Article

# Optimal Allocation Method of Circuit Breakers and Switches in Distribution Networks Considering Load Level Variation 

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#### Abstract

Reliability is a fundamental concept for power systems, and the optimal placement of switchable devices is a valuable tool for improvements in this area. The goal of this paper is to propose an optimal allocation method for circuit breakers and switches that can break the cost-reliability dilemma and simultaneously achieve reliability and economic improvement in terms of the distribution network. Moreover, in view of the fact that variations in the load level can affect the reliability of the distribution network, the variations of different load level scenarios are considered in this paper, where a mixed integer linear programming (MILP) model based on fictitious fault flows is established to derive the optimal allocation scheme that can adapt to the changes of multiple scenarios regarding the load. Meanwhile, due to the constraints of reliability indices, the post-fault reconfiguration scheme of a distribution network under different load level scenarios can also be obtained to enhance its overall reliability. Finally, the applicability and effectiveness of the proposed method are verified by numerical tests on a 54 -node test system.


Keywords: distribution network; load level variation; circuit breakers; switches; allocation; reliability enhancement

## 1. Introduction

According to the definition of reliability, a power system must be capable of consistently providing end users with both the quantity and quality of electricity they require [1]. About $70 \%$ of total electric service interruptions are caused by contingencies in the distribution system [2]. In recent years, the stability of the distribution network has declined due to the large proportion of distributed generation (DG) usage with the continuous development of renewable energy power generation technology [3]. Attaining high reliability for a distribution system is not only important but is also crucial to ensure the uninterrupted supply of electricity to consumers. Reliable distribution systems minimize power interruptions and enhance customer satisfaction. This can be achieved by implementing robust infrastructure, such as redundant lines, automated switching devices, and protective measures against external disturbances.

At present, the main strategies to improve power supply reliability are reducing the equipment failure rate, shortening power restoration time, and improving fault isolation accuracy $[4,5]$. Among the various methods that could improve the reliability of distribution networks, the optimal placement of circuit breakers and switches has a significant impact on enhancing the reliability of the utility grids. It has been demonstrated in earlier studies that remote-controlled switches (RCSs) increase the distribution network reliability indices, such as the system average interruption duration index (SAIDI) and the expected energy not supplied (EENS), by reducing the duration that it takes to restore the power supply as well as speeding up the process of isolating the faulted area from the rest of the distribution
network [6]. Given the fact that installing switchable devices is expensive, it is essential to select the location sites properly to balance benefits and costs [7]. Due to the relatively high investment costs of switchable devices and the budget limitations of utilities to improve the quality of customer services, it is necessary to study the optimal allocation of switchable devices in the distribution network to achieve the maximum improvement to the level of reliability with the lowest investment cost so that the cost-reliability dilemma could be broken.

Moreover, variations in the load level may lead to operating conditions beyond the design limits of the distribution network, which may cause voltage instability, transformer overloads, and other equipment failures leading to outages. Moreover, it can also significantly affect the planning and operation of the distribution network, making it difficult to optimize network utilization and ensure adequate reserve capacity. Overall, the impact of load level variation on the reliability of distribution networks cannot be ignored, and addressing it requires effective planning and operation strategies that take into account the complexity and variability of load levels. Hence, it is worthwhile to study the optimal allocation of circuit breakers and switches that can adapt to load level variations in order to weaken the impact of this on the reliability of the distribution network and maximize the effects of reliability improvements in the distribution network with lower economic investment costs.

The allocation problem of switchable devices has an underlying service restoration problem, consisting of choices about which switchable device must be opened or closed to minimize the unattended area after the isolation of a failure, which is categorized as a complex, combinatorial, and constrained optimization problem [8]. Plenty of pieces of literature have studied the optimal placement of these switchable devices in the distribution network.

The optimal allocation issue in the distribution network is classified as a combinatorial optimization problem that, especially for large cases, can be challenging to solve when optimally utilizing mathematical programming methods. Due to the complexity of the problem, heuristic algorithms are mainly used to solve this [9-11]. The reliability index of EENS was used in [12] to perform the optimal placement of remote-controlled switches, employing the differential search metaheuristic algorithm. In [13], the immune algorithm was used to determine the optimal location of switches by utilizing an objective function that minimizes the expense of investments in line switches and the cost of customer service outages while taking into account the failure rates of the load points concerned. The ant colony optimization algorithm is adopted in [14] to solve the fuzzy multi-objective problem of optimizing the location of sectionalizing switches, with the objectives of reliability improvements and the minimization of the cost of sectionalizing switches. Fuzzy logic and genetic algorithms were employed in a hybrid algorithm in [15] to improve the SAIDI index, which requires many network parameters for its application. A global combination criterion was proposed to simultaneously evaluate the combination performance of multiple switch positions in [16], which avoids the tedious traditional problem of adjusting only one switch position at a time and directly determining the optimal solution. However, some mathematical methods have also been proposed to solve the problem in recent years. In [17], the remote-controlled switch configuration problem is modeled as a mixed-integer linear programming model that divides loads into two categories according to the restoration time, and the configuration scheme is developed with the objective of minimizing the total cost and expected outage losses. A mixed-integer linear programming model for simultaneous switch and tie line placement in distribution systems with complex topologies is presented in the study in [18]. The studies on the allocation of switchable devices usually aim at improving the reliability and economy of the distribution network and mainly focus on the placement of switches with little consideration given to the deployment of circuit breakers. Meanwhile, the influence of load level variation fluctuations on the reliability of the distribution network is usually ignored.

In light of the progress in the above studies, this paper carries the analysis further. In order to break the cost-reliability dilemma and achieve a higher reliability level with a
relatively low investment cost, this paper establishes an optimal allocation model for circuit breakers and switches in distribution networks with the objective function of summing the minimum outage loss and investment cost of circuit breakers and switches while taking the reliability index as the constraint. By considering the fact that changes in load level can have an impact on reliability, variations in load level were considered to enhance the adaptability of the derived optimal configuration scheme to different load level scenarios while improving the practicality of the method proposed in this paper.

The concept of fictitious fault flows is used to realize the linearized calculation of reliability indices and, at the same time, the reconfiguration scheme after the occurrence of faults in different scenarios, which can be solved according to the load level in different scenarios in a targeted manner to realize the effective improvement of distribution network reliability. The proposed model for the optimal allocation of circuit breakers and switches (considering load level variation) is a mixed-integer linear programming model that can achieve efficient solutions for the optimal allocation of switchable devices in distribution networks based on the goal of reliability enhancements and the result can be guaranteed to be the global optimal solution, with good practicality and engineering value.

## 2. Mathematical Model of Optimal Allocation of Circuit Breakers and Switches

The distribution network optimization allocation model for circuit breakers and switches in this study is based on the reliability assessment model put forward in [19], which utilizes the concept of fictitious fault flows to linearize the calculation of the reliability indices of the distribution system. Variations in load level are considered in this paper, and based on different load level scenarios, the optimal circuit breaker and switch allocation scheme and its initial operating state that can adapt to the corresponding load level variations are explored with the goal of improving the reliability and economy of the distribution network, and the reconfiguration scheme of the distribution network after a fault occurs under different load scenarios can be obtained at the same time.

### 2.1. Objective Function

In order to ensure the economic requirements of distribution network planning and operation, this paper minimizes the sum of the outage loss and investment cost of circuit breakers and switches in a distribution network as the objective function, as is shown in (1). For the calculation of outage loss, this is represented by the product of the unit price of outage loss per unit of power and EENS in the distribution network. The improvement of the reliability of the distribution network can be achieved simultaneously by taking the EENS in the system as one of the optimization objectives.

$$
\begin{align*}
f & =\operatorname{minimize}\left(F_{C B} \sum_{i j}\left(x_{i j}^{i, C B}+x_{i j}^{j, C B}\right)+F_{S W} \sum_{i j}\left(x_{i j}^{i, S W}+x_{i j}^{j, S W}\right)+\alpha E E N S\right)=  \tag{1}\\
& =\text { minimize }(\operatorname{Cost}+\alpha E E N S)
\end{align*}
$$

where $F_{C B}$ and $F_{S W}$ are the unit prices of the circuit breaker and switch, respectively. $x_{i j}^{i, C B}$, $x_{i j}^{j, C B}$ and $x_{i j}^{i, S W}, x_{i j}^{j, S W}$ are the binary variables indicating the installation of circuit breakers and switches on side $i$ and side $j$ of the branch $i j$, respectively. If the value of the variable is 1, it means that a circuit breaker or switch is installed on that side; if the value of the variable is 0 , there are no circuit breakers or switches installed on that side. $\alpha$ is the unit price of power outage loss per unit of electricity. The EENS is the expected amount of power not supplied in the distribution network after an outage has occurred.

### 2.2. Logical Constraints on the Installation of Circuit Breakers and Switches and the Corresponding Status

Since the operation status can only be switched between closed and open when a circuit breaker or switch is installed on the branch, it is essential to make constraints to ensure the reasonable operation of circuit breakers and switches. For normal operation
scenarios, the logical restrictions between the installation status and the operating status of the circuit breakers (or switches) are depicted as (2), (3) (or (4), (5)).

$$
\begin{align*}
& b_{i j}^{i, N O} \geq 1-x_{i j}^{i, C B}, \forall i j \in \mathrm{Y}  \tag{2}\\
& b_{i j}^{j, N O} \geq 1-x_{i j}^{j, C B}, \forall i j \in \mathrm{Y}  \tag{3}\\
& s_{i j}^{i, N O} \geq 1-x_{i j}^{i, S W}, \forall i j \in \mathrm{Y}  \tag{4}\\
& s_{i j}^{j, N O} \geq 1-x_{i j}^{j, S W}, \forall i j \in \mathrm{Y} \tag{5}
\end{align*}
$$

where $b_{i j}^{i, N O}$ and $b_{i j}^{j, N O}$ represent the original operation status of circuit breakers placed at end $i$ and $j$ of branch $i j$. If the variable equals 1 , the circuit breaker or switch in this circuit is closed; if both variables equal 0 , the circuit is open. Similarly, $s_{i j}^{i, N O}$ and $s_{i j}^{j, N O}$ represent the original operation status of switches at ends $i$ and $j$ of branch $i j$.

Under outage scenarios, taking into account the variation in load levels, the above logical constraints on the installation of the circuit breakers and switches and their corresponding changeable operating status are rewritten as (6)-(9).

$$
\begin{align*}
& b_{i j}^{i, x y, S C} \geq 1-x_{i j}^{i, C B}, \forall i j \in \mathrm{Y}  \tag{6}\\
& b_{i j}^{j, x y, S C} \geq 1-x_{i j}^{j, C B}, \forall i j \in \mathrm{Y}  \tag{7}\\
& s_{i j}^{i, x y, S C} \geq 1-x_{i j}^{i, S W}, \forall i j \in \mathrm{Y}  \tag{8}\\
& s_{i j}^{j, x y, S C} \geq 1-x_{i j}^{j, S W}, \forall i j \in \mathrm{Y} \tag{9}
\end{align*}
$$

where $x y$ represents the branch where the fault occurs, and $S C$ represents different load level scenarios.

### 2.3. Constraints on Power Flow and the Capacity of Branches

Each normal operating scenario's load demand condition is constrained by (10) and (11). The load demand of a node under normal operating conditions is all the load power connected to that node under the corresponding load level scenario. Constraints (12)-(15) are derived from the linearized power flow equations in [20]. Equation (14) uses a method that combines binary variables with the large $M$ method to constrain the power and voltage in the network to facilitate the linearization of the optimal allocation model. Nodal voltage constraints under normal scenarios are expressed as (16). Constraints (17)-(20) illustrate how the status of switches in the circuit restricts the power flow of branch $i j$; that is, the existence of power flow is possible when and only when the branch is connected.

$$
\begin{gather*}
P_{i}^{S C, N O}=P_{i}^{S C}, \forall i \in \Psi^{L N}  \tag{10}\\
Q_{i}^{S C, N O}=Q_{i}^{S C}, \forall i \in \Psi^{L N}  \tag{11}\\
P_{k i}^{S C, N O}=\sum_{j \in \Psi_{i}} P_{i j}^{S C, N O}+P_{i}^{S C, N O}, \forall k i \in \mathrm{Y}  \tag{12}\\
Q_{k i}^{S C, N O}=\sum_{j \in \Psi_{i}} Q_{i j}^{S C, N O}+Q_{i}^{S C, N O}, \forall k i \in \mathrm{Y} \tag{13}
\end{gather*}
$$

$$
\begin{gather*}
-\left(2-s_{i j}^{i, N O}-s_{i j}^{j, N O}\right) M+2\left(r_{i j} P_{i j}^{S C, N O}+x_{i j} Q_{i j}^{S C, N O}\right) \leq U_{i}^{S C, N O}-U_{j}^{S C, N O} \leq \\
\leq\left(2-s_{i j}^{i, N O}-s_{i j}^{j, N O}\right) M+2\left(r_{i j} P_{i j}^{S C, N O}+x_{i j} Q_{i j}^{S C, N O}\right), \forall i j \in \mathrm{Y}  \tag{14}\\
U_{i}^{S C, N O}=\left(V^{S}\right)^{2}, \forall i \in \Psi \Psi^{S S}  \tag{15}\\
U \leq U_{i}^{S C, N O} \leq \bar{U}, \forall i \in \Psi^{L N}  \tag{16}\\
-M s_{i j}^{i, N O} \leq P_{i j}^{S C, N O} \leq M s_{i j}^{i, N O}, \forall i j \in \mathrm{Y}  \tag{17}\\
-M s_{i j}^{i, N O} \leq Q_{i j}^{S C, N O} \leq M s_{i j}^{i, N O}, \forall i j \in \mathrm{Y}  \tag{18}\\
-M s_{i j}^{j, N O} \leq P_{i j}^{S C, N O} \leq M s_{i j}^{j, N O}, \forall i j \in \mathrm{Y}  \tag{19}\\
-M s_{i j}^{j, N O} \leq Q_{i j}^{S C, N O} \leq M s_{i j}^{j, N O}, \forall i j \in \mathrm{Y} \tag{20}
\end{gather*}
$$

where $P_{i}^{S C, N O}$ and $Q_{i}^{S C, N O}$ describe the active and reactive demand under different load level scenarios at the node under normal operating conditions. The active and reactive power flows through branch $i j$ under different load level scenarios are denoted by $P_{i j}^{S C, N O}$ and $Q_{i j}^{S C, N O}$, respectively. $U_{i}^{S C, N O}$ is the square voltage under different load levels at the node, while $V^{S}$ is the source voltage at a feeder's head end.

In constraints (21)-(23), the branch capacity limitations that are linearized and rely on piecewise relations are offered by [21], which presents a quadratic circular constraint to facilitate dualization in the solving process. Constraints (24) and (25) denote the power of the feeder $f$ provided by the transformer $t r r^{f}$ that connects to it, whereas constraints (26)-(28) indicate the capacity restrictions of the transformers.

$$
\begin{gather*}
-S_{i j}^{C} \leq P_{i j}^{S C, N O} \leq S_{i j}^{C}, \forall i j \in \mathrm{Y}  \tag{21}\\
-S_{i j}^{C} \leq Q_{i j}^{S C, N O} \leq S_{i j}^{C}, \forall i j \in \mathrm{Y}  \tag{22}\\
-\sqrt{2} S_{i j}^{C} \leq P_{i j}^{S C, N O} \pm Q_{i j}^{S C, N O} \leq \sqrt{2} S_{i j}^{C}, \forall i j \in \mathrm{Y}  \tag{23}\\
P_{f}^{S C, N O}=P_{t r f}^{S C, N O}, \forall f \in \Psi^{F}, t r^{f} \in \mathrm{Y}  \tag{24}\\
Q_{f}^{S C, N O}=Q_{t r f}^{S C, N O}, \forall f \in \Psi^{F}, t r^{f} \in \mathrm{Y}  \tag{25}\\
P_{f}^{S C, N O} \leq S_{f}^{C}, \forall f \in \Psi^{F}  \tag{26}\\
Q_{f}^{S C, N O} \leq S_{f}^{C}, \forall f \in \Psi^{F}  \tag{27}\\
\pm Q_{f}^{S C, N O}+P_{f}^{S C, N O} \leq \sqrt{2} S_{f}^{C}, \forall f \in \Psi^{F} \tag{28}
\end{gather*}
$$

where $S_{i j}^{C}$ is the peak transmission capacity of branch $i j$, while $S_{f}^{C}$ represents the transformer's capability in connection to feeder $f$.

### 2.4. Radial Constraints on Distribution Network Topology

Nowadays, a mesh-constructed distribution network architecture is popular in urban areas to improve the reliability of the power supply [22-24]; However, it operates radially. Reconfiguring a distribution network requires changing the topology in order to boost performance while preserving network radiality. The operating status of a switch on either side of the branch, as indicated in (29) for normal operation circumstances, is used to determine the connection status of branch $i j$. Constraint (30) ensures that the distribution network operates in a radial structure.

$$
\begin{gather*}
s_{i j}^{i, N O}+s_{i j}^{j, N O}-1 \leq l_{i j}^{N O} \leq\left(s_{i j}^{i, N O}+s_{i j}^{j, N O}\right) / 2, \forall i j \in \mathrm{Y}  \tag{29}\\
n^{L N}=\sum_{i j \in \mathrm{Y}} l_{i j}^{N O} \tag{30}
\end{gather*}
$$

where $l_{i j}^{N O}$ equals 1 when branch $i j$ is connected under normal operation conditions, and this shows whether branch $i j$ is linked under a specified assignment of switchable devices. The number of load nodes if represented by $n^{L N}$.

### 2.5. Constraints Related to Reliability Assessment

This paper classifies the fictitious fault flows in the network into "RA" and "PF" during the fault recovery after an outage occurs. Just after the outage, a fictitious fault flow identified as "RA" appears and can only be stopped by circuit breakers. Fictitious fault flow, denoted by the symbol "PF," takes place after the fault branch has been isolated and can only be eliminated via switches. The reliability of load points and the power system can be assessed and optimized while the network reconfiguration is being carried out via systematic reliability indices and nodal reliability indices, which are calculated in the distribution network based on the distribution of the two fictitious fault flows in each branch.

The first block of constraints is given in (31)-(38). Constraint (31) sets the location of the branch $x y$ where the fault occurred. As shown in Equations (32) and (33), the spread of the fictitious fault flow "RA" between branches is influenced by the initial switch operating state in the network and can be prevented by tripping the circuit breakers. Constraint (34) guarantees that only one circuit breaker can trip to stop the spread of fault currents under every outage scenario. Constraints (35) and (36) specify the upper and lower limits for representing the fictitious fault flow "RA" variable. Constraint (37) makes sure that the outage does not occur on substation nodes. Equation (38) uses the variable $p_{i}^{x y, S C}$ to represent the state of the power supply after a fault at each node, which is based on the value of the fictitious fault flow "RA" variable at each node.

$$
\begin{equation*}
f_{x y}^{x y, S C, R A}=0 \tag{31}
\end{equation*}
$$

$-\left(2-b_{i j}^{i, x y, S C}-s_{i j}^{i, N O}\right) M+f_{i}^{x y, S C, R A} \leq f_{i j}^{x y, S C, R A} \leq\left(2-b_{i j}^{i, x y, S C}-s_{i j}^{i, N O}\right) M+f_{i}^{x y, S C, R A}, \forall i j \in \mathrm{Y}$
$-\left(2-b_{i j}^{j, x y, S C}-s_{i j}^{j, N O}\right) M+f_{j}^{x y, S C, R A} \leq f_{i j}^{x y, S C, R A} \leq\left(2-b_{i j}^{j, x y, S C}-s_{i j}^{j, N O}\right) M+f_{j}^{x y, S C, R A}, \forall i j \in \mathrm{Y}$
$\sum_{i j \in Y_{I}^{B}} b_{i j}^{i, N O}+\sum_{i j \in \mathrm{Y}_{J}^{B}} b_{i j}^{j, N O}-1=\sum_{i j \in \mathrm{Y}_{I}^{B}} b_{i j}^{i, x y, S C}+\sum_{i j \in \mathrm{Y}_{J}^{B}} b_{i j}^{j, x y, S C}$

$$
\begin{equation*}
0 \leq f_{i}^{x y, S C, R A} \leq 1, \forall i \in \Psi^{L N} \tag{35}
\end{equation*}
$$

$$
\begin{equation*}
0 \leq f_{i j}^{x y, S C, R A} \leq 1, \forall i j \in \mathrm{Y} \tag{36}
\end{equation*}
$$

$$
\begin{gather*}
f_{i}^{x y, S C, R A}=1, \forall i \in \Psi S S  \tag{37}\\
p_{i}^{x y, S C}=1-f_{i}^{x y, S C, R A}, \forall i \in \Psi \Psi^{L N} \tag{38}
\end{gather*}
$$

The second part of the constraints is given by (39)-(67). Constraint (39) sets the outage branch where ' $\mathrm{PF}^{\prime}$ stems from. Constraints (40) and (41) show that the spread of ' $\mathrm{PF}^{\prime}$ ' is restricted by the operating status of the switches. Constraints (42) and (43) restrict the variation in the fictitious fault flow of each node and branch. Constraint (44) makes sure that the outage does not occur on substation nodes. If and only if the load at node $i$ is supplied after post-fault reconfiguration (including unscathed nodes and restored nodes), this condition is designated as $q_{i}^{x y, S C}=1$, which is explained in (45); thus, the demand of load nodes can be given by (46) and (47).

$$
\begin{gather*}
f_{x y}^{x y, S C, P F}=0  \tag{39}\\
-\left(1-s_{i j}^{i, x y, S C}\right) M+f_{i}^{x y, S C, P F} \leq f_{i j}^{x y, S C, P F} \leq\left(1-s_{i j}^{i, x y, S C}\right) M+f_{i}^{x y, S C, P F}, \forall i j \in \mathrm{Y}  \tag{40}\\
-\left(1-s_{i j}^{j, x y, S C}\right) M+f_{j}^{x y, S C, P F} \leq f_{i j}^{x y, S C, P F} \leq\left(1-s_{i j}^{j, x y, S C}\right) M+f_{j}^{x y, S C, P F}, \forall i j \in \mathrm{Y}  \tag{41}\\
0 \leq f_{i}^{x y, S C, P F} \leq 1, \forall i \in \Psi^{L N}  \tag{42}\\
0 \leq f_{i j}^{x y, S C, P F} \leq 1, \forall i j \in \mathrm{Y}  \tag{43}\\
f_{i}^{x y, S C, P F}=1, \forall i \in \Psi^{S S}  \tag{44}\\
q_{i}^{x y, S C}=f_{i}^{x y, S C, P F}, \forall i \in \Psi^{L N}  \tag{45}\\
P_{i}^{x y, S C}=P_{i} q_{i}^{x y, S C}, \forall i \in \Psi^{L N}  \tag{46}\\
Q_{i}^{x y, S C}=Q_{i} q_{i}^{x y, S C}, \forall i \in \Psi^{L N} \tag{47}
\end{gather*}
$$

The demands of the nodes that were not impacted by the outage could not be altered following the post-fault network reconfiguration, as this is assured by constraint (48). The radial structure of the network under outage scenarios is ensured by (49) and (50).

$$
\begin{gather*}
1-p_{i}^{x y, S C} \leq q_{i}^{x y, S C}, \forall i \in \Psi^{L N}  \tag{48}\\
s_{i j}^{i, x y, S C}+s_{i j}^{j, x y, S C}-1 \leq l_{i j}^{x y, S C} \leq\left(s_{i j}^{i, x y, S C}+s_{i j}^{j, x y, S C}\right) / 2, \forall i j \in \mathrm{Y}  \tag{49}\\
\sum_{i \in \Psi^{L N}} q_{i}^{x y, S C}=\sum_{i j \in Y} l_{i j}^{x y, S C} \tag{50}
\end{gather*}
$$

Linearized power flow equations under outage scenarios are shown in (51)-(55). The principles of them are the same as that of (12)-(16) and will not be further elaborated here.

$$
\begin{equation*}
P_{k i}^{x y, S C}=\sum_{j \in \Psi_{i}} P_{i j}^{x y, S C}+P_{i}^{x y, S C}, \forall k i \in \mathrm{Y} \tag{51}
\end{equation*}
$$

$$
\begin{gather*}
Q_{k i}^{x y, S C}=\sum_{j \in \Psi_{i}} Q_{i j}^{x y, S C}+Q_{i}^{x y, S C}, \forall k i \in \mathrm{Y}  \tag{52}\\
-\left(2-s_{i j}^{i, x y, S C}-s_{i j}^{j, x y, S C}\right) M+2\left(r_{i j} P_{i j}^{x y, S C}+x_{i j} Q_{i j}^{x y, S C}\right) \leq U_{i}^{x y, S C}-U_{j}^{x y, S C} \leq \\
\leq\left(2-s_{i j}^{i, x y, S C}-s_{i j}^{j, x y, S C}\right) M+2\left(r_{i j} P_{i j}^{x y, S C}+x_{i j} Q_{i j}^{x y, S C}\right), \forall i j \in \mathrm{Y}  \tag{53}\\
U_{i}^{x y, S C}=\left(V^{S}\right)^{2}, \forall i \in \Psi^{S S}  \tag{54}\\
U \leq U_{i}^{x y, S C} \leq \bar{U}, \forall i \in \Psi^{L N} \tag{55}
\end{gather*}
$$

The status of the switches limits the amount of power that can flow through branch $i j$, as shown in constraints (56)-(59). Similar to the constraints placed on branch power flow by the circuit breakers and switches under normal operation scenarios, the power flow of the branch $i j$ exists under outage scenarios only when the branch is connected.

$$
\begin{align*}
& -M s_{i j}^{i, x y, S C} \leq P_{i j}^{x y, S C} \leq M s_{i j}^{i, x y, S C}, \forall i j \in \mathrm{Y}  \tag{56}\\
& -M s_{i j}^{i, x y, S C} \leq Q_{i j}^{x y, S C} \leq M s_{i j}^{i, x y, S C}, \forall i j \in \mathrm{Y}  \tag{57}\\
& -M s_{i j}^{j, x y, S C} \leq P_{i j}^{x y, S C} \leq M s_{i j}^{j, x y, S C}, \forall i j \in \mathrm{Y}  \tag{58}\\
& -M s_{i j}^{j, x y, S C} \leq Q_{i j}^{x y, S C} \leq M s_{i j}^{j, x y, S C}, \forall i j \in \mathrm{Y} \tag{59}
\end{align*}
$$

The linearized capacity restrictions of the branches, feeders, and transformers are provided by (60)-(67). The principles are the same as that of (21)-(28) and will not be further elaborated here.

$$
\begin{gather*}
-S_{i j}^{C} \leq P_{i j}^{x y, S C} \leq S_{i j}^{C}, \forall i j \in \mathrm{Y}  \tag{60}\\
-S_{i j}^{C} \leq Q_{i j}^{x y, S C} \leq S_{i j}^{C}, \forall i j \in \mathrm{Y}  \tag{61}\\
-\sqrt{2} S_{i j}^{C} \leq P_{i j}^{x y, S C} \pm Q_{i j}^{x y, S C} \leq \sqrt{2} S_{i j}^{C}, \forall i j \in \mathrm{Y}  \tag{62}\\
P_{f}^{x y, S C}=P_{t r f}^{x y, S C}, \forall f \in \Psi^{F}, t r^{f} \in \mathrm{Y}  \tag{63}\\
Q_{f}^{x y, S C}=Q_{t r f}^{x y, S C}, \forall f \in \Psi^{F}, t r^{f} \in \mathrm{Y}  \tag{64}\\
P_{f}^{x y, S C} \leq S_{f}^{C}, \forall f \in \Psi^{F}  \tag{65}\\
Q_{f}^{x y, S C} \leq S_{f}^{C}, \forall f \in \Psi^{F}  \tag{66}\\
\pm Q_{f}^{x y, S C}+P_{f}^{x y, S C} \leq \sqrt{2} S_{f}^{C}, \forall f \in \Psi^{F} \tag{67}
\end{gather*}
$$

### 2.6. Constraints on the Allocation of Circuit Breakers and Switches

In order to reduce the impact of switchable equipment errors on distribution network reliability and power supply continuity, the circuit breakers are generally placed in parallel with the switches in the distribution network, as is shown in (68) and (69).

$$
\begin{align*}
x_{i j}^{i, S W} & \geq 1-x_{i j}^{i, C B}  \tag{68}\\
x_{i j}^{j, S W} & \geq 1-x_{i j}^{j, C B} \tag{69}
\end{align*}
$$

### 2.7. Calculation of Reliability Indices

The indexes $p_{i}^{x y, S C}$ and $q_{i}^{x y, S C}$ concerning whether the load points receive power during different stages of the fault (obtained with the previous constraints) make it possible to linearize the representation of each of the reliability indices of the distribution network under different load level scenarios, as shown in (70)-(75). Among them, the calculation of the outage time index $C I D_{i}^{S C}$ of the load nodes in different scenarios consists of two parts in (70): one is the switching interruption fault time experienced by load points that do not get restored after the action of circuit breakers, and the other part is the time taken for the manual repair of the faults experienced by the load nodes that still cannot receive power after switch action for post-fault reconfigurations. However, Equations (71)-(75) are all common expressions of reliability indices.

$$
\begin{gather*}
C I D_{i}^{S C}=\sum_{x y \in Y} \lambda_{x y} \tau_{x y}^{S W} p_{i}^{x y, S C}+\sum_{x y \in \mathrm{Y}} \lambda_{x y}\left(\tau_{x y}^{R P}-\tau_{x y}^{S W}\right)\left(1-q_{i}^{x y, S C}\right), \forall i \in \Psi^{L N}  \tag{70}\\
C I F_{i}^{S C}=\sum_{x y \in \mathrm{Y}} \lambda_{x y} p_{i}^{x y, S C}, \forall i \in \Psi^{L N}  \tag{71}\\
S A I D I_{S C}=\frac{\sum_{i \in \Psi^{L N}} N C_{i} C I D_{i}^{S C}}{\sum_{i \in \Psi^{L N}} N C_{i}}  \tag{72}\\
S A I F I_{S C}=\frac{\sum_{i \in \Psi^{L N}} N C_{i} C I F_{i}^{S C}}{\sum_{i \in \Psi^{L N}} N C_{i}}  \tag{73}\\
A S A I_{S C}=1-\frac{S A I D I_{S C}}{8760}  \tag{74}\\
E E N S_{S C}=\sum_{h \in H} \frac{\triangle h}{8760} \sum_{i \in \Psi^{L N}} C I D_{i}^{S C} \mu_{h} L_{i}^{S C} \tag{75}
\end{gather*}
$$

where $\lambda_{x y}$ is the failure rate of branch $x y, \tau_{x y}^{S W}$ and $\tau_{x y}^{S W}$ represent the switching-only interruption duration and the repair and switch interruption duration of each branch, respectively. $N C_{i}$ denotes the number of customers at each load node. $\triangle h$ is the duration of load level $h . \mu_{h}$ represents the load factor of load level $h . L_{i}^{S C}$ is the peak demand at node $i$ under different load level scenarios.

After obtaining the reliability of the distribution network under each load level scenario, the final annual reliability indices of the distribution network need to be calculated based on the probability of occurrence of different scenarios, as is shown in (76)-(79).

$$
\begin{align*}
& \text { SAIDI }=\sum_{S C \in \text { Scene }} g_{S C} S A I D I_{S C}  \tag{76}\\
& S A I F I=\sum_{S C \in \text { Scene }} g_{S C} S A I F I_{S C} \tag{77}
\end{align*}
$$

$$
\begin{align*}
& A S A I=\sum_{S C \in S c e n e} g_{S C} A S A I_{S C}  \tag{78}\\
& E E N S=\sum_{S C \in \text { Scene }} g_{S C} E E N S_{S C} \tag{79}
\end{align*}
$$

where $g_{S C}$ is the probability of various load level scenarios.

### 2.8. Constraints of Reliability Indices

The constraints on the reliability indices are necessary to balance the demand for reliability and economy regarding the distribution network and to ensure that the distribution network has a relatively high reliability level while improving the cost-effectiveness of planning. The reliability indices of the distribution network include SAIDI, SAIFI, ASAI, and $E E N S$. Any constraint on any index can ensure the corresponding reliability, so in practice, any constraint from constraints (80)-(83) can be selected.

$$
\begin{gather*}
S A I D I \leq \varepsilon_{S A I D I}  \tag{80}\\
S A I F I \leq \varepsilon_{S A I F I}  \tag{81}\\
A S A I \leq \varepsilon_{A S A I}  \tag{82}\\
E E N S \leq \varepsilon_{E E N S} \tag{83}
\end{gather*}
$$

where $\varepsilon_{S A I D I}, \varepsilon_{S A I F I}, \varepsilon_{A S A I}$, and $\varepsilon_{E E N S}$ are the preset requirements for the reliability indices, respectively.

The entire optimal allocation model of circuit breakers and switches in distribution networks can be described as follows after specifying the objective functions and constraints:

$$
\left\{\begin{align*}
& f=\text { minimize }\left(F_{C B} \sum_{i j}\left(x_{i j}^{i, C B}+x_{i j}^{j, C B}\right)+F_{S W} \sum_{i j}\left(x_{i j}^{i, S W}+x_{i j}^{j, S W}\right)+\alpha E E N S\right)=  \tag{84}\\
&=\text { minimize }(\operatorname{Cost}+\alpha E E N S) \\
& \text { subject to: (2)-(83) }
\end{align*}\right.
$$

## 3. Numerical Test

The proposed model is validated on a 54-node distribution test system. This test system is a 1 MVA, 13.5 kV radial network with four substations, 50 load nodes, eight feeders, and 61 branches, for which the corresponding specific data can be obtained from [25]. The switching-only interruption duration for each branch is 0.5 h , and the repair-and-switching interruption duration is 3 h . The three load levels, with loading factors equal to $70 \%$ ( $2000 \mathrm{~h} /$ year), $83 \%$ ( $5760 \mathrm{~h} /$ year), and $100 \%$ ( $1000 \mathrm{~h} /$ year), of the associated peak demand are used to depict the loading condition [26]. The topological structure of the 54-node test system utilized in this paper, as well as the selection of circuit breaker and switch candidate locations, is shown in Figure 1.


Figure 1. Topological structure of the 54-node distribution test system.
In order to validate the effectiveness of the proposed model in improving the reliability and economy of the distribution network by optimizing the allocation of circuit breakers and switches in the distribution network, this paper selects some of the branches of the system as the candidate locations for circuit breakers or switches and also sets a small number of already-equipped circuit breakers and switches in the system. In this paper, the reliability index SAIDI is constrained by the reliability requirement $\varepsilon_{\text {SAIDI }}$, set as $1.2 \mathrm{~h} /$ year. The outage loss per unit of power is set to RMB 30 yuan/MWh. The investment unit prices of the circuit breakers and switches are set at RMB 42,000 and RMB 15,000, respectively. The sum of outage loss and investment cost is to be minimized as the objective function, and the reliability of the distribution network under the corresponding allocation scheme is evaluated as well.

In a distribution network, the daily load level exhibits peak and off-peak variations, representing the fluctuations in electricity demand throughout the day. The peak and off-peak variations in load level have important implications for the management and operation of the power distribution network. Utilities are needed to ensure that they have enough capacity to meet the peak demand during the day while efficiently utilizing resources during the off-peak period. Therefore, four different load level scenarios, as shown in Figure 2, are set up to reflect the impact of peak and off-peak variations in load level, which are 0.5 times the load level, 1 times the load level, 2 times the load level, and 2.5 times the load level. Therefore, the adaptability of the optimal allocation of circuit breakers and switches to scenario changes, as well as the effectiveness of comprehensive optimization on the reliability and economy of the distribution system, could be optimized. The proposed method is modeled in MATLAB and was solved using the CPLEX solver regarding the optimal allocation scheme of circuit breakers and switches, which takes about 2 min to solve for the 54-node test system used in this paper.


Figure 2. Demand of load nodes in 54-node test systems.
Figure 3 shows the results of the optimal allocation of circuit breakers and switches obtained by solving the model in this paper. Figure 4 illustrates the post-fault reconfiguration strategy of the distribution network under different load level scenarios when a fault occurs at branches 1-9. The reliability indices of each load node in the distribution network under different load level scenarios are shown in Table 1. Table 2 shows the reliability indices of the system under each load level scenario, as well as the overall reliability indices, the investment costs of circuit breakers and switches, and the overall outage losses of the system under the comprehensive consideration of load level variation.


Figure 3. Results of the optimal allocation of circuit breakers and switches.

(a) Post-fault network reconfiguration strategy under Scene 1

(c) Post-fault network reconfiguration strategy under Scene 3

(b) Post-fault network reconfiguration strategy under Scene 2

(d) Post-fault network reconfiguration strategy under Scene 4

Figure 4. Post-fault reconfiguration strategies for distribution network when a fault occurred at branches 1-9 under different load level scenarios.

Table 1. Nodal CID and CIF for the 54 -node test system.

| Node | CID (h/Year) |  |  |  |  | CIF (Interruption/Year) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Scene 1 | Scene 2 | Scene 3 | Scene 4 | Integrated Index | Scene 1 | Scene 2 | Scene 3 | Scene 4 | Integrated Index |
| 1 | 0.077 | 0.348 | 0.302 | 0.206 | 0.933 | 0.077 | 0.342 | 0.291 | 0.203 | 0.913 |
| 2 | 0.077 | 0.348 | 0.302 | 0.206 | 0.933 | 0.077 | 0.342 | 0.291 | 0.203 | 0.913 |
| 3 | 0.111 | 0.468 | 0.376 | 0.284 | 1.238 | 0.082 | 0.344 | 0.283 | 0.208 | 0.916 |
| 4 | 0.111 | 0.468 | 0.376 | 0.284 | 1.238 | 0.082 | 0.344 | 0.283 | 0.208 | 0.916 |
| 5 | 0.158 | 0.668 | 0.561 | 0.402 | 1.789 | 0.082 | 0.344 | 0.283 | 0.208 | 0.916 |
| 6 | 0.158 | 0.668 | 0.561 | 0.402 | 1.789 | 0.082 | 0.344 | 0.283 | 0.208 | 0.916 |
| 7 | 0.068 | 0.290 | 0.231 | 0.170 | 0.759 | 0.087 | 0.380 | 0.306 | 0.225 | 0.998 |
| 8 | 0.119 | 0.518 | 0.417 | 0.307 | 1.361 | 0.087 | 0.380 | 0.306 | 0.225 | 0.998 |
| 9 | 0.132 | 0.578 | 0.486 | 0.342 | 1.537 | 0.077 | 0.342 | 0.291 | 0.203 | 0.913 |
| 10 | 0.080 | 0.360 | 0.309 | 0.214 | 0.962 | 0.077 | 0.342 | 0.291 | 0.203 | 0.913 |
| 11 | 0.058 | 0.264 | 0.214 | 0.155 | 0.691 | 0.051 | 0.228 | 0.190 | 0.133 | 0.602 |
| 12 | 0.090 | 0.391 | 0.327 | 0.227 | 1.035 | 0.051 | 0.228 | 0.190 | 0.133 | 0.602 |

Table 1. Cont.

## CID (h/Year)

CIF (Interruption/Year)

| Node | Scene 1 | Scene 2 | Scene 3 | Scene 4 | Integrated Index | Scene 1 | Scene 2 | Scene 3 | Scene 4 | Integrated Index |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 13 | 0.090 | 0.391 | 0.327 | 0.227 | 1.035 | 0.051 | 0.228 | 0.190 | 0.133 | 0.602 |
| 14 | 0.123 | 0.540 | 0.420 | 0.318 | 1.402 | 0.087 | 0.376 | 0.300 | 0.223 | 0.986 |
| 15 | 0.107 | 0.460 | 0.383 | 0.275 | 1.224 | 0.087 | 0.376 | 0.300 | 0.223 | 0.986 |
| 16 | 0.107 | 0.460 | 0.383 | 0.275 | 1.224 | 0.087 | 0.376 | 0.300 | 0.223 | 0.986 |
| 17 | 0.145 | 0.641 | 0.544 | 0.382 | 1.712 | 0.061 | 0.266 | 0.226 | 0.158 | 0.711 |
| 18 | 0.145 | 0.641 | 0.544 | 0.382 | 1.712 | 0.061 | 0.266 | 0.226 | 0.158 | 0.711 |
| 19 | 0.145 | 0.641 | 0.544 | 0.382 | 1.712 | 0.061 | 0.266 | 0.226 | 0.158 | 0.711 |
| 20 | 0.145 | 0.641 | 0.544 | 0.382 | 1.712 | 0.061 | 0.266 | 0.226 | 0.158 | 0.711 |
| 21 | 0.045 | 0.191 | 0.164 | 0.114 | 0.513 | 0.061 | 0.266 | 0.226 | 0.158 | 0.711 |
| 22 | 0.132 | 0.578 | 0.486 | 0.342 | 1.537 | 0.077 | 0.342 | 0.291 | 0.203 | 0.913 |
| 23 | 0.132 | 0.578 | 0.486 | 0.342 | 1.537 | 0.077 | 0.342 | 0.291 | 0.203 | 0.913 |
| 24 | 0.039 | 0.171 | 0.146 | 0.101 | 0.457 | 0.077 | 0.342 | 0.291 | 0.203 | 0.913 |
| 25 | 0.119 | 0.518 | 0.417 | 0.307 | 1.361 | 0.087 | 0.380 | 0.306 | 0.225 | 0.998 |
| 26 | 0.058 | 0.240 | 0.194 | 0.146 | 0.638 | 0.082 | 0.344 | 0.283 | 0.208 | 0.916 |
| 27 | 0.058 | 0.240 | 0.194 | 0.146 | 0.638 | 0.082 | 0.344 | 0.283 | 0.208 | 0.916 |
| 28 | 0.158 | 0.668 | 0.561 | 0.402 | 1.789 | 0.082 | 0.344 | 0.283 | 0.208 | 0.916 |
| 29 | 0.108 | 0.468 | 0.391 | 0.284 | 1.251 | 0.047 | 0.205 | 0.170 | 0.124 | 0.546 |
| 30 | 0.108 | 0.468 | 0.391 | 0.284 | 1.251 | 0.047 | 0.205 | 0.170 | 0.124 | 0.546 |
| 31 | 0.033 | 0.147 | 0.124 | 0.089 | 0.393 | 0.047 | 0.205 | 0.170 | 0.124 | 0.546 |
| 32 | 0.060 | 0.271 | 0.221 | 0.158 | 0.711 | 0.057 | 0.251 | 0.204 | 0.148 | 0.660 |
| 33 | 0.122 | 0.539 | 0.443 | 0.320 | 1.424 | 0.087 | 0.380 | 0.306 | 0.225 | 0.998 |
| 34 | 0.122 | 0.539 | 0.443 | 0.320 | 1.424 | 0.087 | 0.380 | 0.306 | 0.225 | 0.998 |
| 35 | 0.122 | 0.539 | 0.443 | 0.320 | 1.424 | 0.087 | 0.380 | 0.306 | 0.225 | 0.998 |
| 36 | 0.072 | 0.318 | 0.251 | 0.189 | 0.830 | 0.087 | 0.380 | 0.306 | 0.225 | 0.998 |
| 37 | 0.033 | 0.147 | 0.124 | 0.089 | 0.393 | 0.047 | 0.205 | 0.170 | 0.124 | 0.546 |
| 38 | 0.057 | 0.244 | 0.195 | 0.145 | 0.641 | 0.057 | 0.251 | 0.204 | 0.148 | 0.660 |
| 39 | 0.060 | 0.271 | 0.221 | 0.158 | 0.711 | 0.057 | 0.251 | 0.204 | 0.148 | 0.660 |
| 40 | 0.070 | 0.304 | 0.243 | 0.186 | 0.803 | 0.077 | 0.329 | 0.274 | 0.201 | 0.881 |
| 41 | 0.199 | 0.849 | 0.716 | 0.517 | 2.280 | 0.077 | 0.329 | 0.274 | 0.201 | 0.881 |
| 42 | 0.199 | 0.849 | 0.716 | 0.517 | 2.280 | 0.077 | 0.329 | 0.274 | 0.201 | 0.881 |
| 43 | 0.108 | 0.468 | 0.391 | 0.284 | 1.251 | 0.047 | 0.205 | 0.170 | 0.124 | 0.546 |
| 44 | 0.112 | 0.487 | 0.400 | 0.289 | 1.288 | 0.057 | 0.251 | 0.204 | 0.148 | 0.660 |
| 45 | 0.112 | 0.487 | 0.400 | 0.289 | 1.288 | 0.057 | 0.251 | 0.204 | 0.148 | 0.660 |
| 46 | 0.123 | 0.540 | 0.420 | 0.318 | 1.402 | 0.087 | 0.376 | 0.300 | 0.223 | 0.986 |
| 47 | 0.199 | 0.849 | 0.716 | 0.517 | 2.280 | 0.077 | 0.329 | 0.274 | 0.201 | 0.881 |
| 48 | 0.038 | 0.165 | 0.137 | 0.100 | 0.440 | 0.077 | 0.329 | 0.274 | 0.201 | 0.881 |
| 49 | 0.118 | 0.505 | 0.397 | 0.297 | 1.317 | 0.087 | 0.376 | 0.300 | 0.223 | 0.986 |
| 50 | 0.118 | 0.505 | 0.397 | 0.297 | 1.317 | 0.087 | 0.376 | 0.300 | 0.223 | 0.986 |

Table 2. Reliability indices and economic indices of the 54 -node test system.

|  | SAIFI <br> $(\mathbf{1} /$ Year $)$ | SAIDI <br> $(\mathbf{h} /$ Year $)$ | EENS <br> $(\mathbf{M W h} /$ Year $)$ | ASAI <br> $(\%)$ | Investment <br> (Yuan) | Cost <br> (Yuan) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Scene 1 | 0.0714 | 0.1040 | 0.7640 | - | - | - |
| Scene 2 | 0.3103 | 0.4527 | 6.3444 | - | - | - |
| Scene 3 | 0.2553 | 0.3730 | 9.9622 | - | - | - |
| Scene 4 <br> Integrated <br> Indices | 0.1850 | 0.2697 | 9.8624 | - | - | - |

Under different load level scenarios, the optimal allocation model proposed in this paper can optimize the post-fault distribution network reconfiguration strategy according to the requirements of the required reliability level, as can be seen in Figure 4. The reconfiguration strategy under different load levels is optimized to minimize the outage loss in order to maximize the reliability of the distribution system. For fault scenarios under different load levels, the proposed model can form an optimal post-fault reconfiguration scheme for that load level scenario, which has strong adaptability and practicality and can effectively ensure the level of reliability of the distribution network.

As can be seen from Table 1, different load levels affect the nodal reliability index. From Figure 2, it can be seen that the load levels of scenes 1-4 are distributed from low to high. The higher the load level, the higher the frequency; the longer the duration of the outages at the load nodes, the worse the reliability of the indices of the nodes in the distribution network is. When the load level is low, i.e., the demand for power supply is low, the balance of supply and demand in the distribution network is easier to maintain, and the possibility of outage is relatively low. When the load level is high, i.e., when the customer's demand for electricity is relatively high, the time of electricity consumption is greater, or the load is unstable, then the reliability indices CID and CIF of the distribution network are usually affected to a certain extent. This is because, at this time, the distribution network faces a power load that may exceed the capacity of the design, leading to problems such as overload, excessive current, and aging equipment, which, in turn, can lead to breakdowns and may also lead to problems such as insufficient power supply.

As can be seen from Table 2, the proposed method can effectively maximize the economic benefits while ensuring that the distribution network reliability level meets the requirements. The optimal allocation model of circuit breakers and switches established in this paper takes into account both the economic indices represented by the equipment investment costs and outage losses, and the reliability index represented by EENS of the distribution network. The proposed model can effectively ensure the required reliability level while optimizing the allocation of circuit breakers and switches in the distribution network at the lowest investment cost, effectively reducing the outage losses in the distribution network by optimizing the post-fault reconfiguration scheme under various load levels and successfully achieving a balance between the optimization of reliability and economy.

In summary, the method proposed in this paper can better solve the optimal placement of circuit breakers and switches in the actual distribution network. The model uses faster calculation speed and better calculation quality to obtain an optimal allocation scheme of circuit breakers and switches that ensures the reliability of the distribution network to meet demand at the lowest investment cost, which achieves a balance between the optimization of reliability and economy under the objective function. Furthermore, since changes in load are considered in the model, the model developed in this paper can adapt to the uncertain changes in loads in the distribution network in practical applications, meaning the obtained allocation scheme has stronger adaptability and practicality.

## 4. Conclusions

The number and location of circuit breakers and switches in a distribution network may have a significant impact on the reliability of the distribution network. When the
investment budget is low, the number of switchable devices equipped in the network is usually small, which cannot guarantee satisfactory reliability levels in the distribution network. However, in practice, increasing the investment cost means the utility faces more financial risks, which, to some extent, limits the enhancement of the reliability of the distribution network. Simultaneously, the peak and valley fluctuations in the load levels can also have an impact on the level of reliability of the distribution network. Hence, it is essential to study optimal allocation methods for the placement of circuit breakers and switches in distribution networks and consider the relevant economic and reliability requirements by exploring the establishment of a solution to the cost-reliability dilemma and searching for the best allocation solution that can adapt to load level variations in the distribution network. This will reduce the negative impact of load level variations on the reliability of the distribution system.

This paper presents a linear programming model for the optimal allocation of circuit breakers and switches based on reliability and economic improvements to distribution networks with consideration given to load level variation. In this context, the sum of outage losses and the investment costs of switchable equipment is minimized as the objective function, and the reliability index of the distribution network is taken as the constraint to ensure that the distribution network meets reliability demands while minimizing the investment cost of equipment so as to achieve a balance between the optimization of reliability and the economic indices. In addition, the proposed model takes into account the impact of load level variation on the reliability of the distribution network and considers various load level scenarios to obtain the optimal allocation scheme of switchable equipment that can also adapt to different load fluctuation scenarios as well as target post-fault reconfiguration schemes under different scenarios. By using numerical examples, we verified that the optimal allocation scheme obtained by the proposed model could reduce investment costs and outage loss while ensuring the reliability index of the system meets the requirements. The method provides a practical reference for distribution network planners to carry out distribution network optimization design, and it has strong engineering research significance for optimizing distribution network structures. Last but not least, the linearization of the optimal allocation model was achieved through the concept of fictitious fault flow, which ensured the global optimality of the derived results and also enhanced the computational efficiency and speed of the model in this paper, improving the practicality and scalability of the model.

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