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A Full-State Reliability Analysis Method for Remanufactured Machine Tools Based on Meta Action and a Markov Chain Using an Exercise Machine (EM) as an Example

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Abstract: The reliability of an RMT can be regarded as an important indicator customers can use to recognize its quality; however, it is difficult to implement a full-state reliability analysis of an RMT due to its complicated structural functions. Therefore, a full-state reliability analysis model is proposed herein based on meta action (MA) and a Markov chain for remanufactured exercise machine tools (REMTs). First, an analysis was carried out on individual levels by integrating the MAU decomposition method, and an MAU fault tree model was established layer by layer for the REMT. Second, full-state modeling was performed in view of the MAU characteristics of the REMT, whose operation processes are divided into MAU normal and failure states. A Markov decision-making process was introduced to integrate MAU states and establish our model, which was solved by means of an analytical method for the evaluation of reliability. Finally, an example of a remanufactured machine tool spindle is given to verify the effectiveness of the method.

Keywords: RMT; EM; MA unit (MAU); Markov chain; full state; reliability analysis

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1. Introduction

In the context of the circular economy, resource shortages and environmental pollution have become important issues faced by the current manufacturing industry. As part of the circular economy of the manufacturing industry, remanufacturing has changed the manufacturing mode from linear to closed-loop and enabled the reuse of resources, thus attracting wide attention from society [1]. Machine tools constitute a variety of processing equipment types used in manufacturing and form the basis of the manufacturing industry. Machine tools reflect the development level of manufacturing technology, and with regard to waste machine remanufacturing, reuse is of great significance. The remanufacturing of machine tools mainly consists of the following steps: waste machine tool recycling, the disassembly and cleaning of parts, testing and classification, remanufacturing processing, and machine tool reassembly [2]. Unfortunately, remanufacturing quality management is challenging due to the uncertainty in relation to time, raw materials, paths, and processes during the remanufacturing process. China's remanufacturing market has good prospects, but the low technical and management evaluation in the early stage of remanufacturing development leads to many low-quality products flowing into the market; thus, the development of remanufacturing products has been hindered [3]. Ensuring these products' reliability is an effective way of enhancing customer recognition of RMTs and a necessary means of improving the comprehensive competitiveness of re-manufacturing enterprises [4]. Therefore, the reliability of a remanufacturing machine tool is an important index of the remanufacturing technology implementation process, which needs to be considered. However, remanufactured products are more complex, and the remanufacturing process is full of uncertainties and involves many technologies [5]. Furthermore, the reliability analyses in the existing research on machine tools have not been accurate.

Many scholars at home and abroad have studied the reliability of remanufactured machine tools. Regarding RMTs, a reliability allocation method that combined a neural network with remanufacturing factors and included the concept of reliability decreasing step by step was studied; further, a fault tree model was established, wherein faults were divided into system, subsystem, and component levels, and reliability targets were allocated into various parts [6]. In one study, the reliability of remanufacturing processes was investigated, and a reliability analysis method was set up in light of the effects of processing on reliability, whose remanufacturing processes were based on the GO method. A corresponding GO diagram and probability model of the remanufacturing process were established. [7]. The remanufacturing reliability of mechanical equipment was reviewed in consideration of four aspects: mechatronics, computer simulation, and the interdisciplinary and whole life cycle of the current reliability of mechanical assembly and remanufacturing [8]. In another study, the reliability of RMT sliding guide rails was studied, a reliability prediction was carried out using fuzzy theory, a reliability modeling wad used, and the membership function was selected to determine the wear distribution parameters; further, a few examples were presented to verify the accuracy of our predicted reliability [9]. The future fault times and reliability prediction of RMTs were studied, and a conditional distribution-based AMSAA model was proposed to predict the reliability growth of RMTs [10]. By establishing a combined conditional probability density function for the future failure time and fracture failure time of machine tools, an edge conditional density function for the future failure time was derived. Subsequently, estimates and prediction intervals of their future fault occurrence time, along with reliability points, were provided. Based on the entropy weight fuzzy comprehensive evaluation method, reliability analysis of the CNC system was conducted by collecting fault data of various subsystems in MATLAB. The method of combining subjective and objective weighting based on entropy weight theory and expert experience was used to determine the weight of comprehensive indicators to accurately evaluate their reliability [11]. The accuracy of CNC systems has been studied, in which a motion-accuracy-mapping model based on MAU was established from a dynamic perspective, which was derived using the "FMA" structural decomposition method [12]; further descriptions of its topological structure were given in combination with the multi-body system theory, and CNC errors were modeled by means of the screw theory. The above model was finally solved by means of an intelligent algorithm.

It can be seen from the above research that the reliability analysis of the functions of traditional remanufactured machine tools has mostly been achieved by analyzing the motion of parts. Although some scholars have analyzed reliability from the perspective of parts, they have not considered motion characteristics and have not been able to fully reflect the reliability of motion. At present, the quality requirements of remanufactured machine tools are becoming higher and higher in the market, and the traditional reliability research methods are not suitable for the current development trend and cannot satisfy the market demand. Therefore, this paper proposes a full-state reliability analysis model of a remanufactured machine tool based on the element action from the perspective of kinematics. Combining reliability analysis with the element action, a Markov chain is used to obtain the transition flow of the different states of a machine tool, obtain the transfer probability of the reliability evaluation index, and accurately describe the state of the machine tool system. The model can convert the operation process of the remanufactured machine tool into the system state, and the failure probability of each element action unit can be obtained through failure mode and fault tree analysis. Guarantee measures can be taken in advance for a unit with a high failure rate to improve the reliability of the machine tool.

There are six parts of this paper. The first part introduces the reliability analysis of the remanufactured machine tool and the existing problems and proposes the research content of this paper based on the meta-action unit of the remanufactured machine tools' full-state reliability model. In the second section, the RMT is decomposed into several element action units based on the element action unit decomposition method and the FMA "function-otion-action" structural decomposition method. The third section uses a Markov chain to calculate the state probability of the RMT. The fourth section analyzes and establishes the whole-state model of the RMT. The fifth section uses the reliability analysis of the NC turntable of a remanufactured machine tool as an example for the purpose of verification. The final section summarizes the paper.

2. MA Decomposition Method

It is difficult to perform an overall analysis of an RMT that is a complex mechanical electro-hydraulic integration system. For the convenience of analysis of its quality characteristics, it is usually necessary to simplify the RMT system. Additionally, in order to facilitate reliability analysis, the remanufacturing machine system can be simplified. The commonly used simplification method is "assembly unit component (ACP)" decomposition, that is, employing the "component-component-part" method of decomposing the machine tool system. However, the core of the assembly unit component decomposition method is to decompose the assembly unit into parts, and the decomposition process does not consider the motion characteristics of the remanufactured machine tool. Furthermore, there are still many problems in the application process, such as modeling difficulties caused by multiple parts, a lack of part fault data, etc.; this method cannot be used for the reliability analysis of the dynamic system of a remanufactured machine tool. This paper uses the decomposition method of a meta-action unit (MAU) and the function–motion–action (FMA) structural decomposition method to ascertain the action units of CNC machine tools.

2.1. Conceptual Model of MAUs

Realizing the meta action of a machine tool system requires the joint action of various parts, such as power output parts, power input parts, middleware, support parts, fasteners, etc. According to the assembly relationship, the meta-action unit consists of the entire structure of related parts [13]. According to the movement type, the sportswear unit can be divided into a moving unit and a rotating unit. The moving unit enables the movement of parts, such as cylinder piston movement, rack unit movement, etc. The rotation unit can facilitate the rotation function of parts, such as gear rotation unit, lead screw rotation unit, etc. A meta-action unit consists of three elements, namely, a power input, middleware, and a power output. The conceptual model of the three types of elements (the power input, the power output, and the middle parts) that comprise standard MAUs is shown in Figure 1.

Figure 1 indicates that the power input part of a certain MAU receives the displacement and angle output of its former stage's MAU, which is transmitted to the power output unit through the middleware and then output to the next level of the meta-action unit. According to the movement type, a sportswear unit can be divided into a moving unit and a rotating unit. The moving unit completes the movement of the foundation, and the rotating unit completes the rotation of the foundation. In the traditional method of reliability analysis, the corresponding model is built in the component stage. The component model contains a large number of parts, and there may be hundreds of failure modes. As a result, fault tree analysis is very complex and requires the consideration of hundreds of failure modes. And the traditional method ignores the mutual motion between the parts, only considering the parts themselves. Compared with the traditional method, the element action unit is more hierarchical in its decomposition structure. A remanufactured machine tool can be decomposed into an element action unit according to different levels, and the reliability of the whole machine can be predicted by analyzing the fault coupling degree of the element action unit. It is worth mentioning that after the machine is decomposed into a single action unit, the reliability and importance of the fault between the different units are different, making it easier to find the key fault. Those key fault points can be easily identified during modeling and analysis.

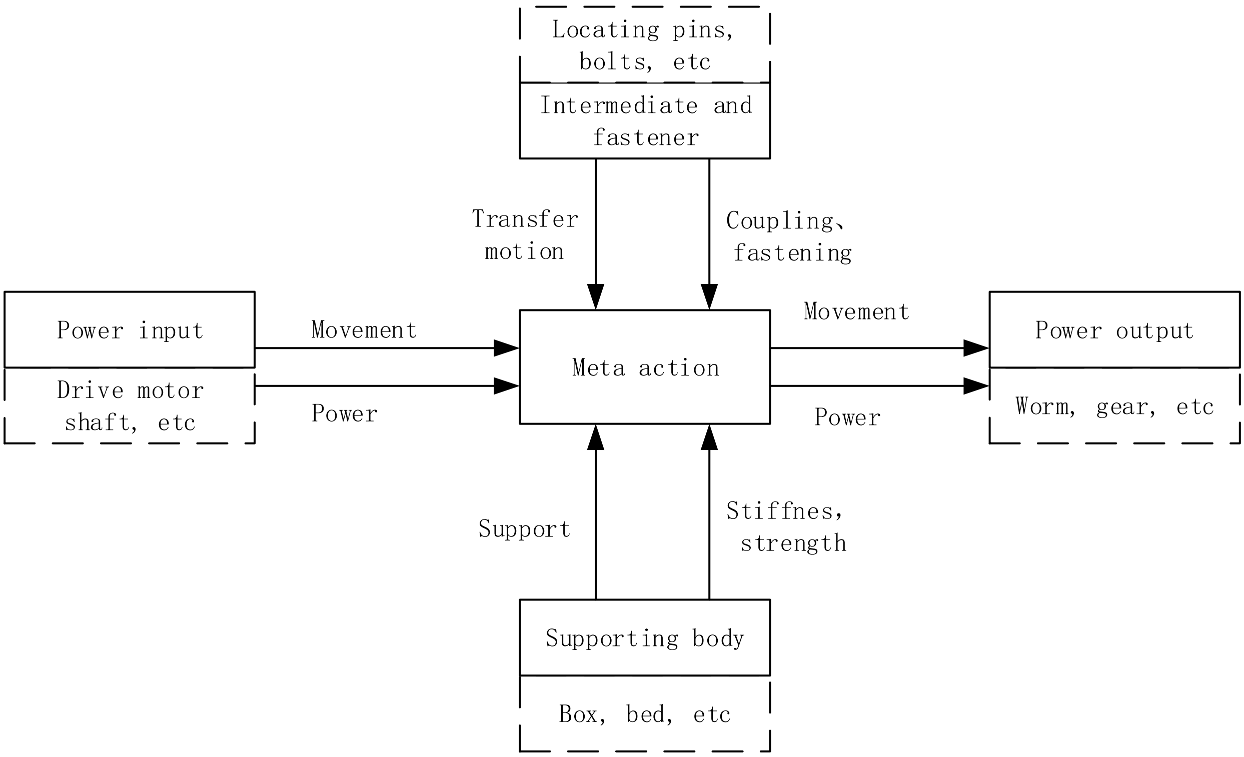

Figure 1. MAU concept model.

2.2. FMA Structural Decomposition

An RMT can achieve multiple functions, such as drilling and milling, which may be achieved by means of some combined movements of mechanisms, such as the rotation of the main axis. In the process of part processing, the movement of each system of the machine tool is gradually realized through the movement transmission of the element action unit, and different mechanism movements are realized via different element actions. The normal operation of the whole machine depends on whether each element action fails. The movement (function) of the operation process of the remanufactured machine tool is very complex layer-by-layer decomposition. The core objective of the FMA structural decomposition method is to decompose the complex movement layer by layer until it is decomposed into the most basic element action unit; this decomposition process is similar to the process of splitting a machine bed into parts. The FMA structural decomposition method is used to study the functioning of a machine tool, and machine tool dismantling is used to study the structure of a machine tool. Using the FMA structural decomposition method, the remanufactured NC machine tool can be decomposed to the element action level, and reliability analysis is carried out at this level. Figure 2 shows an FMA tree for a remanufactured NC machine tool. According to all the dominant functions of the remanufactured machine tool, the functional layer is established; then, the movement process of each function is analyzed to establish the movement layer. Finally, the movement is decomposed into the basic action layer.

As shown in Figure 2, our remanufacturing CNC system has decomposed function levels, such as F_1, F_2, F_3, . . ., F_n, and each function level comprises corresponding movement units (namely, movement levels). The moving layer is composed of the moving units that form each functional layer. For example, the realization of milling function F3 of the machine requires spindle movement M31, feed movement M32, tray exchange movement M33, rotary-table-indexing movement M34, chip-processing movement M35, and tool store tool change movement M36, and the movement layer can continue to be decomposed into one and two movement layers. The indexing movement of turntable M34 is a first-level movement layer, which can be decomposed into the second-level movement layer of turntable-lifting movement M341, turntable-rotating movement M342, pulling-jaw-tightening movement M343, lifting movement M344, etc. The movement layer can be composed of the corresponding movement units, which are in the action layer. In the above FMA decomposition of the structure of the remanufactured machine tool, it can be seen that the realization of the machining function of the machine tool can be decomposed into a series of meta-action units, such as movement and rotation, and the combination of action units constitutes the basic movement of the machine tool.

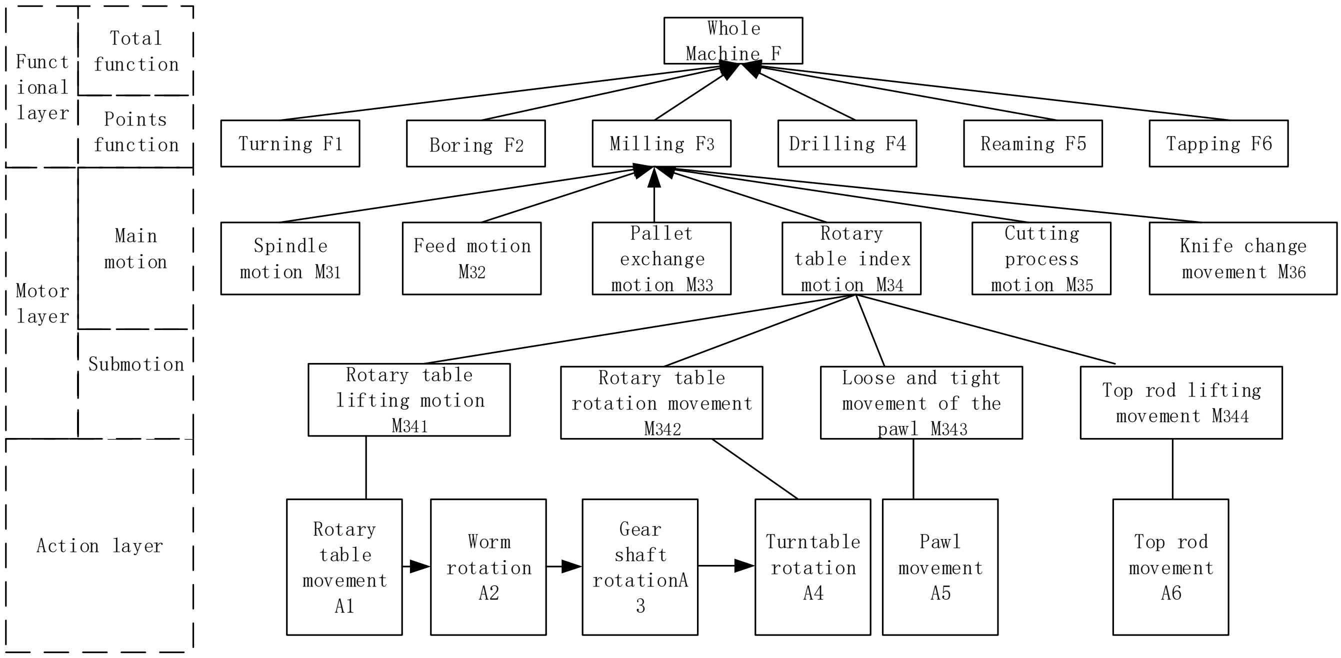

Figure 2. FMA tree of a remanufacturing CNC system.

3. Markov Chain

A Markov chain is a statistical method commonly used in probability theory and mathematical statistics to study event states and transfer laws. Using a Markov chain, the initial probabilities of different states and the probability of transformation between them can be determined so as to predict the future trend of the event state. Markov chains that apply to a continuous set of exponents are called Markov processes, and they are used to describe system state changes. In a Markov chain, both time and state are defined as discrete random processes, and the future state of the event is independent of the past state and instead only relates to the present state [14]. The mathematical expression is as follows:

Considering a random process ($\{X(t), t \in T\}$) and a state space ($S = \{x_1, x_2, \ldots, x_n\}$), the possibility of each state is P. As for $n \geq 2$ and $t_1 < t_2 < \ldots < t_n \in T$, and in the case of $X(t_i) = x_i$ ($x_i \in S, i = 1, 2, \ldots, n-1$), the conditional probability distribution function of $X(t_n)$ is equivalent to $X(t_{n-1}) = x_{n-1}$, that is,

$$\begin{aligned} &P\{X(t_n) \leq x_n || X(t_1) = x_1, X(t_2) = x_2, \ldots, X(t_{n-1}) = x_{n-1}\} \\ &= P\{X(t_n) \leq x_n || X(t_{n-1}) = x_{n-1}\} \end{aligned} \tag{1}$$

$\{X(t), t \in T\}$ is then called a Markov process [15], a theory based on probability statistics that is used to study the states of things, which can also be used for reliability analysis and the evaluation of electromechanical equipment. Generally, while an electromechanical product is undergoing failure, the process of eliminating faults includes searching for fault positions, repairing faults, and carrying out the restoration of a device's normal function. As typical pieces of electromechanical equipment, the failure rate of remanufactured machine tools can be expressed by a "bathtub curve" that changes with time [16], as shown in Figure 3.

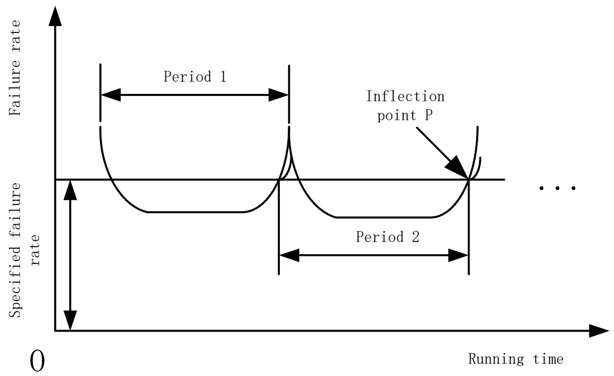

Figure 3. Bathtub curve.

According to Figure 3, the fault state of the remanufactured machine can be divided into three stages: the early failure period, the accidental failure period, and the loss failure period. According to the "bathtub curve" of the failure rate of the remanufactured machine tool, it can be seen that the machine tool is in the random failure period under normal working conditions. The random failure period coincides with the Markov process, during which the failure rate and repair rate are constant and the product's usable life and maintenance time correspond to an exponential distribution [16]. This process satisfies the assumptions of Markov processes. If the operating process of an RMT is divided into m states and State j is obtained by means of the transition of State i, its transition probability is represented by λ_{ij}. In the case that λ_{ij} remains constant, the following relationship exists:

$$P\{X(t+\Delta t)=j|X(t)=i\}=\lambda_{ij}\Delta t+O(\Delta t) \tag{2}$$

where $O(\Delta t)$ is the probability of two or more time offsets occurring within the period (Δt).

The transition probability matrix is as follows:

$$P(\Delta t)=\begin{bmatrix} \lambda_{11}\Delta t & \lambda_{12}\Delta t & \cdots & \lambda_{1n}\Delta t \\ \lambda_{21}\Delta t & \lambda_{22}\Delta t & \cdots & \lambda_{2n}\Delta t \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{n1}\Delta t & \lambda_{n2}\Delta t & \cdots & \lambda_{nn}\Delta t \end{bmatrix} \tag{3}$$

The transition probability density matrix is as follows:

$$A=\lim_{\Delta t\to 0}\frac{P(\Delta t)-E}{\Delta t}=\begin{bmatrix} \lambda_{11}-1 & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22}-1 & \cdots & \lambda_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{n1} & \lambda_{n2} & \cdots & \lambda_{nn}-1 \end{bmatrix} \tag{4}$$

where E represents the identity matrix.

While the probabilistic steady state of an RMT is defined as p_i in the market and the total of the system states is assumed as n, the probability matrix of each steady state can be described as follows:

$$P=[p_1,p_2,\ldots,p_n]^T \tag{5}$$

The solution of the above equation is expressed as

$$\begin{cases} PA=0 \\ \sum p_i=1 \end{cases} \tag{6}$$

The substitution of the relevant equations results in the following description:

$$\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} \begin{bmatrix} \lambda_{11}-1 & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22}-1 & \cdots & \lambda_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{n1} & \lambda_{n2} & \cdots & \lambda_{nn}-1 \end{bmatrix} = 0 \quad (7)$$

$$p_1 + p_2 + \cdots + p_n = 1 \quad (8)$$

In accordance with the above equations, the state probabilities of the remanufactured machine tool can be calculated, followed by the calculation of other relevant indicators.

4. Full-State Modeling of a Remanufactured Exercise Machine Tool

4.1. MAU System

The working state of the analyzed remanufactured machine tool can be divided into two types, namely, normal operation and fault shutdown. The function of the remanufactured machine tool is composed of multiple action units. If one exercise unit fails, the working state of the whole machine tool will fail, and it cannot run normally [17]. Remanufactured machine tools can be decomposed into multiple element action units, where each element action unit constitutes the whole machine tool system, and the working state of the element action unit can be divided into a normal element action unit and a faulty element action unit. The element action structure decomposition method simplifies the operation process of the machine tool into a continuous process and simplifies the uncountable working state of the machine tool into a countable state [18]. By considering changes in these states, our analysis can be made easier. While MAUs are assumed as A_i (i = 1, 2, . . ., n), the corresponding state classification is shown in Figure 4.

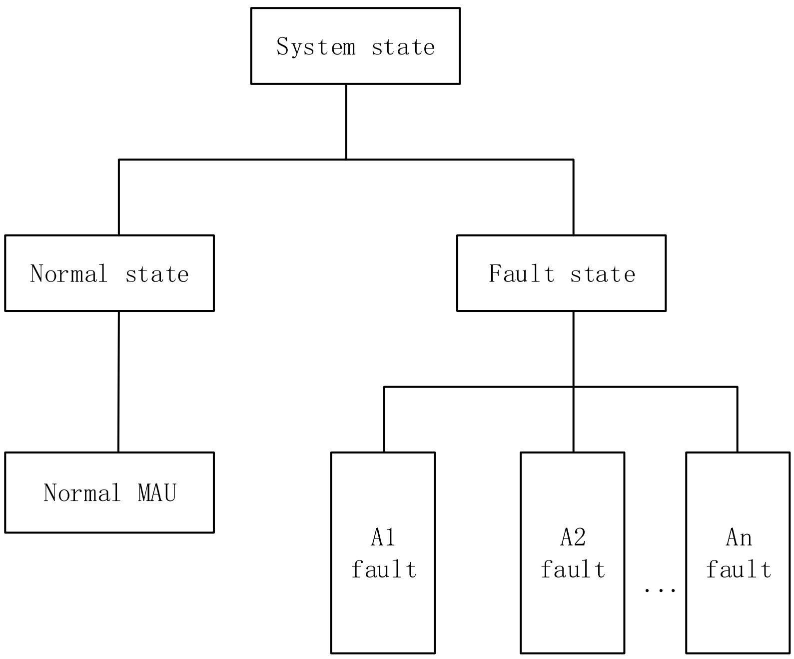

Figure 4. Classification of system states of an RMT.

4.2. Full-State Transition Modeling

If a certain motion unit fails, the other connected element action units cannot operate normally, the whole machine tool will stop functioning, and the next element action failure will reappear after the fault is repaired. Therefore, the fault states will not be converted into each other. If a single action fails, the remanufacturing machine immediately stops working, and the other element action units are not affected, remaining in a normal working state. Then, when the fault state appears, only a single element action unit fails, and the failures of the element action unit are independent of each other. According to the state classification of the remanufactured machine tool system shown in Figure 4, the probability that the remanufactured machine tool is in the normal state is p_0, and the probability that the element action unit is in the fault state is p_i (i = 1, 2, . . . , n). According to the Markov process, the Markov state transfer diagram of the remanufactured machine tool is shown in Figure 5, where α_i and β_i represent the fault and repair rates of an MAU, respectively, whose values are obtained according to the actual work conditions.

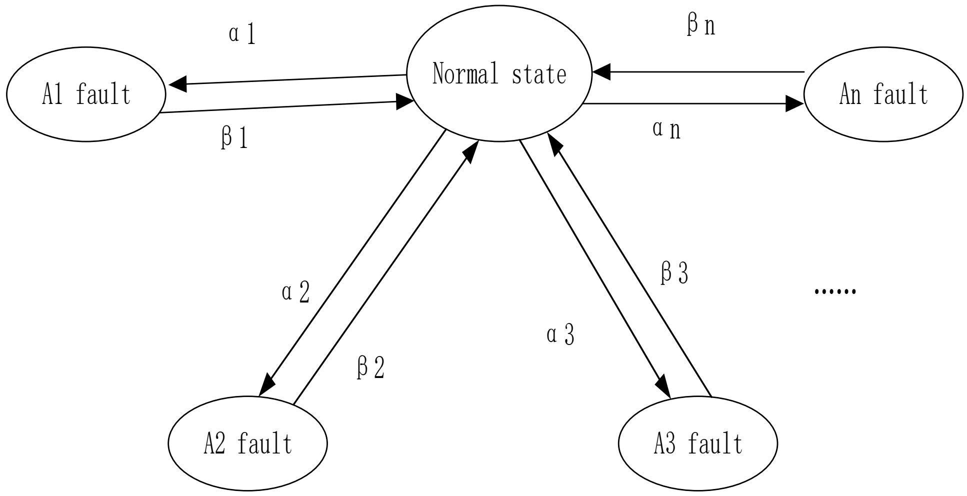

Figure 5. Markov state transition diagram.

While Δt is small enough, Equation (2) can be converted into the following equations:

$$P\{X(t+\Delta t) = j|X(t) = i\} = \lambda_{ij}\Delta t \tag{9}$$

$$P\{X(t+\Delta t) = i|X(t) = i\} = 1 - \sum_{j=1, j\neq i}^{n} \lambda_{ij}\Delta t \tag{10}$$

Thus, the system matrix of the state transfer probability within the period (Δt) is expressed as

$$P(\Delta t) = \begin{bmatrix} 1-\sum\limits_{i=1}^{n}\alpha_i & \alpha_1\Delta t & \alpha_2\Delta t & \cdots & \alpha_n\Delta t \\ \beta_1\Delta t & 1-\beta_1\Delta t & 0 & \cdots & 0 \\ \beta_2\Delta t & 0 & 1-\beta_2\Delta t & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \beta_n\Delta t & 0 & 0 & \cdots & 1-\beta_n\Delta t \end{bmatrix} \tag{11}$$

Further, the transfer density matrix is as follows:

$$A = \begin{bmatrix} -\sum\limits_{i=1}^{n}\alpha_i & \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \beta_1 & -\beta_1 & 0 & \cdots & 0 \\ \beta_2 & 0 & -\beta_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \beta_n & 0 & 0 & \cdots & -\beta_n \end{bmatrix} \tag{12}$$

In accordance with the above modeled PA = 0, the target function can be converted into

$$\begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ \alpha_1 & -\beta_1 & 0 & \cdots & 0 \\ \alpha_2 & 0 & -\beta_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_n & 0 & 0 & \cdots & -\beta_n \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{13}$$

The solution of the above equations can result in the calculations of p_0 and p_i (i = 1, 2, . . ., n). According to the state probability of the system, the working state of the remanufactured machine tool can be effectively determined. By obtaining the fault probability of the element action unit, we can glean that the element action unit is prone to failure, and we can focus on the element action unit, find a fault in time, and ensure the normal operation of the remanufactured machine tool. Research on the reliability technology of remanufactured machine tools based on the element action unit can guide the production process and improve the process technology of remanufactured machine tools. New parts are used as much as possible in the fault prone parts of the unit, and the reliability of the machine tool is ensured via the remanufacturing process's design. In the above model, it is also necessary to establish important indicators for evaluating reliability. The differential equation of the state transition probability of remanufactured machine tools obtained through Figure 5 is as follows:

$$\begin{cases} (\frac{d}{dt} + \sum\limits_{i=1}^{n} \alpha_i) p_0(t) = \sum\limits_{i=1}^{n} \beta_i p_i(t) \\ (\frac{d}{dt} + \alpha_i) p_i(t) = \alpha_i p_0(t) \\ p_0(0) = 1, p_i(0) = 0, i = 1, 2, \cdots, n \end{cases} \tag{14}$$

For the convenience of calculating the above differential equations, the Laplace transform [19] was employed, and its corresponding solution is as follows:

$$\begin{cases} sP_0^*(s) - 1 + \sum\limits_{i=1}^{n} \alpha_i P_0^*(s) = \sum\limits_{i=1}^{n} \beta_i P_i^*(s) \\ sP_i^*(s) + \beta_i P_i^*(s) = \alpha_i P_0^*(s), i = 1, 2, \ldots, n \end{cases} \tag{15}$$

$$\begin{cases} P_0^*(s) = \frac{1}{s + s\sum\limits_{i=1}^{n} \frac{\alpha_i}{s+\beta_i}} \\ P_i^*(s) = \frac{\alpha_i}{s+\beta_i} P_0^*(s), i = 1, 2, \ldots, n \end{cases} \tag{16}$$

Thus, the steady state and fault frequency of our remanufacturing machine system are as follows:

$$\begin{aligned} A_s &= \lim_{x\to\infty} A(t) = \lim_{s\to\infty} sA^*(s) = \lim_{s\to 0} sP_0^*(s) \\ &= \lim_{s\to 0} s \frac{1}{s + s\sum\limits_{i=1}^{n} \frac{\alpha_i}{s+\beta_i}} = \frac{1}{1 + \sum\limits_{i=1}^{n} \frac{\alpha_i}{\beta_i}} \end{aligned} \tag{17}$$

$$\begin{aligned} M &= \lim_{t\to\infty} m(t) = \lim_{s\to 0} sm^*(s) = \lim_{s\to 0} s \sum_{i=1}^{n} \alpha_i p_0^*(s) \\ &= \lim_{s\to 0} s \sum_{i=1}^{n} \alpha_i \frac{1}{s + s\sum\limits_{i=1}^{n} \frac{\alpha_i}{\beta_i}} = \frac{\sum\limits_{i=1}^{n} \alpha_i}{1 + \sum\limits_{i=1}^{n} \frac{\alpha_i}{\beta_i}} \end{aligned} \tag{18}$$

Additionally, the mean time between failures (*MTBF*) and the mean time to repair (*MTTR*) of an RMT are as follows:

$$\begin{cases} MTBF = \frac{A_s}{M} = \frac{1}{\sum\limits_{i=1}^{n} \alpha_i} \\ MTTR = \frac{1-A_s}{M} = \frac{\sum\limits_{i=1}^{n} \frac{\alpha_i}{\beta_i}}{\sum\limits_{i=1}^{n} \alpha_i} \end{cases}$$

In what follows, based on the above analysis, a full-state model of an RMT is established, and reliability evaluation indicators such as the system's steady state (A_s), fault frequency (M), mean time to failure (*MTBF*), and mean time to maintenance (*MTTR*) are presented.

5. Case Analysis

The spindle of a machine tool refers to the axis in the machine tool that causes the workpiece or tool to rotate. It is usually composed of a spindle, bearings, and transmission parts (gears or belt wheels). In a machine tool, it is mainly used to support transmission parts, such as gears and belt wheels, and transfer movement and torque. The spindle system is one of the core functional systems of a CNC machine tool [20]. Compared with other subsystems, it has the characteristics of frequent failure. At the same time, once the spindle system fails, it will affect the quality of the machining parts of the machine tool, so reliability research on the spindle system has important significance in improving the reliability of the entire machine tool. The spindle system plays the role of power transmission in the working process of the CNC machine tool, which is related to the stable operation of the whole machine tool, so the reliability of the spindle system affects the reliability of the whole milling machine. Improving the reliability of the spindle system is the key way in which to improve the reliability of the whole CNC machine tool. The spindle system structure diagram of a THP6513 heavy-duty CNC horizontal milling and boring machine is shown in Figure 6. The main drive system of CNC machine tools usually adopts an AC or DC spindle motor, which induces spindle rotation through a belt drive. The driving force of the spindle motor is transformed into the cutting torque and cutting speed available for cutting through the transmission system.

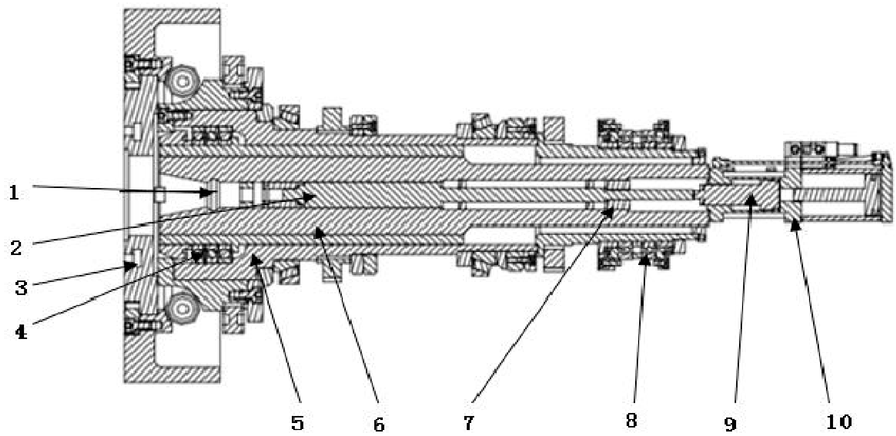

Figure 6. Spindle structure diagram. 1—Clamp cone; 2—Connecting rod; 3—Front cover; 4—Front bearing; 5—Housing; 6—Spindle; 7—Butterfly spring; 8—Rear end bearing; 9—Piston; 10—Broach cylinder.

5.1. Full-State Modeling

According to the modeling process, it is necessary to decompose the spindle of an NC machine tool using the element action decomposition method in order to establish the corresponding element action fault tree. The main shaft of the remanufacturing machine tool was decomposed to the level of the primary action via the structural decomposition of the primary action, and the reliability analysis was carried out at the level of the primary action. Figure 7 shows the FMA structural decomposition of the CNC spindle of the remanufactured machine tool. Firstly, the spindle motion function layer was established, and then the motion layer was determined. The spindle motion function mainly includes spindle rotation, spindle quasi-stop, tool clamping, and relaxation. Finally, a meta-action layer is established, and each mechanical movement is decomposed into a meta-action layer, which is the MAU layer. Spindle rotation can be divided into motor pulley rotation, small circular arc pulley rotation, circular arc pulley rotation, large circular arc pulley rotation, and spindle rotation. Spindle quasi-stop motion can be decomposed into motor code braking and tool clamping, and relaxation motion can be decomposed into the opening and closing of the electromagnetic reversing valve, pull rod piston movement, disc spring expansion, and pull claw movement.

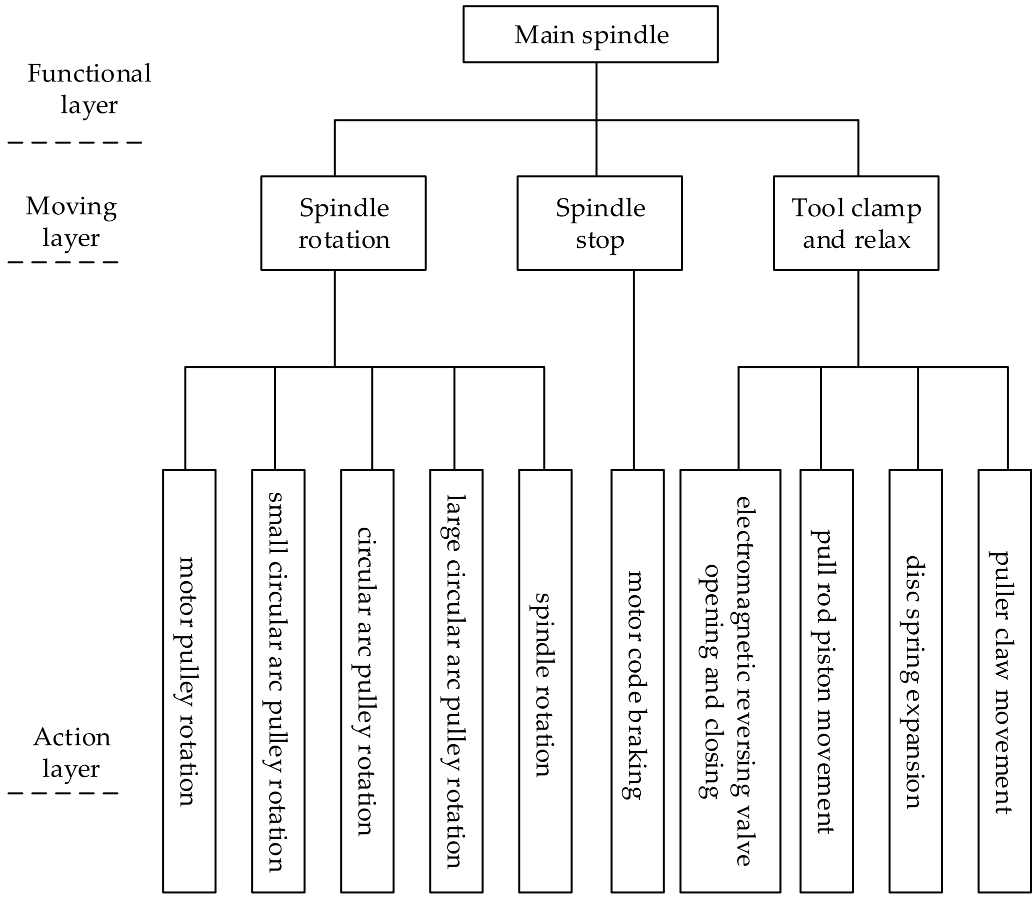

Figure 7. Fault tree of the numerical control turntable.

Figure 7 shows that there are a total of 10 MAUs (A1–A10) in our structurally decomposed numerical control turntable. A1 to A10 represent motor pulley rotation, small circular arc pulley rotation, circular arc pulley rotation, large circular arc pulley rotation, spindle rotation, motor code braking, the opening and closing of the electromagnetic reversing valve, pull rod piston movement, disc spring expansion, and puller claw movement, respectively. To ensure the normal operation of the machine tool spindle, all the element action units must operating normally to ensure the reliability of the machine tool spindle. If one of the element action units fails, the spindle will stop moving and maintain this state. Therefore, the machine tool spindle can be divided into 11 states in total, including 10 fault states and 1 normal state. Accordingly, this turntable can work under a total of 11 states (10 fault states + 1 normal state), whose combinations are given in the following equation.

$$\begin{bmatrix} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & A_9 & A_{10} \\ + & + & + & + & + & + & + & + & + & + \\ - & & & & & & & & & \\ & - & & & & & & & & \\ & & - & & & & & & & \\ & & & - & & & & & & \\ & & & & - & & & & & \\ & & & & & - & & & & \\ & & & & & & - & & & \\ & & & & & & & - & & \\ & & & & & & & & - & \\ & & & & & & & & & - \end{bmatrix} \Rightarrow \begin{bmatrix} SC \\ + \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \end{bmatrix} \tag{19}$$

In the above formula, the left side of the equation is the state of the element action unit, and the right side of the equation is the state of the system. MAU and system states are on the left and right sides, respectively; "+" and "−" represent normal and fault modes.

5.2. Equation Solution

For each element action unit of the machine tool spindle, by monitoring the corresponding failure rate and maintenance rate data of various states, the machine tool spindle Markov state transfer corresponds to the failure rate and maintenance rate of the corresponding 10-element action unit A_i, as shown in Table 1.

Table 1. Statistics of the fault and repair rates.

MAU No.	MAU Name	Failure Rate α_i	Maintenance Rate β_i
A1	Motor with wheel rotation	0.002	0.21
A2	Small circular arc pulley rotation	0.001	0.34
A3	Circular arc pulley rotation	0.001	0.43
A4	Large circular arc pulley rotation	0.002	0.26
A5	Spindle rotation	0.002	0.21
A6	Motor code braking	0.002	0.19
A7	Opening and closing of the electromagnetic reversing valve	0.002	0.20
A8	Pull rod piston movement	0.001	0.25
A9	Disc spring expansion	0.002	0.24
A10	Puller claw movement	0.001	0.19

The turntable transition density matrix can be determined by means of Equation (12), which is expressed as

$$A = \begin{bmatrix} -\sum_{i=1}^{n} \alpha_i & \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \beta_1 & -\beta_1 & 0 & \cdots & 0 \\ \beta_2 & 0 & -\beta_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \beta_n & 0 & 0 & \cdots & -\beta_n \end{bmatrix}$$
$$= \begin{bmatrix} -0.0016 & 0.0002 & 0.0001 & 0.0001 & 0.0002 & 0.0002 & 0.0002 & 0.0002 & 0.0001 & 0.0002 & 0.0001 \\ 0.21 & -0.21 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.34 & 0 & -0.34 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.43 & 0 & 0 & -0.43 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.26 & 0 & 0 & 0 & -0.26 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.21 & 0 & 0 & 0 & 0 & -0.21 & 0 & 0 & 0 & 0 & 0 \\ 0.19 & 0 & 0 & 0 & 0 & 0 & -0.19 & 0 & 0 & 0 & 0 \\ 0.20 & 0 & 0 & 0 & 0 & 0 & 0 & -0.20 & 0 & 0 & 0 \\ 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.25 & 0 & 0 \\ 0.24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.24 & 0 \\ 0.19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.19 \end{bmatrix} \quad (20)$$

PA = 0 is established in light of the above model, and the corresponding probabilities (p_i), which are shown in Table 2, can be calculated per MAU by means of sample data and Equation (11).

Table 2. Steady-state probabilities of all states.

Statas	Probability
Normal operation	0.993
Fault A1	0.000944
Fault A2	0.000292
Fault A3	0.000231
Fault A4	0.000763
Fault A5	0.000944
Fault A6	0.001044
Fault A7	0.000992
Fault A8	0.000397
Fault A9	0.000826
Fault A10	0.000522

Finally, the reliability indicators of our numerical control turntable can be solved using Equations (15)–(17), which are as follows:

$$\left\{ \begin{array}{l} A_S = \lim A(t) = \frac{1}{1+\sum_{i=1}^{n} \frac{\alpha_i}{\beta_i}} = 0.993 \\ M = \lim m(t) = \frac{\sum_{i=1}^{n} \alpha_i}{1+\sum_{i=1}^{n} \frac{\alpha_i}{\beta_i}} = 1.784 \times 10^{-3} \\ MTBF = \frac{A_S}{M} = 556.61 \\ MTTR = \frac{1-A_S}{M} = 3.92 \end{array} \right\} \quad (21)$$

5.3. Result Analysis

According to the solution, the steady-state index of the machine tool spindle is 0.993, indicating that the probability of the machine tool spindle maintaining a normal working state is 99.3%, which indicates the high reliability of the machine tool spindle. Table 2 shows that the motor code brake element action unit has the highest probability of failure in the fault state of the machine tool spindle, and its fault probability is 0.001044. It should be carefully monitored in the process of remanufacturing design, and online monitoring and maintenance can be carried out during the working process of the machine tool spindle to ensure the normal operation of the machine tool spindle and improve the reliability of operation.

The mean time between failure (MTBF) and the mean time to repair (MTTR) of the remanufactured machine tool spindle were calculated to be 556.61 h and 3.92 h respectively. The reliability of the remanufactured machine tool spindle is relatively weak, so it must be improved. The results of the analysis regarding the spindle of the machine tool prove that FMEA is a universal reliability method that can help to analyze the failure mode and impact of remanufactured machine tools. In addition, FMEA can be used to perform fault tree analysis of a remanufactured machine spindle to determine the specific location of a fault. As shown in Table 2, the failure state probabilities of the motor code brake element action unit and the opening and closing of the electromagnetic directional valve are 0.001044 and 0.000992, respectively. According to the Pareto principle, the motor coded brake element action unit and the action unit of the opening and closing of the solenoid reversing valve are the weakest links in the reliability of CNC, which means that the motor coded brake element action unit and the opening and closing of the solenoid reversing valve determine the reliability of a machine tool spindle. Therefore, it is necessary to focus on these two meta-action units and take effective measures to address any corresponding issues. The full-state model based on the element action unit has proven to be effective, helping to evaluate the reliability performance of the analyzed remanufactured machine tool. We have verified that our MAU-based full-state modeling method is effective in identifying weak links and guiding the process design and maintenance of the remanufacturing process.

6. Conclusions

It is difficult to perform a reliability analysis from the overall perspective of an RMT as a complex mechanical electro-hydraulic system. The stability and reliability of the remanufacturing process form the basis for ensuring and improving the reliability of remanufactured machine tools. The machine tool remanufacturing process is different from the initial machine tool manufacturing process in that it concerns the completion of a service cycle, making its reliability behavior significantly different from that of the manufacturing process for new machine tools. In this study, a remanufactured machine tool was decomposed into a series of remanufactured machine tools, and an FMA tree was established to render the reliability analysis of the remanufactured machine tool more accurate.

The operation process of the remanufactured machine tool was divided into a normal state and a fault state, and the working process of the machine tool was transformed from continuous to discontinuous so as to simplify the reliability analysis.

By combining the state of the element action unit with the Markov decision process, the different working states of the remanufacturing machine tool system were transformed through the Markov process. Taking the spindle of the remanufactured machine tool as an example, the FMA tree of the spindle of the machine tool was established, and the differential equation of its state transition probability was calculated through the Markov process. The steady state probability of each action unit of the spindle of the machine tool under stable conditions was solved, and the relevant reliability index was obtained. The reliability performance of the spindle of the remanufactured machine tool was evaluated, the overall state was determined, and the weak links of the machine tool were accurately identified. Thus, the overall reliability of the RMT was effectively improved.

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