and enters the water phase (Eq. 24). Thus, two sets of sand particles are dispersed in water, particles initially dispersed in water (Fd_w) and particles that entered the water from oil ($F_{o,ew}$). Sand dispersed in the water fraction with residence time less than sand settling time (ts_w) is carried away by water. Thus, a fraction of sand initially dispersed in water, and another fraction that entered the water from oil gets carried by water, $F_{w,cw}$ and $F_{o,cw}$, respectively (Eqs. 25 and 26). The remaining fraction of sand in the water settles to the separator bottom (F_s) (Eq. 27).

$$F_{o,co} = Fd_o \int_0^{ts_o} E(t_o)dt_o \quad (23)$$

$$F_{o,ew} = Fd_o - F_{o,co} \quad (24)$$

$$F_{w,cw} = Fd_w \int_0^{ts_w} E(t_w)dt_w \quad (25)$$

$$F_{o,cw} = F_{o,ew} \int_0^{ts_w} E(t_w)dt_w \quad (26)$$

$$F_s = \left(Fd_w - F_{w,cw}\right) + \left(F_{o,ew} - F_{o,cw}\right) \quad (27)$$

The void fraction of the sand particles settled at the separator bottom is estimated using a semi-empirical relation [7] developed for a loosely packed bed.

The purchased equipment cost of the separators (Ce) is approximated using the cost model for vertical pressure vessel [8] (Eq. 28). The equipment cost depends on the shell mass of the separator (SM), which is a function of the separator diameter, height, and shell thickness. The equipment cost is in the United States Gulf Coast basis, January 2006 (Chemical Engineering (CE) index = 478.6 and Nelson-Farrar Cost Index (NFCI) = 1961.6). The cost is converted to 2023.

$$Ce = -10000 + 600\, SM^{0.6} \quad (28)$$

RESULTS AND DISCUSSIONS

In the test problem, the total fluid production rate from the two wells is 50 m^3/h of oil, 50 m^3/h of water, and 900 m^3/h of gas. Sand is produced along with wellbore fluid at a rate of 26.5 kg/h. The sand concentration in the liquid is 0.01 vol%. The particles are spherical and have diameters of 150 microns and 50 microns. The sand density is 2650 kg/m^3. The produced oil, water, and gas densities are 700 kg/m^3, 990 kg/m^3, and 23.6 kg/m^3, respectively. The viscosities of the oil and water are 0.77 x 10^{-3} kg/m/s and 0.55 x 10^{-3} kg/m/s. The lower and upper bounds for the separator diameter (D_S) are 1 m and 3.5 m, respectively, and for the mean residence time of production fluids in the separator (τ), they are 10 mins and 20 mins. The height-to-diameter ratio (r) for the separator is 3. The maximum liquid level in the separator ($H_{S,lmax}$) is 50% of the separator height. The shell thickness is assumed to be 5 mm.

The optimization models are formulated in Python V3.8.6 using PYOMO V6.4.1. The MINLP models are solved using BARON V23.11.13 to 5% optimality gap, all on an Intel Xeon Gold 6248R 3 GHz processor with 48 cores and utilizing a maximum of 100 GB RAM.

Firstly, the separator network optimization model is solved for the three objectives ((i) maximizing sand separation, (ii) minimizing equipment cost, and (iii) minimizing accumulation rate) separately to obtain the bounds for the Pareto front. The results for separator design specifications, separation extent, accumulation rate, and solution time for solving for each objective are given in Table 1. (i) For the given production conditions, a maximum of 84% of the produced sand can be separated from the production fluid with the separator network under study. One large separator (D=2.98 m) and another small separator (D=1.18 m) are selected for the first-layer separation, and the residence time of production fluids is 20 mins in both separators. For second-layer separation, two more equally sized separators are selected to separate sand from the oil and water phases further, with a residence time of 20 mins in the separator. (ii) To minimize the total cost of the network ($43,000), a single separator (D=2.4 m) with a residence time of 10 mins is selected to separate the production fluids to their constituents. (iii) A single large separator (D=3.04 m) is selected to ensure that the bed height increase rate is minimal (0.93 m/week). Only 56% of the sand is separated with this network design. The highest sand separation (84%) is achieved with a total network cost of $107,000 and a weekly total sand accumulation rate of 1.77 m in the separators in the network. The solution time is the highest while solving for the equipment minimization objective (6958 CPU s).

Table 1: Sand separation extent, accumulation rate, network cost, network design specifications, and solution times for three cases: (i) maximum sand separation, (ii) minimum equipment cost, and (iii) minimum total accumulation rate.

Design spec. / Solution time	Results
(i) Maximum sand separation	
Fraction of sand settled*	0.84
Total cost (thousand $)	107
Total accumulation rate	1.77
Layer 1	
Diameter (m)	1.18 2.98
Residence time (min)	20 20
Layer 2	
Diameter (m)	2.37 (O) 2.37 (W)
Residence time (min)	20 (O) 20 (W)
Solution time (CPU s)	355
(ii) Minimum equipment cost	
Total cost* (thousand $)	30
Fraction of sand settled	0.56
Total accumulation rate	1.26
Layer 1	
Diameter (m)	2.4
Residence time (min)	10
Layer 2	Not selected
Solution time (CPU s)	6958
(iii) Minimum accumulation rate	
Total accumulation rate* (m/week)	0.93
Fraction of sand settled	0.65
Total cost (thousand $)	43
Layer 1	
Diameter (m)	3.04
Residence time (min)	20
Layer 2	Not selected

* denotes the model objective for each case
O and W denote the two-phase separators for oil/sand and water/sand separation

A Pareto front plot has been developed by plotting the total equipment cost and total accumulation rate in the network against the fraction of total sand settled in Figure 3. For low sand separation extent, a single separator is selected in the network. A larger separator is used to decrease the accumulation rate, while a smaller residence time is used to decrease the equipment cost. To achieve higher sand separation, a second separator is added in the second layer to separate the sand from the oil phase further while minimizing accumulation. Cost is reduced for a higher separation extent by using two smaller separators in the first layer and a second-layer separator for the water phase, in which the sand settling rate is higher than that in oil. But this arrangement significantly increases the sand accumulation rate, as seen for 71% separation extent. Maximum sand separation is achieved by using all the separators in the network. A high residence time and smaller diameter are preferred for these separators.

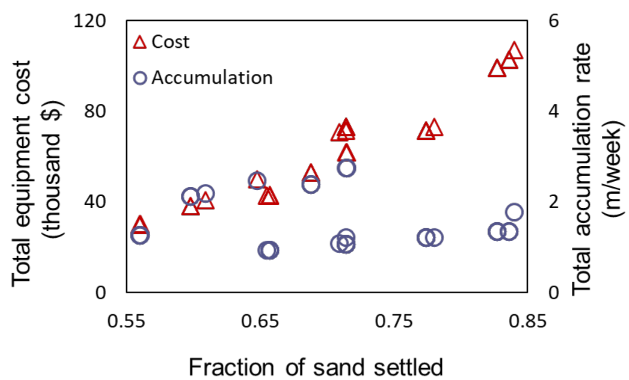

Figure 3. Pareto front for separator network design.

CONCLUSION AND FUTURE WORK

A multi-objective separator network optimization model was developed to maximize the sand separation while minimizing the total equipment cost and total sand accumulation rate in the separators. The developed model was used to obtain a Pareto front for a separator network with two separation stages containing two separators each. The Pareto front can be used for the selection and design of the separators to achieve a desired sand separation extent with the separator network, without high accumulation in the separators and at less equipment cost.

This work will be extended to use the developed model for solving larger separator networks with more design and decision flexibility, such as optimizing the height-by-diameter ratio and determining the conditions under which the production fluids from two wells should/should not be separated in the same separator. Reformulation approaches will be investigated for reducing the solution time of the developed MINLP model.

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