processes

Article

Optimal Discrete Element Parameters for Black Soil Based on Multi-Objective Total Evaluation Normalized-Response Surface Method

Zhipeng Wang [†], Tong Zhu [*,†], Youzhao Wang, Feng Ma, Chaoyue Zhao and Xu Li

Institute of Process Equipment and Environmental Engineering, School of Mechanical Engineering and Automation, Northeastern University, Shenyang 110819, China
[*] Correspondence: tongzhu@mail.neu.edu.cn; Tel.: +86-024-83679926
[†] These authors contributed equally to this work.

Abstract: The lack of accurate black soil simulation model parameters in the design and optimization of soil remediation equipment has led to large errors in simulation results and simulation outcomes, which to some extent restricts the development of soil remediation equipment. Accurate discrete element parameters can improve the efficiency of soil remediation equipment. To improve the reliability of the discrete element contact parameters for black soil, a set of optimal discrete element contact parameters was found that could comprehensively represent a variety of particle sizes and minimize error. In this paper, the best discrete element contact parameters were selected by using a multi-indicator total evaluation normalization method combined with the response surface method, combined with black soil solid and simulated stacking tests. First, the physical parameters of the black soil and the accumulation angle were determined. Next, Plackett–Burman tests were carried out for each grain size in turn to obtain the contact parameters that had a significant effect on the black soil accumulation angle. The important parameters obtained for different particle sizes are all as follows: black soil–black soil static friction coefficient, black soil–black soil rolling friction coefficient, and black soil–stainless steel rolling friction coefficient. In conjunction with the Plackett–Burman test screening results, the steepest climb test was designed for six grain sizes to optimize the range of values. To find the optimal contact parameters for the different particle sizes based on the final results of Box–Behnken experiments, the discrete element parameters of black soil were optimized for the different particle sizes of black soil by using the multi-indicator total evaluation normalization method and response surface method. The results showed that the black soil–black soil static friction coefficient was 1.045, the black soil–black soil rolling friction coefficient was 0.464, and the black soil–stainless steel rolling friction coefficient was 0.215. The errors for each particle size were reduced by 0.89%, 0.7%, 0.84%, 0.57%, 0.71%, and 0.76% for the best combination of parameters before and after normalization, with an average error reduction of 0.745%. This data provides some reference value for the design and optimization of soil remediation equipment.

Keywords: black soil; stacking angle; discrete element methodology; parameter calibration; response surface methodology; multi-objective homogenization method

check for updates

Citation: Wang, Z.; Zhu, T.; Wang, Y.; Ma, F.; Zhao, C.; Li, X. Optimal Discrete Element Parameters for Black Soil Based on Multi-Objective Total Evaluation Normalized-Response Surface Method. *Processes* **2023**, *11*, 2422. https://doi.org/10.3390/pr11082422

Academic Editor: Fabricio Napoles-Rivera

Received: 18 July 2023
Revised: 4 August 2023
Accepted: 9 August 2023
Published: 11 August 2023

1. Introduction

Due to the complex characteristics of the inter-particle contact mechanics of black soils in northeastern China, it is often necessary to simulate and analyze equipment–soil interactive processes for the design and optimization of equipment [1]. Because of the continuous development and improvement in the theory of discrete elements, it has been commonly applied to studying the interactions between the contact parts of soil with related equipment and the mobility of bulk particles [2–4]. The study of black soil–equipment interaction processes using discrete element systems can improve the efficiency of agricultural equipment [5] and is important for soil protection in black soil areas. The overall particle size of black soil particles is small, and tens of millions or even billions of soil quantities need to be set in the process of the design and optimization of related agricultural equipment, which cannot be effectively simulated due to the limited capacity of ordinary computers. The particle scaling method is currently a more feasible treatment method and has been widely used in engineering research. The method scales up the particles in the original system and reduces the number of discrete units in the model so that the original physical model problem can be solved in a reasonable and effective time. However, different scaling ratios are set due to the different purposes and needs of the researcher and different soil scaling can lead to errors between simulation results, which in turn affects the design and optimization of agricultural equipment. The construction of a simulation model for black soil requires the setting of the intrinsic and contact parameters of the particles. The intrinsic parameters of black soil particles can be obtained experimentally, while some of the discrete elemental simulation parameters are not easily obtained. Therefore, the discrete element contact parameters of black soil were optimized for different particle sizes of black soil particles by using the multi-indicator total evaluation normalization method and response surface method [6–9].

At present, a great deal of research has been carried out by scholars both nationally and internationally on the calibration of bulk particle parameters. In discrete element simulation parameter calibration, computational efficiency is an essential consideration in engineering applications. For example, Guo et al. [10] used a central integrated design to evaluate the extraction process of the normalized ethanol formulation using the ethanol concentration, solvent amount, extraction time, and the number of extractions as the investigating factors to determine the optimal ethanol extraction process of dulcimer and to predict its extraction parameters. Based on the discrete element method and response surface method (RSM), Li et al. [11] used the uniformity evaluation index to determine the effect of uniformity among the parameters. Shi et al. [12] established models for discrete elements of deciduous date fruits and conducted Plackett–Burman tests and steepest climb tests with response surface optimization for the optimal discrete element contact parameters for deciduous date fruits. Hu et al. [13] showed that the amplified particle size can be used to simulate the original particle size in uniaxial compression experiments based on a particle bed. Lommen et al. [14] verified the validity of the particle scaling theory within a certain range based on penetration tests and rest angle experiments. Hu et al. [15] calibrated and validated the cotton seed contact parameters using the cotton seed stacking angle as a target, combined with practical and simulated tests. Chen et al. [16,17] verified that the filling effect of the scaled particles was not significantly different from the original particles based on simulation experiments. Sakai et al. [18] performed numerical simulations and found that the coarse-grained model had similar results to the original model. Dai et al. [19] performed a calibration of their simulated parameters using the difference between the physical and simulated values for lily bulb stacking angles as a response. Ma et al. [20] calibrated the contact parameters of shotcrete wet blocks with a combination of physical as well as simulated tests. Finally, Xia et al. [21] performed a calibration of parameters for the discrete elements of wet bulk coal simulation with the goal of determining the stacking angles for wet bulk coal.

There are few studies on the calibration system of particle simulation tests in black soil areas using discrete element theory, and there are almost no studies on the calibration of discrete element contact parameters in black soil using the multi-indicator total assessment normalization method combined with the response surface method. Thus, the black soil discrete element parameters require recalibration.

In this study, the parameters of the discrete element contact parameters of black soil were optimized by combining the black soil solid and simulated stacking tests with the multi-indicator total evaluation normalization method combined with the response surface method. The simulated black soil particle contact parameters were optimized for different particle sizes (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm) using the Plackett–Burman test, the steepest climb test, and the Box–Behnken test to obtain the optimal contact parameters for different particle sizes. The angle of repose of the black soil particles was used as the target. The discrete elemental contact parameters of black soils were optimized for different particle sizes by using a multi-indicator general evaluation normalization method. The optimum mix of discrete element parameters for black soils was determined and their accuracy was verified.

2. Materials and Methods

2.1. Definition of the Intrinsic Characteristics of Black Soil

(1) Size distribution of black soil

Black soil samples were selected from the Dalian region in northeast China. The sampling area was Dalian City (39°0′10.58″ N, 121°27′17.02″ E), which has a temperate monsoon climate with an annual rainfall of 550–950 mm, and the soil type is brown loam, black in color, and rich in humus. The five-point method was used to obtain soil samples from 0 to 400 mm of the soil layer in the park, with a sample mass of 500 g per sampling point, and the soil was allowed to dry naturally after sampling. It is relatively common and convenient to apply the sieving method to measure the particle size and particle size distribution of black soils. Thus, the black soil samples were sieved using standard sieves with different apertures. During the test, 300 g of black soil was weighed and placed in the top sieve, the sieve was shaken horizontally and tapped from time to time, and the mass of lime powder in each layer of the sieve was finally weighed. The results are shown in Figure 1. Sample sizes for the black soil particles after sieving were <1 mm, 1–2 mm, 2–3 mm, 3–4 mm, 4–5 mm, and $\geq$5 mm. The mass ratios corresponding to the various particle sizes are 51.4%, 23%, 5.4%, 8%, 5.2% 0.7%, and 7%, respectively. They are provided for the subsequent establishment of the discrete meta-model of black soil particles.

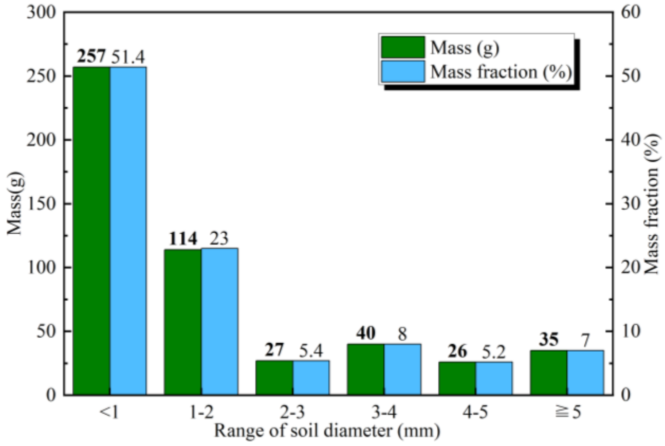


Figure 1. The size distribution of black soil.

(2) Black soil density, Poisson's ratio, and shear modulus

Density is an important simulation base parameter for modeling the discrete elements of black soil. In this paper, the density of the black soil was averaged over five replicates using the hydrometer method, and the density of the black soil was measured to be 2000 kg/m^3. The Poisson's ratio and shear modulus of the black soil particles were chosen to be 0.46 and 1×10^6 pa, taking into account the properties of the black soil [22,23].

(3) Black soil stacking angle

The accumulation angle for black soil particles was measured by the injection method regarding the GB/T 16913.5-1997 national standard and combined with the relevant studies on the rest angle in the existing literature [24]. The black soil particle accumulation experimental setup is shown in Figure 2. The lower end of the funnel has an inner diameter of 50 mm and the cylindrical sump has a diameter of 130 mm, with a distance of 100 mm between them. Before starting the measurement, a sample of black soil particles that were prepared in advance was slowly poured into the center of the funnel from the top center of the funnel.

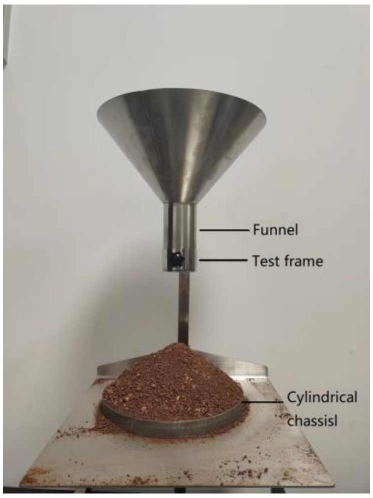


Figure 2. Black soil particle accumulation experimental setup.

The black soil was no longer added to the funnel when the spillage of black soil particles began to occur at the edge of the sump. Until there was no change in the pile height of black soil particles, the pile height, H, of the black soil particles was measured using a steel ruler. The resting angle of the black soil particles was calculated according to Equation (1) and averaged by repeating the experiment five times. The stacking angle of the black soil particles was measured as 36.99°.

$$\theta = \arctan\frac{2\mathrm{H}}{\mathrm{D}} \tag{1}$$

2.2. Black Soil Simulation Contact Model

At present, the object of discrete element studies is generally bulk particles, and, therefore, particle contact models are extremely valuable. In this work, black soil particles were taken from the heavy metal contaminated site and the cohesive force between the particles is small, so the Hertz Mindlin no-slip contact model is used for simulation. It is accurate and efficient in calculating forces. It can accurately calculate the forces between particles and is easy to use. When the granules with a particle radius of R^1 and R^2 are elastically connected, the interparticle nuclear force, F_n, is expressed as [25–27]:

$$F_n = \frac{4}{3}E^*(R^*)^{1/2}\alpha^{3/2} \tag{2}$$

where α is the normal overlap quantity; E^* is the modulus; R^* is an equivalentradius, and E^* and R^* can be expressed as [28]:

$$\frac{1}{E^*} = \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \tag{3}$$

$$\frac{1}{R^*} = \frac{1}{R^1} + \frac{1}{R^2} \tag{4}$$

E_1 and E_2 are the moduli of elasticity of particles 1 and 2; v_1 and v_2 are the Poisson's ratios; and R^1 and R^2 are the equivalent particle radiuses.

The normal damping force F_n^d can be expressed as:

$$F_n{}^d = -2\sqrt{\frac{5}{6}}\beta\sqrt{k_n m^*}v_n{}^{rel} \tag{5}$$

where $v_n{}^{rel}$ is the normal component speed; β is the effect for restitution; k_n is the normal stiffness; and m^* is the equivalent weight.

β, k_n, and m^* are defined as:

$$\beta = \frac{\ln e}{\sqrt{\ln^2 e + \pi^2}} \tag{6}$$

$$k_n = 2E^*\sqrt{r^*\delta_n} \tag{7}$$

$$m^* = \frac{m_1 m_2}{m_1 + m_2} \tag{8}$$

where e is the coefficient of restitution; and m_1 and m_2 are the masses of particles 1 and 2.

The normal damping force, $F_n{}^t$, is expressed as:

$$F_t = -S_t\delta \tag{9}$$

$$S_t = 8G^*\sqrt{R^*\alpha} \tag{10}$$

where δ is tangent overlap quantity; S_t is the tangent stiffness; R^* is the equivalent shear modulus, and G^* is equivalent to the shear modulus.

The normal damping force, F_t^d, can be expressed as:

$$F_t{}^d = -2\sqrt{\frac{5}{6}}\beta\sqrt{k_t m^*}v_t{}^{rel} \tag{11}$$

where $v_t{}^{rel}$ is the relative tangential velocity.

The rolling friction in the simulation can be expressed based on the moment on the particle surface:

$$T_i = -\mu_r F_n R_i \omega_i \tag{12}$$

where μ_r is the coefficient of rolling friction; R_i is a center point and contact point spacing; and ω_i is the vector of unit angular velocity.

2.3. Black Soil Discrete Element Parameters

The simulation parameter for black soil particles and stainless steel were set concerning the relevant domestic and international references and the built-in database of the software. The black soil discrete element parameter used in this paper is shown in Table 1 [29] and was used due to the need for the different scaling of particle sizes and setting different particle sizes for different demand simulations. To minimize the negative effects of the simplified particles and to meet different simulation purposes, the particle size range was enlarged to a maximum of 10 mm. The representative particle sizes were selected for the simulation and were set to 0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm.

The particle density is 2000 kg/m^3, the black soil Poisson's ratio is 0.46, and the black soil shear modulus is 1 MPa. The simulated contact parameters of the materials are relatively different in terms of particle size, density, etc., and cannot be obtained from the relevant references and manuals. In addition, the particle simulation parameters of a single particle size cannot meet the current research needs. Therefore, virtual simulation tests were used for the calibration of discrete element model parameters for black soil.

Table 1. Discrete element simulation parameter table.

Parameter	Value
Density of black soil (kg/m^3)	2000
Poisson's ratio of black soil	0.46
Shear modulus of black soil/Pa	1×10^6
Density of stainless steel/(kg/m^3)	7800
Poisson's ratio of stainless steel	0.3
Shear modulus of stainless steel/Pa	7×10^{10}
Black soil–black soil restitution coefficient	0.2–0.6
Black soil–black soil static friction coefficient	0.2–1.16
Black soil–black soil rolling friction coefficient	0.01–0.7
Black soil–stainless steel restitution coefficient	0.2–0.5
Black soil–stainless steel coefficient of static friction	0.4–0.8
Black soil–stainless steel coefficient of rolling friction	0.05–0.25

2.4. Black Soil Discrete Element Model

The overall particle size of black soil particles is small, and tens of millions or even billions of soil quantities need to be set in the process of the design and optimization of related agricultural equipment, which cannot be effectively simulated due to the limited capacity of ordinary computers. The particle scaling method scales the black soil particle size in the original system and reduces the number of discrete units in the model, thus solving the original physical modeling problem in a reasonably efficient time. Due to the different purposes and needs of researchers, different soil scaling ratios are set, resulting in some errors in the simulation results, which in turn affect the design and optimization of agricultural equipment. Therefore, the discrete elemental contact parameters of black soil particles with different grain sizes were optimized using the normalized method of the overall evaluation of multiple indicators and the response surface method. Based on the measured particle size parameters, considering the usual maximum simulated particle size of 10 mm for black soil, combined with the computer's minimum simulated particle size of 0.75 mm for black soil, the range of simulated particle sizes for black soil was finally determined to be 0.75–10 mm. The particle size range was uniformly set to six groups in preparation for the multi-objective total evaluation normalization method. The discrete element model of black soil particles is illustrated in Figure 3. The black soil creation process was then set to Dynamic. The simulation model of black soil particle accumulation is shown in Figure 4. The lower end of the funnel has an inner diameter of 50 mm, the cylindrical sump diameter is 130 mm, the distance between them is 100 mm and the 0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm particle generation rates were adjusted to 50,000 particles/s, 10,000 particles/s, 10,000 particles/s, 5000 particles/s, 1000 particles/s, and 40 particles/s, respectively. The particle generation quantity was adjusted to infinite. The particle simulation time varied depending on particle size; the fixed time step was set to 22% and the analysis data were saved every 0.01 s. When the particles received at the bottom of the cylinder below the funnel reached the complete overflow state, the particle generation rate was set to 0 particles/s until the end of the complete drop of black soil.

The simulation process for the accumulation of black soil particles of different particle sizes is depicted in Figure 5. The post-processing measurement angles of the black soil accumulation model are shown in Figure 6.

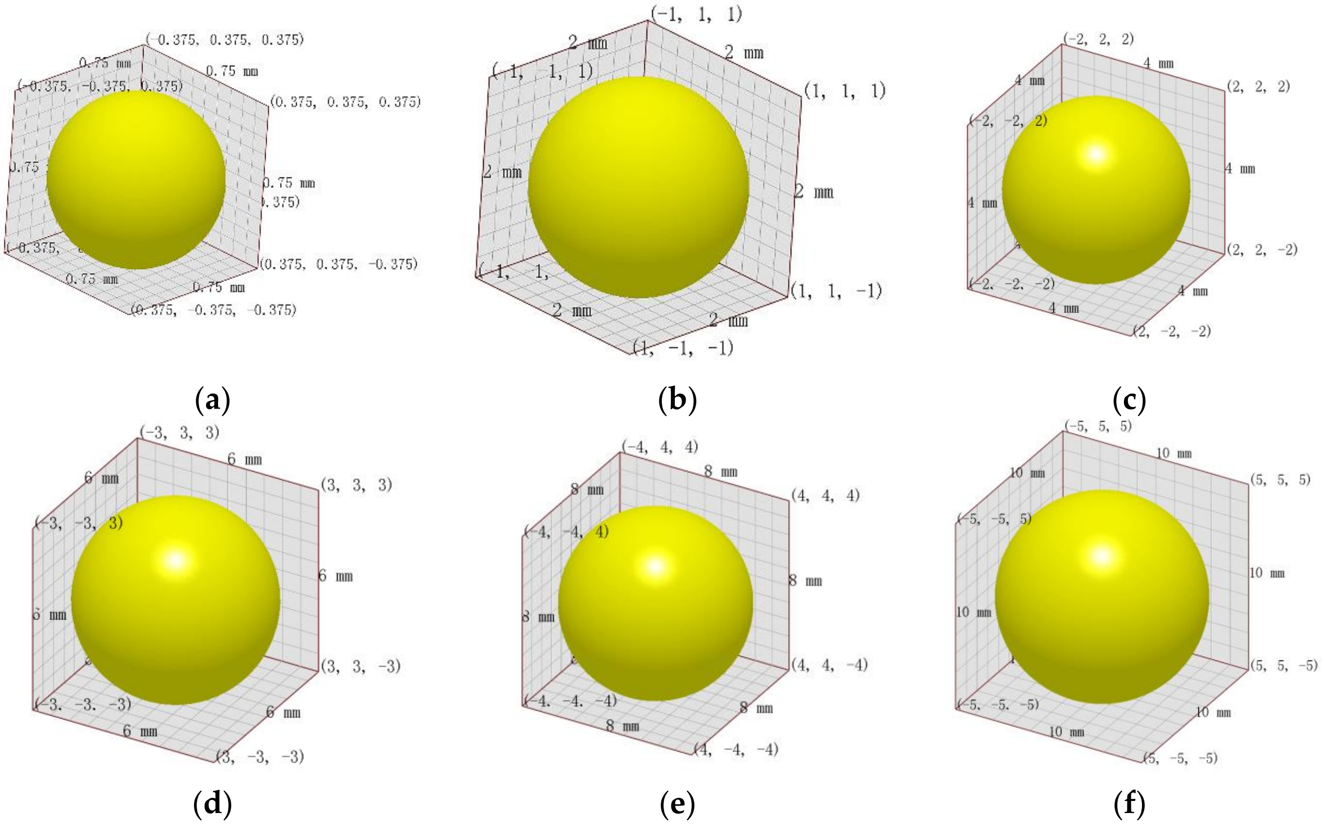


Figure 3. The discrete element model of black soil particles. (**a**) D = 0.75 mm, (**b**) D = 2 mm, (**c**) D = 4 mm, (**d**) D = 6 mm, (**e**) D = 8 mm, and (**f**) D = 10 mm.

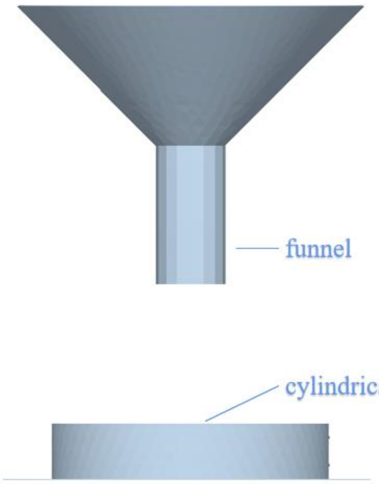
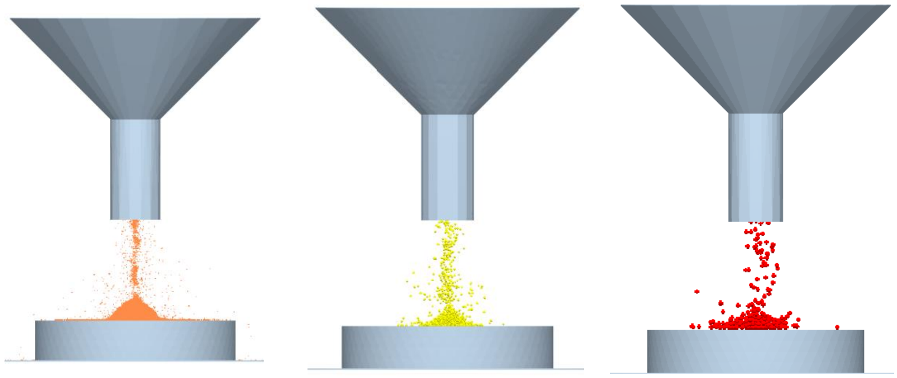
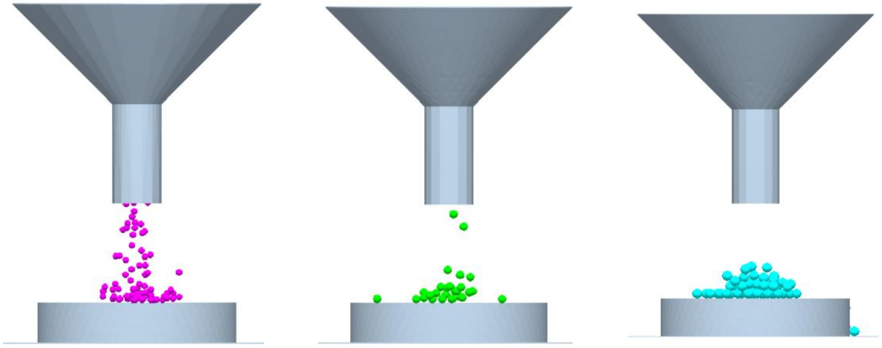


Figure 4. The simulation model of black soil particle accumulation.

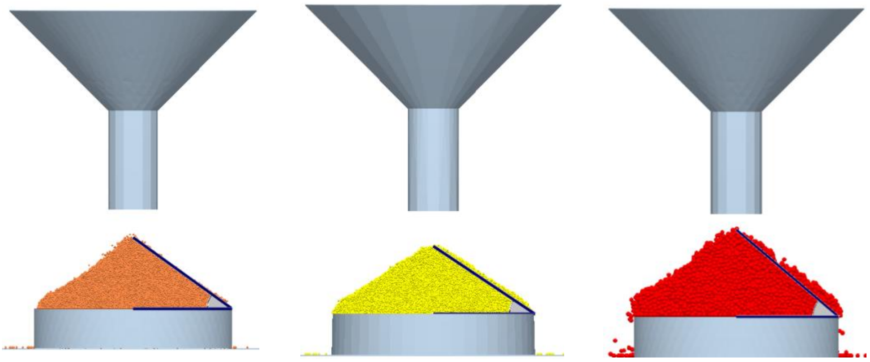


0.75 mm simulation process 2 mm simulation process 4 mm simulation process

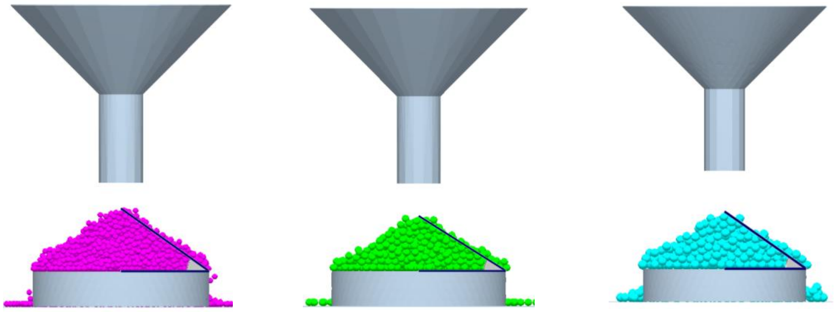


6 mm simulation process 8 mm simulation process 10 mm simulation process

Figure 5. The simulation process of black soil particle accumulation.



0.75 mm post-processing angle measurement 2 mm post-processing angle measurement 4 mm post-processing angle measurement



6 mm post-processing angle measurement 8 mm post-processing angle measurement 10 mm post-processing angle measurement

Figure 6. The post-processing measurement angles of the black soil accumulation model.

2.5. The Calibration Method of the Simulation Parameter

In this study, the contact parameter for black soil was calculated by applying the discrete element and response surface method in combination with the black soil physical and simulated accumulation tests. The simulation process for the accumulation of different black soil particle sizes (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm) is shown below:

(1) The Design-Expert software-based black soil rest angle Plackett–Burman experimental design was repeated for each particle size according to the experimental design steps. The Plackett–Burman test was conducted a total of six times. The parameters of the Plackett–Burman test for different particle sizes are given in Table 2. The experiment was designed with the black soil stacking angle as a target as well as six real variables, A–F, and five virtual parameters, V1–V5. Each simulated contact characteristic had a minimum and maximum value following a certain scale of values, denoted as numbers (−1) and (+1), to filter out the important parameters in the simulated contact parameters of black soil particles [30]. At the same time, a center spot was created for thirteen groups within the simulation trials and each group of trials had five repetitions.

Table 2. Parameters of the Plackett–Burman test for different particle sizes.

Symbol	Parameter	Low Level (−1)	High Level (+1)
A	Black soil–black soil restitution coefficient	0.2	0.4
B	Black soil–black soil static friction coefficient	0.2	0.4
C	Black soil–black soil rolling friction coefficient	0.05	0.1
D	Black soil–stainless steel restitution coefficient	0.2	0.4
E	Black soil–stainless steel coefficient of static friction	0.4	0.8
F	Black soil–stainless steel coefficient of rolling friction	0.05	0.1
V1	Virtual parameter	−1	1
V2	Virtual parameter	−1	1
V3	Virtual parameter	−1	1
V4	Virtual parameter	−1	1
V5	Virtual parameter	−1	1

(2) Based on the findings of the Plackett–Burman test, three simulated parameters of high importance for different particle sizes (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm) were screened and the steepest climbing test was developed. The steepest climbing experiment was conducted six times in total. Every steepest climbing test allows for the error in modeling and practical accumulation of black soil to be calculated. The steepest climb test quickly enter enters the range of meaningful parameters and reaches the target optimum value, and the number of climbing steps is usually taken to be large. The relative error of the black soil particle accumulation angle is calculated as:

$$\zeta = \frac{(\theta - \theta_1)}{\theta_1} \tag{13}$$

where θ—physical of stacking angle, (°) and θ_1—simulation of stacking angle, (°).

(3) From the findings in the steepest climb test and the principle of response surface optimization, the Box–Behnken for black soil rest angle test was developed. Tests were conducted for all (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm) particle sizes. A total of six Box–Behnken tests were performed and the test ranked the screened significance parameters into three classes of low, medium, and high and expressed them in the form of codes (−1), (0), and (+1), using these three classes to carry out the experimental design, while three centroids were chosen for assessing the judgment bias.

(4) After conducting the Plackett–Burman test, steepest climb test, and Box–Behnken test for various black soil grain sizes (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm), the optimum contact parameters for each grain size were obtained. Since the optimal choice of the stacking angle index was not the same for each grain size (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm), Hassan's method was chosen to find the best contact parameters for any black soil grain size from the optimal contact parameters for each grain size normalized by Hassan's method. The formula for OD is:

$$\mathrm{OD} = (\mathrm{D}_1 \times \mathrm{D}_2 \times \mathrm{D}_\mathrm{x}......)^{1/\mathrm{n}} \tag{14}$$

$$D_i = (Y_i - Y_{\min})/(Y_{\max} - Y_{\min}) \tag{15}$$

where $Y_{\max}$ is the maximum value of the stacking angle for each particle size and $Y_{\min}$ is the minimum value of the stacking angle for each particle size.

(5) The Box–Behnken test was designed again based on the discrete meta-stacking angle of the normalized black soil. The test ranked the screened significance parameters into three classes of low, medium, and high, and expressed them in the form of codes (−1), (0), and (+1) and used these three classes to carry out the experimental design, while three centroids were selected to evaluate the judgment error.

3. Results and Discussion

3.1. Plackett–Burman Test

The experimental design and results of the Plackett–Burman test for each black soil particle size (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm) are shown in Table 3. As can be seen from Table 3, in the range of (0.75–6 mm), the black soil accumulation angle becomes larger as the black soil particle size becomes larger; in the range of (6–10 mm), the black soil accumulation angle decreases as the black soil particle size becomes larger. The black soil accumulation angle is maximized at a 6 mm particle size and minimized at a 10 mm particle size. For the same set of contact parameters, the average error of the black soil accumulation angle is in the range of 2–4.5°.

Table 3. Design and results of the Plackett–Burman experiments with different particle sizes.

Number	A	B	C	D	E	F	V1	V2	V3	V4	V5	0.75 mm Angle	2 mm Angle	4 mm Angle	6 mm Angle	8 mm Angle	10 mm Angle
1	−1	1	1	−1	1	1	1	−1	−1	−1	1	21.38	26.9	27.71	27.47	24.03	22.39
2	0	0	0	0	0	0	0	0	0	0	0	20.6	24.72	25.81	25.64	23.88	22.51
3	1	−1	−1	−1	1	−1	1	1	−1	1	1	17.11	21.56	22.31	20.95	20.89	19.68
4	1	1	1	−1	−1	−1	1	−1	1	1	−1	21.85	25.1	27.14	27.1	24.47	23.57
5	1	−1	1	1	1	−1	−1	−1	1	−1	1	19.09	22.38	24.03	24.23	22.56	21.39
6	−1	−1	−1	1	−1	1	1	−1	1	1	1	20.35	22.18	23.03	22.78	21.8	20.75
7	1	−1	1	1	−1	1	1	1	−1	−1	−1	21.4	23.49	24.52	25.3	23.35	22.27
8	−1	1	1	1	−1	−1	−1	1	−1	1	1	22.92	24.16	25.99	26.2	25.64	24.52
9	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	17.1	20.52	21.8	21.31	20.41	20.2
10	1	1	−1	−1	−1	1	−1	1	1	−1	1	20.4	24.35	25.41	26.12	24.23	22.28
11	1	1	−1	1	1	1	−1	−1	−1	1	−1	21.5	25.8	25.99	26.79	25.33	23.16
12	−1	1	−1	1	1	−1	1	1	1	−1	−1	18.1	23.58	24.23	23.75	22.35	21.06
13	−1	−1	1	−1	1	1	−1	1	1	1	−1	19.1	24.12	24.52	24.7	23.99	22.24

Note: Variables A–F and V1–V5 have the same meaning as in Table 2.

The significance of each simulation parameter is presented in Table 4, and the test results for each particle size (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm) were analyzed comparatively using Design-Expert software 11.

As can be seen in Table 4, all particle sizes (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm) differed in their P-ranking for their significant contribution to the black soil particle accumulation angle. Among them, the contribution of contact parameters corresponding to the stacking angle was ranked equally for the 4 mm and 6 mm particle sizes.

Table 4. Significance analysis of Plackett–Burman test parameters for different particle sizes.

Parameters	0.75 mm Contribution/ Significance	2 mm Contribution/ Significance	4 mm Contribution/ Significance	6 mm Contribution/ Significance	8 mm Contribution/ Significance	10 mm Contribution/ Significance
A	1.1807% (6)	0.33% (6)	0.99% (4)	2.85% (4)	1.83% (5)	0.28% (6)
B	29.52% (1)	54.77% (1)	58.64% (1)	51.31% (1)	45.72% (1)	2.31% (3)
C	25.62% (2)	14.90% (3)	27.52% (2)	25.72% (2)	21.89% (2)	14.05% (2)
D	8.45% (5)	0.21% (5)	0.27% (5)	0.30% (5)	2.43% (4)	1.08% (4)
E	12.28% (4)	4.61% (4)	0.18% (6)	0.13% (6)	0.15% (6)	0.58% (5)
F	12.99% (3)	20.38% (2)	7.16% (3)	14.4% (3)	11.03% (3)	42.13% (1)

The three factors that contribute significantly to the black soil particle accumulation angle for all particle sizes (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm) are the black soil–black soil static friction coefficient, black soil–black soil rolling friction coefficient, and black soil–stainless steel rolling friction coefficient. The influence of the above three parameters on the angle of repose of the black soil is extremely significant. The remaining parameters with less influence were taken according to other relevant refs. [25,26] as black soil–black soil restitution coefficient of 0.6, black soil–stainless steel restitution coefficient of 0.38, and black soil–stainless coefficient of 0.8. Therefore, the steepest climbing test and response surface test design for the black soil accumulation angle were carried out, depending on the magnitude and order of contributions from the significance tests, in conjunction with the remaining simulated contact parameters.

3.2. The Steepest Climb Test

Using results from the Plackett–Burman test, three simulated contact parameters of high importance were selected for all particle sizes (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm) and the steepest climbing test was designed. Each important black soil stacking angle parameter was gradually increased from a low level according to the respective step size to determine whether the black soil particle stacking angle was reached. The results of the steepest climb test for different grain sizes are presented in Table 5. From Table 5, it can be seen that the black soil accumulation angle becomes larger with increasing black soil grain size in the range of (0.75–4 mm) and smaller with increasing black soil grain size in the range of (4–10 mm). The black soil accumulation angle is maximized at the 4 mm particle size and minimized at the 10 mm particle size. The angle of repose produces the smallest relative error at level 4, and the trend of deviation generated by the rest angle is from large-to-small to large-to-small in the interval from level 1 to 5. Therefore, the response surface is designed with level 4 as the central location point.

Table 5. Results of the steepest climb test for different grain sizes.

Serial Number	Black Soil–Black Soil Static Friction Coefficient	Black Soil–Black Soil Rolling Friction Coefficient	Black Soil–Stainless Steel Coefficient of Rolling Friction	0.75 mm Repose Angle $\theta/(^\circ)$	2 mm Repose Angle $\theta/(^\circ)$	4 mm Repose Angle $\theta/(^\circ)$	6 mm Repose Angle $\theta/(^\circ)$	8 mm Repose Angle $\theta/(^\circ)$	10 mm Repose Angle $\theta/(^\circ)$
1	0.2	0.01	0.05	14.86	15.2	15.6	15.3	16.6	15.64
2	0.55	0.13	0.13	26.5	29.28	31.49	28.37	27.82	27.47
3	0.9	0.25	0.21	33.05	36.03	36.11	35.75	34.12	33.42
4	1.25	0.37	0.29	36.7	37.63	37.69	37.4	37.23	36.63
5	1.6	0.49	0.37	38.69	39.5	39.83	39.01	38.92	38.75

3.3. Box–Behnken-Test Response Surface Methodology to Optimize the Optimal Contact Parameters for Different Particle Sizes

For each particle size (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm), the Box–Behnken test was designed based on the first two tests. Every experiment ranked the screening significance parameters at three levels each and selects three central points. A total of 15 experiments were designed for each total particle size (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm), each with five replicates, and averaged. The Box–Behnken test protocol and results after multi-objective normalization for different particle sizes are presented in Table 6. As shown in Table 6, the stacking angles for different particle sizes were obtained after the Box–Behnken test. As can be seen from Table 6, the black soil accumulation angle became larger with increasing black soil grain size in the range (0.75–4 mm) and smaller with increasing black soil grain size in the range (4–10 mm). The black soil accumulation angle was the greatest at the 4 mm grain size and smallest at the 10 mm grain size. For the same set of contact parameters, the average error in the black soil accumulation angle is between 0.5 and 2.5°.

Table 6. Box–Behnken test protocol and results after multi-objective normalization for different particle sizes.

Serial Number	**B**	**C**	**F**	**0.75 mm Repose Angle θ/(°)**	**2 mm Repose Angle θ/(°)**	**4 mm Repose Angle θ/(°)**	**6 mm Repose Angle θ/(°)**	**8 mm Repose Angle θ/(°)**	**10 mm Repose Angle θ/(°)**	**OD Value**
1	0	0	0	36.35	37.45	37.69	37.58	37.23	36.45	0.47
2	1	0	1	37.47	38.56	40.25	39.98	37.85	37.65	0.89
3	1	−1	0	36.15	38.1	39.55	38.93	36.67	36.22	0.6
4	0	−1	1	35.9	37.64	39.08	38.12	36.61	36.01	0.51
5	1	0	−1	37.33	38.16	39.96	39.81	37.85	37.43	0.83
6	−1	0	1	35.5	36.96	38.11	37.91	36.23	35.75	0.38
7	−1	0	−1	34.12	35.61	37.51	37.01	35.48	34.52	0.06
8	0	1	1	37.85	38.85	39.93	39.55	38.1	38.05	0.92
9	0	0	0	36.75	37.96	38.19	37.98	37.63	36.85	0.512
10	0	−1	−1	35.8	36.51	38.23	38.13	36.88	36.01	0.43
11	0	0	0	36.88	38.15	38.44	38.18	37.83	36.96	0.54
12	1	1	0	38.09	38.85	40.33	40.13	38.62	38.11	1
13	0	1	−1	36.52	37.8	39.9	38.62	37.55	36.87	0.69
14	−1	−1	0	34.02	35.55	36.72	36.13	35.31	34.35	0
15	−1	1	0	34.86	36.95	38.055	37.35	36.13	35.26	0.28

Based on the final results of the Box–Behnken experiments, the optimum contact parameters for different grain sizes were found. The multi-objective normalized value OD was obtained using the multi-indicator total assessment normalization method and the response surface method to optimize the discrete element parameters for black soils of different grain sizes.

3.3.1. Regression Model Interaction Effect

The Box–Behnken test optimization regression model results are shown in Table 7. Based on the above experimental results, the second-order regression equation of stacking angles for black soil particles and significant factor were developed using Design-Expert software:

$$\begin{aligned}\theta = \quad & -0.84 + 2.71A - 2.17B - 6.54C + 0.71AB - \\ & 2.32AC + 3.91BC - 0.55A^2 + 2.09B^2 + 15.64C^2\end{aligned} \tag{16}$$

The results of the Box–Behnken test model ANOVA are presented in Table 7. The fitted model with $p < 0.0001$ shows that regression analysis of black soil particle stacking angle is extremely significant. The effects of the black soil–black soil static friction coefficient (B), black soil–black soil rolling friction coefficient (C), and black soil–stainless steel rolling friction coefficient (F) on the black soil stacking angle are extremely significant.

Table 7. Box–Behnken test optimization regression model results.

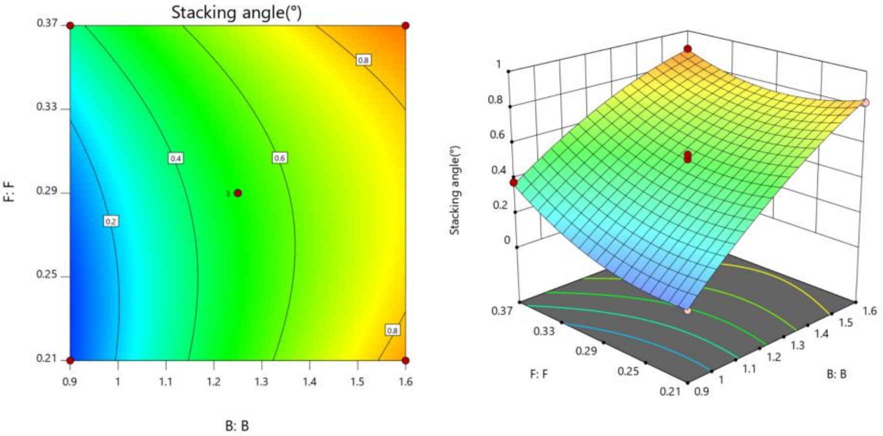
Source of Variation	**Mean Square**	**Freedom**	**Quadratic Sum**	**F Value**	***p* Value**
Model	1.22	9	0.1355	204.79	<0.0001
B	0.8450	1	0.8450	1277.34	<0.0001
C	0.2278	1	0.2278	344.37	<0.0001
F	0.0595	1	0.0595	89.96	0.0002
BC	0.0036	1	0.0036	5.44	0.0670
CF	0.0169	1	0.0169	25.55	0.0039
B^2	0.0056	1	0.0056	8.50	0.0332
F^2	0.0168	1	0.0168	25.37	0.0040
Residual	0.0033	1	0.0033	5.05	0.0745
Lack of fit	0.0370	1	0.0370	55.91	0.0007
Pure error	0.0033	5	0.0007		
Sum	0.0008	3	0.0003	0.2215	0.8754

Note: ($p < 0.01$) indicates that the item is extremely significant and ($p < 0.05$) indicates that the item is significant.

The coefficient of determination $R^2 = 0.9973$, adjusted $R^2 = 0.9924$, and the predicted $R^2 = 0.9846$, values are all >0.9, indicating that the model is closer to the actual situation. The adept precision = 47.0226, indicating a relatively high precision of the model.

3.3.2. Regression Model Interaction Effect Analysis

The influence both interaction terms have upon the angle of the rest of the black soil can be visualized in Figures 7 and 8. It is observed that the interaction effect of (BF) and (CF) have an important influence on the black soil stacking angle. The Design-Expert software was used to draw two interactive 3D response surfaces with a significant effect of accumulation angle. From the (BF) surface in Figure 7, it can be seen that the effect surface curve of the black soil–black soil rolling friction coefficient (B) is steeper than that of the black soil–black soil static friction coefficient (F), suggesting that it exerts a greater influence upon the angle of rest. From the (CF) surface in Figure 8, it is evident that the influence surface profile for the black soil–stainless steel coefficient of friction (C) is steeper than that for the black soil–black soil coefficient of friction (F), indicating that it has a greater influence on the angle of rest.



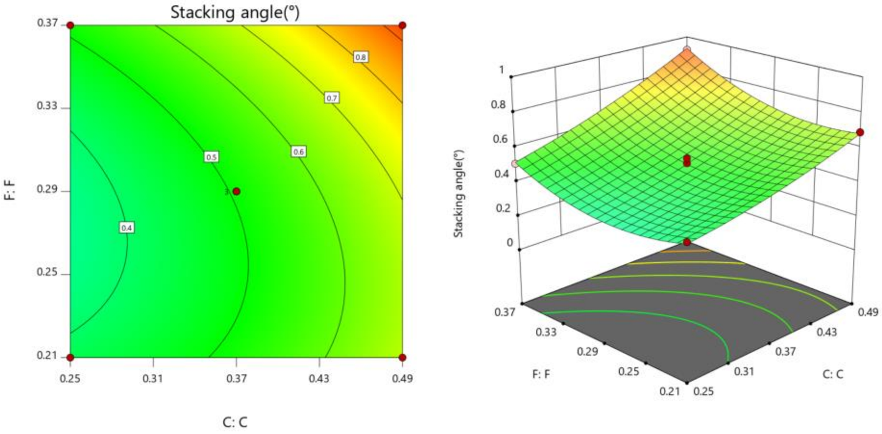
B: black soil–black soil static friction coefficient

F: black soil–stainless steel rolling friction coefficient

Figure 7. Interaction effect of BF.

3.3.3. Determination and Validation of Optimal Parameter Combinations

The best choice of the stacking angle index for the grain sizes (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm) is not the same. Therefore, to obtain more accurate discrete meta-simulation parameters for black soil, the best contact parameter for any black soil grain size needs to be found from the best contact parameter for each grain size. The regression equation for the normalized black soil particle accumulation angle OD was optimized using software with a target black soil particle accumulation angle of 36.95°. To reduce the deviation between the normalized black soil particle accumulation angle OD and the accumulation angle obtained from physical tests, the optimal parameters of the black soil accumulation angle were optimized again. The optimal contact parameters for the black soil normalized accumulation angle OD are a black earth–black earth static friction of 1.045, black earth–black earth rolling friction coefficient of 0.464, and black earth–stainless steel rolling friction coefficient of 0.215.



C: black soil–black soil rolling friction coefficient

F: black soil–stainless steel rolling friction coefficient

Figure 8. Interaction effect of CF.

The optimal contact parameters of the black soil normalized stacking angle OD were analyzed with different particle sizes (0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm) to observe the stacking results and errors before and after the optimization of each particle size. The comparison of the stacking angle before and after normalization for each grain size of black soil is shown in Figure 9. The errors of the simulation test and physical experiment before and after normalization for each particle size of black soil were 2.92%, 2.35%, 1.84%, 1.27, 2.6%, and 2% and 2.03%, 1.65%, 1%, 0.7%, 1.89%, and 1.24%, respectively. The errors of the simulation and physics experiments before and after normalization were reduced by 0.89%, 0.7%, 0.84%, 0.57%, 0.71%, and 0.76% for each particle size of 0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm of black soil, respectively, with an average error reduction of 0.745%.

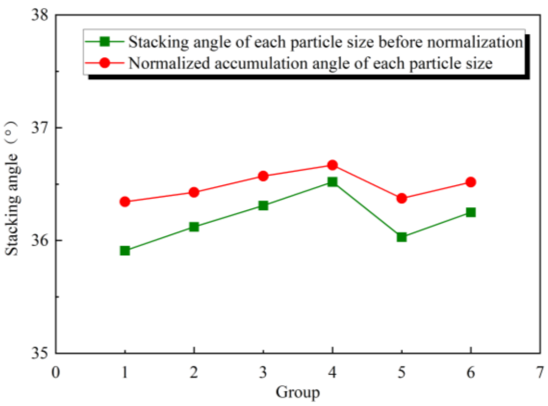


Figure 9. Comparison of the stacking angle before and after normalization for each grain size of black soil.

A comparison of the stacking angles before and after the normalization of each particle size of black soil is shown in Figure 10. After the normalized-response surface optimization, the stacking angle profile of each particle size improved, which is closer to the physical experimental stacking type. Among them, the effect of the stacking angle of the black soil particles sized 4 mm and 6 mm is the closest to the actual physical effect.

Normalization to optimize the stacking angle of the 0.75 mm grain size before the stacking tests.

Stacking angle of the 0.75 mm grain size after normalization to optimize the stacking tests.

Normalization to optimize the stacking angle of the 2 mm grain size before the stacking tests.

Stacking angle of the 2 mm grain size after normalization to optimize the stacking tests.

Normalization to optimize the stacking angle of the 4 mm grain size before the stacking tests.

Stacking angle of the 4 mm grain size after normalization to optimize the stacking tests.

Normalization to optimize the stacking angle of the 6 mm grain size before the stacking tests.

Stacking angle of the 6 mm grain size after normalization to optimize the stacking tests.

Normalization to optimize the stacking angle of the 8 mm grain size before the stacking tests.

Stacking angle of the 8 mm grain size after normalization to optimize the stacking tests.

Normalization to optimize the stacking angle of the 10 mm grain size before the stacking tests.

Stacking angle of the 10 mm grain size after normalization to optimize stacking tests.

Figure 10. Comparison of the stacking angles before and after the normalization of each particle size of black soil.

A comparison of the simulated and actual physical experiments for the optimal particle size of 6 mm is shown in Figure 11. Combining the normalized treatment stacking angle effect and error comparison, the 6 mm particle size is most suitable for the best discrete element contact parameter after normalization. The results show that the calibrated parameter could be a good guide to selecting the parameters for the simulation of discrete elements in black soils and the design and optimization of related equipment.

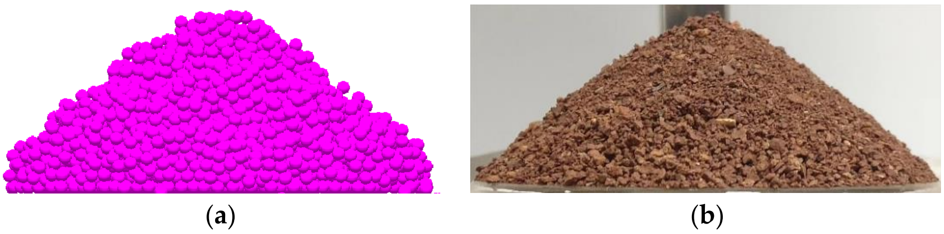

(**a**) (**b**)

Figure 11. A comparison of the simulated and actual physical experiments for the optimal particle size of 6 mm. (**a**) Optimal particle size 6 mm simulation test and (**b**) physical test.

4. Conclusions

In this study, to improve the reliability of the discrete element contact parameters of black soils, the best discrete element contact parameters were selected by combining the black soil solids and simulated stacking tests using the multi-indicator total assessment normalization method combined with the response surface method.

For different particle sizes (0.75–10 mm) of black soil particles, the Plackett–Burman test, the steepest climb test, and the Box–Behnken test were sequentially applied to obtain the optimal contact parameters of black soil with different particle sizes, targeting the stacking angle of black soil particles. Based on the optimal contact parameters for different black soil grain sizes, a set of optimal contact parameter combinations of black soil that can comprehensively represent different grain sizes and minimize error was determined using a multi-objective comprehensive evaluation normalization method; then, its accuracy was verified. The following conclusions were drawn:

(1) The Plackett–Burman experiments with different particle sizes showed that the variables affecting the accumulation angle of black soil particles were the black soil–black soil static friction coefficient, black soil–black soil rolling friction coefficient, and black soil–stainless steel rolling friction coefficient.

(2) The optimum parameters for the black earth contact parameters were 1.045 for the black soil–black soil static friction coefficient, 0.464 for the black soil–black soil rolling friction coefficient, and 0.215 for the black soil–stainless steel rolling friction coefficient.

(3) The errors of the simulation test and physical experiment before and after normalization for each particle size of 0.75 mm, 2 mm, 4 mm, 6 mm, 8 mm, and 10 mm of black soil were 2.92%, 2.35%, 1.84%, 1.27, 2.6%, and 2%, and 2.03%, 1.65%, 1%, 0.7%, 1.89%, and 1.24%, respectively. The errors of the simulation and physics experiments before and after normalization were reduced by 0.89%, 0.7%, 0.84%, 0.57%, 0.71%, and 0.76% for each particle size of black soil, with an average error reduction of 0.745%.

(4) The 6 mm particle size is most suitable for the best discrete element contact parameter after normalization. The simulation results did not differ significantly from the physical measurements. Thus, it can give a guide value to the design and improvement of related agricultural equipment.

Author Contributions: Z.W.: Conceptualization, Software, Funding acquisition, Validation, Writing—Original Draft, Writing—Review and Editing, and Investigation. T.Z.: Writing—Original Draft, Writing—Review and Editing, Investigation, Funding acquisition, Supervision, and Project administration. C.Z.: Data Curation, Visualization. F.M.: Resources and Software. Y.W.: Visualization and Supervision. X.L.: Visualization and Supervision. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by The National Key Research and Development Program of China (grant number: 2020YFC1806402).

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Ucgul, M.; Fielke, J.M.; Saunders, C. 3D DEM tillage simulation: Validation of a hysteretic spring (plastic) contact model for a sweep tool operating in a cohesionless soil. *Soil Till Res.* **2014**, *144*, 220–227. [CrossRef]
2. Ucgul, M.; Fielke, J.M.; Saunders, C. Three-dimensional discrete element modelling (DEM) of tillage: Accounting for soil cohesion and adhesion. *Biosyst. Eng.* **2015**, *129*, 298–306. [CrossRef]
3. Lenaerts, B.; Aertsen, T.; Tijskens, E.; De Ketelaere, B.; Ramon, H.; De Baerdemaeker, J.; Saeys, W. Simulation of grain-straw separation by Discrete Element Modeling with bendable straw particles. *Comput. Electron. Agric.* **2014**, *101*, 24–33. [CrossRef]
4. Li, B.; Chen, Y.; Chen, J. Modeling of soil-claw interaction using the discrete element method (DEM). *Soil Till Res.* **2016**, *158*, 177–185. [CrossRef]
5. Binelo, M.O.; de Lima, R.F.; Khatchatourian, O.A.; Stransky, J. Modelling of the drag force of agricultural seeds applied to the discrete element method. *Biosyst. Eng.* **2019**, *178*, 168–175. [CrossRef]
6. Qi, L.; Chen, Y.; Sadek, M. Simulations of soil flow properties using the discrete element method (DEM). *Comput. Electron. Agric.* **2019**, *157*, 254–260. [CrossRef]
7. Zhu, J.Z.; Zou, M.; Liu, Y.S.; Gao, K.; Su, B.; Qi, Y.C. Measurement and calibration of DEM parameters of lunar soil simulant. *Acta Astronaut.* **2022**, *191*, 169–177. [CrossRef]
8. Bu, H.R.; Yu, S.Y.; Dong, W.C.; Wang, Y.Q.; Zhang, L.X.; Xia, Y.Q. Calibration and Testing of Discrete Element Simulation Parameters for Urea Particles. *Processes* **2022**, *10*, 511. [CrossRef]
9. Zhao, L.; Zhou, H.P.; Xu, L.Y.; Song, S.Y.; Zhang, C.; Yu, Q.X. Parameter calibration of coconut bran substrate simulation model based on discrete element and response surface methodology. *Powder Technol.* **2022**, *395*, 183–194. [CrossRef]
10. Guo, Y.X.; Deng, Z.H.; Guo, S.Y.; Li, Y.J.; Mao, S.; Zhang, T.; Yang, Q.; Chen, Y.; Wang, Y.J.; Li, Q.; et al. Optimization of Ethanol-Based Extraction Process for Duliang Formula by Central Composite Design and Response Surface Methodology. *Nat. Prod. Commun.* **2022**, *17*. [CrossRef]
11. Li, Y.; Ye, M.; Zhang, Y.; Sun, Y.D.; Hao, C.F. Optimization of Particle Distribution for Asphalt Mixtures in Screw Conveyer of Paver Based on Discrete Element Method and Response Surface Methodology. *Adv. Theor. Simu.* **2023**. [CrossRef]
12. Shi, G.K.; Li, J.B.; Ding, L.P.; Zhang, Z.Y.; Ding, H.Z.; Li, N.; Kan, Z. Calibration and Tests for the Discrete Element Simulation Parameters of Fallen Jujube Fruit. *Agriculture* **2022**, *12*, 38. [CrossRef]
13. Hu, Y.Z.; Chan, E.L.; Tsuji, T.; Tanaka, T.; Washino, K. Geometric similarity on interparticle force evaluation for scaled-up DEM particles. *Powder Technol.* **2022**, *404*, 117483. [CrossRef]
14. Lommen, S.; Schott, D.; Lodewijks, G. DEM speedup: Stiffness effects on behavior of bulk material. *Particuology* **2014**, *12*, 107–112. [CrossRef]
15. Hu, M.J.; Xia, J.F.; Zhou, Y.; Luo, C.M.; Zhou, M.K.; Liu, Z.Y. Measurement and Calibration of the Discrete Element Parameters of Coated Delinted Cotton Seeds. *Agriculture* **2022**, *12*, 286. [CrossRef]
16. Chen, S.; Liu, W.W.; Li, S.Q. A fast adhesive discrete element method for random packings of fine particles. *Chem. Eng. Sci.* **2019**, *193*, 336–345. [CrossRef]
17. Roessler, T.; Katterfeld, A. Scaling of the angle of repose test and its influence on the calibration of DEM parameters using upscaled particles. *Powder Technol.* **2018**, *330*, 58–66. [CrossRef]
18. Sakai, M.; Takahashi, H.; Pain, C.C.; Latham, J.P.; Xiang, J.S. Study on a large-scale discrete element model for fine particles in a fluidized bed. *Adv. Powder Technol.* **2012**, *23*, 673–681. [CrossRef]
19. Dai, Z.W.; Wu, M.L.; Fang, Z.C.; Qu, Y.B. Calibration and Verification Test of Lily Bulb Simulation Parameters Based on Discrete Element Method. *Appl. Sci.* **2021**, *11*, 749. [CrossRef]
20. Ma, G.G.; Sun, Z.J.; Ma, H.; Li, P.C. Calibration of Contact Parameters for Moist Bulk of Shotcrete Based on EDEM. *Adv. Mater. Sci. Eng.* **2022**, *2022*, 6072303. [CrossRef]
21. Xia, R.; Li, B.; Wang, X.W.; Li, T.J.; Yang, Z.J. Measurement and calibration of the discrete element parameters of wet bulk coal. *Measurement* **2019**, *142*, 84–95. [CrossRef]
22. *GB/T 16913.2-1997*; Methods of Test for Dust Properties—Part 2: Determination of Effective Density. Standards Press China: Beijing, China, 1997.
23. Tian, X.; Cong, X.; Qi, J.; Guo, H.; Li, M.; Fan, X. Parameter Calibration of Discrete Element Model for Corn Straw-Soil Mixture in Black Soil Areas. *Nongye Jixie Xuebao/Trans. Chin. Soc. Agric. Mach.* **2021**, *52*, 100–108+242. [CrossRef]
24. *GB/T 16913.5-1997*; Methods of Test for Dust Properties—Part 5: Determination of Angle of Repose. Standards Press China: Beijing, China, 1997.
25. Zhao, X.F.; Zheng, P.F.; He, L.; Tao, M. Cutting edge preparation using the discrete element software EDEM. *J. Braz. Soc. Mech. Sci.* **2020**, *42*, 163. [CrossRef]

26. Wang, N.; Yang, S.Q.; Zhao, T.T.; Cao, B.; Wang, C.W. Amending Research on the Expression of the Contact Force of the Spindle Barrel Finishing Based on EDEM Simulation. *Chin. J. Mech. Eng.* **2020**, *33*, 83. [CrossRef]
27. Sun, J.; Liu, Q.; Yang, F.; Liu, Z.; Wang, Z. Calibration of Discrete Element Simulation Parameters of Sloping Soil on Loess Plateau and Its Interaction with Rotary Tillage Components. *Trans. Chin. Soc. Agric. Mach.* **2022**, *53*, 63–73. [CrossRef]
28. Weinhart, T.; Hartkamp, R.; Thornton, A.R.; Luding, S. Coarse-grained local and objective continuum description of three-dimensional granular flows down an inclined surface. *Phys. Fluids* **2013**, *25*, 7147–7151. [CrossRef]
29. Li, Y.; Li, F.; Xu, X.; Shen, C.; Meng, K.; Chen, J.; Chang, D. Parameter calibration of wheat flour for discrete element method simulation based on particle scaling. *Trans. Chin. Soc. Agric. Mach.* **2019**, *35*, 320–327. [CrossRef]
30. Yuan, Q.; Xu, L.; Xing, J.; Duan, Z.; Ma, S.; Yu, C.; Chen, C. Parameter calibration of discrete element model of organic fertilizer particles for mechanical fertilization. *Trans. Chin. Soc. Agric. Mach.* **2018**, *34*, 21–27. [CrossRef]