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# Efficient Approaches for Layout Problems of Large Chemical Plants Based on MILP Model

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**Abstract:** This paper presents two novel solution approaches for addressing large-scale multi-floor process plant layout problems. Based on the mixed integer linear programming (MILP) model, the first solution approach employs a multi-directional search strategy while the second improves solution efficiency by reducing model size through an iterative framework. Both approaches determine the spatial arrangement of the plant equipment considering equipment-floor allocation, non-overlapping constraints, tall equipment penetrating multiple floors, etc. The computational results indicate that the proposed approaches achieved potential cost savings for four illustrative examples when compared to the previous studies. Finally, engineering experience constraints were included to represent a more complex industrial situation, and their applicability was tested with the last example.

**Keywords:** plant layout; MILP; solution efficiency; iterative framework; engineering experience

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## 1. Introduction

The plant layout is an indispensable step in process plant design, which requires finding a suitable spatial arrangement of the specified equipment in a certain space under various constraints [1]. The layout design minimizes the total investment while ensuring production efficiency and safety. Reasonable plant layout can reduce the material handling cost of the chemical plant by 10–30% [2] and the total investment by 50% [3]. Improper plant layout design will lead to risks, as in the survey, 79% of chemical process accidents are caused by design errors, and the most critical design errors are poor layouts [4]. The equipment layout problem of a chemical plant involves the comprehensive consideration of complicated factors such as equipment spatial location, material transportation planning, determination of workshop area and floor number setting, as well as the trade-off between safety and cost. Additionally, there has developed a tendency to design and construct chemical processing plants of greater sophistication, investment cost and of larger scale [5,6]. Under such circumstances, the development of effective designing methodologies for chemical plant layouts becomes pivotal.

Studies regarding plant layout can be classified into two categories, namely, the development of methodologies with more and more practical considerations, and the establishment of solution strategies that can handle large-scale complex cases.

Originally, the computer-aided layout design approach was presented by industrial engineers for the study of facility layout and location [7]. Subsequently, the layout problem of chemical plants was addressed by chemical engineers by considering system costs, and preferences as well as safety [8–10]. These early-stage models were based on heuristic rules, which may lead to sub-optimal solutions. Georgiadis and Macchietto [11] developed a mixed integer linear programming (MILP) approach for process plant layouts. Problem simplification was made by discretization of the layout area, which may result in overestimation of the required area and thus lead to suboptimal solutions. To tackle this, Papageorgiou and Rotstein [12] presented a mathematical programming model for the optimal process plant layout based on a 2-dimensional (2D) continuous spatial domain.

Thereafter, Patsiatzis and Papageorgiou [13] extended the model to multi-floor scenarios and added the floor construction cost as well as the land price to the objective function. The model can reasonably handle the layout of processes with 11 units. Park et al. [14] incorporated the TNT equivalency method into the multi-floor plant layout model to calculate the safety distance between the equipment. Conflict between expected damage costs and layout costs can be well-balanced and applications to process the layout of 10 units were illustrated. Latifi et al. [15] provided a comprehensive model considering the leakage risk of toxic substances, fire, explosion and other scenarios as well as the domino effect corresponding to these hazards. After this, Ejeh et al. [16] presented a more efficient MILP model for safe layout configuration using the Domino Hazard Index (DHI). Anbari and Parvini [17] conducted an investigation into the safety device of a chemical plant layout. In addition to safety concerns, practical aspects were also considered during the design of the chemical plant layout. O'Neill et al. [18] further investigated the implementation of a modular plant layout, which involves conducting work off-site before assembly on-site, to improve operational efficiency. In an effort to reduce costs associated with the pipe network in layout design, J et al. [19] introduced a MILP model that simultaneously optimizes the pipe network topology while selecting the most suitable station location. Besides, the multi-objective layout problem was taken into account recently. Saraswat et al. [20] presented a framework to solve the layout problem based on three objectives: flow distance, average work-in-process, and the number of required material handling devices. Ejeh et al. [21] presented a bi-objective MILP model designed for addressing the multi-floor plant layout problem. The model considers two crucial objectives, including the total monetary cost and financial risk. The financial risk is evaluated as the maximum probable property damage cost, determined by employing Dow's Fire & Explosion Index (F&EI) evaluation procedure. Furthermore, Liu et al. [22] developed mathematical models to achieve a balance between minimizing risk and maximizing rental profit.

The evolution of plant layout models with more and more practical considerations enabled a closer approximation to the problem itself, whereas the model complexity was also raised. In order to derive desirable solutions for large-scale problems, solution strategies were also developed by the community.

In the realm of large-scale modeling, various optimization methods have been proposed, such as decomposition-based, construction-based, improvement-based methods, and meta-heuristic algorithms [23]. Among them, decomposition-based methods, including Lagrangian relaxation and its variations such as the Alternating Direction Method of Multipliers (ADMM), Progressive Hedging (PH), Augmented Lagrangian Method (ALM), Dantzig-Wolfe Decomposition Method (DDM), and Benders Decomposition Method (BDM) are effective tools in tackling large-scale optimization problems [24,25]. These methods offer parallelization, which can substantially decrease computation time, and have been successfully applied in the design of material flow networks for a specific block layout with a single floor [26]. Researchers in the field of chemical plant layout have also employed decomposition-based techniques to propose efficient solutions for large-scale problems. Patsiatzis et al. [27] proposed the rigorous decomposition approach and iterative solution scheme to reduce the computational time. Subsequently, Xu et al. [28] put forward a construction-based algorithm, which first selects a portion of equipment for layout calculation, and then iterates continuously to insert the remaining equipment. Through the novel insertion schemes, the single-floor layout problem with 36 units can be solved. Following this, they employed an improvement-based method to devise an iterative algorithm that yielded superior solutions while also realizing computational efficiencies via a sequence of release and allocation strategies [29]. In order to address large-scale layout problems, various commercial solvers have been introduced into this domain. For example, CPLEX solvers have been utilized to resolve MILP models corresponding to chemical equipment layout problems by leveraging its internal hybrid algorithm primarily based on the branch and bound method [30]. Additionally, DICOPT has been employed to tackle the corresponding MINLP models [31]. From a modeling perspective, Ejeh et al. [32] presented an effective multi-floor layout model based on a continuous domain. This model includes an extensive cost function and equipment layout constraints that consider the tall equipment traversing multiple floors. To reduce computational time, symmetry breaking constraints were implemented and tested in a 17-unit case to showcase the computational efficiency of the model.

A meta-heuristic algorithm was also applied to address the computational intractability of the layout problem based on continuous domain. Kulturel-konak and Konak [33] put forward a hybrid approach that integrated the linear programming method with a genetic algorithm to solve the layout problem with unequal area departments. Leno et al. [34] developed an elitist strategy genetic algorithm using a simulated annealing algorithm as a local search to improve the solution quality. After that, the computational performance was improved by a layered coding genetic algorithm which successfully handled single-floor layout problems with up to 62 units [35]. Many of the heuristic approaches are prone to fall into local optimum. To deal with it, Garcia-Hernandez et al. [36] developed a CRO–VNS algorithm, that hybridizes the coral reefs optimization algorithm with variable neighborhood search and reached better solutions than the previous algorithms. Then, the CSE algorithm has been developed with the aim of enhancing the efficiency of resolving multi-objective layout optimization problems by effectively finding the Pareto optimal solution [37].

The past decades have seen some progress made in the methodology of plant layout problems. However, the current works still have some limitations. For instance, in the multi-floor plant layout research based on continuous domain, the capacities of current methods are not able to meet the design requirement as the computational time increases sharply with the expanding scale of the problem [38]. Therefore, it is of significant importance to develop efficient solution approaches to deal with large-scale ones.

To address the issue, this paper proposed two novel approaches to enhance solution efficiency. The first one improves the quality of the solution by introducing search branches in multiple directions, while the second approach reduces the model size by eliminating some infeasible regions through an iteration framework. Finally, some engineering experience constraints were introduced to achieve a more realistic representation of the complex industrial situation.

Compared to existing studies, this study makes the following contributions:

(a) Two general and efficient solution approaches are provided for the large-scale multi-floor plant layout problem, which can obtain better solutions than current research under the same solution resources.
(b) The layouts of multiple large-scale multi-floor plant layout cases in the literature were optimized. The amount of equipment in the cases reached a maximum of 62, and the optimized solutions in this study achieved cost savings compared to current research, with up to 25.23% cost reduction.
(c) The design of land specifications, workshop structures, and equipment layout plans for a real LNG plant design project were conducted, and based on engineering experience, a MILP model was established. This optimization considered the workshop stability, relative position of equipment, and reserved space for production construction, making the previously idealized layout more practical.

The remaining part of the paper proceeds as follows: Section 2 describes the multi-floor process plant layout problem. The two proposed solution approaches are presented in Section 3, and their computational efficiency is demonstrated by four examples in Section 4, which also discussed the engineering experience constraints for the last example. Finally, concluding remarks on the main highlights are made in Section 5.

## 2. Problem Description and Mathematical Formulation

The process plant layout problem frequently arises in practical projects involving the construction of chemical plants. This research aims to address the multi-floor layout problem of chemical equipment based on the continuous domain. Specifically, the study focuses on determining the area of the workshop, the number of floors, the specifications of the floors, and the positions of the equipment on each floor while adhering to engineering constraints and minimizing costs.

During the optimization process, it is crucial to consider the interactions between various constraints, including physical feasibility, economic viability, and design requirements. Physical feasibility is essential to ensure that the design and actual construction scenarios are consistent, and this involves constraints such as the non-overlapping placement of equipment, the avoidance of edges exceeding workshop floor limits, and the non-disconnection of cross-floor equipment.

In addition to physical feasibility, economic factors must be considered to minimize the total cost while meeting design requirements. Economic factors include connection costs between equipment, material transportation costs, installation costs of workshop floors, area-dependent costs of floors, and land costs. Achieving the optimal equipment layout involves accommodating all equipment while considering the area of each floor, which directly affects the construction cost of the floors. The spatial layout of equipment also affects the connection relationships between equipment, which in turn affects connection cost, horizontal pumping cost, and vertical pumping cost. As a result, there are mutual constraints among various cost factors. To minimize the total investment while meeting the design requirements, it is necessary to identify a suitable equipment spatial layout scheme.

The problem with the process plant layout is described as follows.

Given:

- A set of equipment items and their dimensions;
- A set of potential floors for layout;
- Connectivity relation among equipment items;
- Cost data (connection, pumping, land, and construction);
- Floor height;
- Space and equipment allocation limitations;
- Minimum safety distances between equipment items;
- The specifications and positions of the reserved construction space;
- to determine:
- the total number of required floors for the layout;
- the base land area occupied;
- the area of floors;
- the equipment-floor allocation;

so as to: minimize the total plant layout cost.

The following assumptions are made in the mathematical formulation. Every piece of equipment is described in cubic shape. In the x–y plane, the distance between the units is indicated by the straight-line distance from their geometric center and can be rotated at 90° angles. The vertical distance between the units is represented by the geometric distance between the inlet and outlet connectivity points. All floors are of equal height and the equipment higher than the floor height can pass through the floor. In this study, we employ a continuous-domain mixed-integer linear programming (MILP) formulation [32] to address the multi-floor process plant layout problem. The proposed formulation is presented herein, and we subsequently leverage it to develop two solution methodologies for the layout design needs of large-scale chemical plants, as discussed in Section 3.

### *2.1. Floor Constraints*

The number of floors occupied by each equipment item is expressed as:

$$\sum_{k} V_{ik} = M_i \quad \forall i \tag{1}$$

If equipment $i$ and $j$ occupy the same floor, the variable $N_{ij}$ takes the value of 1, otherwise 0. When both items $i$ and $j$ occupy floor $k$, the value of $n_{ijk}$ is 1, otherwise 0, the relation is given as:

$$n_{ijk} \geq V_{ik} + V_{jk} - 1 \quad \forall i, j > i, k \tag{2}$$

$$N_{ij} \geq n_{ijk} \quad \forall i, j > i, k \tag{3}$$

If a piece of equipment starts on a certain floor, then the floor exists.

$$S^s_{ik} \leq W_k \quad \forall i, k \tag{4}$$

A floor should exist only if the preceding floor is occupied.

$$W_k \leq W_{k-1} \quad \forall k > 1 \tag{5}$$

The minimum number of floors required is expressed as follows:

$$NF \geq \sum_k W_k \tag{6}$$

*2.2. Equipment Orientation Constraints*

Equipment items can be rotated 90° in the x-y plane, which is represented by the constraints of Equations (7)–(9):

$$l_i = \alpha_i O_i + \beta_i (1 - O_i) \quad \forall i \tag{7}$$

$$d_i = \alpha_i + \beta_i - l_i \quad \forall i \tag{8}$$

$$h_i = \gamma_i \quad \forall i \tag{9}$$

*2.3. Multi-Floor Equipment Constraints*

In the layout, each piece of equipment will start on a certain floor ($S^s_{ik} = 1$) and end on a higher floor ($S^f_{ik} = 1$). Equipment with a height greater than the floor height will occupy multiple floors continuously. This relation is represented by the constraints of Equations (10)–(12):

$$-V_{ik} + V_{i,k-1} + S^s_{ik} \geq 0 \quad \forall i, k \tag{10}$$

$$-V_{ik} + V_{i,k+1} + S^f_{ik} \geq 0 \quad \forall i, k \tag{11}$$

$$V_{ik} - V_{i,k-1} = S^s_{ik} - S^f_{i,k-1} \quad \forall i, k \tag{12}$$

*2.4. Non-Overlapping Constraints*

In the design process, in order to prevent the two equipment items from overlapping when they are on the same floor, two binary variables $E1_{ij}$, $E2_{ij}$ and a sufficiently large upper bound $BM$ are introduced to form the constraints of Equations (13)–(16).

$$x_i - x_j + BM(1 - N_{ij} + E1_{ij} + E2_{ij}) \geq \frac{l_i + l_j}{2} + D^{\min}_{ij} \quad \forall i, j > i \tag{13}$$

$$x_j - x_i + BM(2 - N_{ij} - E1_{ij} + E2_{ij}) \geq \frac{l_i + l_j}{2} + D^{\min}_{ij} \quad \forall i, j > i \tag{14}$$

$$y_i - y_j + BM(2 - N_{ij} + E1_{ij} - E2_{ij}) \geq \frac{d_i + d_j}{2} + D^{\min}_{ij} \quad \forall i, j > i \tag{15}$$

$$y_i - y_j + BM(3 - N_{ij} - E1_{ij} - E2_{ij}) \geq \frac{d_i + d_j}{2} + D^{\min}_{ij} \quad \forall i, j > i \tag{16}$$

### 2.5. Distance Constraints

The distances in the $x$, $y$ and $z$ coordinates between equipment $i$ and $j$ are described as follows:

$$R_{ij} - L_{ij} = x_i - x_j \quad \forall (i,j) : f_{ij} = 1 \tag{17}$$

$$A_{ij} - B_{ij} = y_i - y_j \quad \forall (i,j) : f_{ij} = 1 \tag{18}$$

$$TD_{ij} = R_{ij} + L_{ij} + A_{ij} + B_{ij} + U_{ij} + D_{ij} \quad \forall (i,j) : f_{ij} = 1 \tag{19}$$

$$U_{ij} - D_{ij} = FH \sum_k (k-1)(S_{ik}^s - S_{jk}^s) + OP_{ij} - IP_{ij} \quad \forall (i,j) : f_{ij} = 1 \tag{20}$$

### 2.6. Layout Design Constraints

During the layout process, the location coordinates of the equipment must meet some constraints to ensure that the entire equipment is placed inside the floor. The constraints are expressed as follows:

$$x_i \geq \frac{l_i}{2} \quad \forall i \tag{21}$$

$$y_i \geq \frac{d_i}{2} \quad \forall i \tag{22}$$

$$x_i + \frac{l_i}{2} \leq X^{\max} \quad \forall i \tag{23}$$

$$y_i + \frac{d_i}{2} \leq Y^{\max} \quad \forall i \tag{24}$$

$$z_i = \frac{h_i}{2} \quad \forall i \tag{25}$$

### 2.7. Area Constraints

In the process of layout, the floor specifications available for selection are determined by specific cases. Each candidate floor specification includes the area, length and width of the floor.

$$FA = \sum_s AR_s Q_s \tag{26}$$

$$\sum_s Q_s = 1 \tag{27}$$

$$X^{\max} = \sum_s \overline{X_s} Q_s \tag{28}$$

$$Y^{\max} = \sum_s \overline{Y_s} Q_s \tag{29}$$

The variable $NQ_s$ is introduced to linearize the cost term related to $NF$

$$NQ_s \leq K \cdot Q_s \quad \forall s \tag{30}$$

$$NF = \sum_s NQ_s \tag{31}$$

where $K$ represents the number of potential floors.

### 2.8. Objective Function

The objective function minimizes the total connection cost, pumping cost, land area cost, floor construction cost, and floor area-dependent cost:

$$\min \sum_{i} \sum_{j \neq i: f_{ij}=1} [C^c_{ij} TD_{ij} + C^v_{ij} D_{ij} + C^h_{ij}(R_{ij} + L_{ij} + A_{ij} + B_{ij})] + FC1 \cdot NF + FC2 \sum_{s} AR_s \cdot NQ_s + LC \cdot FA \quad (32)$$

subject to Equations (1)–(31). This constitutes model A.3.

## 3. Solution Approaches

The multi-floor process plant layout problem is currently addressed by simultaneous models that demand a considerable amount of computational resources, rendering them difficult to apply in large-scale problems. The MILP model A.3 [32] exemplifies this issue, as its equations and variables exhibit a dramatic increase in complexity as the problem scale expands, as illustrated in Figure 1. To circumvent the potential waste of space and resources in the design of large-scale chemical plants, this section presents two solution approaches that prioritize efficiency and differ primarily in their underlying principles.

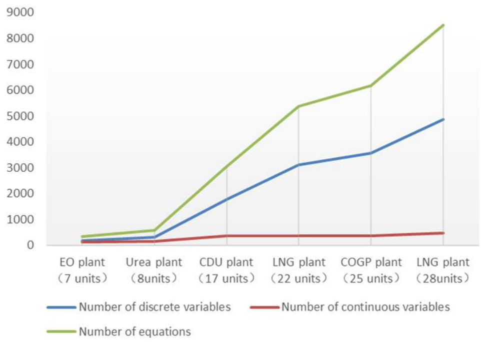


**Figure 1.** Model statistics of different scale cases [13,32,39,40].

### 3.1. Multi-Directional Search Approach

This study presents a solution to the multi-floor process plant layout problem in the continuous domain based on Model A.3, which belongs to the Mixed-Integer Linear Programming (MILP) model. The solver CPLEX was employed in previous literature to solve this model, which primarily includes solving the relaxation problem, branching and bounding steps. Due to the high complexity of the model and the numerous branches involved in solving large-scale layout problems, it is difficult to conduct a complete search for all branches. Therefore, only a feasible solution with deviations from the relaxation solution can be obtained. As the problem scale increases, the deviation tends to be more significant, which can greatly impact the economic efficiency of the solution. To address this issue, this study proposes a multi-directional search approach that increases the number of branches searched without extending the solving time. This approach aims to enhance the probability of obtaining a more optimal solution.

In order to enhance the efficiency and effectiveness of search algorithms, the multi-directional search (MDS) approach utilizes $N$ parallel computing tasks to explore more branches within a fixed time. Unlike directly setting up parallel computing, MDS sets up a unique subproblem $A_n$ for each parallel task $n$, which changes the model's relaxation domain and constraint conditions by adding a certain size of virtual data to each subproblem. This allows each parallel task to branch out in different directions during the solving process, leading to a broader search range.

The MDS approach consists of two main parts: setting up subproblems and solving them. Before solving the subproblems, each subproblem $A_n$ is established based on the original problem, with the number of subproblems equal to the number of parallel tasks. The general formula for all subproblem models adopts the MILP model proposed in Section 2 (A.3). To ensure that each subproblem has different domains and exclusive constraints during the solving process, MDS adds auxiliary data $\overline{X_s}$, $\overline{Y_s}$ and $AR_s$ to the subproblem models based on certain rules. Specifically, the first subproblem $A_1$ does not add auxiliary data to ensure that the overall solution quality is not worse than before. For sub-problems $A_n$ with $n > 1$, a certain amount of auxiliary floor data $s$ are added based on the original case data, and the amount of auxiliary data $S_n$ is defined by the expression in Equation (33):

$$S_n = S_1 + GP(n-1) \quad \forall n \tag{33}$$

where $S_1$ denotes the quantity of candidate floor specifications $s$ from original case data and $GP$ represents the difference in the number of auxiliary data between the subproblem $A_n$ and $A_{n-1}$. Each auxiliary data $s$ contains three types of parameters: $\overline{X_s}$, $\overline{Y_s}$ and $AR_s$, and their rules are defined by expressions Equations (34)–(36),

$$\overline{X_s} = \max\left\{\overline{X_s} \middle| s \in [1, S_1]\right\} + STEP(s - S_1) \quad \forall s \in (S_1, S_n] \tag{34}$$

$$\overline{Y_s} = \max\left\{\overline{Y_s} \middle| s \in [1, S_1]\right\} + STEP(s - S_1) \quad \forall s \in (S_1, S_n] \tag{35}$$

$$AR_s = \overline{X_s Y_s} \quad \forall s \in (S_1, S_n] \tag{36}$$

where $STEP$ represents the length of $\overline{X_s}$ or $\overline{Y_s}$ that is longer than the previous auxiliary data $s-1$. The settings for the number and parameters of subproblems ensure that each subproblem model has different numbers of constraint inequalities and variables, as well as different constraint equation parameters, all of which affect the branching strategy during the solving process. The added auxiliary data $s$ directly increases the number of variables $Q_s$ in constraint Equations (27) and (31), as well as the number of constraint inequalities in Equation (30). Meanwhile, the parameters $\overline{X_s}$, $\overline{Y_s}$ and $AR_s$ in the auxiliary data change the dimensions of the added variables in Equations (26), (28), and (29), which in turn changes the constraint range of the equations. All of these changes affect the initial relaxation solution of the MILP during the solving process, thereby allowing each sub-problem to branch out in different directions during the search. Figure 2 depicts the original solution branch as denoted by the blue arrow, and the newly added solution branch resulting from this approach represented by the green arrows. The feasible solutions discovered during the solution process are indicated by the red circles.

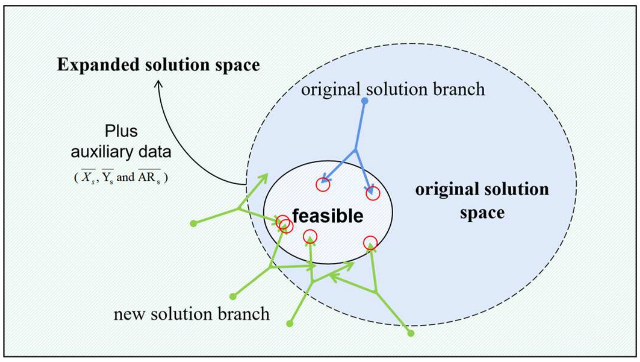


**Figure 2.** Schematic diagram of the multi-directional search approach.

The solution procedure is described by solving subproblems $A_1$–$A_n$ simultaneously and adopting the best feasible solution. Algorithm 1 describes the detailed procedure of the multi-directional search approach. To begin, a set of subproblems is established in accordance with rules Equations (33)–(36), with parameters $S_n$, $\overline{X_s}$, $\overline{Y_s}$ and $AR_s$ defined. Subsequently, within the time limit, the parameter values for the subproblems are input into Model A.3 and solved using the CPLEX solver. The optimal solution of the full-space problem corresponds to the feasible layout with the lowest objective function value among subproblems $A_1$–$A_n$.

**Algorithm 1: Solution Procedure of the Multi-Directional Search Approach**

1. The objective function value of subproblem $A_n$ and the best objective function value of the master problem are denoted as $obj_n$ and $obj\_best$, respectively.
2. Initialize the subproblem number: $n = 1$.
3. **While** $n < N + 1$ **do**
   set $S_n = S_1 + GP(n - 1)$
   **while** $\forall s \in (S_1, S_n]$ **do**
   set $\overline{X_s} = \max\left\{\overline{X_s}\middle|s \in [1,\ S_1]\right\} + STEP(s - S_1)$
   set $\overline{Y_s} = \max\left\{\overline{Y_s}\middle|s \in [1,\ S_1]\right\} + STEP(s - S_1)$
   set $AR_s = \overline{X_s Y_s}$
   $n = n + 1$
4. Set CPU time limit.
5. Solve subproblems $A_n$ simultaneously and obtain optimal feasible values $obj_n$.
6. $obj_{best} = \min obj_n$
7. **Return** $obj\_best$.

In accordance with the preceding exposition, the similarities between the subproblems are manifested in the general formula of the model, which is A.3. The differences are reflected in the inequality constraint coefficients of each sub-problem model based on the general formula A.3, such as the parameters $AR_s$, $\overline{X_s}$, and $\overline{Y_s}$ in the constraint Equations (26), (28), and (29). Additionally, each sub-problem has a different amount of auxiliary data. As deduced from the analysis above, the differences among the subproblems with respect to parameters, dimensions, and constraint expressions allow for branching towards distinct directions during the problem-solving process. By solving multiple subproblems in parallel, the multi-directional search approach achieves the search for multiple branches within the same time limit, thus increasing the probability of obtaining better solutions.

### 3.2. Iterative Downsizing Approach

The previous section outlined how the MDS method improved the efficiency of solving the multi-floor process plant layout problem by expanding the search branch. Given that reducing model complexity is also a viable strategy for improving the efficiency of solving this problem, this section proposes an alternative approach with the primary goal of reducing model complexity to optimize the efficiency of solving the problem.

This section introduces an approach called the Iterative Downsizing (ID) approach, which, on the basis of ensuring comprehensive constraints, decomposes the overall mathematical model to eliminate the domain that does not meet the requirements of the problem in advance, avoiding invalid searches during the optimization process and thus improving the optimization efficiency to obtain a better equipment layout solution. To achieve this goal, this approach first transforms the original problem into a set of subproblems with equal meaning, then constructs low-dimensional models for each subproblem, so that the complexity of the mathematical model set corresponding to the subproblem set is lower than that of the original model. Finally, a targeted iterative optimization strategy is used to solve the subproblem model set to achieve a reasonable allocation of solving resources. In the following sections, we will elaborate on the three components of this method, namely, problem partitioning, low-dimensional model construction, and iterative optimization strategy, based on their objectives, principles, and steps.

#### 3.2.1. Problem Partitioning

The original problem's model A.3 integrated optimized selections of floor parameters and equipment layout for different workshop floor specifications. This relationship was established through variables $Q_s$, $NQ_s$, $FA$, $X^{\max}$, $Y^{\max}$, parameters $\overline{X_s}$ and $\overline{Y_s}$, and constraint expressions (26)–(31), which occupied a large dimension of the model and thus presented significant optimization potential. To achieve the goal of reducing the model dimension, this study replaces the original problem with multiple subproblems that are equivalent to optimizing the layout design for a fixed workshop floor specification. The number of subproblems equals the number of candidate floor specifications in the case data, with each subproblem having a fixed and unique floor specification. Through this transformation, the information content of the original problem is preserved, but the models in the subproblem set can eliminate variables $Q_s$, $NQ_s$, $FA$, $X^{\max}$, $Y^{\max}$, parameters $\overline{X_s}$ and $\overline{Y_s}$, and constraint expressions (26)–(31), thus reducing optimization complexity and improving efficiency.

### 3.2.2. Low-Dimensional Model Construction

To construct a low-dimensional model, the mathematical model of the subproblem (referred to as PL_B) needs to be transformed from variables, parameters, constraints, and the objective function based on model A.3. Firstly, since each subproblem corresponds to only one type of floor specification, variables $FA$, $X^{\max}$, and $Y^{\max}$ related to model A.3 are transformed into parameters in subproblem model PL_B, and are assigned values based on floor specification data, resulting in $X^{\max} = \overline{X_s}$, $Y^{\max} = \overline{Y_s}$, and $FA = X^{\max}Y^{\max}$ for the corresponding subproblem data. Secondly, after fixing a single floor specification data for each subproblem, the optimization of floor specifications is no longer required for each individual subproblem model. Therefore, the area constraint (26)–(31) and related variables $Q_s$ and $NQ_s$ are removed from model PL_B based on model A.3. Finally, since $FA$ is a parameter in model PL_B, the linearization process in model A.3 can be eliminated. The $FC2\sum_s AR_s \cdot NQ_s$ term in the objective function of model A.3 is transformed into $FC2 \cdot FA \cdot NF$ in model PL_B. With these transformations, the objective function of the sub-problem becomes:

$$\min \sum_i \sum_{j \neq i: f_{ij}=1} [C_{ij}^c TD_{ij} + C_{ij}^v D_{ij} + C_{ij}^h (R_{ij} + L_{ij} + A_{ij} + B_{ij})] + FC1 \cdot NF + FC2 \cdot FA \cdot NF + LC \cdot FA \quad (37)$$

subject to Equations (1)–(25). This constitutes model PL_B.

By constructing the low-dimensional subproblem model PL_B, the entire problem model A.3 can be equivalently replaced by $S$ subproblem models PL_B, where $S$ represents the number of element $s$ in the candidate floor specification set. The set of equivalent replacement models PL_B reduces 3 + 2$S$ variables, including $Q_s$, $NQ_s$, $FA$, $X^{max}$, and $Y^{max}$, and 5 + $S$ constraint equations (inequalities) (26)–(31), compared to the original model A.3. This transformation reduces the complexity of the original problem model without missing feasible solutions, which in turn reduces the solution space and promotes the efficiency of the solution, as shown in Figure 3.

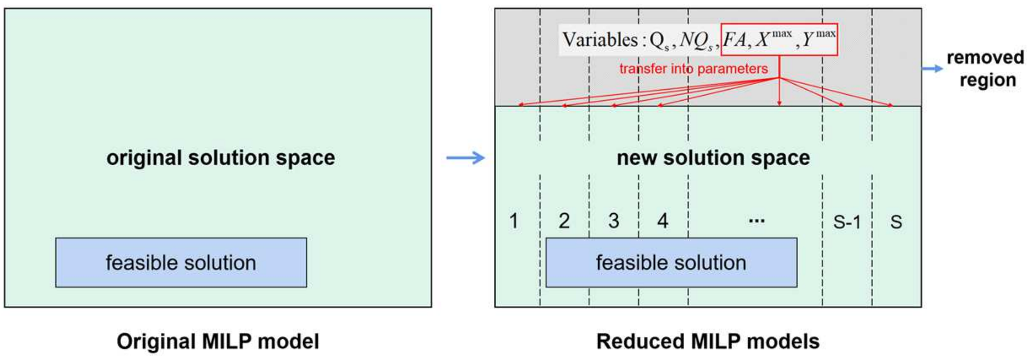


**Figure 3.** The reduced models of the Iterative Downsizing approach.

### 3.2.3. Iterative Optimization Strategy

In this section, we present a two-stage iterative strategy to obtain the optimal solution for the global problem by solving the PL_B subproblem model set. The software GAMS is used to call the CPLEX solver to optimize and solve the model set. Prior to the first stage, we define *obj_best* as the current optimal objective value and set parameters *obj_set*($s$) and *time_set*($s$) to record the sub-optimal objective function value obj and solve time etsolve obtained for each subproblem during the iterative loop.

In the first stage, all subproblems are screened and those with greater optimization potential are selected for branching. The initial screening model is defined by setting a lower limit for the relative gap and an upper limit for the solve time. The relative gap is used to measure the distance between the current optimal integer solution and the optimal relaxed solution [39]. For each set element $s$, the $\overline{X_s}$, $\overline{Y_s}$, and $\overline{X_s\,Y_s}$ values from the case data are passed to the parameters $X^{\max}$, $Y^{\max}$, and $FA$ of the PL_B model. The CPLEX solver is then called to solve the PL_B model, and the sub-optimal objective value and solve time corresponding to the current set element $s$ are recorded. If the current objective function value obj is less than the minimum objective function value *obj_best* in the previous loop, *obj_best* is assigned the value of obj; otherwise, *obj_best* is not assigned a new value. After this judgment and assignment action is completed, we return to the second step and perform corresponding operations on the remaining set elements $s$ until all set elements have been traversed, at which point we exit the loop and proceed to the second stage.

In the second stage, the sub-branch with the minimum objective function value identified during the initial screening is subject to further in-depth solutions, in order to obtain the globally optimal solution to the entire problem. In the first step, we select the set element $s$ corresponding to the minimum objective function value in the target function value set obtained in the first stage. The $\overline{X_s}$, $\overline{Y_s}$, and $\overline{X_s\,Y_s}$ values from the case data are then passed to the parameters $X^{\max}$, $Y^{\max}$, and $FA$ of the PL_B model for the corresponding set element $s$. The solve time limit is then redefined based on the time resource setting, with the aim of allocating most of the time resources to the specific sub-problem selected during the initial screening in the first stage. After solving the PL_B model, the optimal objective value corresponding to the current set element $s$ is recorded. If the current objective function value obj is less than the minimum objective function value *obj_best* in the previous loop, *obj_best* is assigned the value of obj; otherwise, *obj_best* is not assigned a new value. At the end of the second stage, the value of the variable *obj_best* is the optimal objective function value for the entire problem, corresponding to the total cost of the optimal layout solution. The set element $s$ associated with the variable *obj_best* is the optimal floor plan specification for the optimal layout solution. The optimal layout solution for the entire problem, including the workshop floor space specifications, number of floors, equipment spatial coordinates, and equipment orientation, is associated with the subproblem optimal solution corresponding to the set element $s$.

The key points of this two-stage iterative optimization strategy lie in the construction of the low-dimensional model set and the redistribution of solve time resources. The entire process is programmed using the GAMS software language to achieve automated execution, and the pseudocode is shown in Algorithm 2.

**Algorithm 2: Solution Procedure of the Iterative Downsizing Approach**

$obj\_best = +\infty$
Set the parameter $obj\_set(s)$ and $time\_set(s)$ for a record.
Set the relative gap lower bound and the CPU time limit.
**For** set $s$ **do**
  $X^{max} = \overline{X_s}, Y^{max} = \overline{Y_s}$
  $FA = X^{max}Y^{max}$
  Solve the model PL_B, obtaining the suboptimal value $obj$ and the solving time $etsolve$.
  $objset(s) = obj, time\_set(s) = etsolve$
  **If** $obj < obj\_best$ **then**
   $obj\_best = obj$.
**For** set $s$ **do**
 **If** $obj\_set(s) = obj\_best$ **then**
  $X^{max} = \overline{X_s}, Y^{max} = \overline{Y_s}$
  $FA = X^{max}Y^{max}$
  Set the new CPU time limit.
  Solve the model PL_B and obtain the optimal value $obj$.
  **If** $obj < obj\_best$ **then**
   $obj\_best = obj$.
Print $obj\_best$.

## 4. Case Studies

In this section, the proposed solution approaches are applied to four examples of process plant layout problems based on model A.3 [32] in Section 2. Each example was modeled using GAMS [41] modeling system v32.2.0 with the CPLEX 12.10.0.0 solver on an Intel(R) Core(TM) i7-8550U CPU with 8GB RAM. For the floor specification, five alternative sizes (10, 20, 30, 40 and 50 m) were used in examples 1, 2 and 4, resulting in 25 candidate floor specifications. For all case studies, data on equipment dimensions, connectivity, land price and other details are given in the Supplementary Materials.

All of the examples were also solved with the simultaneous approach [32] to provide a measure of the solution quality obtained by both proposed approaches. The CPU time limit was set to 1000 s. Due to the reliance of the MDS approach on parallel resources during the solving process, in order to ensure the rigor of the approach comparison, all examples were once again solved using the simultaneous approach, utilizing parallel computing resources identical to those employed by the MDS approach, as a control group for the MDS approach's solution outcomes.

### *4.1. Example 1*

Example 1 is a liquefied natural gas (LNG) plant [40], whose process flow diagram is shown in Figure 4. The plant consists of 22 units, with six exceeding the floor height of 8 m. The minimum distance between each equipment item is set to 4 m.

The computational performance of the simultaneous approach and the two proposed approaches is shown in Table 1.

**Table 1.** Computational Performance for example 1.

| | Simultaneous | Simultaneous (Parallel) | Multi-Directional Search | Iterative Downsizing |
|---|---|---|---|---|
| Total cost (rmu) | 1,224,638 | 1,203,655 | 1,075,003 | 1,173,208 |
| CPU (s) | 1000 | 1000 | 1000 | 918 |

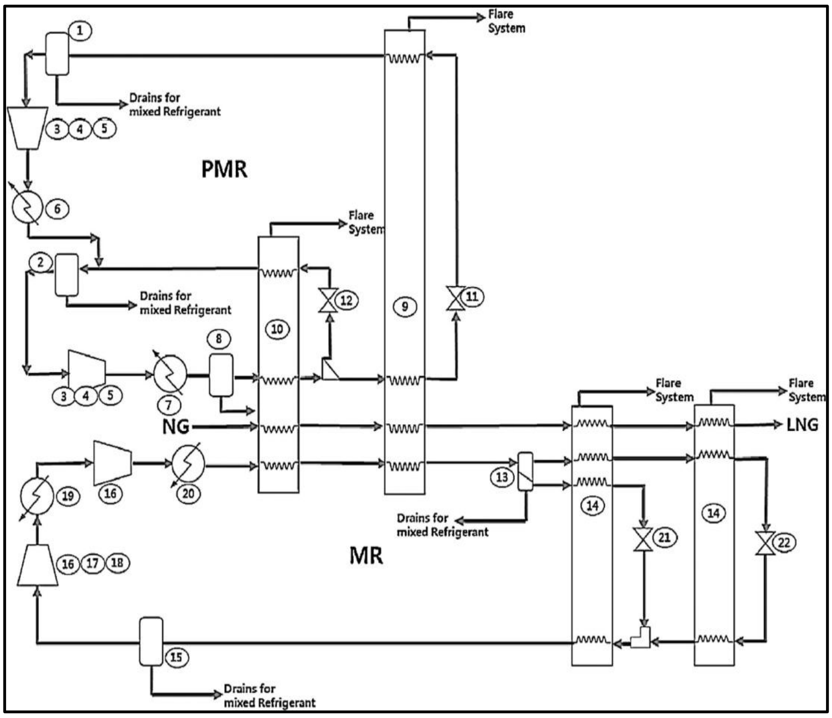


**Figure 4.** Flow diagram for example 1 [40].

Table 1 depicts the outcomes of the proposed approaches, showcasing a lower total cost achieved within the prescribed time limit. Specifically, both approaches have yielded reductions in the total cost of 12.22% and 4.20% compared to the simultaneous approach. Furthermore, when examined against the simultaneous approach utilizing the same parallel computational resources, the MDS approach demonstrated a savings of 10.69% in total cost, thereby establishing the computational efficiency of the proposed approaches.

The parameters of the MDS approach, including *N*, *GP*, and *STEP*, can be varied to affect the efficiency of the solution and the consumption of computational resources. The value of *N* determines the number of subproblems in the approach, and larger *N* values lead to a wider search range for the main problem within the same time frame, increasing the probability of finding optimal solutions. However, this also results in higher consumption of parallel computational resources. *GP* determines the difference in the amount of auxiliary data between subproblems and thereby affects the complexity of the subproblem models. *STEP* determines the growth rate of $\overline{X_s}$ and $\overline{Y_s}$ in the auxiliary data of each subproblem, and a larger *STEP* value results in a greater difference in $\overline{X_s}$ and $\overline{Y_s}$ values between adjacent auxiliary data, which has a greater impact on the domain and constraint expression parameters of the subproblem models. Figure 5a indicates that as *N* increases, the number of subproblems increases and the objective function value of the main problem gradually decreases to a plateau. Combined with subplots (b), (c) and (d) in Figure 5, it is shown that the values of *STEP* and *GP* fluctuate considerably within a certain range, and there are multiple parameter values that lead to MDS solutions that are superior to the original solutions.

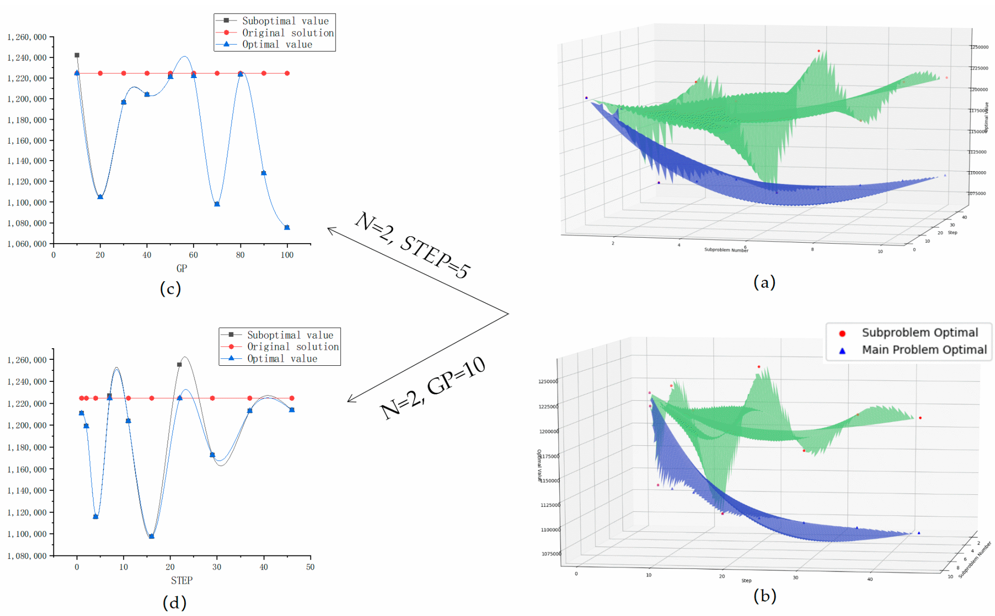


**Figure 5.** Solving time composition of Iterative Downsizing approach: (**a**–**d**).

In this case study, the subproblem quantity $N$ for the MDS approach was fixed at 5, the $GP$ value was set to 10, and the $STEP$ in the definition Equations (34) and (35) was assigned values of 2 and 3 for $\overline{X_s}$ and $\overline{Y_s}$, respectively. The solution results of the five subproblem branches for the MDS approach are shown in Table 2. It can be seen from the results that under the same time limit, subproblems 1 to 5 have different feasible solutions, which is due to the improvement of the solution strategy of the multi-directional search approach for subproblems. In the result, the objective function value obtained by subproblem 2 is the minimum, so it is the optimal solution for this solution.

**Table 2.** Computational results of the multi-directional search approach.

| **Parallel Branch** | **Subproblem $A_1$** | **Subproblem $A_2$** | **Subproblem $A_3$** | **Subproblem $A_4$** | **Subproblem $A_5$** |
|---|---|---|---|---|---|
| Objective function value | 1,228,640 | 1,075,003 | 1,222,235 | 1,188,081 | 1,229,656 |

Figure 6 shows the objective function values obtained from all subproblems of the ID approach. From the objective function values of subproblems, it can be seen that the data corresponding to subproblems 1 to 7 are unable to obtain a feasible solution, and the objective function values of some subproblems (i.e., 20, 21 and 25) after optimization are far greater than those of other subproblems. It can also be seen from the solving time composition shown in Figure 7 that the subproblem without a feasible solution only consumes a very short time resource, and the subproblem with a large gap from the global optimal solution also only consumes a limited time resource, which shows that the ID approach optimizes the allocation of time resources by dividing the full-space problem.

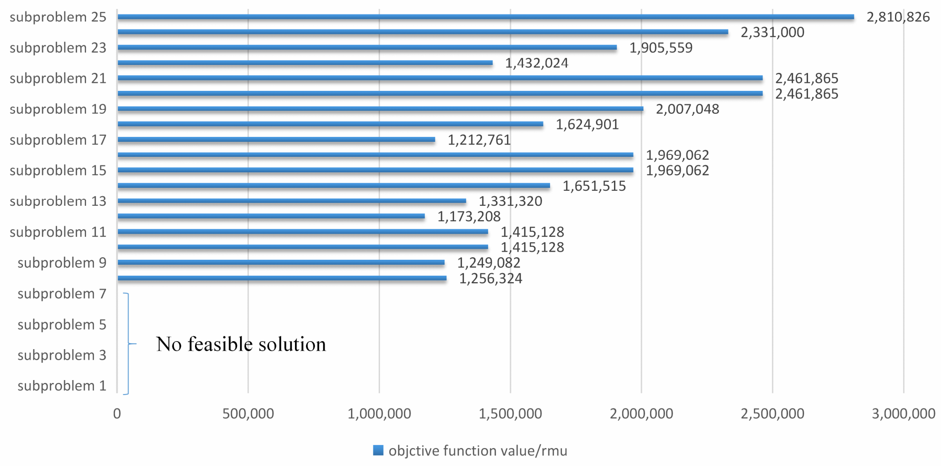


**Figure 6.** Objective function values of Iterative Downsizing approach.

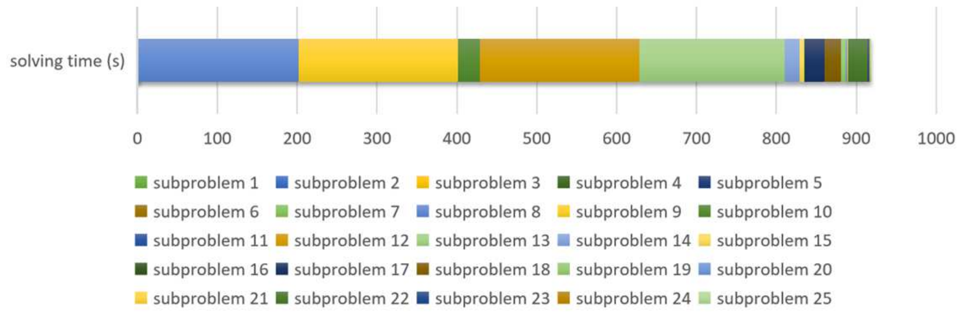


**Figure 7.** Solving time composition of Iterative Downsizing approach.

The above analysis and comparison of the two solution approaches demonstrate the reasons for the different results obtained by the MDS and ID approaches. When the model complexity is high, and it is not possible to search all solution branches within a reasonable time, the MDS approach increases the probability of finding the optimal solution by adding search branches in different directions. In contrast, the ID approach improves the efficiency of a solution by reducing the dimensionality of the model and reallocating the solving time resources. Therefore, the different search strategies result in different solution outcomes.

Based on the characteristics of the approaches, the MDS approach is suitable for the layout problems with many candidate floor specifications due to its inclusion of auxiliary data, resulting in higher feasibility. Conversely, the ID approach does not require parallel computing resources but is better suited for cases with fewer candidate floor specifications, as increased specifications prolong the solving time in the two-stage iterative solving process.

The optimal layouts determined by the three approaches are shown in Figures 8–10. The three layout schemes are different because the three solutions approaches are not capable of obtaining the global optimal solution within the time limit when the scale of the problem is too large. However, the two proposed solution approaches have found solutions with lower total costs under the same time limit.

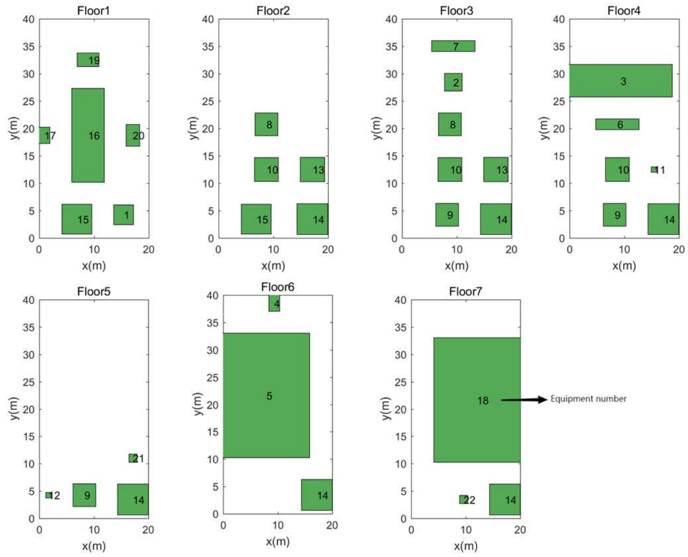


**Figure 8.** Optimal layout for example 1: Simultaneous Approach.

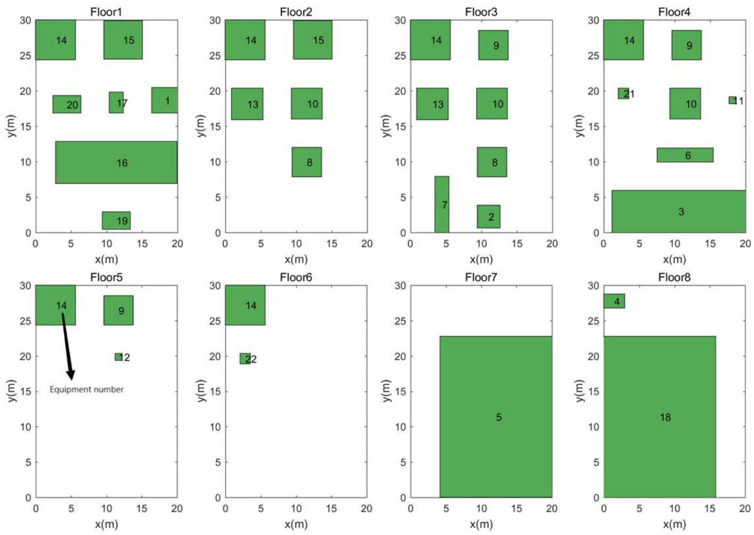


**Figure 9.** Optimal layout for example 1: Multi-directional Search Approach.

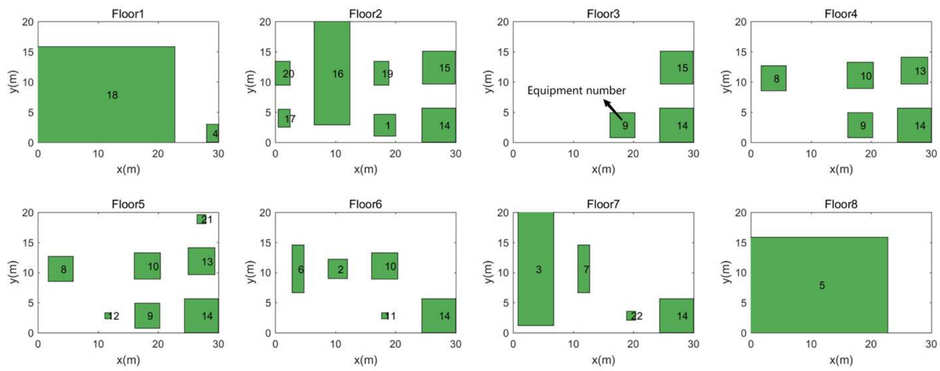


**Figure 10.** Optimal layout for example 1: Iterative Downsizing Approach.

For a better comparison, the main differences in layout details including the number of floors, floor specifications, land area, total connection length, and vertical pumping distance are shown in Table 3.

**Table 3.** Details of layouts for example 1.

| | Number of Floors | Floor Specification | Land Area | Connection Length | Vertical Pumping Distance |
|---|---|---|---|---|---|
| Simultaneous | 7 | 20 × 40 | 800 | 373 | 37 |
| Multi-directional Search | 8 | 20 × 30 | 600 | 373 | 37 |
| Iterative Downsizing | 8 | 30 × 20 | 600 | 410 | 51 |

It can be concluded that the layout schemes of both proposed approaches adopt floor specifications with a smaller area than that of the simultaneous approach, so as to save the investment on area-dependent cost (floor area-dependent cost and land cost). Although the number of floors for the two proposed approaches is higher, greater savings on total cost are achieved through the cost tradeoffs, which can be outlined in the cost details in Table 4.

**Table 4.** Details of costs for example 1.

| | Connection Cost | Pumping Cost | Floor Construction Cost | Floor Area-Dependent Cost | Land Cost | Total Cost |
|---|---|---|---|---|---|---|
| Simultaneous | 52,964 | 420,194 | 32,200 | 186,480 | 532,800 | 1,224,638 |
| Multi-directional Search | 52,898 | 425,865 | 36,800 | 159,840 | 399,600 | 1,075,003 |
| Iterative Downsizing | 57,098 | 519,870 | 36,800 | 159,840 | 399,600 | 1,173,208 |

### 4.2. Example 2

Example 2 is a crude oil and gas processing (COGP) plant [39] consisting of 25 units, nine of which have heights greater than the floor height of 5 m. Figure 11 displays the process flow diagram of the plant, with the identification numbers of the equipment indicated for reference.

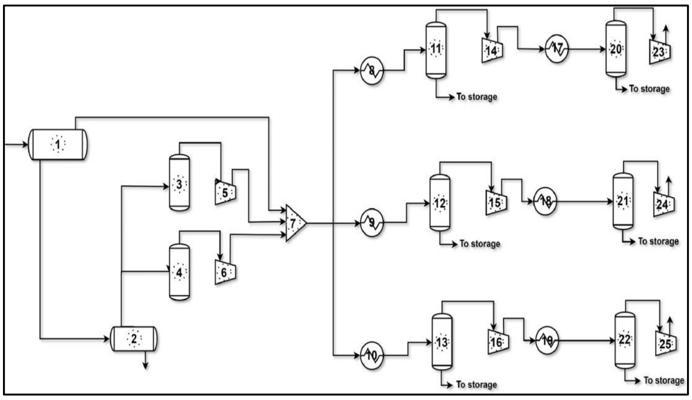


**Figure 11.** Flow diagram for example 2 [39].

The computational performance of all approaches is displayed in Table 5 for comparison.

**Table 5.** Computational Performance for example 2.

| | Simultaneous | Simultaneous (Parallel) | Multi-Directional Search | Iterative Downsizing |
|---|---|---|---|---|
| Total cost (rmu) | 342,025 | 341,537 | 334,292 | 322,142 |
| CPU (s) | 1000 | 1000 | 1000 | 912 |

Upon comparison, the proposed approaches have yielded superior solutions in comparison to the simultaneous approach, within the time limit. The total cost has also been observed to decrease by 2.26% and 5.81%, respectively, with the implementation of the proposed approaches. Notably, the MDS approach has been observed to achieve cost savings of 2.12% when compared to the simultaneous approach, under similar computational resources. The optimal layouts are available in the Supplementary Materials.

Layout details are used to make comparisons for a deeper analysis of the factors contributing to the cost differences among the three solutions. As is shown in Table 6, compared with the simultaneous approach, the proposed two approaches obtain solutions with lower total costs from two different aspects.

**Table 6.** Details of layouts for example 2.

| | Number of Floors | Floor Specification | Land Area | Connection Length | Horizontal Pumping Distance | Vertical Pumping Distance |
|---|---|---|---|---|---|---|
| Simultaneous | 6 | 10 × 30 | 300 | 174 | 92 | 7.38 |
| Multi-directional Search | 6 | 30 × 10 | 300 | 180 | 110 | 1.67 |
| Iterative Downsizing | 7 | 20 × 10 | 200 | 218 | 101 | 22.24 |

The layout of the multi-directional search approach cuts down the total cost by reducing the vertical pumping cost, which is reflected in the distribution of the material transportation. For instance, in the layout of the multi-directional search approach, the output point of unit 19 is higher than the input point of unit 22, so the logistics from unit 19 to 22 does not require vertical pumping costs according to the definition of the model A.3, and it is displayed in Figure 12.

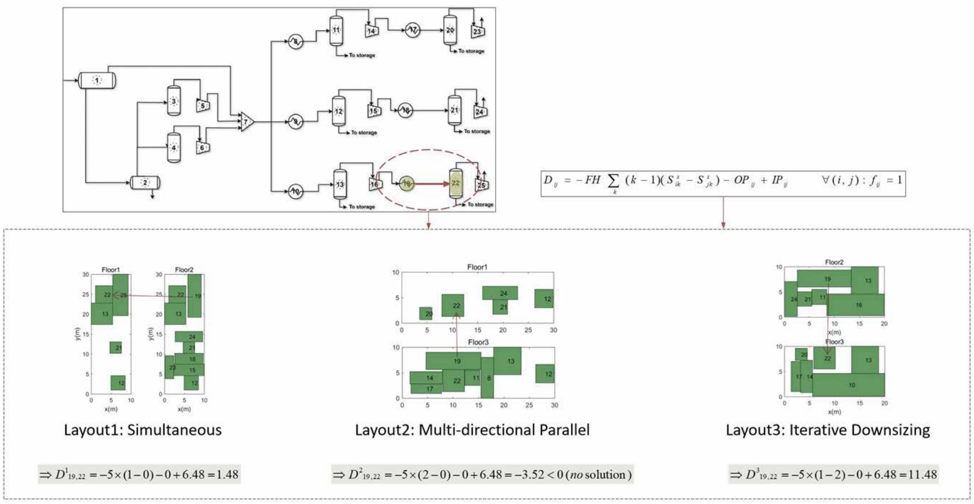


**Figure 12.** Detail comparison of vertical pumping distance (The green frames represent the equipment and the red arrows denote the material flow).

As for the layout of the Iterative Downsizing approach, it adopts a floor specification with a smaller area, thus achieving potential savings on the area-dependent cost, which can be reflected in Figure 13, in which the red number highlighted the higher value of the Iterative Downsizing approach and the blue number represents the lower value for a clear comparison.

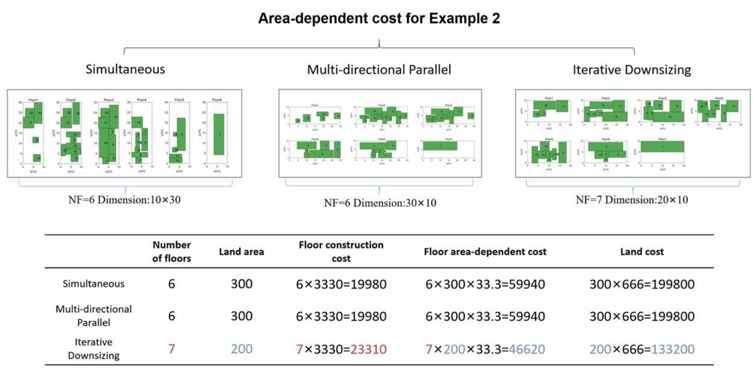


**Figure 13.** Comparison of area-dependent cost for example 2.

The components of the total cost are shown in Table 7. It is obvious from Figure 14 that the area-dependent cost and pumping cost contribute a large proportion of the total cost, thus having the greatest impact on potential savings. Therefore, it is the driving force behind the cost difference between the layouts of the simultaneous approach and the two proposed approaches.

**Table 7.** Details of costs for example 2.

| | Connection Cost | Pumping Cost | Floor Construction Cost | Floor Area-dependent Cost | Land Cost | Total Cost |
|---|---|---|---|---|---|---|
| Simultaneous | 12,618 | 49,687 | 19,980 | 59,940 | 199,800 | 342,025 |
| Multi-directional Search | 13,110 | 41,462 | 19,980 | 59,940 | 199,800 | 334,292 |
| Iterative Downsizing | 16,037 | 102,975 | 23,310 | 46,620 | 133,200 | 322,142 |

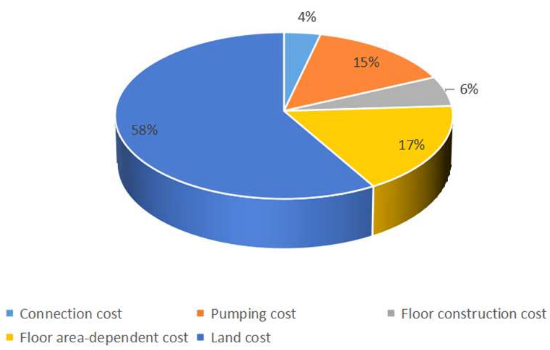


**Figure 14.** Cost proportion of the layout.

### *4.3. Example 3*

To assess the efficacy of the proposed approaches in addressing larger-scale multi-floor layout problems, we present Example 3 Du62, an adapted problem derived from Komarudin [42]. In order to extend the case study problem to a three-dimensional scenario, random heights were assigned to each module. The case study consists of 62 modules to be arranged, with a minimum safety distance of 5 m set between modules. The relative data, including the size of each module, the material transport relationships, and the workshop parameters, are presented in the Supplementary Materials.

Table 8 presents a comparison of the computational performance of all the approaches considered in this study.

**Table 8.** Computational Performance for example 3.

| | Simultaneous | Simultaneous (Parallel) | Multi-Directional Search | Iterative Downsizing |
|---|---|---|---|---|
| Total cost (rmu) | 55,277,646 | 54,138,617 | 51,644,810 | 41,333,091 |
| CPU(s) | 1000 | 1000 | 1000 | 847 |

Our study has revealed that the proposed approaches outperform the simultaneous approach in terms of solution quality, while still staying within the designated time limit. Furthermore, the implementation of the proposed approaches has led to significant cost reductions of 6.57% and 25.23%, respectively. Notably, the ID approach was able to achieve even greater cost savings than previous examples due to its ability to reduce the dimensionality of the complex model and allocate solving time resources more effectively. The MDS approach, in contrast, has been observed to achieve cost savings of 4.61% when compared to the simultaneous approach, while using similar computational resources. Figures 15–17 depict the optimal layouts obtained through these approaches (The equipment is indicated by the green frames in the figures, with the corresponding equipment numbers depicted inside them for ease of reference).

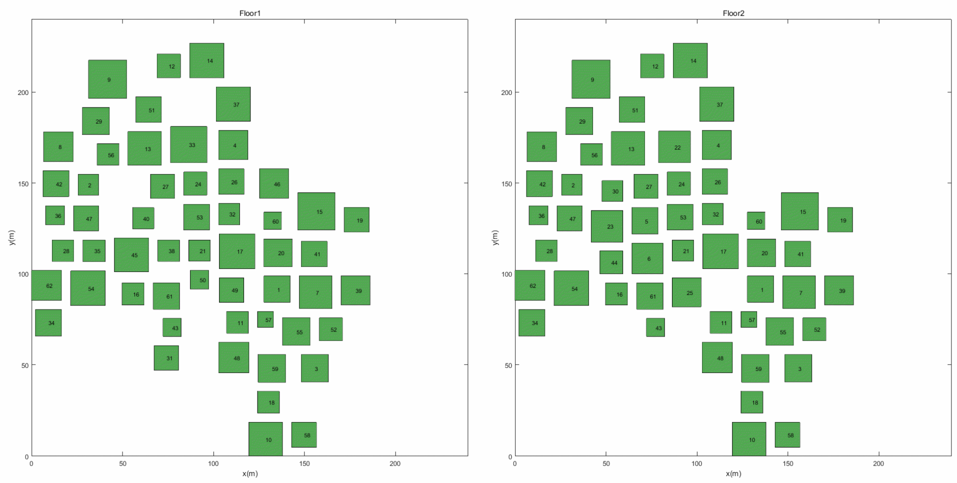


**Figure 15.** Optimal layout for example 3: Simultaneous Approach.

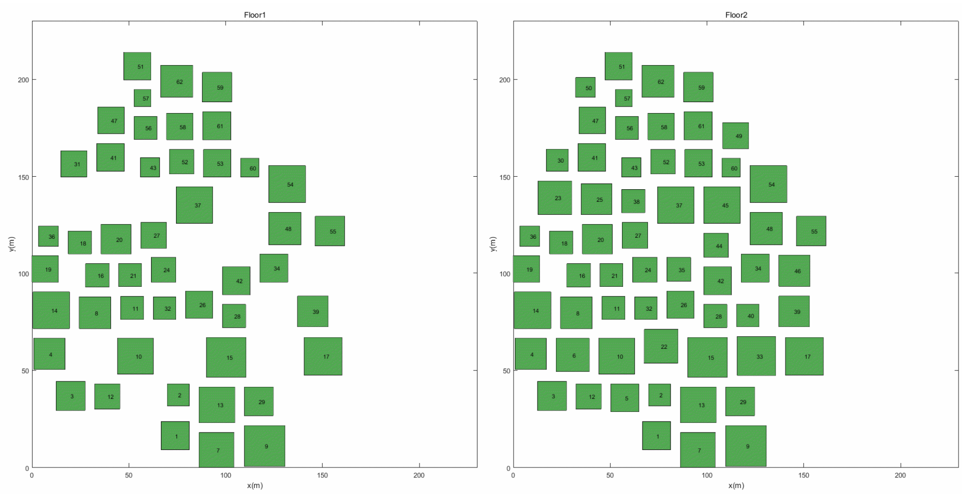


**Figure 16.** Optimal layout for example 3: Multi-directional Search Approach.

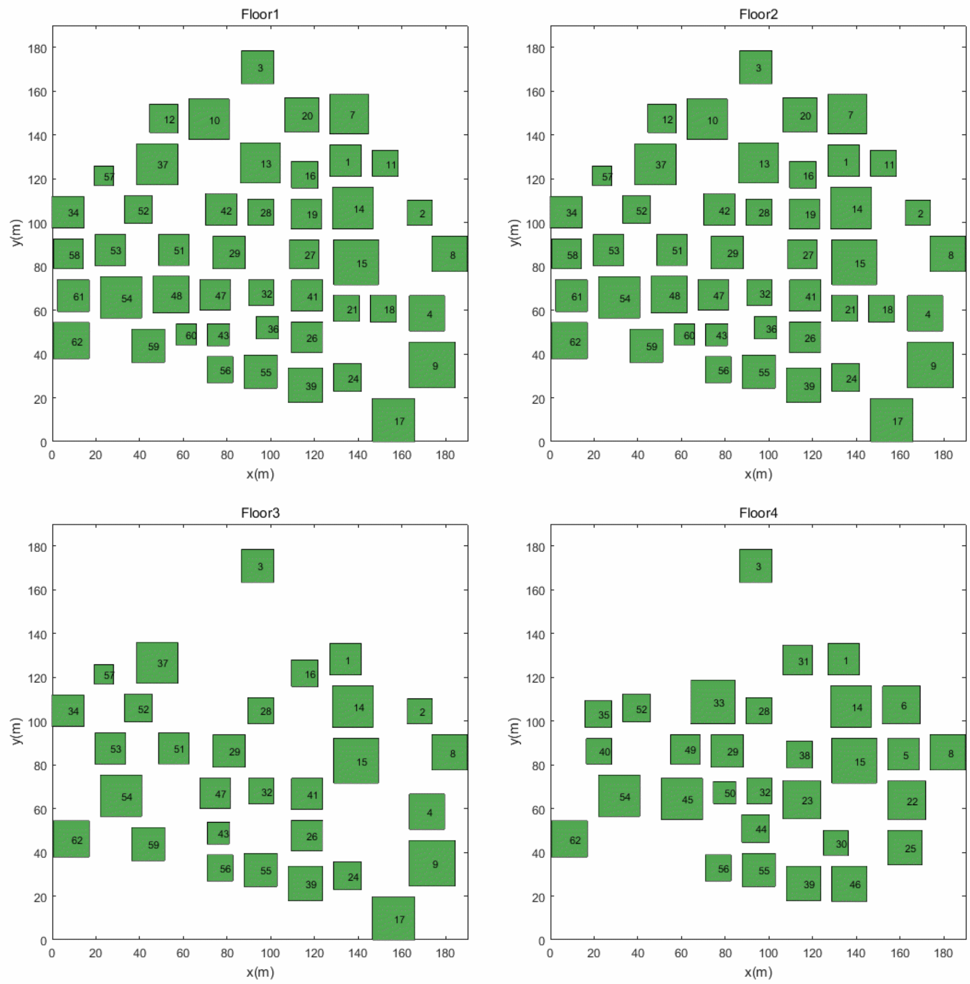


**Figure 17.** Optimal layout for example 3: Iterative Downsizing Approach.

The three layout approaches exhibit significant differences in terms of the number of floors, floor area, and module layout. Specifically, the ID approach results in a greater number of floors, but with the least amount of floor space occupied by the entire workshop. Compared to other approaches, the ID approach demonstrates the highest utilization of vertical space resources. Table 9 displays layout details corresponding to each approach for the quantitative comparison and analysis.

**Table 9.** Details of layouts for example 3.

| | Number of Floors | Floor Specification | Land Area | Connection Length | Horizontal Pumping Distance | Vertical Pumping Distance |
|---|---|---|---|---|---|---|
| Simultaneous | 2 | 240 × 240 | 57,600 | 128,491 | 127,226 | 425 |
| Multi-directional Search | 2 | 230 × 230 | 52,900 | 126,469 | 124,329 | 1130 |
| Iterative Downsizing | 4 | 190 × 190 | 36,100 | 123,409 | 116,704 | 3600 |

The layout of workshop modules determines the distances between modules for vertical and horizontal material transportation, which determines the costs associated with module connectivity and material handling. The choice of workshop floor numbers and specifications can also have an impact on construction costs, floor area-dependent costs, and land costs. As a result, the three schemes result in differences in cost composition, as shown in Table 10. The largest cost difference is reflected in the investment related to area.

**Table 10.** Details of costs for example 3.

| | Connection Cost | Pumping Cost | Floor Construction Cost | Floor Area-Dependent Cost | Land Cost | Total Cost |
|---|---|---|---|---|---|---|
| Simultaneous | 6,558,290 | 6,514,936 | 6660 | 3,836,160 | 38,361,600 | 55,277,646 |
| Multi-directional Search | 6,466,420 | 6,417,190 | 6660 | 3,523,140 | 35,231,400 | 51,644,810 |
| Iterative Downsizing | 6,309,933 | 6,158,718 | 13,320 | 48,085,20 | 24,042,600 | 41,333,091 |

The research and comparative analysis of the above examples 1–3 demonstrate the effectiveness of the two proposed solution approaches for large-scale multi-floor layout problems. The next section will utilize the approaches proposed in this paper to design and optimize the layout of workshop equipment for a chemical plant design project related to LNG technology in China.

## 4.4. Example 4

Example 4 is a liquefied natural gas liquefaction (LNG) plant with 28 units, and the process flow diagram is given in Figure 18, with the identification numbers of the equipment indicated for reference. A total of 19 units have heights greater than the floor height of 5 m, and eight floors are made available for layout. The minimum distance between each equipment item is set to 1 m.

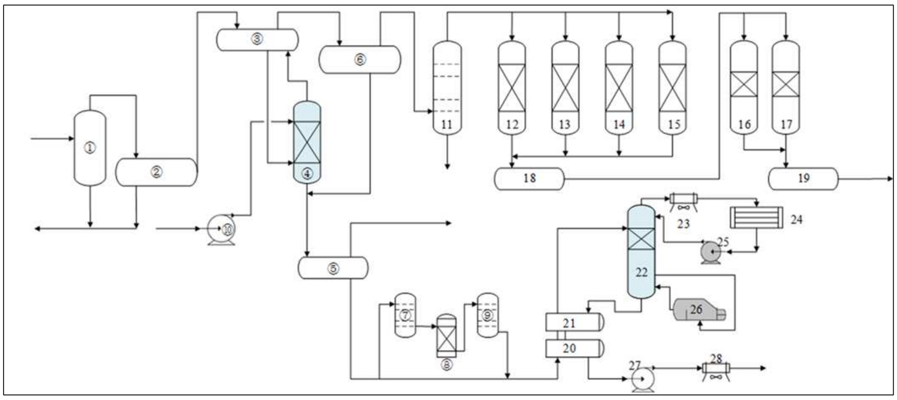


**Figure 18.** Flow diagram for example 4.

The computational performance of all approaches is shown in Table 11.

**Table 11.** Computational Performance for example 4.

| | Simultaneous | Simultaneous (Parallel) | Multi-Directional Search | Iterative Downsizing |
|---|---|---|---|---|
| Total cost (rmu) | 444,829 | 442,768 | 428,455 | 422,142 |
| CPU (s) | 1000 | 1000 | 1000 | 952 |

The results show a better computational performance of the proposed approaches, which get solutions with lower total costs within the time limit. Compared with the simultaneous approach, the proportions of the cost reduction are 3.68% and 5.10%, respectively. Moreover, when evaluated under identical parallel resource allocation, the MDS approach resulted in a cost reduction of 3.23% in comparison to the simultaneous approach for the total cost. The optimal layouts are available in the Supplementary Materials.

The details of the layouts are shown in Table 12 to illustrate the reasons behind the cost differences. The cost savings are majorly attributed to the fact that the layouts of the two proposed approaches are with less connection and horizontal pumping cost due to the optimization of the relevant equipment locations. For instance, the lower horizontal distance between units 4 and 5 leads to a lower pumping distance, thus reducing the corresponding cost and it is detailed in Figure 19.

**Table 12.** Details of layouts for example 4.

| | Number of Floors | Floor Specification | Land Area | Connection Length | Horizontal Pumping Distance | Vertical Pumping Distance |
|---|---|---|---|---|---|---|
| Simultaneous | 7 | 20 × 10 | 200 | 260 | 124 | 5 |
| Multi-directional Search | 7 | 10 × 20 | 200 | 247 | 111 | 5 |
| Iterative Downsizing | 7 | 20 × 10 | 200 | 242 | 106 | 5 |

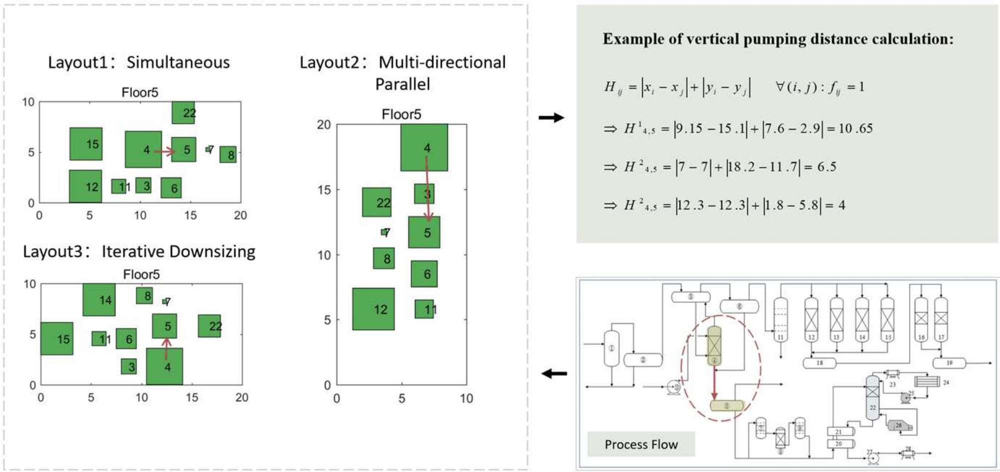


**Figure 19.** Detail comparison of horizontal pumping distance (The green frames represent the equipment and the red arrows denote the material flow).

Engineering Experience Constraints

For examples 1, 2 and 3, the methodologies based on model A.3 can obtain appropriate solutions. However, as the types of layout equipment and engineering conditions become more complex, the gap between the solution based on the current model and the actual situation becomes larger, which can be reflected in example 4. In this case, the solution based on model A.3 did not consider the stability of the workshop structure, nor did it consider the constraint of the fixed relative position of the tower equipment and its associated equipment [43]. For example, the condenser should be placed on the top of the tower equipment, and the reboiler should be installed on the bottom of the tower equipment. However, in the layout solution based on Model A.3, the tower equipment 22, condenser 23 and reboiler 26 do not conform to this principle, which will reduce the practicability of the scheme. In addition, for the overhaul and maintenance work in actual production, certain spaces will be reserved in the workshop to install stairs, which is not considered in the existing model.

In order to get closer to the real industrial situation, additional constraints on engineering experience are introduced:

Big equipment is one of the ten highest frequencies of design failure elements [44]. For the stability of the workshop structure, big equipment higher than the floor height, $i \in MF$, starts from the first floor except for condensers, $i \in cond$. For this, variable $S^s_{ik}$ is introduced taking value of 1 if a piece of equipment *i* starts on floor *k*. Thus:

$$S^s_{ik} = 1 \quad \forall i \in MF \backslash cond, k = 1 \tag{38}$$

In order to maintain the original structure of the tower equipment, $i \in tower$, the overhead condenser shall be placed at the top of the corresponding tower, and the reboiler, $i \in rebo$, at the bottom of it. Variable $S^f_{ik}$ is introduced taking the value of 1 if a piece of equipment *i* terminates on floor *k*. Thus:

$$S^f_{ik} = S^s_{jk} \quad \forall i \in tower, j \in cond, f_{ij} = 1 \tag{39}$$

$$S^s_{ik} = S^s_{jk} \quad \forall i \in rebo, j \in tower, f_{ij} = 1 \tag{40}$$

Certain spaces are reserved for the construction of workshop stairs, and this relation is formulated by introducing the concept of virtual equipment which does not represent real equipment but a reserved space. $I'$ means the set of virtual equipment, which represents the reserved space in the model. The number of the virtual equipment items equals $max(M_i)$, where $M_i$ represents the number of floors required by item *i*. The positions of the virtual equipment are set as follows,

$$x_i = x'_i \quad \forall i \in I' \tag{41}$$

$$y_i = y'_i \quad \forall i \in I' \tag{42}$$

where $x'_i$ and $y'_i$ are the pre-defined geometrical centers of the virtual item *i*.

Each virtual equipment item is allocated to a designated floor. The relation is given as:

$$\sum_{k \in RF} S^s_{ik} = 1 \quad \forall i \in I' \tag{43}$$

$$\sum_{i \in I'} S^s_{ik} = 1 \quad \forall k \in RF \tag{44}$$

where *RF* represents the set of designated floors for the construction of workshop stairs.

The above engineering experience constraints were applied to example 4 by incorporating Equations (38)–(44) in model A.3, and the resulting model is named PL, which was solved within the same time limit as model A.3 for a comparison. It is worth noting that the model PL with the engineering experience constraints has obtained a more realistic layout when compared with the original model A.3 without that. As is shown in Figure 20a, before incorporating the constraints into the model, some tall equipment items were placed on the high floor, which is unfavorable to the stability of the workshop. Meanwhile, equipment 23 (overhead condenser), equipment 24 (overhead reflux drum) and equipment 25 (reflux pump) were not placed on the top of equipment 22 (regenerator), which violates the equipment structure of the regenerator. In addition, there is no workshop stair space for manual operation and maintenance work on different floors. It is clear from Figure 20b, model PL has eliminated these deficiencies and obtained a more reasonable layout, therefore the applicability is proved.

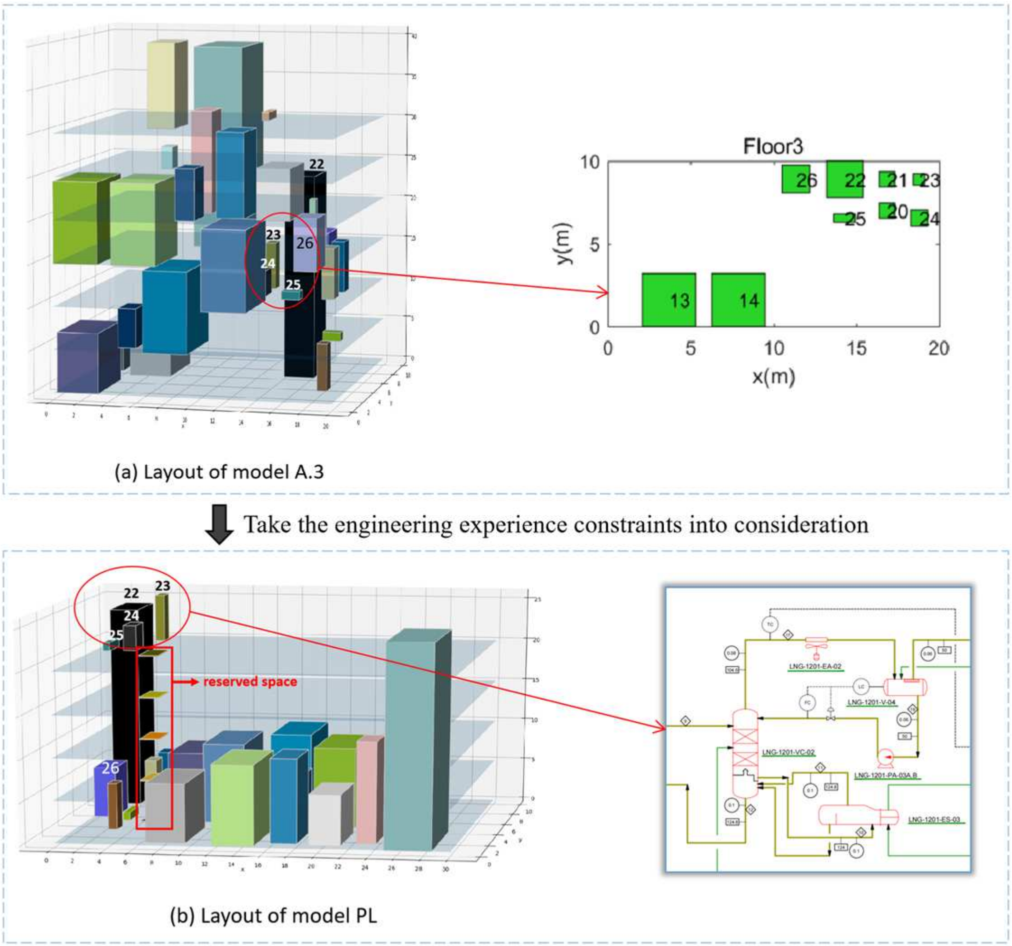


**Figure 20.** Comparison of the engineering experience constraints: (**a**) layout of model A.3, (**b**) layout of model PL (The equipment is indicated by the green frames in the figures, with the corresponding equipment numbers depicted inside them for ease of reference).

## 5. Conclusions

In this paper, two novel solution approaches have been proposed to solve the large-scale, multi-floor process plant layout problem. The first one is based on a multi-directional search strategy, and the second one reduces model size through an iterative solution scheme. Both approaches determine the number of floors, land area, and equipment layout so as to minimize the total cost. The applicability of the proposed solution approaches has been demonstrated by four illustrative examples. Results show that both the proposed approaches are able to obtain a better solution within the same time limit when compared with the current simultaneous approach.

Finally, engineering experience constraints were introduced taking more realistic situations into consideration. These constraints mainly include three parts: firstly, the stability of the workshop structure is considered from the perspective of large equipment location; secondly, the relative positions of tower equipment, condenser and reboiler are fixed; thirdly, the workshop is designed with reserved spaces based on production operation requirements. A more reasonable layout was obtained for example 4, as compared to the layout without engineering experience constraints.

Future work will focus on the development of a more efficient algorithm so that the large-scale problem can be handled well. Furthermore, the trade-off between safety and economy is also worthy of further study in the problem of process plant layout.

**Supplementary Materials:** The following supporting information can be downloaded at: https://www.mdpi.com/article/10.3390/pr11061595/s1, Figure S1: Optimal layout for example 2: Simultaneous Approach; Figure S2: Optimal layout for example 2: Multi-directional Search Approach; Figure S3: Optimal layout for example 2: Iterative Downsizing Approach; Figure S4: Optimal layout for example 4: Simultaneous Approach; Figure S5: Optimal layout for example 4: Multi-directional Search Approach; Figure S6: Optimal layout for example 4: Iterative Downsizing Approach; Table S1: Equipment dimensions for liquefied natural gas plant; Table S2: Parameters of connection and pumping for example 1; Table S3: Other parameters for example 1; Table S4: Equipment dimensions for crude oil and gas processing plant; Table S5: Parameters of connection and pumping for example 2; Table S6: Other parameters for example 2; Table S7: Equipment dimensions for example 3; Table S8: Parameters related to connection for example 3; Table S9: Other parameters for example 3; Table S10: Equipment dimensions for LNG plant; Table S11: Parameters of connection and pumping for example 4; Table S12: Other parameters for example 4; Table S13: Set of equipment for example 4.

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