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Dynamic Characteristics Analysis of Metallurgical Waste Heat Radiative Drying of Thin Layers of Sewage Sludge

Hongyan Guo [1], Zhiwei Tan [1], Haiying Li [1], Yue Long [1], Aimin Ji [2,*] and Liangxu Liu [1,*]

[1] School of Metallurgy and Energy, North China University of Science and Technology, Tangshan 063210, China
[2] Ocean College, Tangshan Normal University, Tangshan 063210, China
* Correspondence: jiaimin2022@126.com (A.J.); liuliangxu@126.com (L.L.); Tel.: +86-152-3055-9182 (A.J.); +86-183-2270-3883 (L.L.)

Abstract: The utilization of metallurgical waste heat for urban sludge drying and dewatering not only affects the subsequent cost of sludge treatment but also provides a pathway for the rational utilization of metallurgical waste heat. The influence of different experimental conditions on sludge drying characteristics, such as drying temperature and thickness, was analyzed based on metallurgical waste heat. Based on the analysis and evaluation of the drying kinetics parameters of commonly used drying mathematical models, a modified Midilli drying kinetic model is proposed. The kinetic parameters and effective diffusivity of sludge drying were analyzed in three stages of sludge drying: rising rate, constant rate, and falling rate. By utilizing the Arrhenius equation, the relationship between the effective diffusion coefficient and thermodynamic temperature is established, revealing the apparent activation energies for the three stages of urban sludge drying as 29.772 kJ·mol^{-1}, 37.129 kJ·mol^{-1}, and 39.202 kJ·mol^{-1}, respectively. This is closely related to the migration, diffusion, and mass transfer resistance of sludge moisture, indicating that the thickness of sludge accumulation affects the drying time of sludge during the treatment of municipal sludge.

Keywords: municipal sludge; dry model; effective diffusion coefficient; activation energy; metallurgical waste heat

Citation: Guo, H.; Tan, Z.; Li, H.; Long, Y.; Ji, A.; Liu, L. Dynamic Characteristics Analysis of Metallurgical Waste Heat Radiative Drying of Thin Layers of Sewage Sludge. *Processes* **2023**, *11*, 2535. https://doi.org/10.3390/pr11092535

Academic Editor: Carlos Sierra Fernández

Received: 12 July 2023
Revised: 9 August 2023
Accepted: 17 August 2023
Published: 24 August 2023

1. Introduction

With the advancement of industrialization and the rapid growth of the urbanization population, the output of municipal sludge is increasing day by day. According to statistics, the average sludge yield during the operation of domestic sewage treatment plants is 1.62 tDS/10^4 m^3 [1]. Conventional drying methods are inadequate to address the current situation, and the drying technology for sludge in China lags significantly behind. Proper treatment and timely follow-up disposal of sludge are very difficult. Therefore, scholars must study the drying theory and develop treatment technology to solve the situation of sludge increasing efficiently. Currently, China produces over 750 million tons of metallurgical slag annually, with the discharged slag temperature typically exceeding 1200 °C and carrying a substantial amount of heat [2]. The recovery and utilization of this immense metallurgical waste heat is of utmost importance.

Sludge is a typical solid sediment, which is mainly obtained after the treatment of sewage produced by human production and life. It is extremely complex in composition and is present in large amounts, making it very difficult to treat [3]. To facilitate the transportation and logistics of sludge during the treatment process, sludge generally needs to be thickened or mechanically dewatered, which can reduce the moisture content of sludge to approximately 75% [4]. It will not only cause great damage to the natural environment but also endanger the physical and mental health of human beings and the survival of other animals and plants. Traditional municipal sludge disposal methods, such as landfill, composting, sea reclamation, incineration, etc. [5–7], can play a certain role in the harmless treatment of municipal sludge. But these traditional disposal methods cause secondary pollution and big energy consumption problems. At the same time, gradual low-temperature drying not only solves the issues of poisonous gases emanating from the pyrolysis of organic pollutants in sewage sludge and big issues such as energy consumption but also reduces the moisture content of sludge, reduces sludge volume capacity, and solves the problems of sludge and of its difficult transportation [8–10].

The most critical step in wastewater sludge treatment is sludge drying, which can be commonly categorized into mechanical drying, natural drying, and thermal drying methods. Mechanical drying involves using mechanical equipment such as centrifugation, filtration, or evaporation to reduce the moisture content of the sludge [11]. Natural drying refers to methods where moisture is allowed to evaporate naturally or gravity is utilized to separate water, with sludge drying beds being a typical example, where sludge is spread over sand beds until it dries [12]. Thermal drying refers to the utilization of thermal energy to conduct heat to the sludge, resulting in the evaporation of moisture [13]. Thermal drying can be further classified into convection drying, conduction drying, and radiation drying based on the modes of heat transfer [14]. Commonly used heat sources for drying are often low-grade waste heat. After the metallurgical blast furnace slag undergoes heat recovery using the air quenching method, the resulting granulated slag still maintains temperatures above 150 °C [15], which aligns with the requirements for thermal drying of sludge.

Currently, the commonly used drying models in research are mostly empirical or semi-empirical models, which rely on experimental data and may not necessarily be applicable to drying under different conditions. The simple Lewis model is easy to apply and has clear physical meaning [16]. The empirical Logarithmic model exhibits higher correlation coefficients and lower root mean square errors [17]. The semi-empirical Page model, compared to the Lewis model, better captures the descending rate period and fits well with the experimental data [18]. The Modified Page model allows for improved accuracy by adjusting the exponent [19]. The two-term exponential model describes the constant-rate and falling-rate periods with higher precision [20]. The Midilli model captures both the constant-rate and falling-rate periods and exhibits a high degree of fit with experimental data [21]. The Henderson and Pabis models can describe the falling rate period and are characterized by their simplicity [22]. Each of the seven models mentioned above has its advantages, but it is difficult to determine which model is more suitable for municipal sludge drying.

The main objective of this study is to conduct preliminary research for industrial projects involving the drying of metallurgical low-grade waste heat. To achieve this, experimental equipment was employed to simulate metallurgical waste heat, and drying experiments were performed on urban sludge samples with varying thicknesses and temperatures. The simulation of urban sludge thin-layer drying curves was carried out using the aforementioned seven models. Based on the goodness-of-fit, the most suitable drying model was selected, and further improvements were made. The kinetic parameters and effective diffusion coefficients of sludge drying in three different stages are analyzed, and the activation energy is calculated and analyzed to characterize the drying characteristics of urban sludge under different conditions and scenarios.

2. Experimental Materials and Methods

2.1. Experimental Materials

The sludge samples were collected from the wastewater treatment plant in Tangshan City. Before the experiments, solid particles and residual garbage were removed from the wet sludge, which was then stored in a refrigerator at 0–5 °C for 24 h to prevent fermentation. Prior to the experiments, a suitable amount of sludge was taken out and allowed to reach a temperature of 20–25 °C in the environment for two hours.

As the moisture content of the collected sludge was initially unknown in this study, a 105 °C electric blast drying oven was used to heat and dry the sludge. The weight was measured every two hours until a constant weight state was achieved. Eventually, the moisture content of the sludge in the experiment was determined to be approximately 80.26%. The fundamental characteristics of sludge are presented in Table 1.

Table 1. Fundamental characteristics of sludge.

Industrial Analysis (ar [1])/%				Elementary Analysis (d [2])/%					$Q_{net.ad}$ [3]/MJ·kg^{-1}
M	A	V	FC	C	H	O	N	S	
80.26	4.65	12.79	2.30	31.45	5.27	26.40	3.58	0.77	12.78

[1] as received; [2] dry basis; [3] air dry basis.

2.2. Experimental Instrument

DHS-16A halogen moisture tester (Changzhou Heng Positron Instrument Co. Ltd., Changzhou, China): This instrument has a maximum weighing capacity of 100 g, with a display readability of 1 mg and an error of ±1 mg. It uses a halogen lamp as the heating source, with a maximum temperature of 160 °C. The time control range is 0–90 min.

DHG-9070A Electric Blower drying oven (Shaoxing Yicheng Instrument Manufacturing Co. Ltd., Shaoxing, China): This equipment has a temperature control range of RT + 10 °C to 250 °C. The temperature stability is ±1.0 °C, and the temperature resolution is 0.1 °C. At the test point of 100 °C, the temperature uniformity is within ±3%. The volume of the oven is 80 L.

FA-2204 high precision electronic balance (Hangzhou Jingfei Instrument Technology Co. Ltd., Hangzhou, China): This balance has a maximum weighing capacity of 220 g, with a readability of 0.1 mg and a repeatability of ±0.0002 g. The linear error is ±0.0003 g.

2.3. Experimental Methods

The preparation of sludge cakes involves using two glass plates, each with a layer of 100-mesh sieve net on it. The sludge is then pressed between the glass plates to ensure flatness. After the sludge cake is formed, the glass plates are opened, the sieve net is removed, and a small knife is used to modify the sludge cake to meet the experimental conditions. DHS-16A halogen moisture tester was used to complete the drying experiment. The drying temperatures were 45 °C, 55 °C, 75 °C, 105 °C, and 135 °C, and the thicknesses were 0.5 mm, 1 mm, 2 mm, and 4 mm. The drying equipment incorporated a self-made balance for real-time measurement, continuously weighing the sludge cake during the moisture evaporation process. Three experiments were conducted for each sludge cake thickness, and the average value was obtained. During the experiment, the height of the radiant heat source was maintained at 1 cm and no hot air interference was ensured. The radiation temperature error was ±2 °C. Data were recorded once every minute after startup.

3. Analysis of Experimental Results

3.1. Influence of Temperature on Sludge Drying

Taking a thin municipal sludge layer with a thickness of 2 mm as an example, its moisture content changes over time at 45 °C, 55 °C, 75 °C, 105 °C, and 135 °C, as shown in the figure below.

As can be seen from Figure 1, when the thickness of municipal sludge is the same, the drying time of sludge is significantly shortened by increasing the drying temperature. The reason is that by increasing the drying temperature, that is, enhancing the radiation intensity on the surface of the sludge, the adsorbed water on the surface of the sludge evaporates rapidly, the surface humidity is significantly reduced, and there is a temperature gradient in the internal and external water. Under the driving force of the temperature difference in heat transfer, the water in the sludge migrates outward at a faster rate. Therefore, when the temperature rises from 45 °C to 135 °C, the drying time for a 2 mm thick municipal thin sludge layer to equilibrium water content is reduced from 245 min to about 20 min.

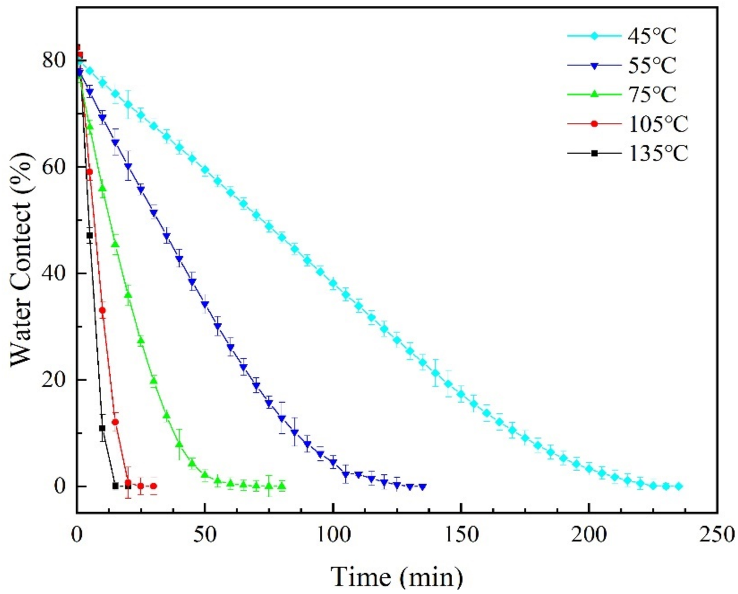

Figure 1. The relationship between sludge moisture content and temperature with a thickness of δ = 2 mm.

3.2. Effect of Thickness on Sludge Drying

Taking the thin sludge layer with a drying temperature of 75 °C as an example, its moisture content changes over time under the four thicknesses of 0.5 mm, 1 mm, 2 mm, and 4 mm, as shown in the figure below.

As can be seen from Figure 2, when the drying temperature is constant, the smaller the thickness of the thin sludge layer, the shorter the drying time of the thin layer. Because the thickness of the sludge is small, the internal water of the sludge is more likely to migrate outward, and the mass transfer resistance is smaller; therefore, the drying time of the sludge is shorter, and the water on the surface evaporates faster. When the drying temperature is constant and the thickness of the thin sludge layer decreases from 4 mm to 1 mm, the drying time to equilibrium moisture content is reduced from about 130 min to nearly 15 min.

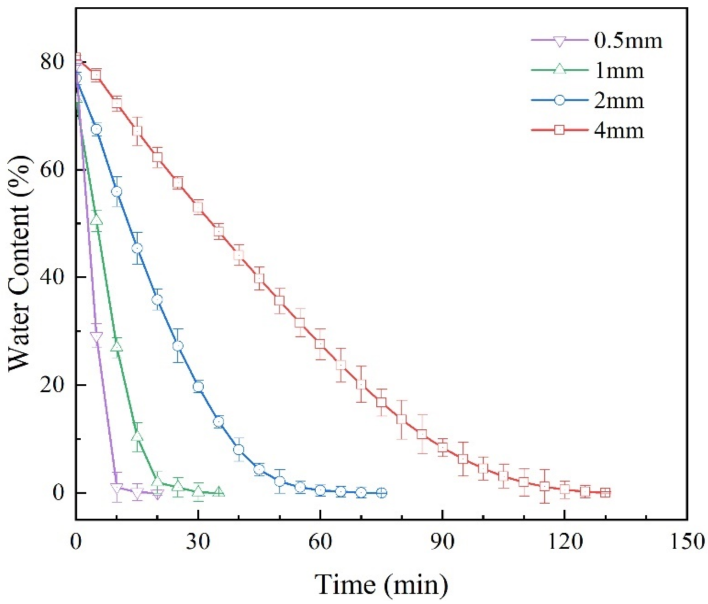

Figure 2. The relationship between moisture content and thickness of sludge at temperature T = 75 °C.

3.3. Effect of Temperature on Sludge Drying Rate

Taking the 2 mm thin sludge layer as an example, its drying rate changes over time at 45 °C, 55 °C, 75 °C, 105 °C, and 135 °C, as shown in the figure below.

As can be seen from Figure 3, when the thickness of municipal sludge is the same and the drying temperature is increased, the total drying time of the sludge is shorter. When the temperature rises from 45 °C to 135 °C, 2 mm increases obviously with the drying rate peak value of thin sludge layer, and the range of isometric drying stage of sludge decreases obviously, and the time of descending drying becomes shorter.

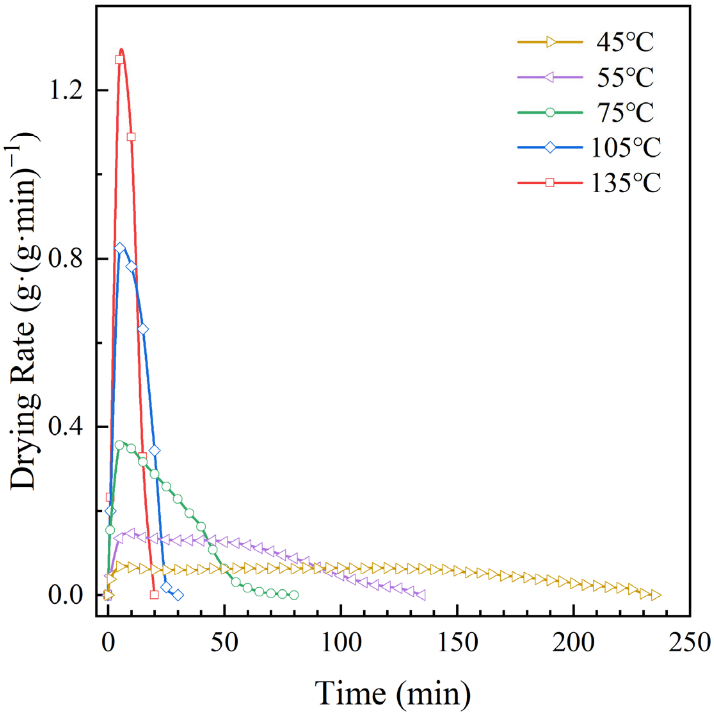

Figure 3. The relationship between sludge drying rate and temperature at a thickness of δ = 2 mm.

3.4. Effect of Thickness on Sludge Drying Rate

Taking the thin sludge layer at a drying temperature of 75 °C as an example, its moisture content changes over time under the four thicknesses of 0.5 mm, 1 mm, 2 mm, and 4 mm, as shown in the figure below.

It can be seen from Figure 4 that, under the same municipal sludge temperature, the thicker the thin sludge layer, the longer the total drying time. The increasing drying time of sludge has no obvious change, the peak value of municipal sludge drying rate increases from 0.0514 g·(g·min)$^{-1}$ to 0.2645 g·(g·min)$^{-1}$, the constant drying time also shows little change, and the decreasing drying time increases with the increase in thickness. The thickness of sludge increased from 0.5 mm to 4 mm, and the desiccating time of sludge decreased from 10 min to about 120 min. From this, it can be observed that, under the same thickness, higher drying temperatures result in a faster change in drying rate and a shorter total drying time. The higher the peak point of the drying rate during the up-rate drying stage, the more negligible the time in the constant rate drying stage, leading to a shortened time in the down-rate drying stage. Likewise, at the same temperature, thinner sludge exhibits a faster change in drying rate and a shorter total drying time. However, there is no significant change in the time of rise-rate drying and iso-rate drying, but the time of down-rate drying is noticeably shortened. Consequently, in the municipal sludge treatment process, measures such as timing agitation should be adopted during the deceleration stage to enhance the drying rate, conserve energy, and reduce costs.

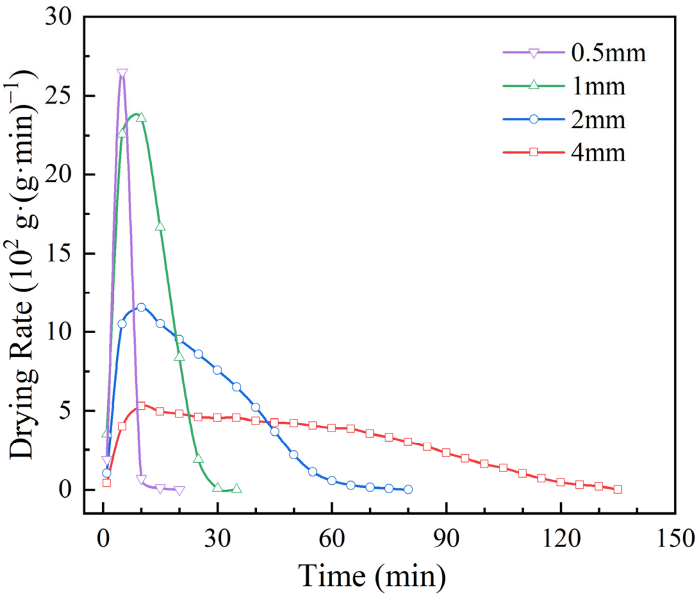

Figure 4. The relationship between sludge drying rate and thickness at temperature T = 75 °C.

4. Model Analysis and Goodness Evaluation

The drying process is a complicated heat and mass transfer process. Through numerous domestic and international scientific research, workers experiment with different materials to uncover the mechanisms and characteristics of the drying process. They summarize the experience of many incomplete experiences and theories, experiment with a material drying mathematical model, and, based on the characteristics of sludge and on tests on the working condition of municipal sludge, choose several common mathematical models of sludge drying. These are used to quantitatively describe the drying law of municipal sludge and predict and verify the accuracy and reliability of the test data.

The following table uses seven commonly used thin-layer drying models to simulate and analyze the drying curve of the thin municipal sludge layer. The moisture content measured in the drying test of 2 mm thin layer of municipal sludge at five temperatures (45 °C, 55 °C, 75 °C, 105 °C and 135 °C) is taken as the sample value. Origin software was used to fit the thin-layer drying model of sludge at different temperatures, and each parameter value of the drying model was obtained. The measured data obtained from the municipal sludge drying test was fitted and verified by the commonly used drying mathematical model. Standard deviation (S_D), residual sum of squares (R_{SS}), Chi-square (χ^2), and correlation coefficient R^2 were used to evaluate the goodness of fit of the model.

The commonly used drying mathematical model is evaluated using fitting parameter values and the Origin software, as shown in Table 2 below.

Table 2. Fitting parameter values and accuracy evaluation of each model.

Model Name	Value	2 mm					Reference
		135 °C	**105 °C**	**75 °C**	**55 °C**	**45 °C**	
Lewis MR = exp(−kt)	k	0.2195	0.1238	−0.02133	−0.0168	−0.0114	[23]
	R^2	0.9402	0.9493	0.3204	−1.1088	−1.4925	
	S_D	0.0147	0.0059	0.1672	0.0144	0.0047	
	χ^2	0.0048	0.0042	1245.8521	1548.067	1693.049	
	R_{SS}	0.0678	0.0967	21,072.4211	43,345.866	79,573.280	
Logarithmic MR = aexp(−kt) + b	a	1.1630	1.2923	90.6649	106.5904	161.6415	[24]
	b	−0.3160	−0.4061	−8.9063	−25.4111	−78.8582	
	k	0.1048	0.0566	0.0378	0.0120	0.0033	
	R^2	0.9907	0.9909	0.9895	0.9833	0.9852	
	S_D	0.1704	0.1686	5.1016	7.7434	21.7998	
	χ^2	0.0008	0.0008	7.2975	4.9380	3.2936	
	R_{SS}	0.0091	0.0159	94.8676	128.3873	148.2111	

Table 2. *Cont.*

Model Name	Value	2 mm					Reference
		135 °C	105 °C	75 °C	55 °C	45 °C	
Page MR = exp(−kt^n)	k	0.1789	0.0779	−0.0293	−4.6016	−5.5922	[25]
	n	1.1163	1.2015	0.9250	−0.0862	−0.1073	
	R^2	0.9443	0.9586	−0.6839	0.2637	0.4063	
	S_D	0.1682	0.1053	0.5681	0.2415	0.3173	
	χ^2	0.0486	0.0034	1174.7973	540.5235	403.3151	
	R_{SS}	0.0632	0.0755	17,621.9600	14,594.1360	18,552.495	
Modified Page MR = exp(−(kt)^n)	k	0.2140	0.1195	−0.0230	−0.0120	−0.0114	[26]
	n	1.1180	1.2051	1.0000	1.0000	1.0000	
	R^2	0.9401	0.9586	−0.8066	−1.1952	−1.4925	
	S_D	0.1409	0.0930	0.2276	0.1320	0.0465	
	χ^2	0.0049	0.0034	1240.4390	1611.4903	1729.8539	
	R_{SS}	0.0632	0.0755	17,646.1500	43,510.2380	79,573.2800	
Henderson and Pabis MR = aexp(−kt)	a	0.8969	0.9508	84.3280	85.3485	90.7964	[27]
	k	0.1982	0.1180	0.0493	0.0211	0.0103	
	R^2	0.9544	0.9506	0.9751	0.9652	0.9450	
	S_D	0.0620	0.0478	3.3148	2.6224	2.7258	
	χ^2	0.0037	0.0041	17.3562	25.5843	37.3831	
	R_{SS}	0.0480	0.0900	242.9874	690.7748	1719.6243	
Two-term exponential MR = aexp(−kt) + bexp(−$k_1$at)	a	0.3899	0.4129	37.6105	7.0574	93.2083	[28]
	b	0.5070	0.5379	46.7151	78.2889	−13.0433	
	k	0.1982	0.1179	0.0493	0.0211	0.0106	
	k_1	0.5084	0.2858	0.0013	0.0030	1.0001	
	R^2	0.9461	0.9456	0.9732	0.9624	0.9471	
	S_D	0.7100	0.7543	249.0900	489.7758	87.3237	
	χ^2	0.0044	0.0045	18.6914	27.6310	35.9266	
	R_{SS}	0.0480	0.0900	242.9877	690.7754	1580.7692	
Midilli MR = aexp(−kt^n) + bt	a	0.7098	0.7543	76.8406	76.7693	77.5928	[29]
	k	−0.0036	−0.0401	0.0114	0.0028	0.0006	
	n	4.58×10^{-9}	6.41×10^{-11}	1.4097	1.4445	1.5195	
	b	−0.0596	−0.0397	−0.0221	−0.0335	−0.0441	
	R^2	0.9229	0.9503	0.9981	0.9985	0.9985	
	S_D	30.5821	1.5786	1.1165	0.6864	0.6008	
	χ^2	0.0054	0.0041	1.3399	1.1407	1.0414	
	R_{SS}	0.0598	0.0824	16.0783	28.5171	45.8205	

Where k and k_1 are the empirical coefficients of the model; a, b, and n are empirical constants of the model.

It can be seen from Table 2 above that the correlation coefficients R^2 of the Logarithmic Model all exceed 0.99 under the medium temperature drying condition of 105–135 °C. The standard deviation (S_D), Chi-square (χ^2), and residual square (R_{SS}) were also the smallest in all models. It shows that the Logarithmic model is more suitable to describe the change of drying water content and time prediction of thin municipal sludge layer in the range of 105–135 °C. Under low-temperature drying conditions (45~75 °C), R^2 of the Midilli model was more than 0.99, and standard deviation (S_D), Chi-square (χ^2), and residual square (R_{SS}) were the smallest among all models. The results show that the Midilli model is more suitable to describe the variation of drying water content and the prediction of drying time of thin municipal sludge layer at 45–75 °C.

Modeling

Through fitting and verification of the above seven mathematical models and test values, it can be seen that Temperature conditions in the Logarithmic model are suitable for the municipal sludge drying process, and the Midilli model is more suitable for the low-temperature condition of the municipal sludge drying process. To unify the municipal sludge in the moderate temperature drying mathematical model, we simply introduce a Logarithmic model of a zero dimension number n to eliminate the equations describing the drying process in early and late moisture prediction error [30]. After considering the influence of water content change at the initial moment of municipal sludge, a linear term is introduced to modify Logarithmic model into Midilli model. Finally, to improve the description of the single diffusion model of the equation, an additional empirical constant term C is added to obtain a new model, namely the modified model Midilli, whose expression is MR = aexp(−kt^n) + bt + c. The fitting comparison between this model and the measured values of municipal sludge tests at different temperatures is shown below (Figure 5).

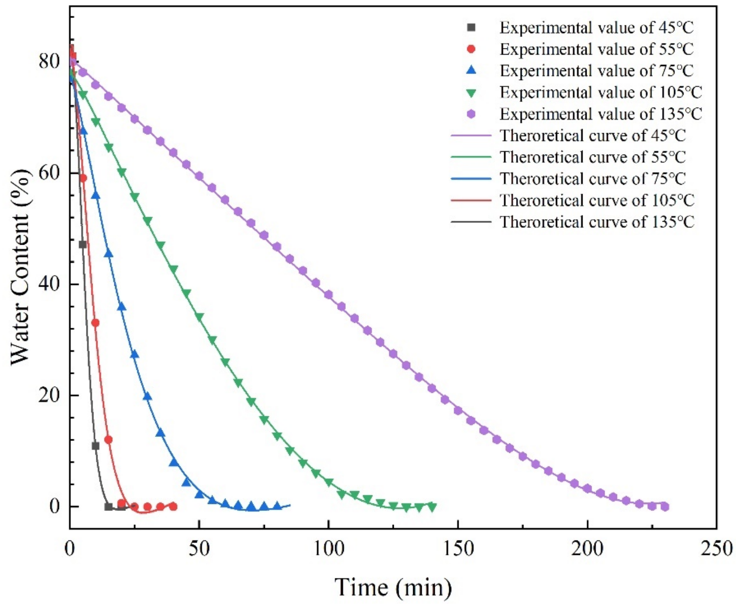

Figure 5. Fitting comparison between experimental and theoretical values.

The goodness of fit of the model was verified by taking the fitting results under five working conditions of 2 mm thickness and 45~135 °C drying temperature as an example, as shown in Table 3 below.

Table 3. Parameter values and evaluation excellence of modified Midilli model fitting.

Model Name	Value	135 °C	105 °C	75 °C	55 °C	45 °C
Modified Midilli MR = aexp(−kt^n) + bt + c	a	84.0776	87.9204	96.3962	60.6362	0.0006
	k	0.0283	0.0224	0.0143	−0.0307	−0.4738
	n	1.8481	1.6068	1.2798	0.7834	0.5681
	b	0.0405	0.1383	0.2061	−2.0067	−0.4305
	c	−1.2315	−5.5717	−18.7135	17.0443	80.5924
	R^2	0.9998	0.9983	0.9992	0.9996	0.9998
	S_D	2.2793	4.5312	6.7908	0.8545	0.1309
	χ^2	0.2509	1.9180	0.6295	0.2722	0.1757
	R_{SS}	1.0036	11.5079	9.4428	7.3486	8.2559

The comparison between the fitting value of the modified Midilli model in Table 3 and the fitting value of the traditional model in Table 2 shows that the correlation coefficient $R^2 > 0.998$, and the standard deviation S_D, Chi-square χ^2, residual square, and R_{SS} are also smaller. The accuracy of the fitting degree of this evaluation is better than that of the traditional drying model. It shows that the modified Midilli model can simulate municipal sludge drying well at 45~135 °C. Because the new model can accurately describe the variation trend of the water content of municipal sludge, it has a certain guiding significance for the subsequent treatment and disposal of sludge.

5. Kinetic Analysis of Sludge Water Transfer

Based on Fick's second law and Arrhenius law of liquid diffusion theory, kinetic characteristics of sludge drying were analyzed from two aspects of the effective water diffusion coefficient and sludge apparent activation energy.

5.1. Effective Water Diffusion Coefficient Calculation

Water transfer during sludge drying is a complex process of heat and mass transfer, which is reflected in Fick's Second Law. The integral expression of Fick's second law is as follows [31]:

$$MR = \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\exp\left(-(2n+1)^2\frac{\pi^2 D_{eff} t}{4\delta^2}\right) \quad (1)$$

where D_{eff} is the effective water diffusion coefficient of the material in $m^2 \cdot s^{-1}$; t is the drying time in the unit s; δ is the thickness of material drying in the unit m; n is the number of experimental samples.

When the drying time is long, only the first term of the above equation [32] can be taken, and then logarithmic deformation can be taken on both sides of the equation at the same time, so that:

$$\ln MR = \ln\frac{8}{\pi^2} - \frac{\pi^2 D_{eff}}{4\delta^2}t \quad (2)$$

It can be seen from Formula (2) above that there is a linear relationship between the logarithm of the effective water diffusion coefficient and drying time. Therefore, this line can be fitted according to the relationship between water content measured in the test and time, and its slope can be obtained from the fitted $\ln D_{eff}$-t function graph, and then the effective water diffusion coefficient D_{eff} can be obtained according to the formula below.

$$D_{eff} = -\frac{4k\delta^2}{\pi^2} \quad (3)$$

Considering the different degrees of water migration in the sludge, the sludge drying rate curve should be divided into three main parts, namely, the ascending stage, the constant stage and the descending stage, and the effective diffusion coefficient of each stage can be calculated, respectively. In the up-rate drying stage, the adsorbed water on the surface of sludge is mainly removed. In the constant rate drying stage, the free water needs a certain amount of heat as a driving force to diffuse to the surface before evaporation and removal. In the down-rate drying stage, part of the bound water in the sludge is removed, which requires more energy to destroy the molecular structure. Taking the drying temperature of 75 °C and the test value of drying conditions of municipal sludge with different thicknesses as an example, the effective water diffusion coefficient can be calculated as 2.63×10^{-9} $m^2 \cdot s^{-1}$ according to the Expression (3). Similarly, the effective water diffusion coefficient $\ln D_{eff}$ of municipal sludge with other thicknesses can be calculated. The results are shown in Table 4.

Table 4. Effective water diffusivity of the thin sludge layer with different thicknesses at a drying temperature of 75 °C.

Thickness	Slope	D_{eff}	R^2
0.5 mm	-7.43×10^{-3}	7.53×10^{-10}	0.86467
1 mm	-4.33×10^{-3}	1.75×10^{-9}	0.92945
2 mm	-1.62×10^{-3}	2.63×10^{-9}	0.82413
4 mm	-6.55×10^{-4}	4.25×10^{-9}	0.96312

It can be seen from Table 4 above that in the sludge drying process at 75 °C, when the thickness decreases from 4 mm to 0.5 mm, the effective diffusion coefficient of municipal sludge moisture increases from -7.43×10^{-3} $m^2 \cdot s^{-1}$ to -6.55×10^{-4} $m^2 \cdot s^{-1}$. Therefore, when the temperature is constant, the path of water diffusion in the sludge is reduced and the mass transfer resistance is small. Therefore, the thinner the sludge thickness is, the larger the D_{eff} of the effective water diffusion coefficient in the sludge drying process is.

Taking the test values of the municipal sludge with a thickness of 2 mm at different temperature drying conditions as an example, the effective water diffusion coefficient

D_{eff} at three stages at different temperatures can be calculated. The results are shown in Table 5 below.

Table 5. Effective water diffusivity of a 2 mm thick thin sludge layer at different temperatures.

Temperature	Drying Stage	D_{eff}	R^2
45 °C	Acc phase	1.535×10^{-10}	0.99975
	Constant phase	2.3442×10^{-10}	0.98333
	Slow downstage	9.926×10^{-10}	0.93221
55 °C	Acc phase	3.664×10^{-10}	0.99757
	Constant phase	6.391×10^{-10}	0.99136
	Slow down stage	1.669×10^{-09}	0.97264
75 °C	Acc phase	9.157×10^{-10}	0.99445
	Constant phase	5.642×10^{-9}	0.94621
	Slow downstage	3.842×10^{-9}	0.98297
105 °C	Acc phase	1.413×10^{-9}	0.974597
	Constant phase	3.015×10^{-9}	0.99546
	Slow downstage	1.271×10^{-8}	0.94625
135 °C	Acc phase	2.351×10^{-9}	0.94549
	Constant phase	5.755×10^{-9}	0.99057
	Slow downstage	2.479×10^{-8}	0.95809

As can be seen from Table 5 above, in the sludge drying process of sludge with a thickness of 2 mm, the effective diffusion coefficient of municipal sludge moisture ranges from 1.535×10^{-10} to 2.351×10^{-9} $m^2 \cdot s^{-1}$ at 45~135 °C in the speed-increasing stage. At the constant velocity stage, the effective diffusion coefficient of municipal sludge moisture ranges from 2.344×10^{-10} $m^2 \cdot s^{-1}$ to 5.755×10^{-9} $m^2 \cdot s^{-1}$ at 45~135 °C. In the deceleration stage, the effective diffusion coefficient of water in municipal sludge is 9.926×10^{-10}~2.479×10^{-8} $m^2 \cdot s^{-1}$ at 45~135 °C. Therefore, when the thickness is constant, the higher the temperature is, the larger the temperature difference between the heat transfer of water inside and outside the sludge is, and the stronger the driving force of water diffusion outward. Therefore, the diffusion coefficient in the process of sludge drying is greater.

5.2. Apparent Activation Energy Calculation

The functional relation between the effective diffusion coefficient and the reciprocal of the thermodynamic temperature T can be established by the Arrhenius equation. The Expression (4) is as follows:

$$D_{eff} = D_0 \exp\left(-\frac{E_a}{RT}\right) \quad (4)$$

where D_0 is the diffusion factor of the equation, and its unit is $m^2 \cdot s^{-1}$; E_a is the apparent activation energy in $kJ \cdot mol^{-1}$; T is the thermodynamic temperature of drying in K; R is the gas constant, and its value is generally 8.314×10^{-3} $kJ \cdot (mol \cdot k)^{-1}$. By taking logarithmic deformation of both sides of Equation (4), the following Expression (5) can be obtained:

$$\ln D_{eff} = \ln D_0 - \frac{E_a}{R}\frac{1}{T} \quad (5)$$

According to the Expression (5), we can know where the apparent activation energy E_a was put forward by Arrhenius when he explained his empirical formula. The logarithm of the effective water diffusion coefficient is in a linear relationship with the reciprocal thermodynamic temperature corresponding to the test drying, so the apparent activation energy can be obtained by fitting the slope of the line according to $\ln D_{eff} - 1/T$. The diffusion factor D_0 of the equation can be obtained by fitting the intercept of the line according to $\ln D_{eff} - 1/T$. When the sludge thickness is 2 mm, the slope, intercept and correlation coefficient of the linear fitting of the three drying stages $\ln D_{eff} - 1/T$ are shown in the table below.

According to the slope and intercept in Table 6 above, there is a linear relationship between the logarithm of the effective diffusion coefficient of sludge and the inverse of the thermodynamic temperature of drying. Through the calculation of Expression (5), it is easy to calculate that the diffusion factor of municipal sludge in the rating-up drying stage is 1.789×10^{-5} $m^2 \cdot s^{-1}$, and the apparent activation energy is 29.772 $kJ \cdot mol^{-1}$. In the same way, the diffusion factors in the constant drying stage and the reduced drying stage were 3.487×10^{-4} $m^2 \cdot s^{-1}$ and 2.876×10^{-4} $m^2 \cdot s^{-1}$, respectively, and the apparent activation energies were 39.202 $kJ \cdot mol^{-1}$ and 37.129 $kJ \cdot mol^{-1}$, respectively. When the thickness is constant, the apparent activation energy of sludge from large to small is the falling rate drying process, the constant speed drying process, and the higher speed drying; the reason is that the acc drying stage is mainly performed to remove sludge adsorption on the surface of the water, the constant-speed drying stage is mainly performed to remove sludge clearance of free water. Free water needs a certain amount of heat as a driving force to spread to the surface and evaporation, so the constant-speed drying stage of the activation energy is higher than acc drying stage. In the down-rate drying stage, to remove part of the water bound to the sludge, more energy is needed to destroy the molecular structure, so the activation energy in the down-rate drying stage is higher than that in the constant rate drying stage.

Table 6. The slope and intercept of three stages of linear fitting of 2 mm sludge $\ln D_{eff} - 1/T$.

Drying Stage	Slope	Intercept	R^2
Acc phase	−0.358	−10.931	0.94348
Constant phase	−0.447	−7.961	0.97157
Slow down stage	−0.472	−5.851	0.99516

As evident from the results, the diffusion coefficient increases with the temperature but decreases with thickness. The apparent activation energy is related to the extent of moisture diffusion in each drying stage. The apparent activation energy for urban sludge increases sequentially during the up-rate drying stage, constant rate drying stage, and down-rate drying stage, with values of 29.772 $kJ \cdot mol^{-1}$, 37.129 $kJ \cdot mol^{-1}$, and 39.202 $kJ \cdot mol^{-1}$, respectively. This relationship is closely associated with the moisture migration, diffusion, and mass transfer resistances in the sludge, indicating that when treating urban sludge, spreading the sludge to reduce its thickness would be beneficial in shortening the drying time.

6. Conclusions

Based on experimental data from drying municipal sludge samples of different thicknesses and temperatures using simulated residual metallurgical heat, improvements were made to traditional drying models. The new model provides better descriptions of the moisture content, effective diffusion coefficient, and apparent water diffusion coefficient during the sludge drying process. The activation energy analysis of urban sludge drying under different conditions and scenarios led to the following conclusions:

1. In the urban sludge treatment process, methods such as timed stirring can be employed when the drying rate is low to enhance the sludge drying rate.
2. The modified Midilli model accurately describes the moisture content and drying equilibrium time within the range of 45–135 °C while also allowing for energy consumption calculations. This is of practical significance for sludge treatment and disposal.
3. The diffusion coefficient increases with temperature but decreases with thickness. The apparent activation energy is related to water diffusion in each stage, increasing successively in the ascending rate, constant rate, and falling rate stages. This is attributed to moisture migration, diffusion, and mass transfer resistances.

Author Contributions: Conceptualization, A.J., L.L., H.L. and Y.L.; Data curation, Z.T.; Formal analysis, H.G.; Funding acquisition, A.J., H.L. and Y.L.; Investigation, Z.T.; Methodology, H.G.; Project administration, A.J. and L.L.; Resources, H.G.; Software, Z.T.; Supervision, Y.L.; Validation, H.G. and Z.T.; Visualization, H.L.; Writing—original draft, H.G.; Writing—review and editing, H.G., Z.T., A.J. and L.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research is funded by the Soft Science Research Program (Project No.: 22554005D), project name: Carbon Emission Status and Carbon Neutral Technology Innovation Roadmap of Hebei Iron and Steel Industry. And the Natural Science Foundation of Hebei Province, project number E2022209138.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: All the data generated or analyzed in this study are included in this published article.

Conflicts of Interest: The authors declare that they have no conflict of interests.

References

1. Lei, W. Investigation on the Sludge Yield of Municipal Wastewater Treatment Plants in Key Watershed of China. *China Water Wastewater* **2018**, *34*, 23–27. [CrossRef]
2. Barati, M.; Jahanshahi, S. Granulation and Heat Recovery from Metallurgical Slags. *J. Sustain. Metall.* **2020**, *6*, 191–206. [CrossRef]
3. Zhang, W.; Qi, D.; Wu, S.; Dong, J.; Zhao, W. Effect of Different Pretreatment Methods on Sewage Sludge Anaerobic Digestion. *Trans. Chin. Soc. Agric. Mach.* **2014**, *45*, 187–198. [CrossRef]
4. Oladejo, J.; Shi, K.; Luo, X.; Yang, G.; Wu, T. A Review of Sludge-to-Energy Recovery Methods. *Energies* **2019**, *12*, 60. [CrossRef]
5. Werther, J.; Ogada, T. Sewage sludge combustion. *Prog. Energy Combust.* **1999**, *25*, 55–116. [CrossRef]
6. Fourti, O.; Jedidi, N.; Hassen, A. Humic substances change during the co-composting process of municipal solid wastes and sewage sludge. *World J. Microbiol. Biotechnol.* **2010**, *26*, 2117–2122. [CrossRef]
7. Schnell, M.; Horst, T.; Quicker, P. Thermal treatment of sewage sludge in Germany: A review. *J. Environ. Manag.* **2020**, *263*, 110367. [CrossRef]
8. Wang, W.; Luo, Y.; Qiao, W. Possible solutions for sludge dewatering in China. *Front. Environ. Sci. Eng. China* **2010**, *4*, 102–107. [CrossRef]
9. Werle, S.; Wilk, R.K. A review of methods for the thermal utilization of sewage sludge: The Polish perspective. *Renew. Energy* **2010**, *35*, 1914–1919. [CrossRef]
10. Praspaliauskas, M.; Pedišius, N. A review of sludge characteristics in Lithuania's wastewater treatment plants and perspectives of its usage in thermal processes. *Renew. Sustain. Energy Rev.* **2017**, *67*, 899–907. [CrossRef]
11. Rodrigo, D.F.B. Composition, Production, and Treatment of Sewage Sludge. In *Sustainable Sewage Sludge Management and Resource Efficiency*; Başak, K.T., Ed.; IntechOpen: Rijeka, Croatia, 2020; p. 3. ISBN 978-1-83962-707-1.
12. Elbaz, A.A.; Aboulfotoh, A.; ElGohary, E.H.; Reham, M. Review Classification of sludge drying beds SDB (conventional sand drying beds CSDB, Wedge-wire, Solar, and Vacuum assisted and paved drying beds PDB). *J. Mater. Environ. Sci.* **2020**, *11*, 593–608.
13. Tic, W.J.; Guziałowska-Tic, J.; Pawlak-Kruczek, H.; Woźnikowski, E.; Zadorożny, A.; Niedźwiecki, Ł. Novel Concept of an Installation for Sustainable Thermal Utilization of Sewage Sludge. *Energies* **2018**, *11*, 748. [CrossRef]
14. Bennamoun, L.; Arlabosse, P.; Léonard, A. Review on fundamental aspect of application of drying process to wastewater sludge. *Renew. Sustain. Energy Rev.* **2013**, *28*, 29–43. [CrossRef]
15. Dai, X.T.; Qi, Y.H.; Zhang, C.X.; Xu, H.C.; Yan, D.L.; Hong, Y.C. Analysis of Blast Furnace Slag Quenching Dry Granulation Process. *J. Iron Steel Res.* **2007**, *19*, 14–19. [CrossRef]
16. Chen, X.D.; Putranto, A. *Modelling Drying Processes: A Reaction Engineering Approach*; Cambridge University Press: Cambridge, UK, 2013; ISBN 9781107012103.
17. Ambawat, S.; Sharma, A.; Saini, R.K. Mathematical Modeling of Thin Layer Drying Kinetics and Moisture Diffusivity Study of Pretreated Moringa oleifera Leaves Using Fluidized Bed Dryer. *Processes* **2022**, *10*, 2464. [CrossRef]
18. Ramachandran, R.P.; Akbarzadeh, M.; Paliwal, J.; Cenkowski, S. Computational Fluid Dynamics in Drying Process Modelling—A Technical Review. *Food Bioprocess Technol.* **2018**, *11*, 271–292. [CrossRef]
19. Jayasuriya, H.; Pathare, P.B.; Al-Attabi, Z.; Al-Hamdani, A. Drying Kinetics and Quality Analysis of Coriander Leaves Dried in an Indirect, Stand-Alone Solar Dryer. *Processes* **2023**, *11*, 1596. [CrossRef]
20. Bennamoun, L.; Ndukwu, M.C. Modeling and simulation of drying kinetics/curves: Application to building materials. *J. Build. Pathol. Rehabil.* **2021**, *7*, 8. [CrossRef]
21. Zia, S.; Khan, M.R.; Aadil, R.M. Kinetic modeling of different drying techniques and their influence on color, bioactive compounds, antioxidant indices and phenolic profile of watermelon rind. *J. Food Meas. Charact.* **2023**, *17*, 1068–1081. [CrossRef]
22. Doymaz, I. Thin-Layer Drying of Spinach Leaves in a Convective Dryer. *J. Food Process. Eng.* **2009**, *32*, 112–125. [CrossRef]

23. Lewis, W.K. The Rate of Drying of Solid Materials. *J. Ind. Eng. Chem.* **1921**, *13*, 427–432. [CrossRef]
24. Ruixun, J.; Aimin, L.; Wang, W. Thin layer drying characteristics and kinetics model of dewatered sludge. *China Environ. Sci.* **2009**, *29*, 22–25. [CrossRef]
25. Page, G.E. *Factors Influencing the Maximum Rates of Air Drying Shelled Corn in Thin Layers*; ProQuest Dissertations Publishing: Ann Arbor, MI, USA, 1949.
26. Yin, L.B.; Xu, Q.S.; Peng, X.W.; Yu, Z.S.; Liao, Y.F.; Ma, X.Q. Dynamic analysis on thin layer drying of dyeing sludge. *Therm. Power Gener.* **2015**, *4*, 62–66. [CrossRef]
27. Henderson, S.M. Grain drying theory, I. Temperature effect on drying coefficient. *J. Agric. Eng. Res.* **1961**, *6*, 169–173.
28. Sharaf-Eldeen, Y.; Blaisdell, J.; Hamdy, M. A Model for Ear Corn Drying. *Trans. Asae* **1980**, *23*, 1261–1265. [CrossRef]
29. Midilli, A.; Kucuk, H.; Zhao, J. A New Model for Single-Layer Drying. *Dry Technol.* **2002**, *20*, 1503–1513. [CrossRef]
30. Kong, L.; Yang, X.; Dong, J.; Jingyi, Z. Research Progress of Thin-layer Drying Model for Paper Sludge. *China Pulp Pap.* **2019**, *38*, 70–75. [CrossRef]
31. Akgun, N.A.; Doymaz, I. Modelling of olive cake thin-layer drying process. *J. Food Eng.* **2005**, *68*, 455–461. [CrossRef]
32. Koua, K.B.; Fassinou, W.F.; Gbaha, P.; Toure, S. Mathematical modelling of the thin layer solar drying of banana, mango and cassava. *Energy* **2009**, *34*, 1594–1602. [CrossRef]