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Price Discovery of Consignment Auctions for Emission Permits

Jae-Do Song [1] and Young-Hwan Ahn [2,*]

[1] College of Business Administration, Chonnam National University, 77, Yongbong-ro, Buk-gu, Gwangju 61186, Korea; sjaedo@chonnam.ac.kr
[2] Department of Climate, Environment and Energy Studies, Sookmyung Women's University, Cheongpa-ro 47-gil 100, Yongsan-gu, Seoul 04310, Korea
* Correspondence: yh.ahn@sm.ac.kr; Tel.: +82-2-710-9207

Abstract: A consignment auction aims to increase political feasibility by reducing the financial burden of initial permits allocation and to do the role of price discovery. However, previous analytical models presented contradictory results for the price discovery function of a consignment auction. Thus, this study reexamines whether a consignment auction can perform its price discovery function. The study uses a simple game model with several assumptions differentiated from previous analytical models: explicit consideration of the secondary market and firms as price-takers with various behaviors to respond to uncertainty about the price in the secondary market. Firms are classified into three types: speculators who seek arbitrage, doctrinarians who determine a permit demand based on an estimation of their marginal abatement cost, and neutralists who keep a permit demand the same as initial emission endowments. The results reveal that when a consignment auction was introduced, the expected equilibrium price was identical to that of the secondary market price, demonstrating that the auction could deliver the price discovery function. This is because speculators and doctrinarians provide information about their price expectations and marginal abatement cost through their estimated demand functions. Additionally, the smaller number of neutralists is, and the higher the risk-seeking propensity of speculators is, the more effective the price discovery function is.

Keywords: emissions trading scheme; consignment auction; price discovery; climate change mitigation

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1. Introduction

The marketable emission permit system (or transferable discharge permit system) is designed to provide a cost-effective way for firms to reduce emissions via economic incentives. Coase [1] argued that the private sector could allocate resources, such as clean air and water, in cost-effective ways if they were recognized as property with corresponding rights and were tradable in a marketplace. Thus, emission permits are allocated against certain rules in emissions trading schemes and are traded among firms. Montgomery [2] showed that a market for transferable permits could theoretically achieve a cost-effective allocation of control, irrespective of the initial allocation of permits. Additionally, the system is more likely to reach government-set environmental goals than a carbon tax—another type of carbon pricing—because the marketable emission permit system sets a cap on total emissions before being enforced.

However, it is not easy to realize the theoretical advantages of the system in practice [3]. Price uncertainty and market inefficiency may arise due to lack of trading, market concentration, and so on. These problems may distort long-term investment and undermine the current choice of optimal mitigation activity [4]. If trading does not occur due to a lack of liquidity, the market cannot achieve static and dynamic cost-effectiveness, which is incentivized by synchronizing participants' marginal abatement cost to provide long-term investment incentives [5]. The problems could be more serious at the early stage of the emission market. For instance, the EU emissions trading system (ETS) introduced in 2005 experienced an inefficient market in its first phase [6,7]. In China and Korea, where emissions trading was introduced relatively recently, a severe lack of liquidity has already been observed partly due to uncertain permit prices [8–11]. In alleviating the problem of uncertain permit prices, the price discovery function is an important issue that generates price information in the initial permits allocation to boost permit trading.

To introduce the emissions market, an initial allocation of permits is necessary. Generally, there are two methods for initial allocations: grandfathering and auctioning. Grandfathering is the free allocation of permits, usually based on a firm's historical emissions or outputs. This is more politically feasible than auctioning because participants do not pay for the permits in the initial allocation phase. Whereas auctioning requires expenditures from firms, it can generate revenue for the government and establish a clear price signal that can enhance the functioning of the market [12,13]. The EU, several US states, and Québec introduced a mixed-method, proportional auction where a significant amount of emission permits is distributed through grandfathering and the remainder allocated through auctioning. The Regional Greenhouse Gas Initiative, the EU ETS, and Québec's cap and trade system allocate more than 50% of the emission permits through auctioning. Korea also introduced the proportional auction in 2018. By contrast, emission permits allocated in China, Kazakhstan, and Tokyo (Japan) are mostly based on grandfathering.

The consignment auction, also known as the "revenue-neutral" auction, was developed to allow price discovery function, which is the main advantage of auctioning and increasing political feasibility as in grandfathering [14–16]. A consignment auction works as follows. First, emission permits are allocated to firms based on their historical emissions for free, and they immediately consign the emission permits to a third-party auctioneer. Then, all firms submit a demand schedule to the auctioneer depending on the price level, and the auctioneer determines the market-clearing price by adding these demand schedules. The determined market-clearing price delivers price information and plays the role of price discovery. Finally, the auctioneer gives the auctioned emission permits back to the participants. The amount collected or paid by the firms is calculated by multiplying the market-clearing price by the difference between the number of emission permits consigned and the number of auctioned emission permits. This peculiar settlement process reduces the financial burden of participants and increases the political feasibility of emissions trading.

The price discovery function of consignment auctions could reduce the problem of uncertain permit prices. It greatly enhances the efficiency of the marketable emission permit system. Consignment auction is also expected to encourage interest in emission permit trading within organizations, motivate advanced planning for emission permit trading, and increase political feasibility [16–18]. In reality, California's cap and trade program introduced the consignment auction in 2012. In California, the consignment auction is multipurpose; it is used for price discovery and to mitigate adverse economic impacts on sectors vulnerable to greenhouse gas emission reduction policies by recycling the auction revenue [12].

However, these expected effects are largely inferred from qualitative reasoning. They need to be analyzed using more rigorous methods. Dormady and Healy [19] and Franciosi et al. [17] used experimental approaches but had a limited number of experiment participants. To the best of our knowledge, there have been only two studies with analytic models. Khezr and MacKenzie [15] showed that a consignment auction with a constant marginal value of permits would neither boost emission permit trading nor deliver price discovery. However, Liu and Tan [13] showed that the consignment auction could play the role of price discovery and the equilibrium in Khezr and MacKenzie [15] occurs only with the assumption of the constant marginal value of permits.

In this study, using a game–theoretic model, we reinvestigate whether a consignment auction and a proportional consignment auction can play the role of price discovery. The model identifies the price discovery equilibrium in a consignment auction and a proportional consignment auction. The result is similar to Liu and Tan [13]. However, Liu and Tan [13] used a single-period model with given values of permits. Under their assumptions, the expectation of the price in the secondary market is meaningless, and the price discovery function cannot be analyzed explicitly. Thus, we explicitly reflect the existence of the secondary market after the consignment auction by establishing a two-period model. Further, we assume various behaviors of firms to respond to uncertainty about the price in the secondary market by classifying firms into three types: firms seeking arbitrage in a consignment auction (speculators), firms determining a permit demand based on an estimation of their marginal abatement cost (doctrinarians), and firms keeping a permit demand the same as initial emission endowments (neutralists).

By introducing the secondary market and various behaviors of firms explicitly, we can better describe how and when the price discovery function of consignment auctions works well. The price discovery function works because speculators provide price expectations in the secondary market, and doctrinarians provide information about their marginal abatement cost through their estimated demand functions. The consignment auction aggregates this information in the determined market-clearing price. However, neutralists do not contribute to this aggregation of information. Thus, the smaller number of neutralists is, and the higher the risk-seeking propensity of speculators is, the more effective the price discovery function is.

This paper is structured as follows. Section 2 reviews literature about a consignment auction. Section 3 explains the assumptions of the model in this study. Sections 4 and 5 derive equilibrium in a consignment auction and a proportional consignment auction. Finally, Section 5 provides some conclusions.

2. Literature Review of Consignment Auction

Auctions for emission rights are multiunit auctions where the auctioneer sells goods or assets to bidders who demand multiple units [20]. Single-item auctions have traditionally been applied in many real cases, and many theoretical studies on them have been carried out since Vickrey [21]. Studies on multiunit auctions began relatively late, although Wilson [22] showed that auctions of shares could generate significantly lower bid prices than unit-item auctions could. Ausubel [23] pointed out the importance of price discovery in the auction process of divisible goods and presented simultaneous clock auctions as a proper auction format. A group of researchers, including Ausubel et al. [24], Back and Zender [25], Krishna [26], and Wang and Zender [27], showed bid shading or demand reduction can occur in a multiunit auction with firms that have some level of market power. Meanwhile, Federico and Rahman [28] discussed that situations dealt with in general auction literature are somewhat different from the real electricity market and analyzed that perfect competition situation where firms act as price-takers results in efficient allocation.

There have been studies on what type of auction is efficient in allocating allowances in the context of carbon permits. Lopomo et al. [29] showed that a uniform-price sealed-bid auction is more appropriate than other types of auctions in that it prevents collusion without significant loss of the price discovery function. Meanwhile, Khezr and MacKenzie [30] pointed out that although the allocation of allowances through uniform auctions has several advantages, in reality, these advantages are not realized due to nontruthful bidding. He suggested that as an alternative to correct this, the regulator determines the total amount of allowances after all bids have been submitted. Khezr and MacKenzie [31] also showed that a uniform auction with allowance reserve could lead to distortion by unintentionally raising the clearing price higher. Alvarez et al. [32] performed a theoretical assessment on the efficiency of uniform auctions as a means of allocating allowances. He showed that for a uniform auction to be effective there should be no correlation between the types of bidders as well as many bidders.

A pollution permit auction with a consignment was originally proposed by Hahn and Noll [3], who called it a "revenue-neutral" or "zero-revenue" auction. They stressed the potential benefits of the consignment auction mentioned in the Introduction. Since then, surprisingly, only a few studies have directly examined the consignment auction, theoretically or empirically. Among them, Franciosi et al. [17] used experiments to compare the performance of revenue-neutral and conventional uniform-price auctions and found that the two mechanisms show little difference in price and market efficiency. Meanwhile, Dormady and Healy [19] found that a consignment auction leads to higher prices and a more inefficient allocation than a nonconsignment auction would in a laboratory environment. This may be explained by the fact that consignees have an incentive to boost auction prices by inflating demand and, by extension, increase auction revenue.

Khezr and MacKenzie [15] conducted a theoretical study on the performance of a consignment auction using an analytic model. Their analysis derived the equilibrium wherein emission permits are not traded, irrespective of the price level in a consignment auction. In other words, from consignment auctioning, firms only obtained emission permits corresponding to their initial emission endowments at the equilibrium. Hence, the equilibrium price does not affect firms, and the actual price at equilibrium fails to reflect the market value of the emission permits. Therefore, it implies that a consignment auction would neither boost emission permit trading nor deliver price discovery. They also analyzed a proportional consignment auction where only a proportion of the total permits are endowed to firms, and the endowed permits are consigned to an auctioneer. The nonendowed permits and the consigned permits are allocated through the proportional consignment auction. The equilibrium characteristics in a proportional consignment auction turned out to be similar to those in a consignment auction.

However, Liu and Tan [13] showed that consignment auctions could play the role of price discovery using the assumption of nonincreasing marginal values for the permits. Only when firms have a constant marginal value of the permit as assumed in Khezr and MacKenzie [15], the nontrade equilibrium appears. Liu and Tan [13] argued that a constant marginal value of the permit is not realistic; "firms may have a fairly flat marginal abatement cost curve within a short period of time or a specific technology class, it generally becomes more costly for them to satisfy a higher abatement target." Even though the results in Liu and Tan [13] are similar to this study, the model used in this study is very different from the one in Liu and Tan [13] as described in the Introduction.

3. Model Assumption

To analyze the effect of a consignment auction through a mathematical model, it is first necessary to define the equilibrium price and trading volume without the consignment auction and then compare them with a consignment auction. It is challenging to develop an analytic model reflecting all the details of real emissions trading and define the single market equilibrium price because permits are traded continuously and the price constantly changes to reflect the situation at a particular point in time. Thus, the model simplifies the situation and assumes an emission permit trading scenario wherein demand and supply in the year of implementation are all put together at the market-clearing price. Assuming this trading scenario, this study defines the equilibrium price and equilibrium trading volume before a consignment auction is introduced and then investigates the equilibrium after the introduction of the consignment auction.

Firm i's quantities of unregulated and reduced emissions under the cap-and-trade system are denoted as Q_i and q_i, respectively. Therefore, all firms surrender emission permits corresponding to q_i at the end of the emission permit trading period. The marginal abatement cost function for firms is defined as follows:

$$MAC(q_i) = \frac{(Q_i - q_i)}{K_i} \tag{1}$$

where K_i indicates the efficiency level of a firm's emissions mitigation technology, determined by the probability density function, $k(K_i)$. The expected value of K_i is $\mathrm{E}(K_i) = K$. Under this marginal abatement cost function, when a firm emits $q_i = Q_i$, the marginal abatement cost becomes 0. The marginal abatement cost increases when q_i decreases further, and the degree of this decrease differs across firms depending on K_i. Meanwhile, the average value of Q_i for all firms is denoted as $\mathrm{A}(Q_i) = Q$.

There are two periods. In the 0th period, the same amount of emission permits ω is allocated to firms, and a consignment auction that uses a uniform price rule is conducted. The total sum of permit allocation is less than the total unregulated emissions ($\sum \omega < \sum Q_i$), which implies that the government tries to reduce total emissions through an emission cap and trade. The 1st period refers to the secondary market where emission permits are traded between firms. The assumption of the same amount of emission permits ω might be unrealistic considering that even in a consignment auction grandfathering is common. However, the assumption that the gap between Q_i and ω is different firm by firm sufficiently reflects the important aspect of reality.

Further, we assume imperfect information where firms know only their K_i, Q_i, and whether $Q_i > Q$ or not but do not know information K and Q. Naturally, firms do not know other firms' situation exactly in reality. Whether $Q_i > Q$ or not means that $(Q_i - \omega)$ is higher or lower than average, and we believe that firms can know it approximately based on their experience even though they do not know exactly Q. We also assume a perfect competitive market wherein the total number of firms is so high that the behavior of each firm does not affect the price and firms act as price-takers as described in Cramton [33] and Federico and Rahman [28].

4. Equilibrium Analysis of the Consignment Auction

4.1. Consignment Auction of Perfect Information

To help understand the basic nature of a consignment auction, we start with a case where they already know all the information, including K and Q in the 0th period. Firms' cost function, TC_i, is defined as follows:

$$TC_i = P^0\left(q_i^0 - \omega\right) + P^1\left(q_i^1 - q_i^0\right) + \int_{q_i^1}^{Q_i} \frac{(Q_i - q_i)}{K_i} dq_i \tag{2}$$

where q_i^0 and q_i^1 are the amount of emission permits through the consignment auction in the 0th period and the 1st period, respectively, and P^0 and P^1 are the market-clearing prices in the 0th period and the 1st period, respectively.

We use backward induction. To derive the equilibrium price and quantities in the 1st period: Equation (2) is calculated as the partial differential with q_i^1, and the following condition is identified:

$$\frac{\partial TC_i}{\partial q_i^1} = P^1 - \frac{\left(Q_i - q_i^1\right)}{K_i} = 0 \tag{3}$$

From this equation the following demand function can be derived:

$$q_i^1 = Q_i - K_i P^1 \tag{4}$$

This demand function means that q_i^1 is determined at a level where firm i's marginal abatement cost equals the 1st-period price, P^1. Now, the market-clearing ($\sum q_i^1 = \sum \omega$) condition in the 1st period is described as follows:

$$nQ - nKP^{1*} = n\omega \tag{5}$$

Additionally, as a competitive scenario was assumed, there is no incentive for firms to change the existing strategy (or demand function) when the market-clearing price is defined by Equation (5). Therefore, the equilibrium price in the 1st period is defined as follows:

$$P^{1*} = \frac{1}{K}(Q - \omega) \tag{6}$$

Equation (6) means that a larger difference between the average emissions across the whole economy and the average emission allocations leads to an increase in P^{1*}. Additionally, P^{1*} declines as K, the firms' average efficiency level rises.

Additionally, each firm's emission permits at the end of the 1st period are determined by substituting Equation (6) into Equation (4):

$$q_i^{1*} = Q_i - \frac{K_i}{K}(Q - \omega) \tag{7}$$

Equation (7) means that the amount of each firm's emission permits decreases as the firm's relative efficiency level, K_i/K, and the average emissions reduction requirement, $Q - \omega$, increases.

The results above show that the equilibrium price and quantities of emission permits in the 1st period are independent of the equilibrium in the 0th period in the perfect information case. Furthermore, each firm's emission permits are determined at a level at which each firm's marginal abatement cost equals the 1st-period price P^1, and an efficient allocation is achieved where the emissions abatement cost is minimized across the society. This suggests that the secondary market works efficiently regardless of the introduction of a consignment auction.

Theorem 1. *In the 1st period (i.e., the secondary market), in a perfect information scenario, emission permits are determined at a level at which each firm's marginal abatement cost equals the market-clearing price, therefore achieving an efficient allocation wherein the emissions abatement cost is minimized across society.*

Then, the firms' problem in the 0th period is to choose the amount of emission permits, given the equilibrium price and quantities in the 1st period. By substituting the equilibrium values into TC_i and calculating the partial differential with respect to q_i^0, we can obtain the following equation as the 0th period's equilibrium condition:

$$\frac{\partial \mathrm{TC}_i}{\partial q_i^0} = P^0 - P^{1*} \rightarrow \; P^{0*} - P^{1*} = 0 \tag{8}$$

This equation indicates that when the consignment auction clearing price in the 0th period is lower than the secondary market price in the 1st period ($P^0 < P^{1*}$), firms have an incentive to increase q_i^0 as much as possible and sell $q_i^0 - q_i^{1*}$ of the emission permits in the 1st period to reduce the total cost. In the opposite case ($P^0 > P^{1*}$), firms have no incentive to buy permits through the consignment auction in the 0th period and purchase all the permits they need in the 1st period. In other words, the demand function in the 0th period would be flat at P^{1*}. Given that all firms' demand function is the same, the allocation of permits to each firm will be the same amount, ω. The equilibrium $P^{0*} = P^{1*}$ is achieved due to the incentive to attempt arbitrage.

Theorem 2. *With perfect information, the equilibrium price in the 0th period (i.e., with the consignment auction) is the same as that in the 1st period.*

The above analysis of a perfect information scenario is intended to help us understand the basic nature of the equilibrium in a consignment auction. In a perfect information scenario, the arbitrage incentive makes P^{0*}, the market-clearing price in the 0th period the same as P^{1*}. However, given that the analysis assumes perfect information, firms already know P^{1*}, and discussing the consignment auction's price discovery function becomes meaningless. Hence, we need to analyze an imperfect information case to provide realistic implications.

4.2. Consignment Auction of Imperfect Information

Under imperfect information, we assume that firms only know their Q_i and K_i, and firms' cost function is the same as Equation (2). Even with imperfect information, the equilibrium in the 1st period is the same as those of perfect information. We skip the derivation of the equilibrium because it is clear. Then, in the 0th period with imperfect information, firms as price-takers will form an estimation of P^{1*} and their demand function that reflects their arbitrage incentive based on the estimated P^{1*}, for example, flat one at estimated P^{1*}. However, because firms do not know exactly the P^{1*}, there can be different behaviors according to the level of information and risk aversion. To describe the real-world situation, we consider three types of firms: speculators, doctrinarians, and neutralists. The type of firms is determined randomly. The numbers of speculators, doctrinarians, and neutralists are assumed to be n_S, n_D, and n_N, respectively.

The first type of firm, speculators, explicitly seek arbitrage because they believe they have enough information to estimate P^{1*}. The magnitude of the arbitrage incentive will be based on the speculators' risk-taking propensity. To reflect such firms' behavior, it is assumed that speculators estimate the distribution of P^{1*} of which average is $\hat{P}_i^{1*}$ and symmetric. $\hat{P}_i^{1*}$ follows the distribution of $\mathrm{g}\left(\hat{P}_i^{1*}\right)$ and has expectation $E\left(\hat{P}_i^{1*}\right) = P^{1*}$. Their risk-taking propensity is $R_i\ (> 0)$ and is independent of $\mathrm{g}\left(\hat{P}_i^{1*}\right)$. The average of R_i for the speculators is R. Based on these assumptions and the basic attributes of arbitrage, it is assumed that a speculator has the following demand function:

$$q_i^0\left(P^0\right) = \omega + R_i\left(\hat{P}_i^{1*} - P^0\right) \tag{9}$$

This demand function corresponds to arbitrage behavior with the assumption that $\hat{P}_i^{1*} = P^{1*}$.

The second type of firm, called the "doctrinarians," also has arbitrage incentive, but they do not believe they have a good estimate of P^{1*} in the 0th period. They only know that their demand function in the 1st period will be given as Equation (4), $q_i^1\left(P^1\right) = Q_i - \mathrm{K}_i P^1$. Then, they can utilize this as their demand function in the 0th period, $q_i^0\left(P^0\right) = Q_i - \mathrm{K}_i P^0$. However, with the demand function, the expected q_i^0 of doctrinarians whose Q_i is more (less) than Q tend to purchase more (less) than ω even when $P^0 > P^{1*}$ ($P^0 < P^{1*}$) because $Q - KP^{1*} = \omega$ from Equation (5) and $\mathrm{E}(K_i) = K$. This violates arbitrage incentives and $q_i^0\left(P^0\right) = Q_i - \mathrm{K}_i P^0$ cannot be the best strategy of price-takers. Thus, they should adjust downward (upward) their demand function to be $q_i^0\left(P^0\right) = Q - KP^0$.

However, because they do not know the exact Q, the adjustment cannot be the exact one. Thus, we denote the adjustment of each doctrinarian as δ_i and assume that the average of all δ_i is 0. The assumption means δ_is of firms with $Q_i > Q$ is symmetric with those with $Q_i < Q$, and this assumption is weaker than $\mathrm{E}(\delta_i) = Q_i - Q$. Accordingly, the following demand function is an adjusted form of the demand function given by Equation (4):

$$q_i^0\left(P^0\right) = Q_i - \mathrm{K}_i P^0 - \delta_i \tag{10}$$

This demand function looks similar to the actual demand function identified in the 1st period. However, the doctrinarians' true purpose in the 0th period is not to achieve the equilibrium quantity of the 1st period but to utilize arbitrage. Thus, the important aspect of the demand function defined in Equation (10) is that the expected $q_i^0 = \omega$ when $P^0 = P^{1*}$. Given that they cannot estimate P^{1*}, they just adjust the demand function given by Equation (4) not to violate arbitrage incentive.

The last type of firm, neutralists, submit a fixed demand function equal to the initially endowed ω. They follow the grandfathering rule and give up the chance to minimize cost using arbitrage. They represent a special case of the first type where $R_i = 0$. Neutralists may adopt this strategy due to their very low risk-taking propensity or lack of information. Regardless of the rationale, their resulting behavior is similar to the one described by Khezr and MacKenzie [15].

Then, let us identify the equilibrium of imperfect information. We assumed uncertainty only in the 0th period. Firms have perfect information in the 1st period. Each firm's demand is the same, as in Equation (4), and P^{1*} and q_i^{1*} are also equivalent to those in a perfect information case.

In the 0th period, the total demand for permits can be described as the sum of three groups' demands as follows:

$$\sum q_i^0\left(P^0\right) = \sum_{S,N}\left[\omega + R_i\left(\hat{P}_i^{1*} - P^0\right)\right] + \sum_{D}\left[Q_i - K_iP^0 - \delta_i\right] \quad (11)$$

Given that $E\left(\hat{P}_i^{1*}\right) = P^{1*}$, the average of R_i for the speculators is R, R_i is independent of $\hat{P}_i^{1*}$, and the average of all δ_i is 0, the expected total demand in the 0th period is calculated as follows:

$$E\left[\sum q_i^0\left(P^0\right)\right] = (n_S + n_N)\omega + Rn_S\left(P^{1*} - P^0\right) + n_D\left(Q - KP^0\right) \quad (12)$$

Replacing P^0 with $P^{1*} = \frac{1}{K}(Q - \omega)$ on the right-hand side generates $E\left[\sum q_i^0\left(P^0\right)\right] = \sum\omega$, which balances the demand and the supply of permits in the 0th period. Therefore, the expected market-clearing price in the 0th period, P_e^{0*}, becomes P^{1*}:

$$P_e^{0*} = P^{1*} = \frac{1}{K}(Q - \omega) \quad (13)$$

If $P_e^{0*} = P^{1*}$ is satisfied, firms do not have an incentive to deviate from the equilibrium quantities, and the market-clearing price becomes the equilibrium price. Then, using Equations (9) and (10), the expected amounts of auctioned permits for each group in the 0th period, q_{Se}^{0*} for speculators, q_{Ne}^{0*} neutralists, and q_{De}^{0*} doctrinarians, are as follows:

$$q_{Se}^{0*} = q_{Ne}^{0*} = \omega \text{ and } q_{De}^{0*} = Q_i - \frac{\mathrm{K}_i}{K}(Q - \omega) - \delta_i \quad (14)$$

In the equilibrium, as assumed, neutralists maintain ω, the amount of their initial emission endowments. The amount acquired by each speculator would be higher than ω if its $\hat{P}_i^{1*}$, the estimate of P^{1*}, is higher than the market-clearing price (P^{1*} expected by the market), and vice versa. However, because $E\left(\hat{P}_i^{1*}\right) = P^{1*}$, speculators' expected amount is ω in the equilibrium. The expected amount for an individual doctrinarian is also similar. If the adjustment δ_i is correct, the amount acquired by doctrinarians will be ω. However, due to their lack of information about Q and K, the real amount acquired can be different from ω. However, their total demand is expected to be the same as the sum of the initial allocated amount of emission permits ($\sum\limits_{D} q_{De}^{0*} = \mathrm{n}_D\omega$). Summarily, neutralists do not make actual transactions, and speculators and doctrinarians expect an expense or revenue in the consignment auction.

Theorem 3. *With imperfect information, the expected equilibrium price in the consignment auction in the 0th period is the same as that of the secondary market in the 1st period.*

The equilibrium described in Theorem 3 assumed that firms determine their demand to be ω at $P^0 = P^{1*}$ and purchase more or less than ω when $P^0 \neq P^{1*}$, even though they do not know exactly P^{1*}. However, the choice of q_i^0 at $P^0 = P^{1*}$, denoted as $q_{i\,P^0=P^{1*}}^0$ from now on, is not the decision that minimizes the total cost defined in Equation (2). For example, when $q_i^0\left(P^0\right)$ defined in Equation (9) with $q_{i\,P^0=P^{1*}}^0$ is substituted instead of ω, the total cost of speculator becomes as follows:

$$TC_i = -P^0\omega + P^{1*}q_i^{1*} + \left(P^0 - P^{1*}\right)\left(q_{i\,P^0=P^{1*}}^0 + R_i\left(\hat{P}_i^{1*} - P^0\right)\right) + \int_{q_i^{1*}}^{Q_i}\frac{(Q_i - q_i)}{K_i}dq_i \quad (15)$$

To minimize the total cost with given 1st-period equilibrium, $q_{i\,P^0=P^{1*}}^0$ should be as much as possible if $P^0 < P^{1*}$, as low as possible if $P^0 > P^{1*}$, and any $q_{i\,P^0=P^{1*}}^0$ is indifferent when $P^0 = P^{1*}$. Thus, the choice of $q_{i\,P^0=P^{1*}}^0$ is not the result of cost minimization but an arbitrary choice based on the expectation of P^0. However, it is important that $q_{i\,P^0=P^{1*}}^0 = \omega$

is the unique one that satisfies market clearing at $P_e^{0*} = P^{1*}$ when all firms choose the same $q_i^0{}_{P^0=P^{1*}}$. If firms expect $P_e^{0*} \neq P^{1*}$, their strategy, the demand function, cannot be the equilibrium one. Further, because firms can think of ω as a natural reference point, the demand function with $q_i^0{}_{P^0=P^{1*}} = \omega$ is sufficiently reasonable. Therefore, the equilibrium described in Theorem 3 is meaningful.

Given that the consignment auction equilibrium characteristics in the 0th period show that a consignment auction's equilibrium price equals the expected equilibrium price in the secondary market, the consignment auction can perform the price discovery function. This means that the collective demand functions provided by firms reflect enough of each firm's information to perform the price discovery function.

Notably, the above results depend on the assumptions; speculators have an aggregate expectation $E\left(\hat{P}_i^{1*}\right) = P^{1*}$ and the average of δ_is is 0. If the expectations are incorrect, the expected equilibrium price in the consignment auction will not be the same as the equilibrium price in the secondary market. However, firms have no choice but to act based on the assumption of their expectation about P^{1*} or adjustment of demand (δ_i) is right. The assumptions can be interpreted to imply that firms do their best to guess correctly. The important aspect is that a consignment auction aggregates the information about the expected prices of spectators and the expected average demand function of doctrinarians. The price discovery function does not mean that a consignment auction can accurately predict the secondary market price. It just implies that the clearing price in the consignment auction would act as a reasonable predictor of the market price in the near future.

Meanwhile, the equilibrium implies that $\sum\limits_{N} q_{Ne}^{0*} = \mathrm{n_N}\omega$, and $\sum\limits_{D} q_{De}^{0*} = \mathrm{n}_D\omega$, which means that market-clearing occurs for each group. However, the market-clearing for neutralists happens regardless of the price. That is, neutralists reveal no information through their demand function. Thus, a consignment auction does not perform the price discovery function when only neutralists are in the market. Meanwhile, because the behavior of speculators and neutralists converges if the overall risk-taking propensity of speculators ($\overline{R}$) is close to zero, the higher risk-taking propensity of speculators will increase the effectiveness of the price discovery function in a consignment auction.

Now let us discuss the justification of the three groups. It was previously assumed that firms are randomly assigned to one of the three types—speculators, neutralists, doctrinarians—and the number of firms in each group is n_S, n_N, and n_D, respectively. What needs to be considered first for this discussion is that the equilibrium implies only $P_e^{0*} = P^{1*}$ and does not imply $P^0 = P^{1*}$. P^0 sums the information that firms have at the time of the consignment auction. Hence, speculators can save the total cost through arbitrage only when their prediction is more accurate than the equilibrium price in a consignment auction. Thus, for firms who believe they have sufficient information, it would be a reasonable strategy to become speculators.

By contrast, in the 0th period, doctrinarians try to estimate the average demand function of all firms in the 1st period, $Q - \mathrm{K}P^0$, by adjusting their best-guessed demand function, $Q_i - \mathrm{K}_i P^0$, as much as δ_i. Even though the expected result of this adjustment is not $Q - \mathrm{K}P^0$, it is a way to use the given information to minimize deviation from the arbitrage rule and purchase more (less) than ω when $P^0 < P^{1*}$ $\left(P^0 > P^{1*}\right)$. If firms think that they do not have sufficient information to estimate P^{1*}, they can be doctrinarians. The only necessary condition for this strategy to create the equilibrium is that the average of δ_is is 0. Some firms who believe that their information is insufficient even for the adjustment of δ_i, or have a very high-risk aversion, will become neutralists. It is important to note that speculators need the information to predict the equilibrium price in the secondary market, but doctrinarians need to consider only Q and K. It looks somewhat odd because, in this simplified model, obtaining the expectation of P^{1*} needs only Q and K. However, in practice, estimation of P^{1*} will need much more information than adjusting their demand function. Therefore, the three types of firms could be classified based on the firms' level of information and risk aversion.

The discussion above implies that the number of firms in each group is affected by the level of information the firms have. Therefore, it may be possible to ease the assumption that the number of firms in each group is given exogenously. For example, if governments introduce a policy to provide relevant information to the market, it could increase the number of speculators and doctrinarians, which, in turn, could increase the effectiveness of price discovery in a consignment auction (measured by the standard deviation for $P^0 - P^{1*}$). Furthermore, we can consider the effectiveness of price discovery in a consignment auction even when the level of information is given because the effectiveness could also influence the firms' strategy choice. If more firms behave as speculators or increase their R_i when the standard deviation for $P^0 - P^{1*}$ is expected to be large, the increase in the number of speculators or R_i, in turn, will lower the standard deviation for $P^0 - P^{1*}$. Considering this circular causality, a certain level of effectiveness in a consignment auction can be endogenous. However, if the high standard deviation for $P^0 - P^{1*}$ lowers the number of speculators or decreases R_i, a consignment auction will become more ineffective. Even though we cannot clearly determine which scenario will occur because the speculators pursue arbitrage based on inefficiency in the market, the first scenario looks more realistic.

Finally, it is worth discussing the assumption of price-takers or competitive markets. If some firms have market power, do they have an incentive for bid shading? However, in a consignment auction situation, the firms' main incentive is to utilize the possibility of arbitrage. The arbitrage incentive is different from the bid shading incentive, making firms try to purchase at a lower price or try to sell at a higher price. Further, the arbitrage incentive works differently according to whether the market-clearing price P^0 is higher or lower than P^{1*}. In the $P^0 > P^{1*}$ scenario, firms have an incentive to reduce the number of emission permits obtained from a consignment auction. However, to create the $P^0 > P^{1*}$ scenario intentionally, they need to increase the number of emission permits in demand. Therefore, there is no incentive to intentionally increase the number of emission permits in demand to create the $P^0 > P^{1*}$ scenario. A similar logic applies to the $P^0 < P^{1*}$ scenario. Hence, in a consignment auction scenario, firms do not have an incentive to control the market-clearing price. In this respect, the assumption of price takers which do not consider the possibility of market power is thought not to make a deviation from the results without the assumption.

5. Equilibrium Analysis of a Proportional Consignment Auction

This section changes the previous assumptions to analyze the equilibrium price in a proportional consignment auction. In a proportional consignment auction, only a proportion of the total permits are endowed to firms, and the endowed permits are consigned to an auctioneer. The unallocated permits and the consigned permits are distributed through the proportional consignment auction. For the analysis of a proportional consignment auction, it is assumed that ω', the same amount of emission permits allocated to firms, is smaller than ω ($\omega = \omega' + \alpha$, $\omega > \alpha > 0$). In the auction, all available permits ($\sum\omega$) are traded. Under this assumption, all firms' total costs and demand functions in the 1st-period secondary market are identical to the Equations (2) and (4) in Section 4.1, but ω is substituted by ω', as in the following:

$$TC_i = P^0\left(q_i^0 - \omega'\right) + P^1\left(q_i^1 - q_i^0\right) + \int_{q_i^1}^{Q_i} \frac{(Q_i - q_i)}{K_i} dq_i \tag{16}$$

Given that each firm's demand function in the 1st period defined in Equation (4) is not relevant to ω', the equilibrium in the 1st-period secondary market is the same as that in a consignment auction and fulfills an efficient allocation of emission permits.

Then, let us think about the demand functions in the 0th period. Firms can submit the same demand function defined in Equations (9) and (10). However, as discussed about Theorem 3, the choice of $q^0_{i\ P^0=P^{1*}}$ has an arbitrary aspect, and we should think about what the reference point will be. We can think of ω and ω', but if all firms choose ω' as $q^0_{i\ P^0=P^{1*}}$ cannot clear market at $P_e^{0*} = P^{1*}$, and firms have an incentive to change their $q^0_{i\ P^0=P^{1*}}$

from ω'. Thus, $q^0_{i\,P^0=P^{1*}} = \omega$ is the unique reference point that generates market clearing at $P_e^{0*} = P^{1*}$ and no incentive to deviate, assuming that all firms have the same reference point. Thus, if we assume that three types of firms choose their demand function the same as those in Equations (9) and (10), it results in equilibrium which is the same as consignment auction defined in Equations (13) and (14).

Theorem 4. *In a proportional consignment auction of imperfect information, the expected equilibrium price and the amount of emission permits obtained at the equilibrium are identical to those in a consignment auction scenario.*

A proportional consignment auction is different from a consignment auction because firms purchase emission permits as much as α, which is the gap between ω and ω'. However, this gap could lead to another difference when most firms behave as neutralists, and some firms can exert market power. In a proportional consignment auction, because they have to pay for the additional purchase of α, all firms may have an incentive for bid shading to lower P^0 if they have market power. However, to lower P^0, firms should lower their demand, and this bid shading incentive works in the opposite direction to arbitrage incentive because the firm tries to purchase more when $P^0 < P^{1*}$, as discussed in the last paragraph of Section 4.2, following Theorem 3. Thus, the arbitrage incentive reduces the bid shading incentive. However, there is a possibility that the bid shading incentive might dominate the arbitrage incentive.

6. Conclusions and Policy Implication

The lack of trading and information on permit prices may undermine the advantages of emissions trading. This study examined the effect of introducing a consignment auction as one way to solve this problem. We showed that, with a consignment auction or proportional consignment auction, the equilibrium price in the (proportional) consignment auction is the same as the expected price in the secondary market, when valid is the assumption on the overall expectation of the market about the price and the expectation of average demand function in the secondary market. This means that a consignment auction can perform the price discovery function due to the information provided by some types of firms through their demand functions. This result contrasts with the results in Khezr and MacKenzie [15] but aligns with the results in Liu and Tan [13] and the qualitative studies about consignment auctions [12,14,16–18].

However, this study is differentiated from qualitative studies [12,14,16–18] and experimental studies [17,19] by adopting an analytical model. Further, this study also provides a deeper understanding of the price discovery function in consignment auctions than Liu and Tan [13] by explicitly introducing the secondary market and three types of firms.

The speculator is a firm that pursues arbitrage and presents the demand function based on a prediction on its equilibrium price in the secondary market; it provides market-related information. The doctrinarian estimates the average demand function in the secondary market based on their information about the quantities of unregulated emissions (Q_i) and the efficiency level (K_i); it provides information about abatement cost. Consignment auction aggregates that information from the speculators and doctrinarians and thus, provides price discovery.

However, the neutralist submits the demand function based on the level of initial emission endowments and does not provide any information about their price expectation in the secondary market and their marginal abatement cost. Thus, if there are only neutralists, the consignment auction does not aggregate any information, and emission permits are not traded in the consignment auction, similar to the result of Khezr and MacKenzie [15]. Even when there are many speculators, if their risk-seeking propensity is very low, their behavior becomes similar to neutralists, and they do not contribute to the price discovery function. Thus, only when there is a sufficient number of speculators with high risk-seeking propensity and doctrinarians, consignment auctions can do the role of price discovery.

However, it is also necessary to highlight speculators' role in price discoveries. Doctrinarians and speculators reveal a certain level of information in the market-clearing price of a (proportional) consignment auction. Given that speculators seek arbitrage, they have an incentive to participate in the consignment market more actively as inefficiency in the market increases, that is, when the market-clearing price in a (proportional) consignment auction is more likely to deviate from the predicted equilibrium price in the secondary market (e.g., when there are many neutralists). If the number of speculators or their risk-seeking propensity is determined endogenously, a (proportional) consignment auction will guarantee a certain level of price discovery.

It is worth noticing theoretically that the equilibrium was derived with the assumption of price takers. If the assumption is relaxed, there might be bid shading or demand increment equilibrium as described in Ausubel et al. [24], Krishna [26], and Liu and Tan [13]. We cannot judge whether the real market is closer to a competitive one with price takers or a collusive one with market power. However, to understand the role of consignment auctions, both aspects should be considered. Another aspect that should be noticed is that arbitrage incentives can reduce bid shading incentives as discussed at the end of Sections 4.2 and 5. To lower the market-clearing price rather than the equilibrium price in the secondary market, firms should purchase less, violating arbitrage incentives. Considering this, the effect of the price taker assumption is thought to be limited.

The results in this study provide important practical implications. Previous studies argued that uncertain permit price is one of the main causes of lack of liquidity in permits trading, which in turn results in inefficiency [8–11]. The price discovery function shown in this study means that consignment auctions can improve this price uncertainty problem.

One other contribution of this study is to show that the proportional consignment auction can also perform the price discovery function similar to a consignment auction. Khezr and MacKenzie [15] examine only the proportional consignment auction; in their equilibrium, it does not deliver price discovery. According to the result of this study, emissions trading markets such as those in the EU, several US states, and Korea, where the proportional auction was already introduced, can also adopt a consignment auction scheme.

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References

1. Coase, R. The Problem of Social Cost. *J. Law Econ.* **1960**, *3*, 1–44. [CrossRef]
2. Montgomery, D.W. Markets in licenses and efficient pollution control programs. *J. Econ. Theory* **1972**, *5*, 395–418. [CrossRef]
3. Hahn, R.W.; Noll, R.G. *Designing A Market for Tradable Emissions Permits*; Social Science Working Paper 398; California Institute of Technology: Pasadena, CA, USA, 1981.
4. Aldy, J.; Stavins, R.N. Using the Market to Address Climate Change: Insights from Theory & Experience. *Deadalus* **2012**, *141*, 45–60. [CrossRef]
5. Acworth, W.; Ackva, J.; Haug, C.; Montes de Oca, M.; Fuss, S.; Flachsland, C.; Koch, N.; Kornek, U.; Knopf, B.; Edenhofer, O.; et al. *Emissions Trading and the Role of a Long-Run Carbon Price Signal: Achieving Cost-Effective Emission Reductions under an Emissions Trading System*; ICAP (International Carbon Action Partnership): Berlin, Germany, 2017.
6. Daskalakis, G.; Markellos, R.N. Are the European Carbon Markets Efficient? *Rev. Futures Mark.* **2008**, *17*, 103–128.
7. Montagnoli, A.; de Vries, F.P. Carbon trading thickness and market efficiency. *Energy Econ.* **2010**, *32*, 1331–1336. [CrossRef]
8. Healy, H.; Cames, M.; Matthes, F. *Climate Action and the Emissions Trading System (ETS) in China*; Policy Department A: Economy and Scientific Policy, European Parliament: Brussels, Belgium, 2016.
9. Lee, J.; Yu, J. Market Analysis during the First Year of Korea Emission Trading Scheme. *Energies* **2017**, *10*, 1974. [CrossRef]
10. Song, J.; Ahn, Y. Cognitive bias in emissions trading. *Sustainability* **2019**, *11*, 1365. [CrossRef]

11. Zhao, X.; Jiang, G.; Nie, D. How to improve the market efficiency of carbon trading: A perspective of China. *Renew. Sustain. Energy Rev.* **2016**, *59*, 1229–1245. [CrossRef]
12. Burtraw, D.; McCormack, K. Consignment auctions of free emissions allowances. *Energy Policy* **2017**, *107*, 337–344. [CrossRef]
13. Liu, Y.; Tan, B. Consignment Auctions Revisited. *Econ. Lett.* **2021**, *203*, 1–6. [CrossRef]
14. Borenstein, S.; Bushnell, J.; Wolak, F.; Zaragoza-Watkins, M. Expecting the Unexpected: Emissions Uncertainty and Environmental Market Design. *Am. Econ. Rev.* **2019**, *109*, 3953–3977. [CrossRef]
15. Khezr, P.; MacKenzie, I.A. Consignment auctions. *J. Environ. Econ. Manag.* **2018**, *87*, 42–51. [CrossRef]
16. Peskoe, A. *Designing Emission Budget Trading Programs under Existing State Law*; Harvard Environmental Policy Initiative: Cambridge, MA, USA, 2016. Available online: http://environment.law.harvard.edu/CPPResources (accessed on 31 May 2020).
17. Franciosi, R.; Isaac, M.; Pingry, D.; Reynolds, S. An experimental investigation of the Hahn-Noll Revenue Neutral Auction for Emissions Licenses. *J. Environ. Econ. Manag.* **1993**, *24*, 1–24. [CrossRef]
18. Ledyard, J.O.; Szakaly-Moore, K. Designing organizations for trading pollution rights. *J. Econ. Behav. Organ.* **1994**, *25*, 167–196. [CrossRef]
19. Dormady, N.; Healy, P. The consignment mechanism in carbon markets: A laboratory investigation. *J. Commod. Mark.* **2019**, *14*, 51–65. [CrossRef]
20. Burtraw, D.; Linn, J.; Palmer, K.L.; Paul, A.; McCormack, K.; Yin, H. *Approaches to Address Potential CO_2 Emissions Leakage to New Sources under the Clean Power Plan*; Resources for the Future: Washington, DC, USA, 2016.
21. Vickrey, W. Counterspeculation, auctions, and competitive sealed tenders. *J. Financ.* **1961**, *16*, 8–37. [CrossRef]
22. Wilson, R. Auctions of shares. *Q. J. Econ.* **1979**, *93*, 675. [CrossRef]
23. Ausubel, L.M. An Efficient Ascending-Bid Auction for Multiple Objects. *Am. Econ. Rev.* **2004**, *94*, 1452–1475. [CrossRef]
24. Ausubel, L.M.; Cramton, P.; Pycia, M.; Rostek, M.; Weretka, M. Demand reduction and inefficiency in multi-unit auctions. *Rev. Econ. Stud.* **2014**, *81*, 1366–1400. [CrossRef]
25. Back, K.; Zender, J.F. Auctions of divisible goods: On the rationale for the treasury experiment. *Rev. Financ. Stud.* **1993**, *6*, 733–764. [CrossRef]
26. Krishna, V. *Auction Theory*, 2nd ed.; Academic Press: Cambridge, MA, USA, 2010.
27. Wang, J.J.; Zender, J.F. Auctioning divisible goods. *Econ. Theory* **2002**, *19*, 673–705. [CrossRef]
28. Federico, G.; Rahman, D. Bidding in an electricity pay-as-bid auction. *J. Regul. Econ.* **2003**, *24*, 175–211. [CrossRef]
29. Lopomo, G.; Marx, L.M.; McAdams, D.; Murray, B. 2011. Carbon allowance auction design: An assessment of options for the United States. *Rev. Environ. Econ. Policy* **2011**, *5*, 25–43. [CrossRef]
30. Khezr, P.; MacKenzie, I.A. Revenue and efficiency in pollution permit allocation mechanisms. *Energy Econ.* **2021**, *93*, 105033. [CrossRef]
31. Khezr, P.; MacKenzie, I.A. Permit market auctions with allowance reserves. *Int. J. Ind. Organ.* **2018**, *61*, 283–306. [CrossRef]
32. Alvarez, F.; Mazón, C.; André, F.J. Assigning pollution permits: Are uniform auctions efficient? *Econ. Theory* **2019**, *67*, 211–248. [CrossRef]
33. Cramton, P. Competitive bidding behavior in uniform-price auction markets. In Proceedings of the Hawaii International Conference on System Sciences, Big Island, HI, USA, 5–8 January 2004; IEEE: New York, NY, USA, 2004. [CrossRef]