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Disturbance Observer-Based Terminal Sliding Mode Tracking Control for a Class of Nonlinear SISO Systems with Input Saturation

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Abstract: This paper focuses on the trajectory tracking control for general nonlinear single-input single-output (SISO) systems in which the output is not directly related to the control input. To address the tracking problem with the consideration of possible model uncertainty, external disturbance, and control input saturation, we employ the input-output feedback linearization technique and design a finite-time disturbance observer-based terminal sliding mode controller to improve the tracking performance and enhance the robustness. The stability analysis is carried out by using the Lyapunov method. To alleviate the chattering while achieving an acceptable control performance, a boundary layer method is adopted for the trade-off between the high-frequency control actions and the bounded unavoidable nonzero steady-state error. The proposed method is evaluated on the two typical nonlinear systems, which are fully linearizable and partially linearizable, respectively, and compared to the state-of-the-art method in terms of tracking and robustness through comprehensive numerical simulations. The results show that the proposed method not only renders the estimated disturbance error tends to be zero in finite time , but also has superiority in the fast reaction to disturbance and small tracking error without high-frequency chattering.

Keywords: disturbance observer; sliding mode control; feedback linearization; trajectory tracking control; control saturation

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1. Introduction

Proportional-integral-derivative (PID) is the most common control algorithm that had been successfully applied in the processes industry due to algorithm simplicity and satisfactory control performance [1]. However, PID is mainly used for linear single-input single-output (SISO) systems while it may be intractable for highly nonlinear systems with unpredictable disturbances. In past decades, robust nonlinear control methods were maturely developed and successfully applied to a variety of fields, such as robust nonlinear model predictive control (NMPC) [2–5], operator-based robust right coprime factorization (RRCF) [6–8], and sliding mode control (SMC) control [9–13]. Among them, SMC, also called variable structure control, is one of the most popular robust nonlinear control techniques due to the characteristic of rapid convergence with satisfactory transient performance, insensitivity to model uncertainties and disturbances, and remarkable computational simplicity [10].

Generally, the first and most important step in the design of SMC is to construct an appropriate sliding surface, which determines the control performance of the system to a certain extent [11]. The linear switching hyperplane was commonly adopted such that the state could be asymptotically forced to the equilibrium point with an arbitrary convergence rate. However, it may impose a strong control force and could not provide any information about the prescribed finite-time convergence, which is very desirable for some applications with specific requirements, e.g., fast and high precision trajectory tracking control [12]. In contrast to linear sliding surfaces, terminal sliding mode (TSM) control provides asymptotic stability with finite-time convergence [13]. To further improve the transient performance with faster convergence and achieve finite-time stability, the fast terminal sliding mode control (FTSMC) method was developed, which combines the advantage of linear sliding surface and nonlinear exponential terminal sliding surface such that the fast transient convergence can be achieved regardless of the distance between the system state and equilibrium [14]. To solve the singularity problem caused by the usage of negative fractional power terms in FTSMC, its variants, such as nonsingular terminal sliding mode control (NTSMC), were investigated [15]. In Ref. [16], the authors proposed a global sliding mode (GSM) control scheme where the reaching phase was removed so that the sliding mode exhibited invariants and robustness to system perturbations globally.

Based on the design of the sliding surface, the derived control law of SMC is also of importance, which generally consists of an equivalent control law and a discontinuous switching control law. The former, which is related to the nominal model dynamics, is derived without disturbances to maintain the state (error) on the sliding surface while sliding into the origin. The latter is basically obtained by using Lyapunov stability analysis concerning the sliding surface (e.g, $s(t)\dot{s}(t) < 0$), and deployed in the reaching phase for disturbance rejection at the cost of chattering. However, such chattering representing the oscillations with finite frequency and amplitude in the vicinity of the equilibrium point may result in unwanted wear and tear of the actuators, seriously affect the system performance, and even lead to instability [17]. To effectively alleviate chattering, the commonly used discontinuous $sign(\cdot)$ function that crosses the sliding manifold could be approximatively replaced by using a continuous smoothing function (e.g, hyperbolic tangent function $tanh(\cdot)$) [18], or utilizing the boundary layer method in which the discontinuous switching law is only applied outside a given boundary while the linear feedback control technique is used inside the boundary [19]. By doing so, a nonzero steady-state error would be inevitable. Another popular method for chattering alleviation is high-order SMC. This method intentionally increases the relative degree between the input and output such that the actual control input is the integration of the high-frequency switching terms through imposing the variables and their successive derivative terms to be converged to zero, e.g., $s(t) = \dot{s}(t) = \cdots = s^{r-1}(t)$ for r_{th} order SMC [11,18].

Besides the chattering in the SMC design, actuator physical constraints are also required to be considered in many practical control systems [20]. Input saturation as a nonsmooth nonlinearity severely degrades the closed-loop system performance, which may lead to undesirable inaccuracy or even cause instability. In Ref. [21], the authors proposed an operator-based RRCF method to deal with the accurate position tracking problem for ionic polymer metal composite actuator in the presence of model uncertainty and input saturation. Relying on the extraordinary ability to handle constraints, a robust model predictive control scheme was proposed for a linear uncertain system in the form of polytopic and subject to actuator saturation [22]. In Ref. [23], the authors developed an adaptive fuzzy control method-based small-gain technique for a nonstrict-feedback nonlinear system with the consideration of unmodeled dynamics and input saturation.

In addition to control saturation, the unavoidable external disturbance in practical applications needs to be further considered for performance improvement. A common way to do this is to design a disturbance observer to approximate and compensate for the lumped disturbance caused by the input saturation and external disturbance. In Ref. [24], the authors designed a robust control law in conjunction with a constructed auxiliary system for the dynamic positioning of ships in the presence of unknown time-varying disturbances and input saturation. The estimated error of disturbance and positioning errors were controlled to be globally uniform and ultimately bounded. In Ref. [25], the nonsmooth saturation nonlinearity was firstly approximated by a smooth function. The bounded approximation error and the external disturbance were solved by using an adaptive backstepping approach to guarantee the transient tracking performance. In order to explicitly consider input saturation in the control design of uncertain nonlinear systems [26], the authors developed a finite-time nonlinear terminal SMC disturbance observer in which the model uncertainties, external disturbances, and input saturation effect are included into a single disturbance term acting on the system.

In this paper, we investigate the problem of trajectory tracking control for a class of nonlinear SISO systems where the systems' output is not directly related to control input and may be subject to model uncertainties, external disturbances, and input saturation. To the best of our knowledge, such systems are common in practical applications but rarely discussed on the whole. To this end, we adopt a terminal sliding mode tracking control scheme to achieve a satisfactory tracking control performance while possibly reducing the chattering. The highlights and contributions of this paper can be summarized as follows:

- Input-output feedback linearization technique is applied to explicitly establish the relationship between the system output and the control input.
- Sliding mode disturbance observer is designed to estimate the lumped disturbance including the possible uncertainties, external disturbances, and input saturation, and is able to significantly reduce the chattering while keeping a certain tracking performance.
- Disturbance observer-based terminal sliding mode tracking controller with boundary layer method is further designed for the trade-off between the high-frequency chattering caused by the discontinuous $sign(\cdot)$ function and the nonzero steady-state error caused by the continuous control within the boundary layer.
- The proposed method is evaluated by two typical nonlinear systems that are fully linearizable and partially linearizable, respectively, and compared with the state-of-the-art method in terms of tracking performance and robustness through numerical simulations.

The structure of this paper is organized below. The problem formulation is firstly presented in Section 2. Then, the finite-time sliding mode disturbance observer and the terminal sliding mode tracking controller are designed in Section 3 and 4, respectively. The proposed method is evaluated with illustrated simulations in Section 5. Last, we conclude our work in Section 6.

2. Problem Formulation

In this paper, we consider a general class of n_{th} order SISO nonlinear systems, which is expressed as follows:

$$\begin{cases} \dot{x} = f(x) + g(x)u + d \\ y = h(x) \end{cases} \tag{1}$$

where $x = [x_1, x_2, \cdots, x_n]^T \in \mathcal{R}^n$ is a state vector that is assumed to be available for measurement, $f(x) = [f_1(x), f_2(x), \cdots, f_n(x)]$ and $g(x) = [g_1(x), g_2(x), \cdots, g_n(x)] \neq 0$ are smooth vector functions, $h(x)$ denotes the smooth scalar function, $u \in \mathcal{R}$ is control input, $y \in \mathcal{R}$ is system output, $d = [d_1(x), d_2(x), \cdot, d_n(x)] \in \mathcal{R}^n$ that possibly include the unmeasured disturbances caused by external disturbances and model uncertainty. From Equation (1), it can be seen that the output y is not directly related to control input u, which may require input-output feedback linearization for the nonlinearities cancellation and controllers' design [27]. Before the introduction of the sliding mode tracking controller, we first present the relevant definition below.

Definition 1 (Relative degree). *In nonlinear control, relative degree $r \in \mathcal{Z}$ represents the smallest number of times that the system output y has to be differentiated until the control input* *u explicitly appears in* y^r, $0 < r \leq n$ *[28,29]. That is,* $L_g L_f^j h = 0$ *for* $j = 0, 1, \cdots, r-2$ *and* $L_g L_f^{r-1} h \neq 0$ *with*

$$y^r = \overbrace{L_f^r h(x)}^{:=a(z)} + \overbrace{L_g L_f^{r-1} h(x)}^{:=b(z)} \cdot u + \overbrace{L_{f+gu+d}^{r-1} L_d h(x) + \sum_{\lambda=1}^{r-1} L_{f+gu+d}^{\lambda-1} L_d L_f^{r-\lambda} h(x)}^{d_z} \tag{2}$$

where,

$$\begin{cases} L_f h(x) = \frac{\partial h(x)}{\partial x} f(x) = \sum_{j=1}^{n} \frac{\partial h(x)}{\partial x_j} f_j(x) \\ L_d h(x) = \frac{\partial h(x)}{\partial x} d(x) = \sum_{j=1}^{n} \frac{\partial h(x)}{\partial x_j} d_j(x) \\ L_f^r h(x) = L_f(L_f^{r-1} h(x)) \\ L_f^0 h(x) = h(x) \\ L_g L_f^{r-1} h(x) = \frac{\partial (L_f^{r-1} h(x))}{\partial x} \cdot g(x) \\ L_{f+gu+d} L_d h(x) = \frac{\partial (L_d h(x))}{\partial x} (f(x) + g(x)u + d) \end{cases} \tag{3}$$

According to Equation (2), the system (1) can be therefore transformed as $y^r = v$ via input-output feedback linearization with control law

$$u = \frac{1}{L_g L_f^{r-1} h(x)} (v - L_f^r h(x)) \tag{4}$$

According to Equation (2) and assuming $r = n$, the new state vector is set as:

$$z_n = L_f^{n-1} h(x) + L_g L_f^{n-2} h(x) \cdot u + L_{f+gu+d}^{n-2} L_d h(x) + \sum_{\lambda=1}^{n-2} L_{f+gu+d}^{\lambda-1} L_d L_f^{n-1-\lambda} h(x) \tag{5}$$

which depends not only on the state vector of system (1) but also on high-order derivatives of perturbations. We take the first-order and second-order derivatives with respect to y as an example, then the derivative $\dot{y}$ and $\ddot{y}$ can be expressed as follows:

$$\begin{cases} \dot{y} = \dot{z}_1 = \nabla h(f + g \cdot u + d) = L_f h(x) + L_g h(x) u + L_d h(x) \\ \ddot{y} = \ddot{z}_1 = L_f^2 h(x) + L_g L_f h(x) u + L_d L_f h(x) + L_{f+gu+d} L_d h(x) \end{cases} \tag{6}$$

where $y = z_1 = h(x)$, in accordance with (5), obviously, $z_2 = \dot{z}_1$, $z_3 = \dot{z}_2 = \ddot{z}_1$, using the recursive higher-order Lie derivatives, we obtain the input-output linearizable canonical form

$$\begin{cases} \dot{z}_i = z_{i+1}, i = 1, 2, \cdots, n-1 \\ \dot{z}_n = a(z) + b(z)u + d_z \\ y = z_1 \end{cases} \tag{7}$$

where $z = [z_1, z_2, \cdots, z_n]$. For simplicty but without loss of generality, the referred disturbances $d_j, j = 1, 2, \cdots, n$ in z are assumed to be known in the context of input-output feedback linearization. Further, if the system's control input is subject to input saturation, i.e.,

$$u = \begin{cases} u_{max},\ if\ u_d > u_{max} \\ u_d,\ if\ u_{min} \leq u_d \leq u_{max} \\ u_{min},\ if\ u_d < u_{min} \end{cases} \tag{8}$$

where u_{max} and u_{min} are bounds, and u_d is the designed control signal. Taking (8) into account and substituting $\bar{u} = u - u_d$ into (7), we obtain

$$\begin{cases} \dot{z}_i = z_{i+1}, i = 1, 2, \cdots, n-1 \\ \dot{z}_n = a(z) + b(z)(u_d + \bar{u}) + d_z = a(z) + b(z)u_d + d_L \\ y = z_1 \end{cases} \tag{9}$$

where $d_L := d_z + b(z)\bar{u}$ represents the disturbances caused by the external disturbance and the control saturation. The control object is to steer the system output y to follow a desired bounded signal y_d in the presence of disturbances and input saturation. Note that the effects caused by the disturbance d in Equation (1) can be attenuated by the usage of a feedforward technique if such a disturbance is measurable. However, in most cases, it is impossible to be directly measured or too expensive to measure all the disturbances in d. One intuitive method is to design a disturbance observer to approximately estimate the disturbance from the measurable variables. Moreover, the effects caused by the mismatched disturbance may be impossible to be completely eliminated no matter what kind of control scheme adopted [30].

Remark 1. *In Equation (7), the relative degree r is assumed to be equal to n, which is called fully linearizable. However, in the case of partially linearizable, (i.e., $r < n$), the rest $n - r$ state variables are unobservable, which is called internal dynamics that may not be stable [31]. Since zero dynamics only contains the internal dynamics, the stability analysis of internal dynamics can be simply carried out by analyzing the stability of zero dynamics where the output is constantly kept at zero through changing control actions [32,33] (see Lemma 1). Note that for linear systems, the stability of zero dynamics means global stability of internal dynamics. Yet, in terms of nonlinear systems, only local stability of internal dynamics holds even with global exponential stabilizing of zero dynamics [34].*

Lemma 1. *For system (9), if the the relative degree r is less than the system order n, then define the external dynamics $z = [z_1, z_2, \cdots, z_r]^T = [y, \dot{y}, \cdots, y^{(r-1)}]^T$, and choose the left unobserved system states $\varsigma = [\varsigma_1, \varsigma_2, \cdots, \varsigma_{(n-r)}]^T$, there exists a local homeomorphism $\Xi(x) = [z_1, z_2, \cdots, z_r, \varsigma_1, \varsigma_2, \cdots, \varsigma_{(n-r)}]$ such that [29]*

$$\begin{cases} z^{(r)} = a(z, \varsigma) + b(z, \varsigma)u + d_L \\ \varsigma^{(n-r)} = a'(z, \varsigma) + b'(z, \varsigma)u + d'_L \\ y = z_1 \end{cases} \tag{10}$$

holds. According to (10), we have

$$\varsigma^{(n-r)} = a'(z, \varsigma) + \frac{b'(z, \varsigma)(z^{(r)} - a(z, \varsigma) - d_L)}{b(z, \varsigma)} + d'_L \tag{11}$$

Utilizing the analysis of zero dynamics with $z = 0$, we obtain that

$$\varsigma^{(n-r)} = a'(0, \varsigma) - \frac{b'(0, \varsigma)(a(0, \varsigma) + d_{L(z=0)})}{b(0, \varsigma)} + d'_{L(z=0)} \tag{12}$$

If the stability of zero dynamics (12) is achieved, then the internal dynamics ς is locally stabilizing.

In the next Section 5.2, we will analyze such characteristics through example 2 in which the complex nonlinear system has unmatched disturbances and unobserved internal dynamics.

3. Design of Finite-Time Sliding Mode Disturbance Observer

In order to suppress or attenuate the negative effects caused by disturbances, the disturbance observer is an alternative to achieve the desired performance. In general, the idea behind the disturbance observer is to lump disturbance and uncertainty together for total estimation [35]. Depending on the recursive terminal sliding mode controller based on the disturbance observer, the authors investigated the tracking problem for a class of third-order chained–form nonholonomic systems in the presence of unknown external disturbances [36]. In Ref. [37], a class of SISO nonlinear systems with mismatched uncertainties and disturbances were solved by using the multiple-surface SMC-based disturbance observer.

Apart from the design of the sliding mode controller, the sliding mode disturbance observer has also attracted considerable research attention for the robust control of uncertain nonlinear systems due to its simplicity, transparency, and strong ability of disturbance attenuation and rejection [38]. Since the sliding mode disturbance observer does not rely on the bounded disturbance of the model but is only related to its bounds [39], the observer can be firstly developed for approximately estimating and compensating the unknown disturbance. The approximation error of the disturbance observer could be theoretically proved to be converged to zero in finite time. To effectively compensate for the unmeasured disturbance, auxiliary variables are used for the design of a finite-time sliding mode disturbance observer due to simplicity, transparency, and designability. In this paper, the auxiliary variable s_d and the intermediate variable ψ are introduced, where $s_d = \psi - z_n$ and ψ is given by [40]

$$\dot{\psi} = -ks_d - \beta sign(s_d) - \xi s_d^{\frac{p_0}{q_0}} - |a(z)|sign(s_d) + b(z)u_d \tag{13}$$

The relationship between auxiliary variables and other variables in the control system would be described in detail in Figure 1 in Section 4. In Equation (13), p_0 and q_0 are odd positive integers while $p_0 < q_0$, and the designed parameters k, β and ξ are also positive. Then, Equation (14) is derived as

$$\dot{s}_d = \dot{\psi} - \dot{z}_n = -ks_d - \beta sign(s_d) - \xi s_d^{\frac{p_0}{q_0}} - |a(z)|sign(s_d) - a(z) - d_L \tag{14}$$

Now, the terminal SMC estimated disturbance is given:

$$\hat{d}_L = -ks_d - \beta sign(s_d) - \xi s_d^{\frac{p_0}{q_0}} - |a(z)|sign(s_d) - a(z) \tag{15}$$

Considering Equations (9), (13)–(15), we obtain

$$\begin{aligned} \tilde{d}_L = \hat{d}_L - d_L &= -ks_d - \beta sign(s_d) - \xi s_d^{\frac{p_0}{q_0}} - |a(z)|sign(s_d) - a(z) - \dot{z}_n + a(z) + b(z)u_d \\ &= -ks_d - \beta sign(s_d) - \xi s_d^{\frac{p_0}{q_0}} - |a(z)|sign(s_d) - \dot{z}_n + b(z)u_d \\ &= \dot{\psi} - \dot{z}_n = \dot{s}_d \end{aligned} \tag{16}$$

Lemma 2. *If there exists a positive definite function $V(t)$ such that the differential inequality [26]:*

$$\dot{V}(t) + \varrho V(t) + \Gamma V^{\theta}(t) \leq 0, \forall t > t_0 \tag{17}$$

holds for $t \geq t_0$ and $V(t_0) \geq 0$, then $V(t)$ converges to the equilibrium point in finite time with

$$t_s \leq t_0 + \frac{1}{\varrho(1-\theta)} In \frac{\varrho V^{1-\theta}(t_0) + \Gamma}{\Gamma} \tag{18}$$

where $\varrho > 0, \Gamma > 0, 0 < \theta < 1$.

Assumption 1. *The unknown disturbance d_L in (9) is bounded with $|d_L| < \beta$.*

Theorem 1. *Considering the nonlinear SISO systems (9) with disturbances and input saturation, the finite time terminal sliding mode disturbance observer (15) is designed such that the disturbance approximation error (16) converges to zero in finite time t_s.*

Proof. Considering the Lyapunov function

$$V_d = \frac{1}{2}s_d^2 \tag{19}$$

and utilizing the Equation (14) and Assumption 1, the derivative of V_d is given

$$\begin{aligned} \dot{V}_d = s_d\dot{s}_d &= s_d(-ks_d - \beta sign(s_d) - \xi s_d^{\frac{p_0}{q_0}} - |a(z)|sign(s_d) - a(z) - d_L) \\ &= -ks_d^2 - \beta|s_d| - \xi s_d^{\frac{p_0+q_0}{q_0}} - |a(z)||s_d| - a(z)s_d - s_d d_L) \\ &\leq -ks_d^2 - \beta|s_d| - \xi s_d^{\frac{p_0+q_0}{q_0}} + |s_d||d_L| \\ &\leq -ks_d^2 - \xi s_d^{\frac{p_0+q_0}{q_0}} \\ &\leq -2kV_d - 2^{\frac{p_0+q_0}{2q_0}}\xi V_d^{\frac{p_0+q_0}{2q_0}} \end{aligned} \tag{20}$$

□

According to Lemma 2 and Equation (20), the auxiliary variable s_d converges to the equilibrium point in finite time t_s, i.e.,

$$\begin{aligned} t_s &\leq t_0 + \frac{q_0}{k(q_0 - p_0)} In(\frac{2k(\frac{1}{2}s_d^2)^{\frac{q_0-p_0}{2q_0}}(t_0)}{2^{\frac{p_0+q_0}{2q_0}}\xi} + 1) \\ &< t_0 + \frac{q_0}{k(q_0 - p_0)} In(\frac{ks_0^{\frac{q_0-p_0}{q_0}}(t_0)}{\xi} + 1) \end{aligned} \tag{21}$$

Consequently, $\dot{s}_d$ converges to zero in finite time, which shows that the estimated disturbance error $= \tilde{d}_L = \hat{d}_L - d_L$ in Equation (16) is also convergent to zero in finite time.

4. Terminal Sliding Mode Control

In order to achieve a satisfactory tracking control performance in the presence of disturbance and input saturation, in this section, we adopt a terminal sliding surface that combines the linear surface with the designed sliding mode observer. First, as a linearized stable differential operator, the linear sliding surface $s_l(t)$ in the conventional SMC is concerned with tracking error $e(t)$ and its derivatives, which can be designed as

$$s_l(t) = C^T e \tag{22}$$

Here, $e = [e_1, e_2, \cdots, e_{n-1}, e_n]$, n indicates system's order, $e_1 = y - y_d$, $\dot{e}_i = e_{i+1}$, $C = [c_1, c_2, \cdots, c_{n-1}, 1]^T$ represents the bandwidth of error dynamics and set to be positive constants such that the roots of polynomial $c_1 + c_2 s + \cdots + c_{n-1}s^{n-2} + s^{n-1}$ lie in the open left half (Hurwitz) plane. Then, the terminal sliding surface is designed as

$$s_n(t) = s_l(t) + s_d(t) \tag{23}$$

Theorem 2. *For a class of nonlinear systems* (9) *that is subject to disturbance and input saturation, assume all the states are available, and based on the designed sliding mode observer $\hat{d}_L$ in Equation* (15)*, the system's output tracking control performance can be guaranteed and the desired trajectory can be attained in finite time by the designed control law with*

$$u_d(t) = -\frac{1}{b(z)}(\sum_{j=1}^{n-1} c_j e_{j+1} - y_d^{(n)} + a(z) + \hat{d}_L + \eta s_n + \sigma s_n^{\frac{p}{q}}) \tag{24}$$

Proof. Consider the Lyapunov function

$$V = \frac{1}{2}s_n^2 \tag{25}$$

calculate the time derivative of V, and utilize the system Equation (9), the linear sliding hyperplane $s_l(t)$ in (22) and the control law in (24), we obtain

$$\begin{aligned}
\dot{V} = s_n\dot{s}_n &= s_n(\dot{s}_l(t) + \dot{s}_d(t)) = s_n(\sum_{j=1}^{n-1} c_j\dot{e}_j + \overbrace{\dot{e}_n}^{=y^{(n)}-y_d^{(n)}} + \dot{s}_d) \\
&= s_n(\sum_{j=1}^{n-1} c_j\dot{e}_j + a(z) + b(z)u_d + d_L - y_d^{(n)} + \dot{s}_d) \\
&= s_n(\overbrace{-\hat{d}_L + d_L}^{-\dot{s}_d} + \dot{s}_d - \eta s_n - \sigma s_n^{\frac{p}{q}}) \\
&= -\eta s_n^2 - \sigma s_n^{\frac{p+q}{q}} \\
&\leq -2\eta V - 2^{\frac{p+q}{2q}}\sigma V^{\frac{p+q}{2q}}
\end{aligned} \tag{26}$$

which shows that the desired trajectory can be convergently tracked in finite time according to Lemma (2), i.e.,

$$t_s < t_0 + \frac{q}{\eta(q-p)} In(\frac{\eta s_0^{\frac{q-p}{q}}(t_0)}{\sigma} + 1) \tag{27}$$

where p and q are odd positive integers while $p < q$, $\eta > 0$ and $\sigma > 0$. □

Finally, the block diagram of the proposed method is described in Figure 1.

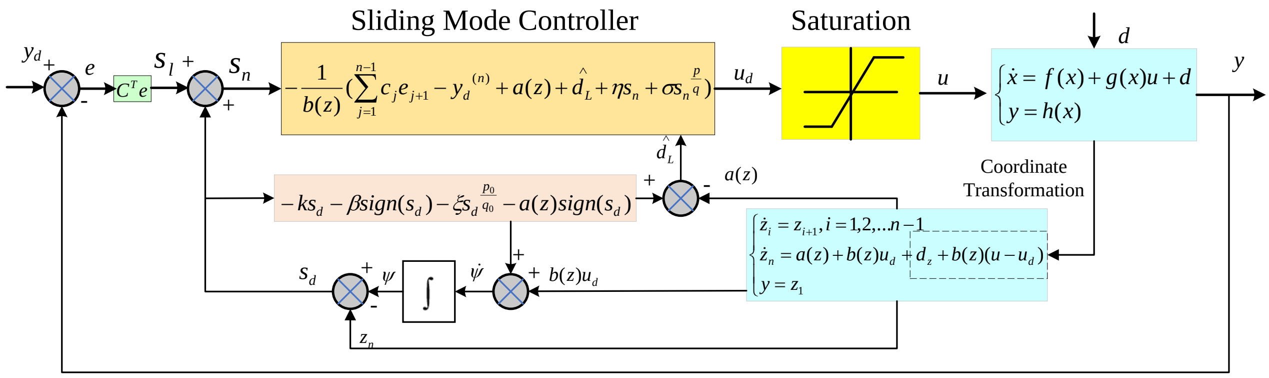

Figure 1. Block diagram of sliding mode tracking control-based finite-time disturbance observer and input-output feedback linearization.

Remark 2. *There are kinds of variants concerning the sliding mode surface* s_l. *For simplicity but without loss of generality, we take the second-order nonlinear systems as an example, e.g., PID-type terminal sliding mode* $s_l(t) = c_1e(t) + \dot{e}(t) + \mu\int_0^t e(\tau)^{q/p}d_\tau$ *in [13], linear adaptive sliding mode* $s_l(t) = (1+\hat{\alpha})(c_1e(t) + \dot{e}(t))$ *in [26], fast terminal sliding mode* $s_l(t) = c_1e(t) + \dot{e}(t) + \beta_1e(t)^{p_1/q_1}$ *in [40]. Note that on the basis of the above sliding hyperplanes, the respective control laws can be derived by meeting the Lyapunov stability. Although introducing the additional parameter may provide more flexibility in the controller's design, it aggravates the burden of parameter tuning. We would like to point out that the usage of a discontinuous sign function in the disturbance observer (15) and control law (24) results in high robustness to disturbance but with possible high-frequency chattering. Such a chattering phenomenon could be relieved by simply using* *the boundary layer method. However, by doing so, a nonzero steady-state error may be unavoidable. We analyze the relationship between the boundary layer and chattering through illustrative examples in Section 5.*

5. Illustrative Examples

Since various systems such as robotics, electronics, and mechanics can be modeled as a nonlinear second-order or third-order structure [41], it is necessary to investigate such systems with respect to stabilization and tracking. Therefore, in this section, we take two typical nonlinear systems as examples and study the trajectory tracking problem with consideration of disturbance and control saturation.

5.1. Example 1

Considering the Van der Pol circuits system [42],

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -2x_1 + 3(1 - x_1^2)x_2 + u + d \\ y = x_1 \end{cases} \tag{28}$$

where $x = [x_1, x_2] \in \mathcal{R}$ is state, $y \in \mathcal{R}$ is output, $u \in [u_{min}, u_{max}]$ is control input, d is external disturbance. The desired trajectory is set as $y_d = 2sin(0.2t)$ and the disturbance signal is chosen as $d = 2sin(0.1\pi t) + 3sin(0.2\sqrt{t+1}) + d_s$, where $d_s = 5, 15 < t < 20$. The control input is limited within $u \in [-20, 15]$, the initial conditions are set as $x_1(0) = x_2(0) = 0.1$. Since y is not directly related to u, we first apply the input-output feedback linearization technique for coordinate transformation. It can be seen that the relative degree is equal to the system order (i.e., $r = n = 2$), and u explicitly appears in $y^{(2)}$. Then, according to Equation (7), $a(z) = -2x_1 + 3(1 - x_1^2)x_2, b(z) = 1$ and $d_z = d$ can be derived, respectively. According to (24), the terminal sliding mode controller based on the disturbance observer is given as

$$u_d(t) = -c_1(\dot{x}_1 - \dot{y}_d) + y_d^{(2)} + 2x_1 - 3(1 - x_1^2)x_2 - \hat{d}_L - \eta s_2 - \sigma s_2^{\frac{p}{q}} \tag{29}$$

where

$$\begin{cases} s_d = \psi - z_2 = \psi - x_2 \\ \dot{\psi} = -ks_d - \beta sign(s_d) - \xi s_d^{\frac{p_0}{q_0}} - \left|-2x_1 + 3(1 - x_1^2)x_2\right| sign(s_d) + u_d \\ s_l(t) = C^T e = c_1 e_1 + c_2 \dot{e}_1 = c_1(y - y_d) + c_2(\dot{x}_1 - \dot{y}_d) \\ s_2(t) = s_l(t) + s_d(t) \\ \hat{d}_L = -ks_d - \beta sign(s_d) - \xi s_d^{\frac{p_0}{q_0}} - \left|-2x_1 + 3(1 - x_1^2)x_2\right| sign(s_d) + 2x_1 - 3(1 - x_1^2)x_2 \end{cases} \tag{30}$$

To achieve a good tracking performance, the parameters of the sliding mode controller are chosen as $c_1 = 66$, $c_2 = 1$, $k = 30$, $\beta = 6$, $\xi = 0.5$, $p_0 = p = 5$, $q_0 = q = 7$, $\eta = 10$, $\sigma = 1$. The simulation results are given below. In Figure 2, it can be seen that although there exists a large estimate error that is mainly caused by the control input saturation and the imposed additional step signal, the designed sliding mode observer is able to approximate the disturbance such that the estimated error tends to zero in finite time. According to Equation (27) and the parameter setting, the finite time is calculated as $t_s = 1.0086$, which can be verified in Figure 3. In Figure 4, we observe that due to the high robustness to disturbance, the perfect tracking performance is achieved but at the expense of high-frequency chattering, c.f., Figures 2 and 3. Such unexpected chattering may seriously aggravate the burden of actuators and therefore affect the closed-loop system performance.

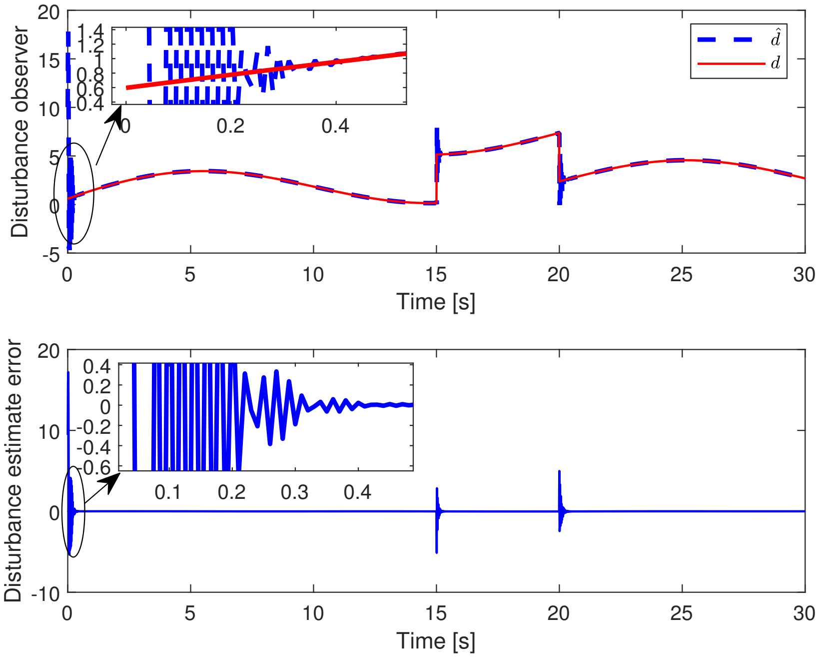

Figure 2. Disturbance estimate under the discontinuous control signal with the resulting high−frequency chattering.

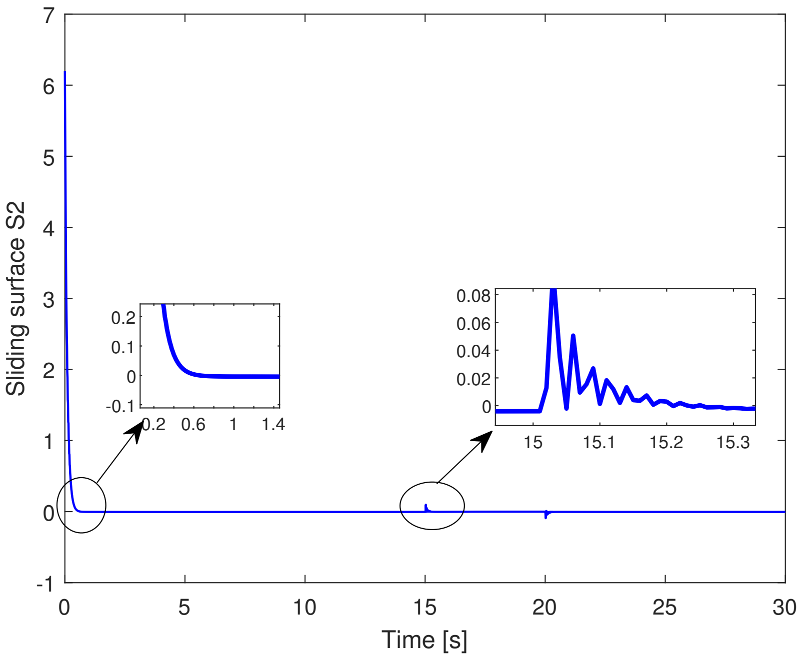

Figure 3. The variation of sliding hyperplane for finite−time convergence.

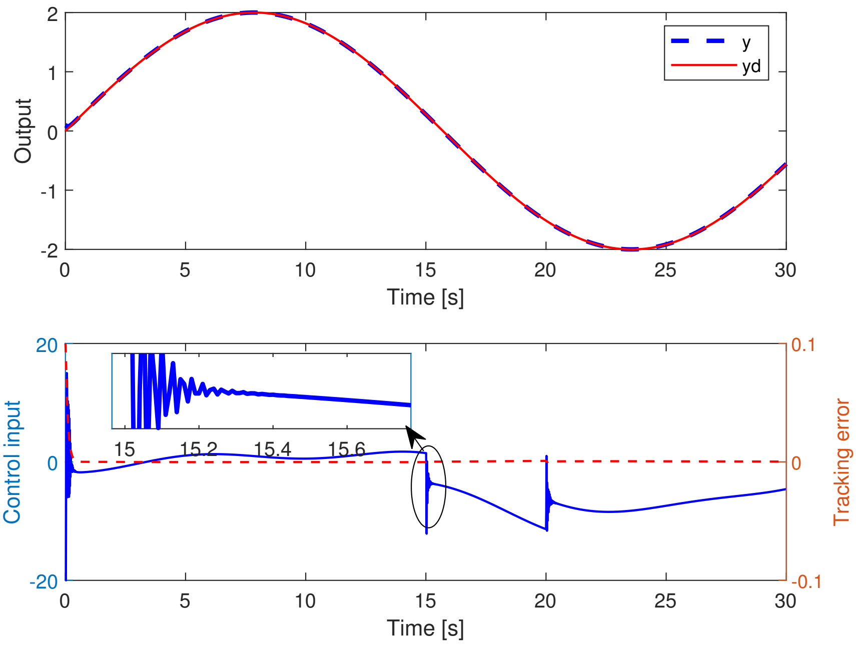

Figure 4. Perfect trajectory tracking with high−frequency switching control actions at the cost of chattering.

To alleviate the high-frequency control actions, the above discontinuous control function $sign(\cdot)$ can be replaced by a saturation function, i.e.,

$$sat(s) = \begin{cases} 1, s > \Delta, \\ \kappa s, |s| \leq \Delta, \kappa = \frac{1}{\Delta} \\ -1, s < -\Delta \end{cases} \tag{31}$$

where $\Delta > 0$ represents the thickness of the boundary layer. In Figure 5, we test three different thicknesses of boundary layer and conclude that the thicker the boundary layer, the smaller the chattering, the slower the reaction to disturbance, and the larger the nonzero steady-state tracking error. To further evaluate the tracking performance and show the robustness of the proposed method, a PID controller and a sliding mode controller under input-output feedback linearization technique are adopted, respectively [43], i.e.,

$$\begin{cases} u_{pid} = k_p e(t) + k_i \int_0^t e(\tau) d_\tau + k_d \dot{e}(t) \\ u_{smc}^I = y_d^{(2)} + c_I \dot{e} + 2x_1 - 3(1 - x_1^2)x_2 + \eta_I sat(s_l) \end{cases} . \tag{32}$$

The controllers' gains are tuned to achieve the minimum possible tracking error through experimental study, where $k_p = 35, k_i = 0.5, k_d = 10$ and $c_I = 20, \eta_I = 15$. In Figure 6, we find that both PID and sliding mode controllers are effective manners to follow the desired trajectory. However, among them, the proposed method (i.e., e_{smc2} marked by the green line) has the smallest tracking error and fastest reaction to unpredictable disturbances.

Remark 3. *Note that the usage of a boundary layer can reduce the chattering to a certain extent, but gives rise to the nonzero steady-state error, which means that the sliding mode controller with a boundary layer does not guarantee asymptotic stability but rather uniform ultimate boundedness. In practical applications, the trade-off between the unwanted high-frequency control actions and the unavoidable bounded steady-state error should be taken into consideration. A simple and possible remedy way is using a time-varying boundary layer* Δ *instead of the traditional fixed values according to the variations of tracking error* e_1*, i.e,* $\Delta = \Delta_1 \chi + \Delta_2(1 - \chi)$*, where* Δ_1 *and* Δ_2 *are* *different boundary layers,* $\Delta_1 > \Delta_2 > 0$*,* $\chi = 1$ *for* $|e_1| > e_0$ *and* $\chi = 0$ *for* $|e_1| \leq e_0$*,* $e_0 < \Delta$ *is a positive value* [44].

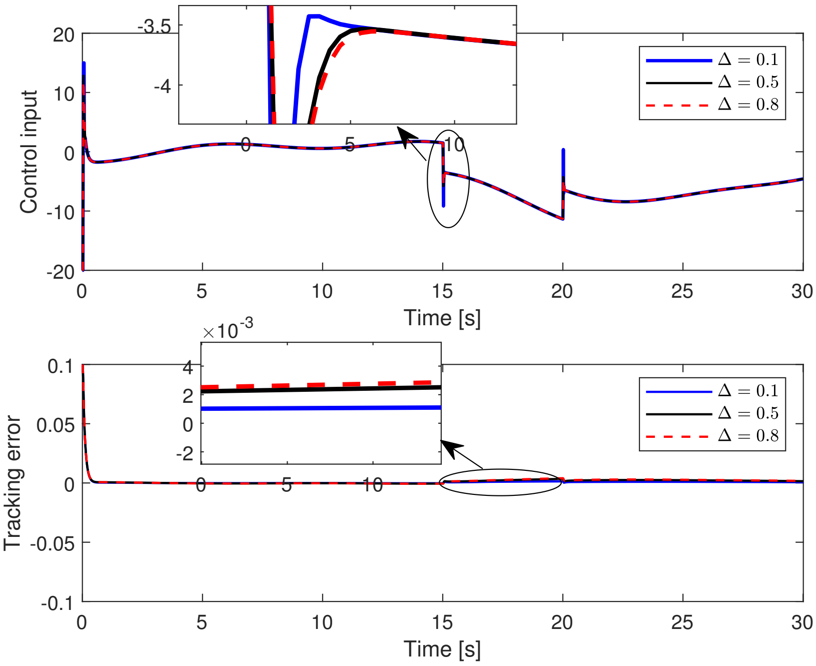

Figure 5. Control input and tracking error under different boundary layer thicknesses.

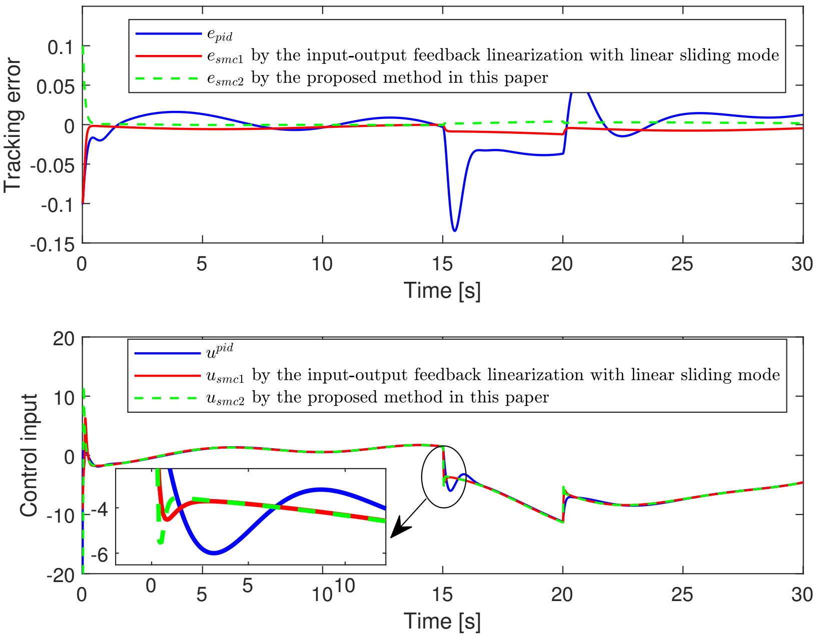

Figure 6. Comparison of tracking performance between PID and SMC for Van der Pol circuits system.

5.2. Example 2

In this part, we consider a classical three-order nonlinear system with time-varying disturbance [29], i.e.,

$$\begin{cases} \dot{x}_1 = sinx_2 + (x_2+1)x_3 + d_1 \\ \dot{x}_2 = x_1^5 + x_3 + d_2 \\ \dot{x}_3 = x_1^2 + u + d_3 \\ y = x_1 \end{cases} \tag{33}$$

where $x = [x_1, x_2, x_3] \in \mathcal{R}^3$ is state, $y \in \mathcal{R}$ is output, $d_1 = sint$ is the imposed disturbance signal while $\{d_2, d_3\} = sint$ represents the unknown disturbances caused by the external disturbances and model uncertainty. Similar to system (28), the control input u also explicitly appears in $y^{(2)}$, but the relative degree is less than the system order, which needs to analyze the stability of internal dynamics. According to Equations (2) and (9), we obtain

$$\begin{cases} \dot{z}_1 = z_2, \\ \overbrace{y^{(2)}}^{\dot{z}_2} = \overbrace{L_f^2 h(x)}^{:=a(z)} + \overbrace{L_g L_f h(x)}^{:=b(z)} u_d + \overbrace{L_d L_f h(x) + L_{f+gu+d} L_d h(x) + b(z)\bar{u}}^{:=d_L} \\ y = z_1 \end{cases} \tag{34}$$

where

$$\begin{cases} a(z) = (cosx_2 + x_3)(x_1^5 + x_3) + (x_2+1)x_1^2 \\ b(z) = x_2 + 1 \\ L_d L_f h(x) = (cosx_2 + x_3)d_2 + (x_2+1)d_3 \\ L_{f+gu+d} L_d h(x) = \dot{d}_1 \end{cases} \tag{35}$$

The terminal sliding mode controller based on the disturbance-observer is given as

$$u_d(t) = -\frac{1}{x_2+1}(c_1(\dot{x}_1 - \dot{y}_d) - y_d^{(2)} + a(z) + \hat{d}_L + \eta s_2 + \sigma s_2^{\frac{p}{q}}) \tag{36}$$

In accordance with (15) and similar to (30), the designed terminal sliding mode surface s_d and disturbance observer $\hat{d}_L$ can also be derived. The parameters of sliding mode controller are chosen as $c_1 = 9$, $c_2 = 1$, $k = 65$, $\beta = 5$, $\xi = 2$, $p_0 = p = 5$, $q_0 = q = 9$, $\eta = 8$, $\sigma = 0.5$. Similar to example 1, the simulation results concerning the disturbance observer, and the trajectory tracking and control input are obtained in Figure 7 and Figure 8, respectively. In Figure 7, we observe that the disturbance estimate error tends to zero in finite time after a short period of oscillation. According to Equation (27) and parameter settings, the finite time is computed as $t_s = 0.7706$, which can be verified in Figure 8. Similar to example 1, the two types of controllers defined by Equation (32) are compared to the proposed method in terms of tracking performance and robustness. $k_p = 50$, $k_i = 0.5$, $k_d = 10$ are chosen for PID setting while $c_I = 10$, $\eta_I = 3$ for SMC. In Figure 9, we observe that SMC-type controllers outperform PID in terms of tracking performance and fast and robust response to time-varying disturbance. On the downside, however, good tracking performance is achieved at the cost of high-frequency control actions for the SMC with feedback linearization and linear sliding surface. On the basis of the designed sliding mode disturbance observer, satisfactory tracking performance is attained while suppressing the chattering to a great extent. For sure, the boundary layer could be further adopted for chattering attenuation but may sacrifice a certain tracking performance. There are other points that we would like to highlight:

- Regarding the parameters in the terminal sliding mode control law based on the disturbance observer, β is determined such that the Assumption 1 holds. The parameters $c_j, j = 1, 2, \cdots$ are chosen such that the roots of polynomial $c_1 + c_2 s + \cdots + c_{n-1} s^{n-2} + s^{n-1}$ lie in the open left half (Hurwitz) plane. The finite-time convergence of disturbance estimate error is subject to the combination of k, ξ, p_0, q_0 while the parameters η, σ, p, q are selected for the finite-time convergence concerning the trajectory tracking error. In these parameters, $p_0 < q_0$ and $p < q$ must hold. As for the left parameters, in general, there is no rule for the deterministic parameters tuning procedure to achieve the perfect tracking with fast convergence and without chattering. Such parameters may be determined by the trial-and-error method according to the accuracy requirement of tracking, allowable chattering frequency for the actuator, and different initial conditions.
- In example 2, since the relative degree is less than the system order, there exists one unobservable state. According to Lemma 1, we choose $\{y, \dot{y}, x_3\}$ as the new state set and utilize the method of zero dynamics analysis to keep the system output and its successive derivatives to be zero through changing control actions. In accordance with Equations (34) and (35), the stability of internal dynamics x_3 can be guaranteed when $x_2 \in (-1, \frac{\pi}{2})$. The conclusion is verified in Figure 10 where the system output and its successive derivatives are kept at zero while the internal dynamics x_3 are bounded along with the variation of external dynamics x_2. Further, the variations of x_2 and x_3 in the context of trajectory tracking are presented in Figure 11, where both x_2 and x_3 are bounded input bounded output (BIBO) stable.

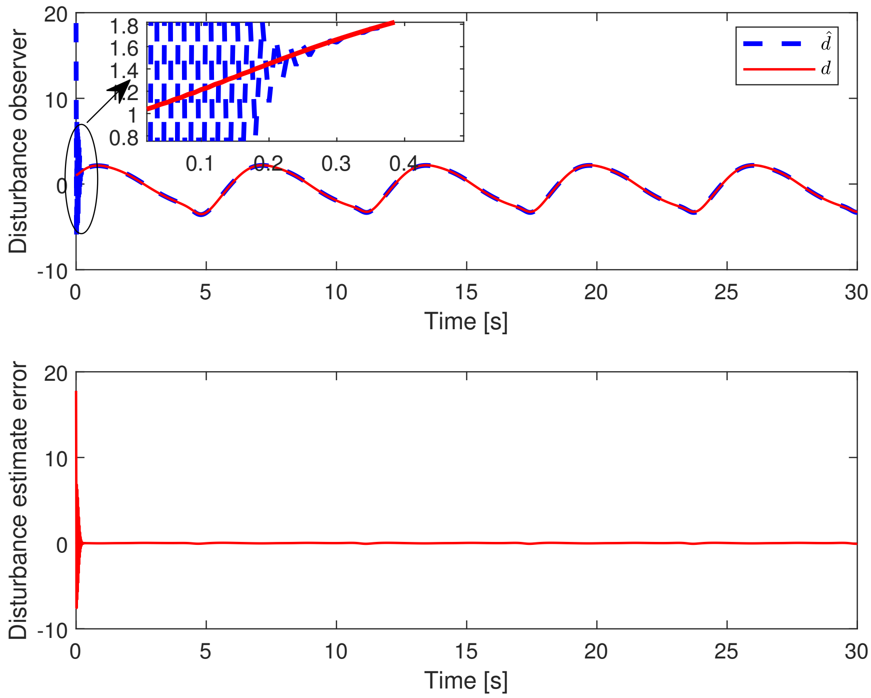

Figure 7. Disturbance estimate in the presence of time−varying disturbance.

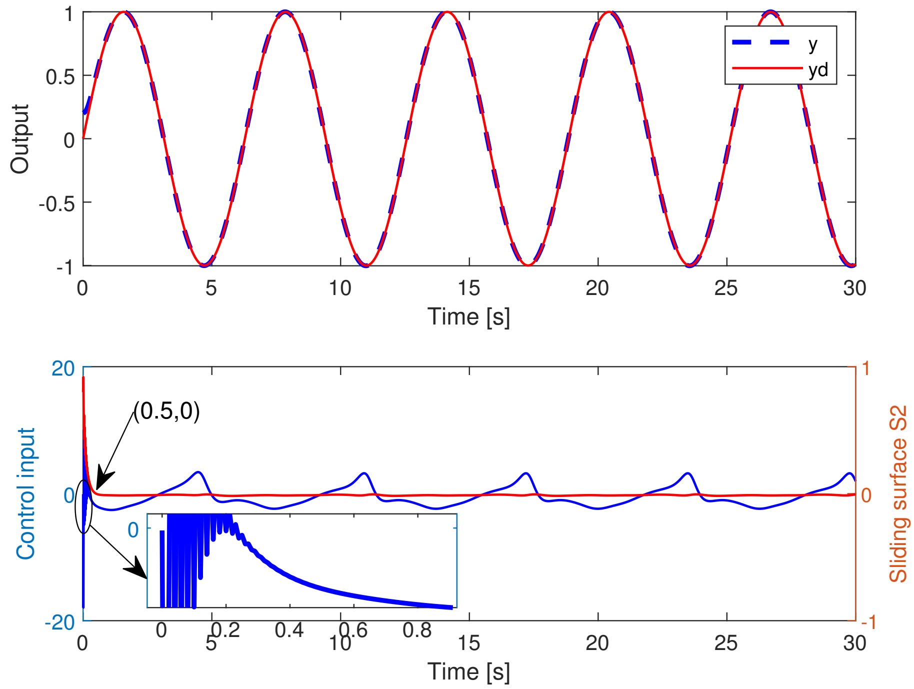

Figure 8. Finite−time trajectory tracking with sliding mode control−based disturbance observer.

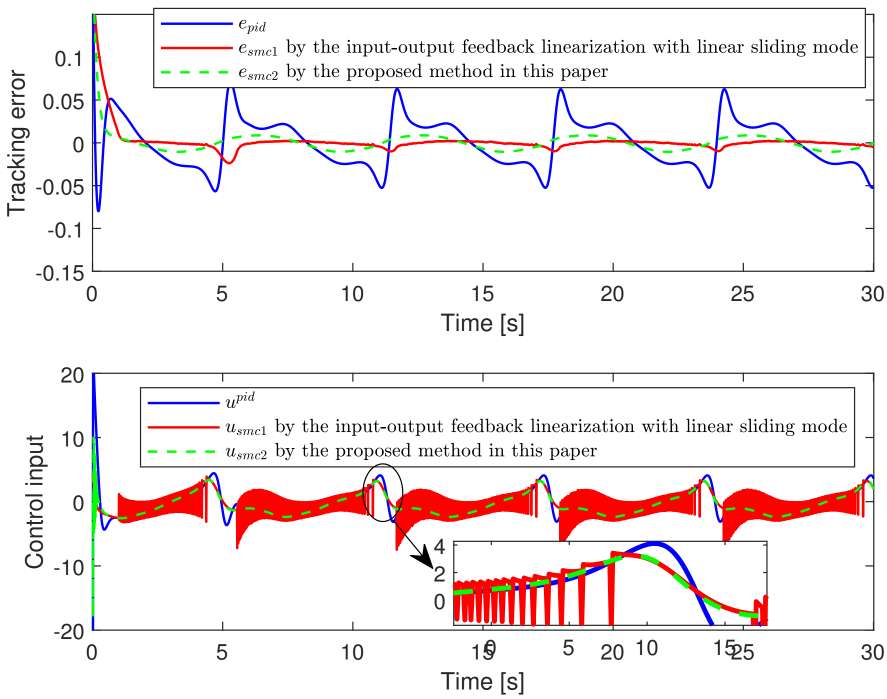

Figure 9. Comparison of tracking performance and control input with different types of controllers.

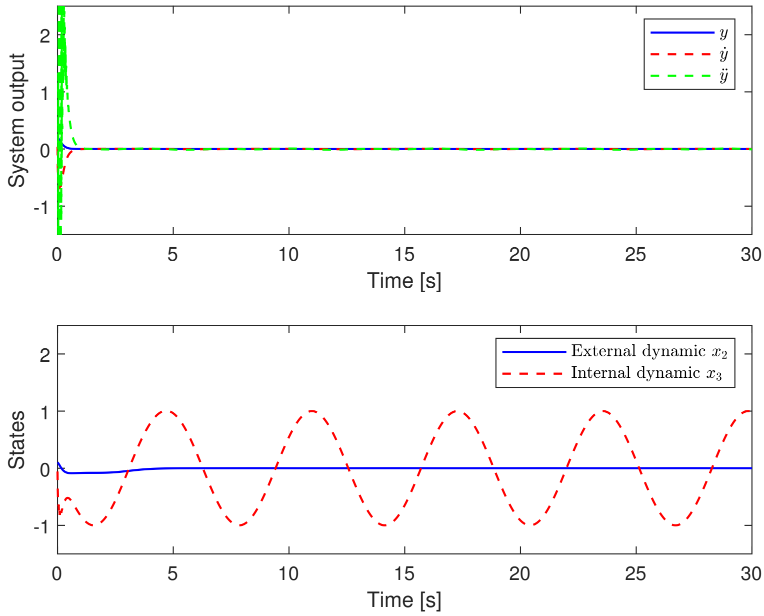

Figure 10. The variations of the bounded external dynamics x_2 and internal dynamics x_3 under zero dynamics.

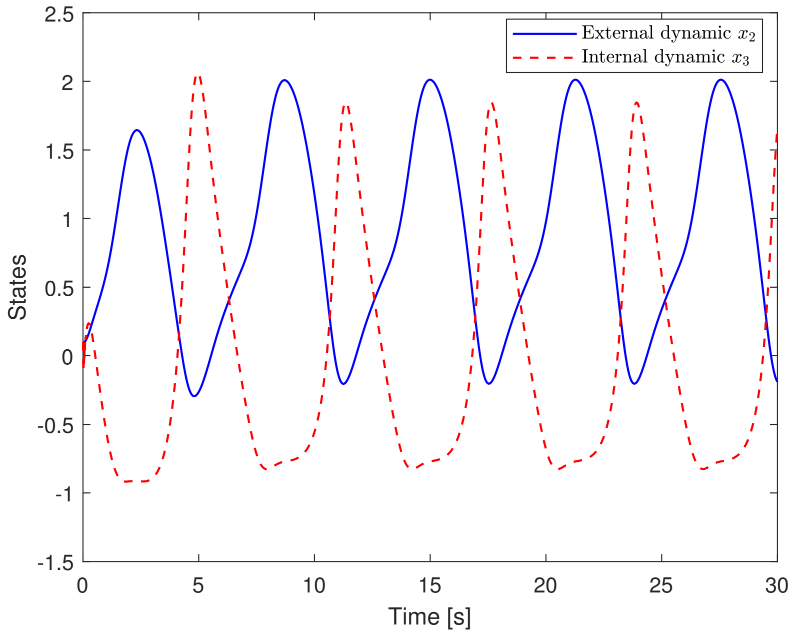

Figure 11. The variations of the bounded external dynamics x_2 and internal dynamics x_3.

6. Conclusions and Outlook

In this paper, we investigated the problem of trajectory tracking for a class of nonlinear SISO systems in which the system output was not directly related to the control input. To acheive a satisfactory tracking performance in the presence of possible model uncertainty, disturbance, and control saturation, a sliding mode disturbance observer-based terminal sliding mode tracking controller was theoretically designed, which combines the advantage of traditional linear sliding hyperplane and nonlinear terminal sliding surface. The finite-time convergence was proved by using Lyapunov stability analysis. The effectiveness of the proposed method was demonstrated through numerical simulations in comparison to

PID and sliding mode controller based on the linear sliding surface and the input-output feedback linearization technique. The results show that the proposed method is superior to the state-of-the-art method on responsiveness and robustness to unpredictable disturbance, resulting in a smaller tracking error. In particular, the designed disturbance observer was able to suppress the chattering caused by the discontinuous control actions to a certain extent. Further, the unexpected chattering could be alleviated by using the boundary layer method but at the expense of a nonzero steady-state error. Moreover, the unobserved internal dynamics may exist in the process of feedback linearization. If so, the stability of internal dynamics and zero dynamics need to be further considered.

In the design of the disturbance observer and terminal sliding mode controller, it refers to some parameters that are of significant importance for guaranteeing and improving tracking performance. Yet, tuning such parameters in an allowable range to accomplish a good performance is time-consuming and challenging, especially for multi-input multi-output (MIMO) systems. Moreover, although the boundary layer method is convenient for chattering alleviating, it deteriorates the tracking performance. Therefore, adaptively tuning the controller's parameters, especially for the boundary layer, could be a future research direction.

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Conflicts of Interest: The authors declare no conflict of interest.

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