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A mathematical programming optimization framework for wind farm design considering multi-directional wake effect

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ABSTRACT

The placement of wind turbines is a crucial design element in wind farms, given the energy losses resulting from the wake effect. Despite numerous studies addressing the Wind Farm Layout Optimization (WFLO) problem, considering multiple directions to determine wind turbine spacing and layout remains limited. However, relying solely on one predominant direction may lead to overestimating energy production, and loss of energy generation. This work introduces a novel mathematical programming optimization framework to solve the WFLO problem, emphasizing the wind energy's nonlinear characteristics and wake effect losses. Comparisons with traditional layout approaches demonstrate the importance of optimizing wind farm layouts during the design phase. By providing valuable insights into the renewable energy sector, this research aims to guide future wind farm projects towards layouts that balance economic considerations with maximizing energy production.

Keywords: Wind, Turbines, Energy Systems, Renewable and Sustainable Energy, Optimization

INTRODUCTION

Due to the escalating energy crisis, there has been a growing inclination towards generating more energy from renewable sources. Consequently, aside from constructing new power generation facilities such as wind or solar plants, studying how to optimize these installations is crucial [1]. Among the various renewable energy options available, wind energy stands out due to its efficient power generation capacity and the ability to produce energy on a large scale, making it an attractive option for expanding electricity generation capacity [2-5].

Wind power is produced by converting the kinetic energy of air in motion using a turbine. Usually, wind turbines are arranged in groups known as wind farms to increase power production and minimize costs. Several factors influence the energy production of a wind farm, including wind speed and direction, and various meteorological conditions. Specifically, energy losses occur due to the wake effect. As wind turbines extract energy from the wind, a wake forms downstream, reducing the wind speed. Consequently, the placement of wind turbines significantly impacts the efficiency of a wind farm [6].

Traditionally, wind farms have followed a rule of thumb of placing wind turbines in rows with 8–12 rotor diameters of spacing parallel to the prevailing wind direction and columns spaced 3–5 rotor diameters apart perpendicular to the wind direction [7]. More recently, several studies have been conducted to determine the optimal positioning of wind turbines within a designated land area [8-11]. The primary objective of these studies has been to minimize wake effects and, consequently, maximize expected power production. These studies consider dividing the domain into a grid that defines possible turbine locations. However, only a limited number of studies have extended their focus to include determining the optimal inter-turbine distance. This problem, known as the wind farm layout optimization (WFLO) problem, has garnered substantial research interest.

Given the nonlinearity inherent in wind energy's characteristics, addressing this issue poses a noteworthy difficulty. Proposed approaches to tackle the WFLO problem predominantly involve using data-driven or metaheuristic algorithms such as genetic algorithms,

Abbreviations		**Variables**	
1D	one wind direction	A	land area used (m^2)
2D	two wind directions	AC	annual costs (USD/year)
WFLO	wind farm layout optimization	C_{inv}	annual investment costs (USD/year)
Indices		C_{OM}	annual operation & maintenance costs (USD/year)
c	columns		
d	day	N_{tot}	total number of turbines (-)
j	wind direction number	$P_{r,c,t,d}$	energy produced at row r, column c at time t of day d (MWh)
r	rows		
t	hour time step	$P_{r,c,t,d}^{curve}$	energy produced according to power curve (MWh)
Parameters			
α	shift coefficient (-)	P_{nom}	nominal capacity of a wind turbine (MW)
A_{max}	land area available (m^2)	v	wind velocity considering wake effect (m/s)
C_{inv}^{cap}	Investment costs per capacity (USD/MW)		
		$v_{r,c,t,d}^{curve}$	wind velocity at row r, column c at time t of day d (m/s)
C_{OM}^{cap}	operation & maintenance costs per capacity (USD/MW)	$v_{wd_j,r,c,t,d}$	wind velocity in direction wd_j at row r, column c at time t of day d (m/s)
CRF	capital recovery factor (%)		
Ct	trusted coefficient (-)	v^0	undisturbed velocity at hub height (m/s)
Di	rotor diameter (m)	$v_{t,d}^0$	undisturbed velocity at hub height time t of day d (m/s)
f_d	frequency of representative day d (-)		
H	hub height (m)	$w_{wd_j,t,d}$	wind direction
H_i	altimeter height (m)	WL	wake effect losses (-)
i	interest rate (%)	x	x-axis distance (m)
k	rate of wake expansion (-)	y	y-axis distance (m)
L	wind turbines lifetime (years)	$y_{r,c}^b$	wind turbine purchase at row r, column c
M	big M (-)	z	characteristic distance (m)
$v_{t,d}^m$	wind velocity at altimeter height at time t of day d (m/s)		

random search, and particle swarm optimization. While these algorithms are practical for providing near-optimal solutions, they often do not supply guarantees of optimality [12]. Furthermore, to our knowledge, none of the studies have considered more than one dominant wind direction to determine the spacing and layout of wind turbines, even though promising locations for wind farm installations, such as the Texas Panhandle, have at least two dominant wind directions, and some studies have suggested that ignoring wind direction could lead to an overestimation of the wind energy production [13].

This study introduces a novel mathematical programming optimization framework designed to determine the optimal position and spacing of wind turbines. It explores various design objective criteria, including costs and energy production. The study considers the intrinsic nonlinear attributes of wind energy, incorporates modeling of wake effect losses, and, for the first time, considers both single and multiple dominant wind directions. To illustrate its efficacy, the proposed model is applied in a case study focusing on the energy transition of the University of Wisconsin-Madison campus.

PROBLEM FORMULATION

Wake effect model

The wind losses due to the wake effect are model using the Jensen model, which is one of the most widely used wake model [14-15]. It assumes that the diameter of the wake increases linearly in proportion to the downstream distance, z. The speed downstream can be calculated as

$$v(z) = v^0 \cdot [1 - (1 - \sqrt{1 - \mathrm{Ct}}) \left(\frac{Di}{Di+2kz}\right)^2] \quad (1)$$

, where v^0 is the undisturbed incoming velocity, k is the rate of the wake expansion and have a value of 0.075 for onshore wind, Di is the diameter of the rotor of the wind turbine and Ct is the trust coefficient, which as a value of 0.8 [16-17].

In this study, the wake effect model was adapted to consider multiple wind directions. For each wind direction wd_j the velocity is calculated considering in characteristic distance z_j. The calculation of the wind velocity for the two directions wd_1 and wd_2 is presented below, while an illustration of the two chosen directions is presented in Figure 1. The main assumption is that the wind velocity reaching the turbines ($v_{wd_j,r,c,t,d}$) that are in the same row (r) or column (c) are not affected by each other. The losses due to the wake effect will be denoted as WL, re-writing equation (1) as the following:

$$v(z) = v^0 \cdot [1 - \mathrm{WL}(z)] \quad (2)$$

For wind direction wd_1, the velocity at each position at time t of day d ($v_{wd_1,r,c,t,d}$) can be calculated as:

- For the first row ($r = 1$)

$$v_{wd_1,1,c,t,d} = v^0_{t,d} \ \ \forall\, c\,, t, d \quad (3)$$

- For any other row ($\forall\, r \neq 1, c\,, t, d$)

$$v_{\mathrm{wd}_1,r,c,t,d} = v_{\mathrm{wd}_1,r,c-1,t,d} \cdot \left[1 - y^b_{r,c} \cdot \mathrm{WL}(y)\right] \quad (4)$$

Similarly, for wind direction wd_2, the velocity ($v_{wd_2,r,c,t,d}$) is calculated as:

- For the first column ($c = 1$)

$$v_{wd_2,r,1,t,d} = v^0_{t,d} \ \ \forall\, r\,, t, d \quad (5)$$

- For any other column ($\forall\, r, c{\neq}1\,, t, d$)

$$v_{\mathrm{wd}_2,r,c,t,d} = v_{\mathrm{wd}_2,r,c-1,t,d} \cdot \left[1 - y^b_{r,c} \cdot \mathrm{WL}(x)\right] \quad (6)$$

The binary variable $y^b_{r,c}$ denotes if the turbine in row r, and column c is installed (1 if it is installed, 0 if not). Lastly, it is assumed that the wind only blows in one direction at each time step, given by equation (6). We denote the parameter $w_{wd,t,d}$ which has a value of 1 if the wind blows in direction wd at time t of day d, and 0 if not.

$$v^{curve}_{r,c,t,d} = \sum_{wd} v_{wd,r,c,t,d} \cdot w_{wd,t,d} \quad (7)$$

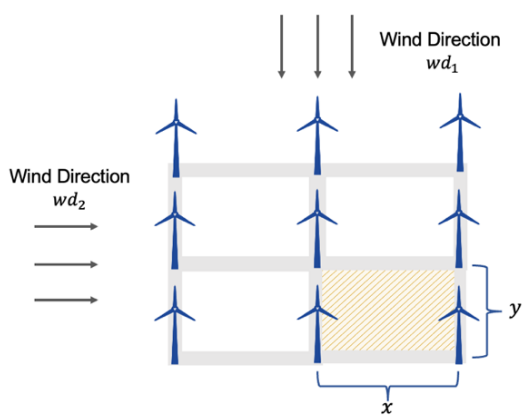

Figure 1. Wind farm layout and turbine spacing.

Wind farm layout optimization model

The power output of the wind turbines ($P_{r,c,t,d}$) was calculated using piecewise linear approximation of the power curve of a wind turbine Vestas V112-3.08, selected considering the average size of wind turbine used in the market [18]. Figure 2 shows the results of the linearization done. Regarding the operation of the wind farm, the turbines only produce energy if they are installed.

$$P_{r,c,t,d} \leq M \cdot y^b_{r,c} \ \forall\, r, c\,, t, d \quad (8)$$

$$P_{r,c,t,d} \leq P^{curve}_{r,c,t,d}\left(v^{curve}_{r,c,t,d}\right) \ \forall\, r, c\,, t, d \quad (9)$$

The area used by each turbine is considered to be a rectangle corresponding to the shaded area in Figure 1, and the total area available is limited (Equation 10 and 11).

$$A = N_{tot} \cdot x \cdot y \quad (10)$$

$$A \leq A_{max} \quad (11)$$

Two objective functions are considered in this study: minimization of annual costs, and maximization of the annual energy produced. The annual costs (AC) include the annual investment costs (C_{inv}) and the annual operation and maintenance costs (C_{OM}), as expressed by equations (12).

$$AC = C_{Inv} + C_{OM} \quad (12)$$

The initial investment costs (C^{cap}_{inv}) are converted into annual investment cost per capacity using the Capital Recovery Factor (CRF) [19], defined as

$$CRF = \frac{i}{1-(1+i)^{-L}} \quad (13)$$

, where i in the interest rate and L the lifetime of the wind turbines. In this study, an interest rate of 7% and a lifetime of 30 years were assumed [20]. The annual investment and operation and maintenance costs can be expressed by equations (14) and (15), respectively.

$$C_{Inv} = \mathrm{N_{tot}} \cdot P_{nom} \cdot C^{cap}_{inv} \cdot \mathrm{CRF} \quad (14)$$

$$C_{OM} = N_{tot} \cdot P_{nom} \cdot C^{cap}_{OM} \quad (15)$$

The annual energy produced (AEP) is calculated as the sum of the energy produced at each hour and day represented by equation (16).

$$AEP = \sum_{r,c,t,d} P_{r,c,t,d} \cdot f_d \quad (16)$$

, where f_d is the frequency of the representative day d.

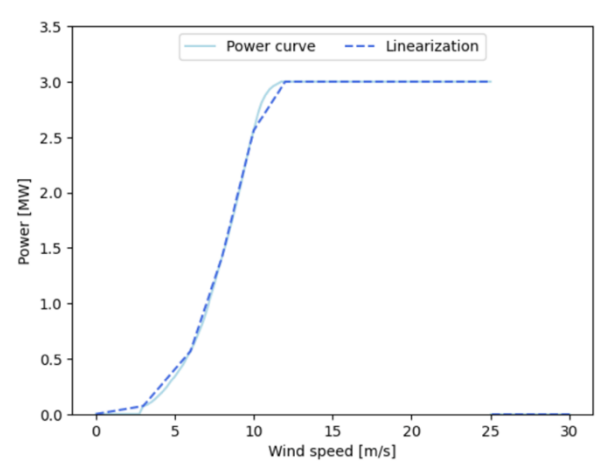

Figure 2. Power curve of the turbine Vestas V112-3.08 [21].

CASE OF STUDY

To demonstrate the applicability of the developed model formulation and framework, a case study was conducted to design a wind farm in Madison, Wisconsin as an option for energy transition in the University of Wisconsin-Madison. The wind data used were obtained from the AOSS Tower located at the University of Wisconsin-Madison [22]. A typical meteorological year was constructed using data from 2013 to 2023. The annual wind source distribution can be observed in Figure 3. There are two predominant wind directions: SW and NW, with an average wind speed of 4.8 m/s at 30 m (altimeter height). It is noteworthy that the wind speed is below 3 m/s only 1% of the year (yellow label), indicating an area with potential for the installation of a wind farm.

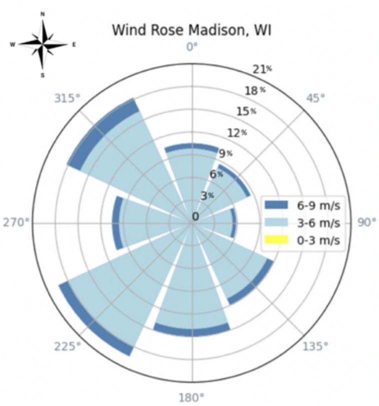

Figure 3. Wind speed and direction distribution in Madison, Wisconsin.

The proposed model is classified as a mixed integer nonlinear problem (MINLP) problem, where the nonlinear terms are associated with the wake effect. To account for the variability and intermittency of the wind resource, and at the same time make the problem tractable, the year wind data was reduced to three representative days using hierarchical clustering. Hierarchical clustering is a machine learning algorithm that groups similar data points into nested clusters based on their proximity, forming a tree-like structure [23]. The three representative days selected are presented in Figure 4. Before using the data in the model, the wind speed inputs were adjusted from the measured height (H_i) to the hub height of the turbine (H) using equation (17) and considering a shift coefficient (α) of 0.14 [21]. The model was implemented in GAMS [24] and solved using the BARON solver [25].

$$v_{t,d}^{o} = v_{t,d}^{m} \cdot \left(\frac{H}{H_i}\right)^{\alpha} \ \forall\, t,d \quad (17)$$

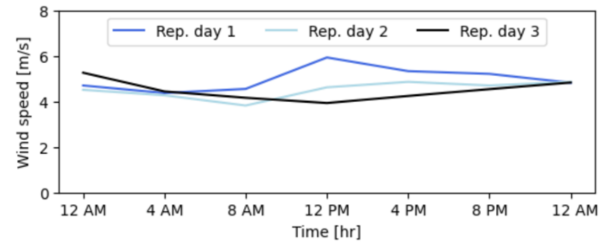

Figure 4. Representative days for wind speed using hierarchical clustering.

RESULTS & DISCUSSION

In this study, we approached the problem by examining one and two wind directions ($wd_1$and wd_2) for up to nine wind turbines. Figure 5 presents the results for minimizing annual costs and maximizing yearly energy production. Utilizing costs as the primary design criterion for a wind farm may not be ideal, as it does not optimize turbine placement and overall area utilization, consequently reducing the energy output. The findings reveal a potential reduction in energy production of up to 13% for identical-capacity wind farms when costs are minimized compared to the maximization of energy production. Maximizing energy output appears more effective in optimizing the wind farm layout, as it reduces energy losses. However, it is essential to evaluate the land area used. The model tends to position turbines further apart to maximize energy, which might require a more extensive area. Figure 6 illustrates different arrangements leading to varying energy production for the installation of six turbines for reference. In both cases, the annual costs are the same, but the configurations differ. Minimizing costs results in layouts with more closely spaced rows and a higher number of rows, increasing wake effect energy losses. On the other hand, maximizing energy production leads to a larger distance between turbines, utilizing more land.

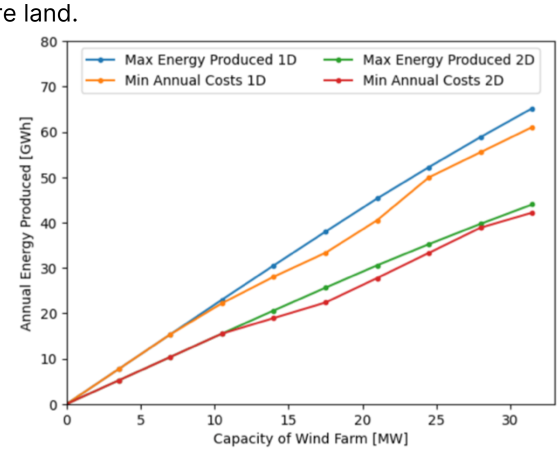

Figure 5. Annual energy produced considering the minimization of annual costs and maximization of energy produced.

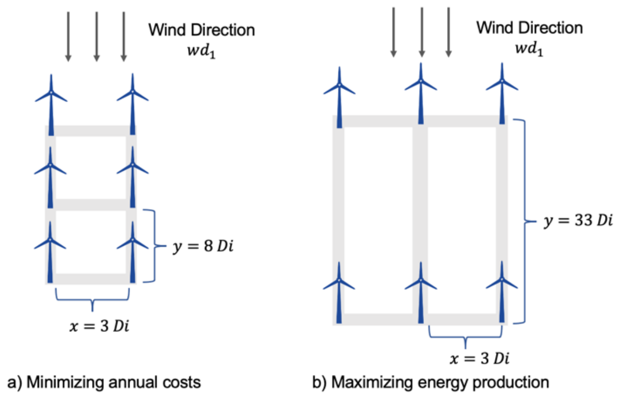

Figure 6. Layout of installing 6 wind turbines considering a) minimization of annual costs b) maximization of annual energy production.

Figure 7 compares energy production when considering the model with one or two wind directions and using Patel's rule of thumb. The findings indicate that adhering to the rule of thumb for turbine placement could lead to lower energy production; therefore, optimizing the layout is a crucial aspect of wind farms and should be evaluated during design. Considering only the predominant wind direction, i.e., one direction, could lead to overestimating energy production, which can affect the project's financial aspects. Taking into consideration multiple wind directions represents the system more accurately, as the wind blows in multiple directions in many locations.

A sensitivity analysis was conducted regarding the land area available. The results, depicted in Figure 8, indicate that the model is sensitive to this value. It is important to note, that in practical scenarios, the land availability will be either already bought or modifying the existing land may involve associated costs or logistical constraints, imposing limitation for its modification. Considering an unlimited area can lead to an overestimated energy production by up to 15%, according to the model developed. Moreover, assuming an unlimited or vast area suggests a linear relationship between capacity and energy produced, neglecting wake effects and energy losses. As the number of turbines increases, so does the wake effect, resulting in a nonlinear increase in energy production.

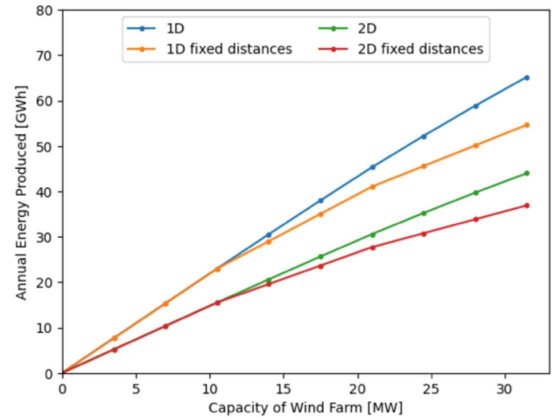

Figure 7. Annual energy production considering different number of wind speed directions and distances between turbines.

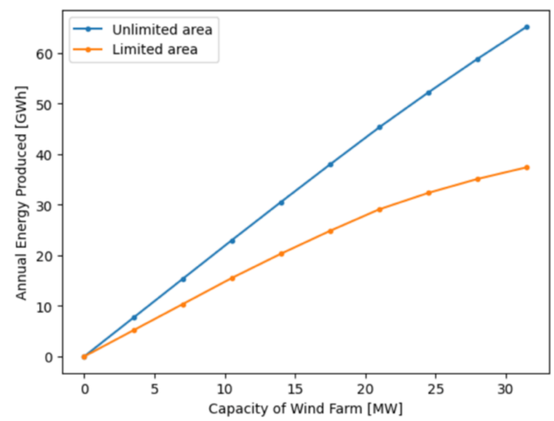

Figure 8. Annual energy produced considering different values for land area available.

CONCLUSION

This study introduced a comprehensive optimization framework to design an optimal layout for a wind farm, accounting for the wake effect in various directions. The energy system was formulated as a MINLP problem with two objective functions: minimizing annual costs and maximizing energy production, showing a difference of up to 13% in the energy produced between both criteria. The results indicate that considering only the predominant wind direction could lead to an overestimation of the energy produced. The model was solved using up to two wind directions, but future work should extend this to evaluate the ideal number of wind directions to consider when designing wind farms in different locations, considering solving time and model accuracy. Additionally, coping with the uncertainties associated with wind farm parameters is an important area for future work. The following steps for this work will also focus on extending the analysis to higher wind farm capacities and evaluate extreme weather scenarios to refine the model's accuracy and applicability.

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