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Configuration Synthesis of Novel Hybrid Transmission Systems Using a Combination of a Ravigneaux Gear Train and a Simple Planetary Gear Train

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Abstract: Thirty-two novel hybrid transmissions consisting of a Ravigneaux gear train and a single planetary gear train are synthesized using a creative design methodology based on graph-theory and the lever analogy method. The design process commences by identifying an existing transmission configuration which meets all of the design requirements. The chosen design is then used to synthesize all possible mechanism permutations which satisfy the design constraints. The feasible mechanisms which satisfy both the design requirements and the design constraints are converted into analogous levers. The levers which fail to provide the required operation modes of the hybrid transmission are eliminated and the remaining levers are assigned brakes and clutches in order to realize the final designs. The responsiveness of the new hybrid transmissions and the feasibility of the proposed design methodology are confirmed by analyzing the power flow and kinematics of one of the designs in all of the operation modes.

Keywords: hybrid transmission; graph theory; lever analogy; design approach; electrically variable transmission (EVT); kinematic analysis

1. Introduction

The Ravigneaux gear train (RGT) is a double planetary gear set consisting of a ring gear, two sun gears and two planet gears [1]. The planet gears share the same carrier, and hence RGTs are smaller and lighter than a combination of simple planetary gear trains (SPGTs) with the same components and functions. Given these advantages, RGTs have been widely used in conjunction with one or more SPGTs to realize transmission systems with various speed levels, including 6-speed [2], 8-speed [3] and 10-speed [4].

Although RGTs have been used in many commercial automatic transmissions, their use in hybrid transmissions (HTs) is still rather limited. Several of the leading car manufacturers, including Toyota [5,6], Ford [7], Hyundai [8,9] and Volkswagen [10], have commercialized HTs based on either RGTs or integrated RGTs and SPGTs. However, as the trend toward zero-emission vehicles accelerates, the need for more efficient and effective HTs is growing. Consequently, the development of new HT configurations based on RGTs and SPGTs has attracted considerable interest in academia and the automobile industry in recent decades.

Among the various techniques available for accomplishing transmission design, graph theory (proposed by Freudenstein and Dobrjanskyj in 1966 [11]) and the lever analogy method (presented by Benford and Leising in 1981 [12]) are two of the most commonly used. Graph theory provides a powerful technique for synthesizing all configurations which meet a particular set of design criteria.

Yan [13] used graph theory to develop a new creative design methodology for mechanisms, while Yan et al. [14,15] employed graph theory to synthesize new parallel and series-parallel HT configurations. Hsu et al. [16] applied graph theory to perform the automated synthesis of all feasible displacement graphs for one-degree-of-freedom (1-DOF) PGTs with eight links. Ding and Yang [17] and Chatterjee and Tsai [18] used a similar approach to synthesize the graph sets of PGTs with nine and ten links, respectively. The lever analogy method has also been widely used for transmission power flow analysis, selection, and control synthesis. For example, Ross and Route [19] formed compound levers consisting of multiple levers connected in parallel based on a consideration of the required number of ratios and their magnitudes. Kim et al. [20] presented a compound lever-based methodology for synthesizing the optimal set of compound-split transmission configurations for hybrid electric vehicles (HEVs). Dagci et al. [21] presented an automated design process based on the analogous lever method for realizing 2-SPGT transmission systems with enhanced fuel economy and performance for light-duty trucks. Esmail [22] used a nomograph approach (one form of analogous lever method) to perform the design and enumeration of 9-link RGT mechanisms. Liao and Chen [23] used the analogy lever method to develop a simple approach for analyzing the gear ratios of multi-speed transmissions and continuous electrically variable transmissions (EVTs).

The literature contains many the examples of using algorithms for synthesizing the set of all possible transmissions which satisfy the given configuration and performance constraints. However, for computational reasons, these algorithms are typically limited to configurations consisting of no more than two SPGTs and eight members since, otherwise, the search space becomes too large. Methods that can create complex configurations with multiple SPGTs are often limited to the number of configurations found, moreover most research focuses on conventional automatic transmissions rather than HTs. Accordingly, the present study considers the problem of synthesizing HTs consisting of a RGT and SPGT and a large number of members.

The design problem is solved using the same method as that used by the present group in [24] to synthesize two-mode HTs consisting of three PGTs with nine links and fourteen joints. Once again, the feasibility of the design methodology is confirmed as they are used to configure new HTs with the same number of links and joints as above. As the result, a total of 32 novel HT configurations are derived.

2. Design Object and Methodology

2.1. Design Object

Figure 1 shows the two gear sets considered in the present design problem, namely a RGT (Figure 1a) and a SPGT (Figure 1b).

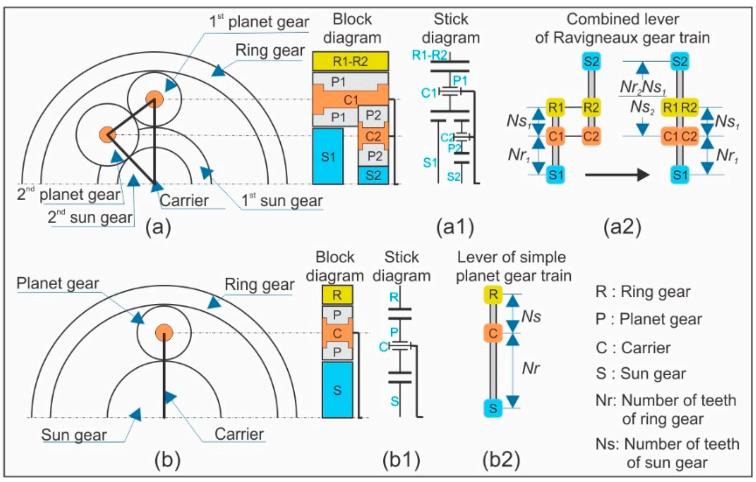


Figure 1. Main components of considered HT system: (**a**) Ravigneaux gear train; (**b**) simple planetary gear train.

As shown, the RGT comprises six components, namely a ring gear, two planet gears, a carrier (shared by both planet gears) and two sun gears (one large and one small). In the analysis which follows, the numbers of teeth on the ring gear, large sun gear and small sun gear are denoted as N_{r1}, N_{s1} and N_{s2}, respectively. Moreover, for convenience in the later calculations, it is assumed that a second ring gear existed and has a number of teeth N_{r2} equal to N_{r1}, that means the RGT is composed of two the SPGTs.

In the RGT, the first planet gear meshes with the ring gear via an internal gear joint and the large sun gear by an external gear joint, and simultaneously revolves around the carrier. Meanwhile, the second planet gear simultaneously meshes with the first planet gear and the small sun gear by two external gear joints, respectively, and revolves around the same carrier. As shown in Figure 1b, the SPGT has four components, namely a ring gear, a planet gear, a carrier and a single sun gear. The planet gear simultaneously meshes with both the ring gear and the sun gear, and revolves around its pin. To facilitate the remaining analysis, the numbers of teeth on the ring gear and sun gear are denoted as N_{r3} and N_{s3}, respectively.

2.2. Design Synthesis Methodology

Yan [13] used graph theory to develop a creative design methodology for synthesizing all possible configurations of the target mechanism based on its topological characteristics. However, for the HT problem considered in the present study, it is necessary to perform a more robust design procedure which not only considers the topological characteristics, but also the placement of the inputs (INs) output (OUT), brakes (Bs) and clutches (CLs) needed to satisfy the required operating modes. Solving this problem using graph theory alone is challenging; particularly for configurations with a large number of members, due to the difficulties involved in monitoring the power flow by observation. As described above, the methodology was originally proposed by Ho and Hwang in [24] for the design of two-mode HTs with three PGTs. The design procedure are illustrated in Figure 2.

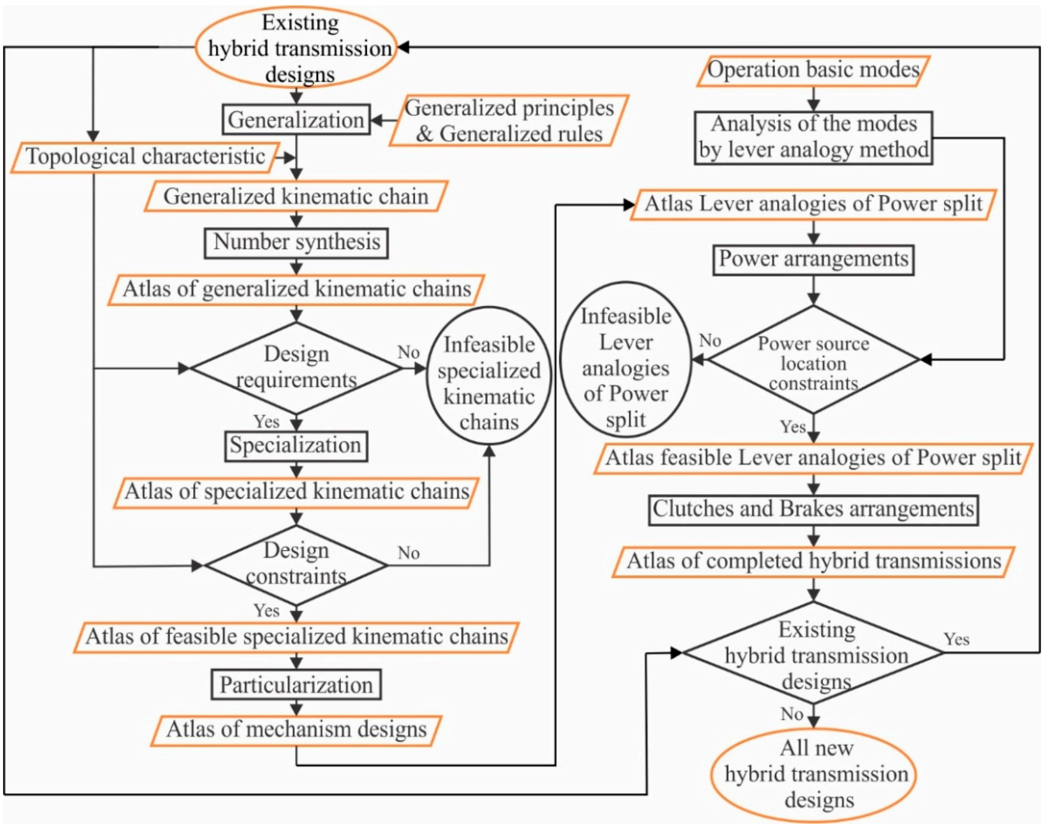


Figure 2. Design procedure for hybrid transmissions.

- The basic steps in the design procedure are described as follows: Based on the design goals and object, a search is conducted for an existing design which satisfies the desired performance and constraints.
- Based on the topological characteristics and design requirements, an atlas of generalized kinematic chains (GKCs) is synthesized.
- Based on the design constraints, the atlas of GKCs is filtered to leave only the atlas of feasible specialized kinematic chains (SKCs).
- The feasible SKCs are converted into equivalent mechanism designs and these designs are then converted into lever analogies.
- Based on the required operation modes, the infeasible lever analogies are identified and removed to leave only the feasible levers.
- Clutches and brakes are assigned to each feasible lever in order to obtain the atlas of completed HT designs.
- Existing HT designs are removed from the atlas of completed designs to leave a set of novel HT designs.

3. Select Existing Design and Analyze Topological Characteristics

Based on the design object described in Section 2.1, the parallel HT system presented in [25] and the conventional automatic transmission system proposed in [26] were selected as starting points for the present configuration synthesis process. The two transmission systems are shown in Figure 3a,b, respectively.

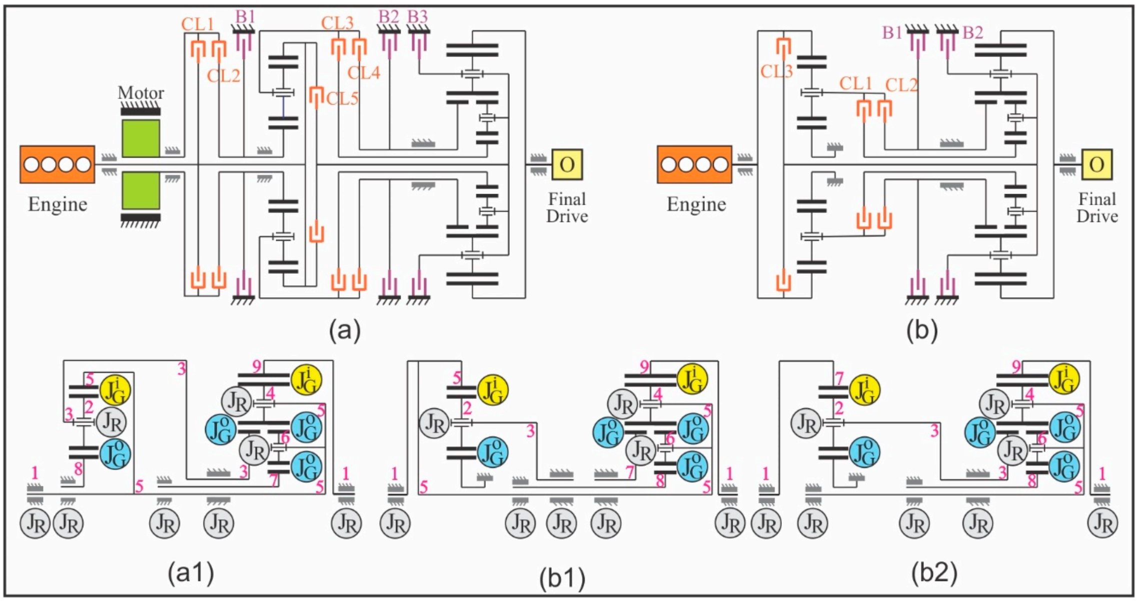


Figure 3. Schematic diagrams of existing HT designs: (**a**) Patent No. US 7,448,975, and (**b**) ZF-6HP26. Note that (**a1**) and (**b1**,**b2**) show the topological characteristics of (**a**) and (**b**), respectively.

To analyze the topologies of the selected transmission designs, it is first necessary to remove all of the inputs (INs), outputs (OUTs), clutches (CLs) and brakes (Bs) from the corresponding diagram. Note that when removing the CLs, it is necessary to retain the interconnecting links between the RGT and SPGT on each transmission system. The resulting topological characteristics of the parallel HT system and automatic transmission system are shown in Figure 3(a1,b1,b2), respectively. As shown, both systems have the following features:

- Each system (Figure 3a,b) consists of two connected gear sets. In Figure 3(a1), the two sets are connected by two links, while in Figure 3(b1,b2), the sets are connected by a single link and the sun gear of the SPGT has a fixed connection with the ground link.

- The both systems comprise nine members, namely a ground link (member 1), three planet gears (members 2, 4 and 6), two carriers (members 3 and 5), and three single links (members 7, 8 and 9).
- Each above system has fourteen joints, including eight revolute joints (J_R) and six gear joints (J_G). As explained in Section 2.1, the RGT has three external gear joints and one internal joint, while the SPGT has one external gear joint and one internal joint. Consequently, each system has four external gear joints (J_G^o) and two internal gear joints (J_G^i).
- In Figure 3(a1), all of the members except the three planet gears (members 2, 4 and 6) are adjacent to the ground link by one revolute joint. In Figure 3(b1, b2), the sun gear of the SPGT is fixed to the ground link, and hence the sun gear (member 2) is adjacent to the ground link (member 1) by one external gear joint, (J_G^o). (All of the remaining links are the same as those shown in Figure 3(a1).
- The number of degrees of freedom (DoF) of the both systems, F_p, is equal to:

$$F_p = 3(N_L - 1) - \sum N_{Ji} C_{Pi} = 2 \tag{1}$$

where C_{Pi} is the number of constraints acting on each i-type joint, N_L is the number of members, and N_{Ji} is the number of i-type joints (note that for the system shown in Figure 3, $N_L = 9, N_{JR} = 8$, $C_{PR} = 2$, $N_{JG} = 6$ and $C_{PG} = 1$).

4. Construct Generalized Kinematic Chains (GKCs)

The process of constructing the generalized kinematic chains (GKCs) involves converting the members of each system into corresponding links (Figure 4) and then, based on an analysis of the topological characteristics described above for the connections between members, connecting the links to form chains.

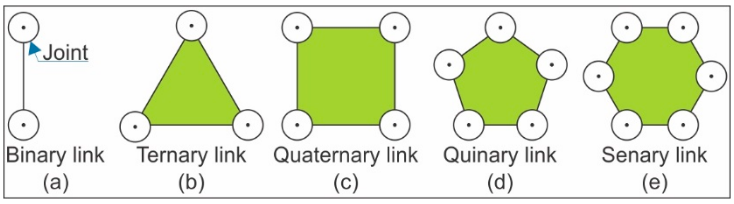


Figure 4. Types of links.

The GKC process has general principles and standards to be followed, which are detailed by Yan [13]. The following is the process of the generalized chain for this design case. For the design case considered in the present study, the main steps in the GKC process are as follows:

- In Figure 3(a1), member 1 (the ground link) is connected with five other members (3, 5, 7, 8 and 9). In Figure 3(b1,b2), member 1 is connected with six other members (2, 3, 5, 7, 8 and 9). Hence, member 1 is generalized into either a quinary link (five joints per links) or a senary link (six joints per links), where this link is referred to as link 1 hereafter.
- Member 3 (the carrier link) Figure 3(a1,b2) connects with members 1, 2 and 4. Meanwhile, member 3 in Figure 3(b1) connects with members 1 and 2. Thus, member 3 can be generalized as either a binary link or a ternary link (referred to as link 3 hereafter).
- In Figure 3(a1,b1), member 5 (the carrier link) is connected with four other members (1, 2, 4 and 6). In Figure 3(b2), member 5 is connected with three members (1, 4 and 6). Therefore, member 5 can be generalized as either a quaternary link or a ternary link (referred to as link 5 hereafter).
- In Figure 3(a1), members 2 and 6 (the planet gears) are connected with three other members (3, 5 and 8, or 4, 5 and 7, respectively). In Figure 3(b1,b2), members 2 and 6 are also connected with three other members (1, 3 and 5, or 4, 5 and 8, respectively). Thus, members 2 and 6 can both be generalized as ternary links (referred to as link 2 and link 6 hereafter).

- In Figure 3(a1,b2), member 4 (the planet gear) is connected with four other members (3, 5, 6 and 9). In Figure 3(b1), member 4 is also connected with four other members (5, 6, 7 and 9). Thus, both links can be generalized into a quaternary link (referred to as link 4 hereafter).
- In both systems, members 7, 8 and 9 are each connected with two other members. Thus, all three members can be generalized as binary links (referred to as links 7, 8 and 9 hereafter).

5. Atlas of Generalized Kinematic Chains (GKCs)

Applying the GKC synthesis algorithm described to [13] to the compound RGT/SPGT system described above with nine links and fourteen joints generates a total of 4176 GKCs (equivalent to 29 link assortments [24]). However, as described in the following, not all of these GKCs satisfy the design requirements and are therefore discarded.

5.1. Design Requirements

Based on the topological analysis presented in Section 3, the transmission system consists of nine members (one ground link, three planet gears, two carriers, three sun gear(s)/ring gear(s)) and 14 joints (eight revolute joints and six gear joints). Thus, as described above, the GKC for the system comprises 9 links and 14 joints. Based on the results of the GKC process described in Section 4, the following design requirements are identified:

5.1.1. Ground Link (Link 1)

- Link 1 must be a quinary link or senary link.

5.1.2. Planet Gear Links (Links 2, 4 and 6)

- Links 2 and 4 must be ternary links, while link 6 must be a quaternary link.

5.1.3. Carrier Links (Links 3 and 5)

- Link 3 must be a binary link or ternary link.
- Link 5 must be a ternary link or quaternary link.

Furthermore, the numbers of links in the GKC are constrained as follows: (1) a maximum of one quinary link or senary link; (2) a maximum of two quaternary links and a minimum of one; and (3) a minimum of two ternary links.Based on these design requirements, the following link assortments should be eliminated:

- Link assortments 1–5, 10–13 and 18–29 (since they fail to meet the ground link requirements given in Section 5.1.1);
- Link assortments 6, 9, 14 and 17 (since they fail to meet the requirement for the number of quaternary links).

In other words, out of the 29 link assortments in the atlas of GKCs, only four link assortments meet the design requirements. As shown in Table 1, these link assortments correspond to a total of 1812 GKCs.

Table 1. Link assortments with nine links and 14 joints which satisfy design requirements.

No.	Link Assortment	Number of Links						Number of GKCs
		Binary	Ternary	Quaternary	Quinary	Senary	Septenary	
1	7th link assortment	2	5	1	1	0	0	464
2	8th link assortment	3	3	2	1	0	0	939
3	15th link assortment	3	4	1	0	1	0	218
4	16th link assortment	4	2	2	0	1	0	191
		Total number of GKCs						1812

Figure 5 shows some typical GKCs among the 1812 GKCs shown in Table 1. While all of these GKCs satisfy the design requirements specified above, they do not all meet the design constraints (shown by the red lines in Figure 5), as described in the following section.

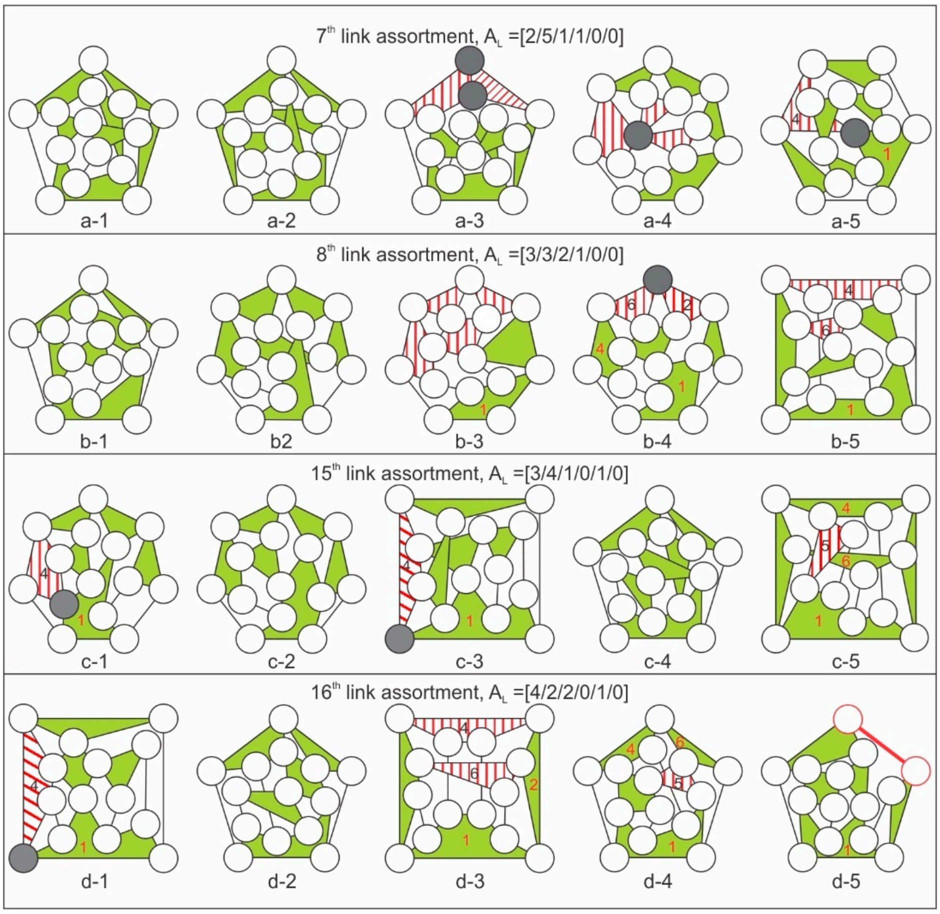


Figure 5. Illustrative GKCs with 7th, 8th, 15th and 16th link assortments.

6. Atlas of Feasible Specialized Kinematic Chains

The process of generating the feasible specialized kinematic chains involves positioning the links and joints in each GKC in such a way as to satisfy the design constraints identified in the topological analysis of the selected designs.

6.1. Design Constraints

Based on the topological analysis presented in Section 3, the specialized kinematic chains must satisfy the following design constraints.

6.1.1. Common Constraints

- Two adjacent links can be separated by only one joint.
- One joint only connects two links.

6.1.2. Ground Link (Link 1)

- All binary links must be adjacent to the ground link.
- For a ground link which is a quinary link, only three links are not adjacent to the ground link (i.e., two ternary links and one quaternary link). All of the other links must be adjacent to the ground link.

- For a ground link which is a senary link, only two links are not adjacent to the ground link (i.e., a ternary link and a quaternary link). All of the other links must be adjacent to the ground link.

6.1.3. Planet Gear Links (Links 2, 4 and 6)

- Link 2 (ternary link) is not adjacent to link 4 (quaternary link) or link 6 (ternary link).
- Link 4 must be adjacent to link 6 by one joint.

6.1.4. Carrier Links (Links 3 and 5)

- Link 5 (quaternary link) must be adjacent to two planet gear links (links 4 and 6).
- Link 3 must be adjacent to planet gear link (link 2).

6.1.5. Revolute Joints

- Each GKC must have eight revolute joints.
- Revolute joints are used to connect the planet gear link and carrier link, and the ground link and all its adjacent links (except when link 2 is adjacent to the ground link).

6.1.6. Gear Joints

- Each GKC has six gear joints, including four external gear joints and two internal gear joints.
- Each planet gear link has two gear joints (except for link 4 which has three gear joints). In particular, link 2 has one external gear joint and one internal gear joint, link 4 has two external gear joints and one internal gear joint, and link 6 has two external gear joints.
- The joint between link 4 and link 6 is an external gear joint.

6.2. Specialized Chains

The process of generating the specialized chains and identifying the feasible GKCs involves the following eight-step procedure:

- Step 1: Check all of the GKCs found in Section 5 and eliminate the GKCs which do not satisfy the common constraints given above.
- Step 2: Allocate the ground link (link1), as shown in Figure 6(a1). All of the GKCs found in Step 1 that satisfy the ground link binding condition are assigned a ground link. Any GKCs which do not satisfy the design constraint are removed.
- Step 3: Allocate the planet gear links (links 2, 4 and 6), as shown in Figure 6(a2). Any GKCs found in Step 2 which meet the planet gear link requirements are assigned planet gear links. The other GKCs are considered infeasible GKC, and they are eliminated.
- Step 4: Allocate the carrier link(s). The links found in Step 3 which satisfy the carrier link conditions are assigned as carrier links. Note that when attaching the carrier link for a GKC, it is necessary to list all of the locations in the GKC at which the carrier link can be assigned. For example, in the configuration shown in Figure 6, three positions are available which satisfy the carrier link condition (link 3), as shown in Figure 6(a3-1–a3-3).
- Step 5: Allocate the revolute joints. Based on the ground link conditions, each GKC found in Step 4 is assigned revolute joints. Note that each GKC has eight revolute joints, as shown in Figure 6(a4-1–a4-3).
- Step 6: Allocate the gear joints. Each GKC found in Step 5 is assigned six gear joints, including four external gear joints and two internal gear joints. Each pair of different gear joints on a link is permutated. As described in Section 2.1, the system has three planetary gears. However, there are only two pairs of different gear joints. Consequently, four results are obtained, as shown in Figure 6(a5-1-1–a5-1-4).
- Step 7: Convert the specialized kinematic chains (SKCs) obtained in Step 6 to corresponding transmission mechanism designs, where these designs are opposite to the generalized chains.

That is, the links adjacent to the internal gear joints are ring gears, while those adjacent to the external gear joints are sun gears (with the exception of the external gear joint between the two planet gear links (links 4 and 6)), as shown in Figure 6(a6-1-1–a6-1-4).

- Step 8: During the assignment and permutation of the gear joints in Step 6, if both gear joints relate to binary links, the new structure (after permutation) will not be different from the original structure (before permutation) and should therefore be removed.

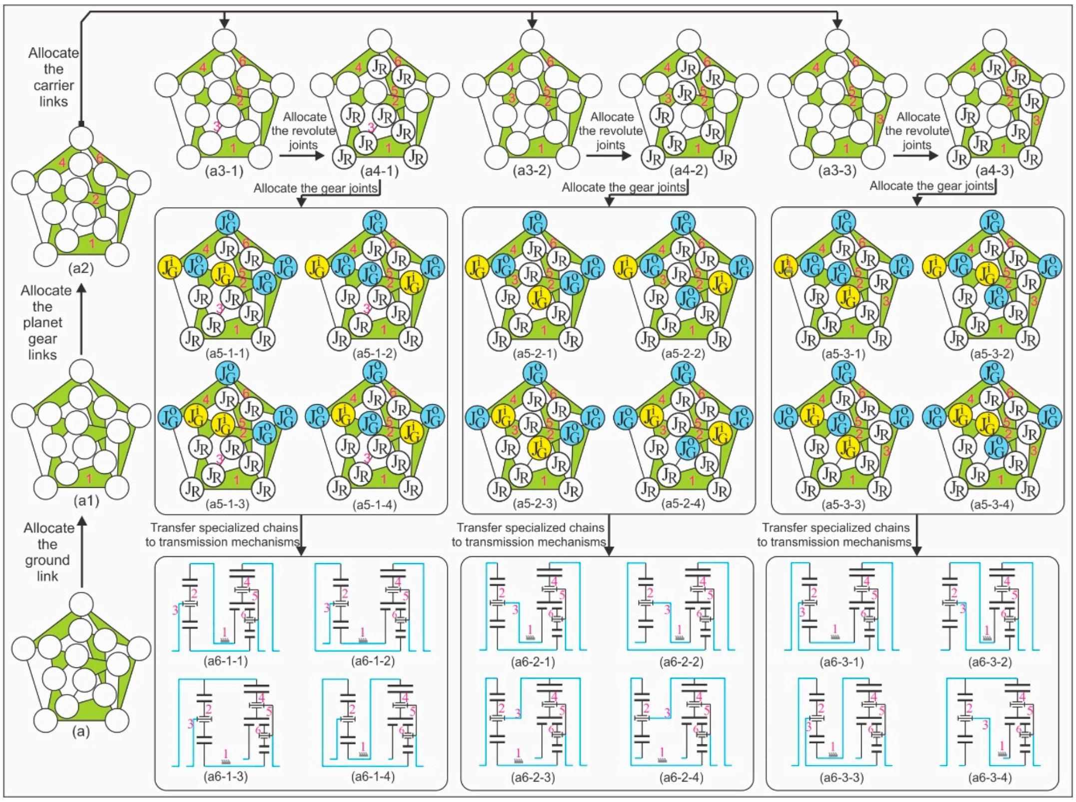


Figure 6. Specialization process for GKCs.

Overall, Steps 1–4 in the process described above aim to identify all the GKCs which do not satisfy the design constraints (i.e., the so-called infeasible GKCs). For example, referring to Figure 7, Step 1 results in the removal of all GKCs with the forms shown in Figure 7a,b, while Step 2 (assigning the ground link) removes GKCs with the forms shown in Figure 7c–e. Similarly, Step 3 removes GKCs with the forms shown in Figure 7f,g, while Step 4 removes GKCs with the form shown in Figure 7h.

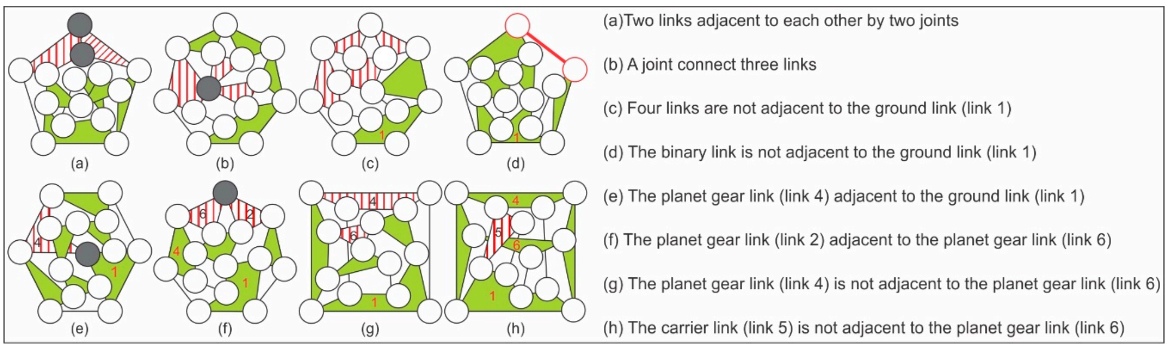


Figure 7. Infeasible generalized kinematic chains.

As shown in Figure 8, the specialization process identifies just seven GKCs (referred to as feasible GKCs hereafter) which satisfy all the design constraints.

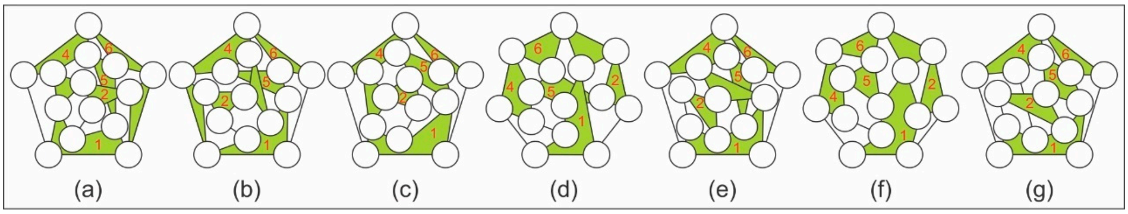


Figure 8. Feasible generalized kinematic chains.

Taking the feasible GKC shown in Figure 8a for illustration processes, 12 feasible SKCs can be synthesized, as shown in Figure 6. Twelve feasible SKCs are obtained for every feasible GKC, so in total 84 SKCs are obtained. Within the scope of this study, the goal is to design the HTs, so to approach existing design only those SKCs containing links with the highest level of 5 (i.e., quinary links) are considered. Hence, just 48 feasible SKCs are retained for further analysis, as shown in Figure 9.

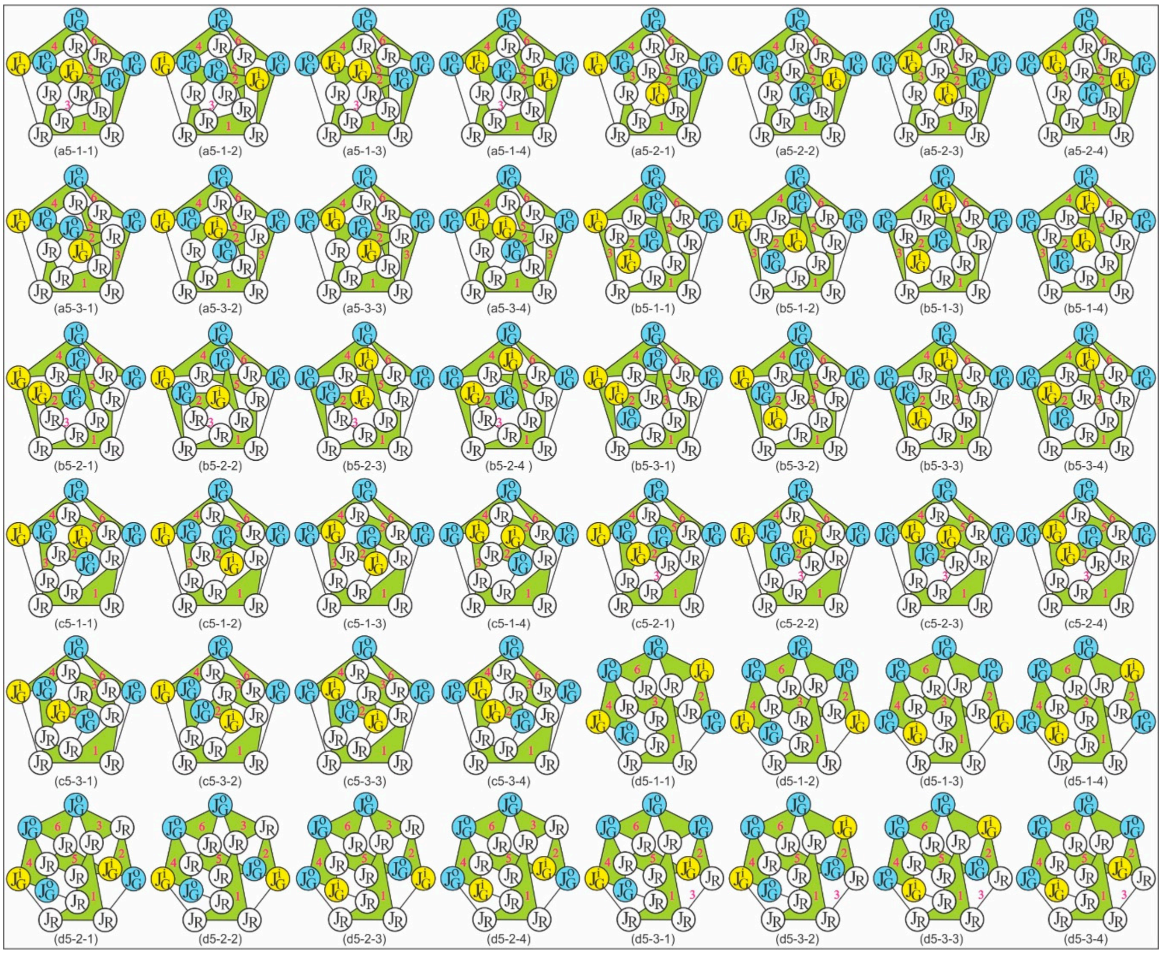


Figure 9. Atlas of feasible specialized kinematic chains.

7. Atlas of Transmission Mechanism Designs

Applying Step 7 (Section 6.2) of the specialized chain process to the atlas of feasible SKCs, 48 transmission mechanism designs are obtained, as shown in Figure 10.

In accordance with Step 8 (Section 6.2) of the specialized chain process, the 48 transmission mechanisms are inspected in order to identify any duplicated structures. A detailed observation of Figure 10 reveals 12 duplicate pairs, namely Figure 10(b6-1-1,b6-3-3), Figure 10(b6-1-2,b6-3-4), Figure 10(b6-1-3,b6-3-2), Figure 10(b6-1-4,b6-3-1), Figure 10(b6-2-1,b6-2-3), Figure 10(b6-2-2,b6-2-4),

Figure 10(d6-1-1,d6-1-4), Figure 10(d6-1-2,d6-1-3), Figure 10(d6-2-1,d6-2-4), Figure 10(d6-2-2,d6-2-3), Figure 10(d6-3-1,d6-3-4), and Figure 10(d6-3-2,d6-3-3). Removing these duplicated mechanisms, 36 mechanisms are retained for further analysis.

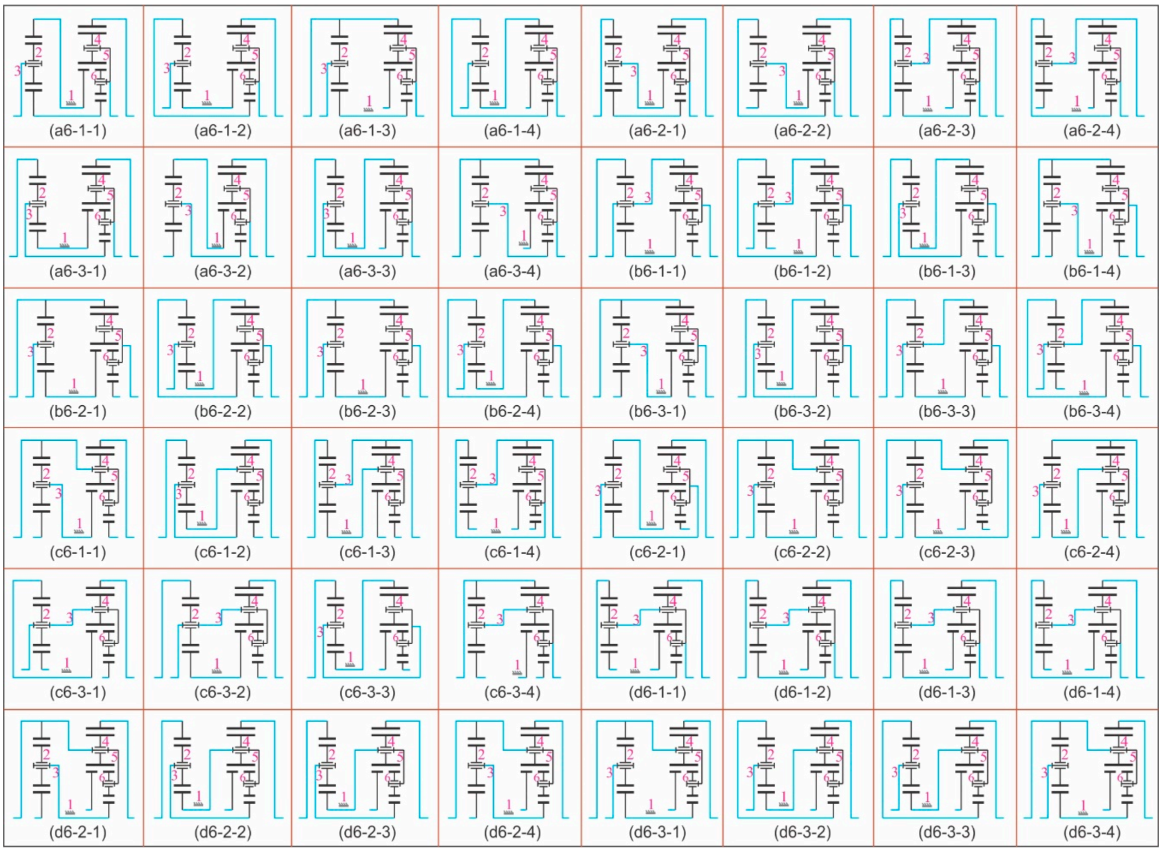


Figure 10. Atlas of transmission mechanism designs.

8. Atlas of Feasible Lever Analogies for New Hybrid Transmissions

8.1. Lever Analogy

For a SPGT, the analogous lever has the form of a single vertical lever with three nodes (R, C and S) representing the ring gear, carrier and sun gear, respectively (see Figure 1(b2)). Benford and Leising [12] showed that the length of the lever depends on the number of teeth on the ring gear and sun gear. In particular, the distance from the R node to the C node is proportional to the number of teeth on the sun gear, while the distance from the C node to the S node is proportional to the number of teeth on the ring gear.

For transmission systems formed by multiple SPGTs, the positions of the interconnect links between the components are used to convert the transmission into a set of corresponding analogous levers using the method described in [12]. RGTs are a good example of a combination of two SPGTs, in which the ring gear and carrier of PGT1 connect with the ring gear and carrier of PGT2, respectively, where PGT2 is double-pinion PGT. The corresponding lever analogy of the RGT is shown in Figure 1(a2).

8.2. Atlas of Lever Analogies for Transmission Mechanisms (TMs)

In accordance with the design methodology shown in Figure 2, the 36 transmission mechanisms found in Section 7 are converted into corresponding levers. The transmission system considered in the present study comprises a single RGT and a single SPGT coupled by two interconnect links. As shown in Figure 1(a2), the analogous lever of the RGT consists of 4 nodes, while that of the SPGT consists of three nodes (see Figure 1(b2)). Consequently, the analogous lever of the transmission mechanism consists of 5 nodes, where the distance ratio between the nodes depends on the positions of the interconnect links between the RGT and SPGT.

In combining the analogous levers of the SPGT and RGT, respectively, let the symbols on the original analogous levers be denoted as follows: R1–R2, C1, C2, S1 and S2 for the common ring gear, first carrier, second carrier, first sun gear and second sun gear of the RGT, respectively, and R3, C3 and S3 for the ring gear, carrier and sun gear of the SPGT, respectively. In addition, let k_1 and k_2 be the ratios of the teeth number on the common ring gear to those on the first sun gear and second sun gear, respectively, in the RGT. As described in Section 2.1, the RGT is assumed to have two ring gears, which are axially mounted and have the same number of teeth $N_{r2} = N_{r1}$, aims to make creating analogous lever for RGT easier. Finally, k_3 is the ratio of the number of teeth on the ring gear to the number of teeth on the sun gear of the SPGT. The following relations can thus be obtained:

$$k_1 = N_{r1}/N_{s1} \quad (2)$$

$$k_2 = N_{r2}/N_{s2} \quad (3)$$

$$k_3 = N_{r3}/N_{s3} \quad (4)$$

Figure 11 shows the general process of converting the transmission mechanisms into analogous lever systems. A total of 45 analogous levers are synthesized from the 36 transmission mechanisms (note that for each transmission mechanism, there are three correlation of value of k_1, k_2 and k_3 are considered when converting the transmission mechanisms into analogous levers ($k_1 < k_2 < k_3$, $k_1 < k_3 < k_2$ and $k_3 < k_1 < k_2$).

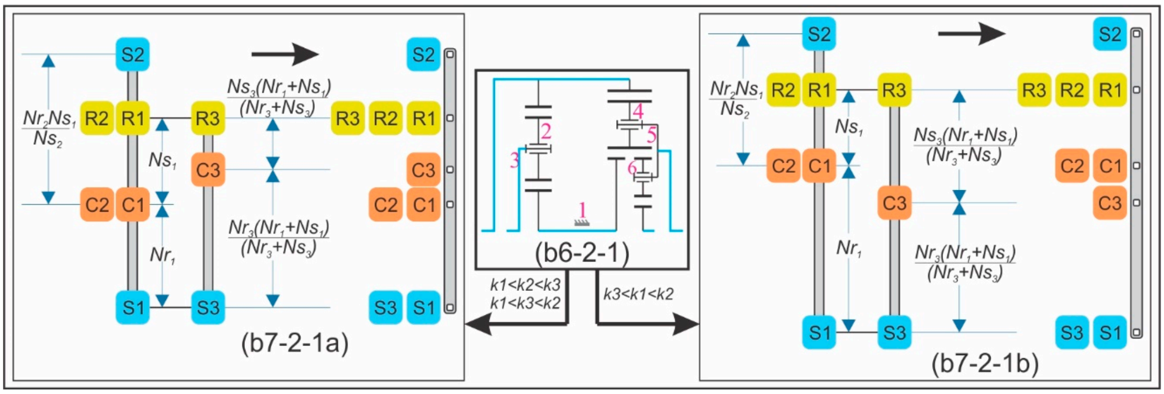


Figure 11. Conversion of transmission mechanisms into compound analogous levers.

8.3. *Configure Feasible Levers*

To evaluate the practicality of the analogous lever mechanisms described above, it is first necessary to define the operation modes which the transmission system must support. Any mechanisms which cannot support these modes should then be ignored in the further analysis.

8.3.1. Operation Modes of Hybrid Transmission

The HT considered in the present study comprises two electrical motors (MG1 and MG2), an input (IN) from the engine (EN), an output (OUT) to the final drive, an arrangement of clutches (CLs) and brakes (Bs), and the indispensable component is a PGT(s) system. Moreover, the HT is required to support three basic operating modes, namely a battery mode, an electrically variable transmission mode (EVT mode), and a fixed gear mode (FG mode). The EVT mode comprises two separate modes, namely an input-split mode and a compound-split mode, while the FG mode has four gear ratios, corresponding to (i) under drive (UD), (ii) direct drive (DD), and (iii) over drive (OD). The working principles of the various modes can be described as follows:

- Battery mode: The battery mode operates when the vehicle is moving only very slowly, and intermittently. In this mode, the EN is free and MG2 uses the energy supplied by the battery alone to propel the vehicle (provided that the battery has sufficient capacity).
- EVT modes: In the input-split mode, the power from the EN is divided into two paths, namely an electrical path and a mechanical path, where the rate of division between them depends on the speed of MG1. The electrical path is used for either charging, or to assist MG2 in driving the vehicle. By contrast, in the mechanical path, the power from the EN is used to drive the vehicle directly. However, a large power loss is incurred under high vehicle speeds (due to the high energy conversion on the electrical path). Consequently, the input-split mode functions well only at low speeds. To overcome this limitation, the compound-split mode uses both MG1 and MG2 to adjust the power distribution ratio between the electrical and mechanical paths. In particular, the engine capacity is split by the first PGT (next the engine), and they are compounded by the final PGT (next the Output) of the system. In contrast to the input-split mode, the compound-split mode works well at high speeds since the power conversion on the electrical path is low, and hence the energy loss is also low.
- Fixed gear modes: The fixed gear modes are used to meet the need for a greater output torque when the vehicle accelerates. Depending on the specific configuration employed, the HTs may have four, five or even six gear ratios in the fixed gear mode. Moreover, the two electric motors may either spin freely or work as motors to provide additional power to the EN when the vehicle accelerates.

8.3.2. Set Up Levers for EVT Modes and FG Modes

According to [27,28], the optimum point of performance for hybrid transmissions occurs when the vehicle (working in either the input-split mode or the compound-split mode) operates around the mechanical points (MPs), i.e., the points at which the electric motor speed is equal to zero. In other words, the entire EN power is transmitted through the mechanical path to drive the vehicle directly, and hence no energy loss occurs in the electrical path. (It is noted that the input-split mode has one MP, while the compound-split mode has two MPs.) In order for the MPs to appear in the modes in which the system operates, the four components that provide and absorb power in the HT (i.e., EN, MG1, MG2 and OUT) need to be positioned at appropriate locations within the system. In terms of the lever analogy, the relative positions between the four components (EN, MG1, MG2 and OUT) in the EVT be described as follows [29]:

- Input-split mode: MG2 and OUT are placed in the same position. Consequently, the lever for the input-split mode has only three nodes, corresponding to the positions of EN, MG1 and OUT/MG2, respectively. As shown in Figure 12a, two layouts are possible, namely one layout for the high ratio mode (Figure 12(a1)) and another layout for the low ratio mode (Figure 12(a2)). As described above, the input-split mode is generally used when the vehicle operates at low speed. Consequently, in the present study, the high ratio layout (Figure 12(a1)) is chosen for further analysis.
- Compound-split mode: As shown in Figure 12b, three layouts are possible for the compound-split mode depending on the particular ratio required, namely high, low or compound (a combination of high and low ratios). As described above, the compound-split mode is employed when the vehicle operates at moderate to high speeds. Accordingly, in the present study, the compound ratio layout (Figure 12(b3)) is selected for further analysis purposes. (Note that in assigning the components of the HT to the lever, MG1 and MG2 are always placed at the outermost positions, while OUT is placed next to MG1 and EN is placed next to MG2.
- Fixed-gear mode: To ensure that the HT can function as a conventional automatic transmission when operating in the FG mode, the transmission must provide both forward and reverse capabilities. The reverse gear is typically provided by MG2. Hence, only the forward gear is

considered here. A transmission system usually has many UDs, but only one DD and one OD. Consequently, as shown in Figure 12, the analogous lever of the transmission mechanisms after being combined has five nodes. In the present study, different gear ratios are created by adjusting the position of the brake [19]. Through this process, three different layouts are obtained, as shown in Figure 12(c1–c3), where both UD and OD exist. The first layout (Figure 12(c1)) is deemed to be preferable since it provides two UDs and one OD. However, if such a design configuration cannot be obtained, then layout Figure 12(c3), or finally layout Figure 12(c2), is chosen. In assigning the positions of EN and OUT on the lever for the preferred layout (Figure 12(c1)), EN and OUT are taken as the 2nd and 3rd nodes of the lever, respectively.

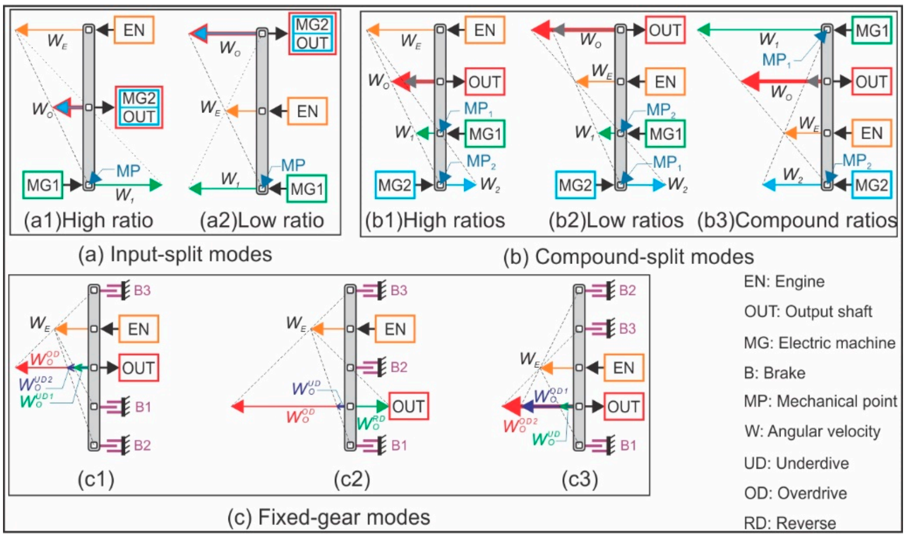


Figure 12. Lever diagrams of different operating modes.

8.3.3. Constraints on Positions of Input and Output

As described above, the transmissions designed in the present study are required to provide three main operation modes, namely battery, EVT and fixed gear. To implement the battery mode, the second electric motor (MG2) and OUT must be on the same train. Similarly, to perform the input-split mode, EN and the first electric motor (MG1) must be mounted on the same train. By contrast, to implement the compound-split mode, EN and OUT must be on different trains. (Note that in the present designs, SPGT is designated as the output of the transmission system.)

To avoid overloading MG2 when it works as a motor in driving the vehicle directly (in the battery mode), and to maximize the function of MG2 when it acts as a generator (in the regenerative braking process), MG2 and OUT must be attached to two of the three components of the SPGT such that when the remaining components is fixed, the gear ratio from MG2 to OUT is the highest. In other words, the ratio when transmitting from OUT to MG1 is the lowest. Similarly, to maximize the performance of MG1 and increase its rotational speed when it works as a generator in the input-split mode, the assigned positions of EN and MG1 on the same train need to be satisfy such that the gear ratio from EN to MG1 is the smallest (i.e., the speed of MG1 is many times higher than that of EN).

The kinematic equation of the SPGT has the form [30]:

$$W_s N_s + W_r N_r = W_c (N_r + N_s) \quad (5)$$

In accordance with Equation (5), the gear ratio of the SPGT can be determined simply from a knowledge of the locations of the IN component, OUT component, and braked component.

Finally, the mounting positions for MG2 and OUT, respectively, are given in order of decreasing priority as (S3, C3), (S3, R3) and (R3, C3); while those for MG1 and EN, respectively, are given as (S1, C1), (S2, C1), (S1, R1), (S2, R1) and (R1, C1) (Note that S3, C3 and R3 are the sun gear, carrier and ring gear of the SPGT, respectively, while S1, S2, C1 and R1 are first sun gear, second sun gear, carrier and ring gear of the RGT, respectively).

8.3.4. Atlas of Feasible Levers

The process of assigning the IN and OUT components is performed in accordance with both the lever modes shown in Figure 12 and the constraints on the IN and OUT positions described above. For the 45 analogous levers generated in Section 8.2, 35 feasible levers which satisfy all of the design conditions are found after assigning the IN and OUT components, as shown in Figure 13.

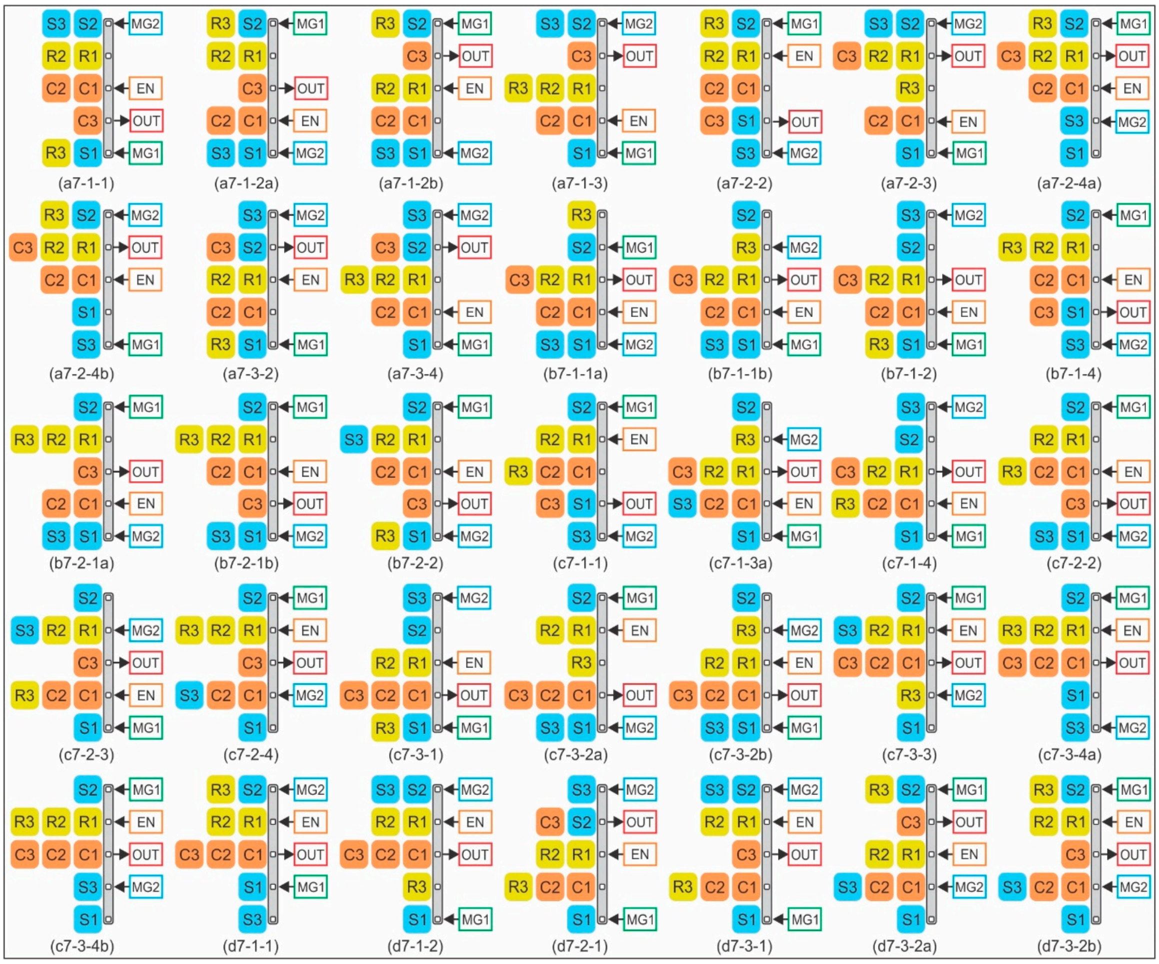


Figure 13. Atlas of feasible levers.

9. Completed Hybrid Transmission Designs

As shown in Figure 2, the design methodology concludes by transforming the feasible levers into complete HTs by assigning additional CLs and Bs in such a way as to enable them to support all of the required operation modes. The process of assigning the CLs and Bs involves a 7-step process. For the sake of clarity, the feasible lever shown in Figure 13(b7-2-1a) is chosen for illustrative purposes here in describing the details of the 7-step process.

- Step 1: Convert analogous lever into compound lever diagram (see Figure 14(b7-2-1a1)).
- Steps 2 and 3: Depending on the operating conditions of the Battery mode, the EN may either turn freely or be turned off; meaning that it does not participate in the driving process. It should therefore be separated from the rest of the system via the placement of the first clutch (CL1), as shown in Figure 14(b7-2-1a2). In addition, for the power to be transmitted from MG2 to OUT

when MG2 works as a motor to drive vehicle, the third component in the SPGT must be locked. Hence, the first brake (B1) is assigned to lock R3, as shown in Figure 14(b7-2-1a3). To function in the input-split mode, one end of the SPGT must be separated from the connection with the RGT. In the design considered here, this is achieved using CL1 (note that when CL1 is engaged, the transmission operates in the compound-split mode).

- Step 4: To create a direct ratio (DD), a second clutch (CL2) is added to the HT to connect any two components on the same train such that all of the components on the train rotate at the same speed as the EN. For example, as shown in Figure 14(b7-2-1a4), when CL2 connects C1 and S1, all of the components of the RGT rotate at the same speed. Furthermore, when CL1 is also engaged, the power produced by EN is transmitted directly to OUT.
- Step 5: Through Step 4, the first fixed gear (FG1) is created when CL2 and B1 are both active. To create the second fixed gear (FG2), a second brake (B2) is added to the position of S2, as shown in Figure 14(b7-2-1a5). FG2 is then obtained when B1 and B2 are both active. The third fixed gear (FG3) and fourth fixed gear (FG4) are similarly achieved when B1, CL1, and B2, CL1 are engaged, respectively (note that all four ratios are UD ratios).
- Step 6: Though Steps 1–5, all of the required transmission modes other than the low gear ratio in the fixed gear mode are achieved. Consequently, it is sufficient to add only a third brake (B3) at the position of S1 to provide a low gear ratio (OD ratio) when both CL1 and B3 are engaged.
- Step 7: The compound lever diagram generated in Steps 1–6 is converted back to a transmission block diagram Figure 14(b8-2-1).

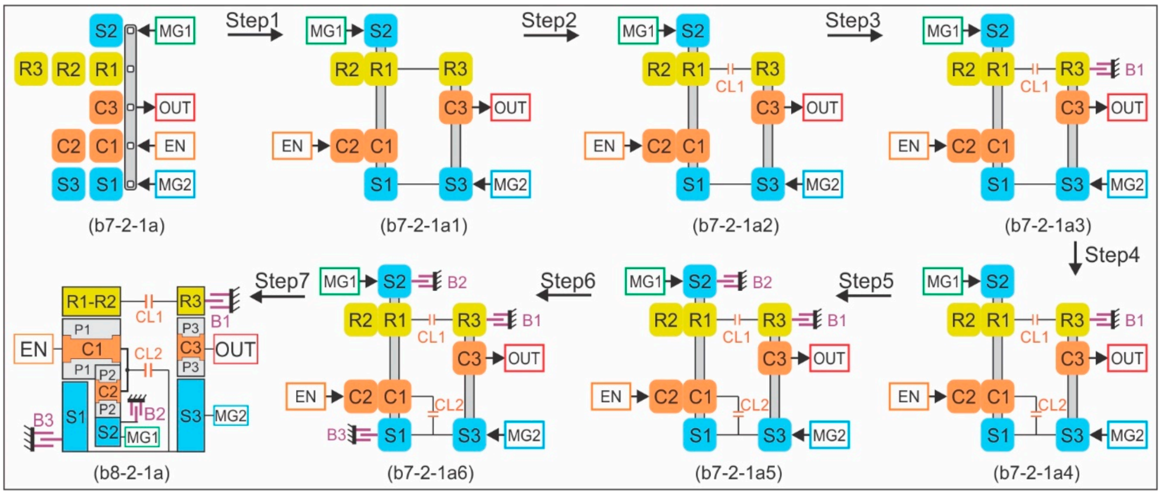


Figure 14. Illustrative example of clutch and brake assignment to feasible levers.

Applying the procedure described above to the 35 feasible levers found in Section 8.3 yields a total of 32 transmission block diagrams (note that three pairs of feasible levers result in the same transmission block diagram since even though the levers differ in terms of the positions of the nodes, the positions of the interconnect links and components EN, OUT, MG1, MG2, Bi and CLi are similar). Comparing the 32 HTs with the existing designs, it is found that none of the designs (including three Bs and FG modes with up to five or even six ratios) are the same as the existing designs (with only two Bs and four ratios in the FG mode). In other words, the design process results in a total of 32 novel HTs consisting of a RGT and SPGT, where each design provides one battery mode, two EVT modes, and a fixed gear mode with five or six different gear ratios.

10. Kinematic Analysis of Novel Hybrid Transmissions

To confirm the correctness of the design methodology, this section performs a systematic kinematic analysis of the HT design shown in Figure 15(b8-2-1a), which provides six gear ratios in the FG mode (namely four UD ratios, one DD ratio and one OD ratio) in addition to the battery mode and EVT modes. The corresponding mode switching diagram is shown in Figure 16(b8-2-1-a2) (note that a schematic diagram of the transmission system is also provided in Figure 16(b8-2-1-a1) to facilitate the discussions).

For convenience in calculating and evaluating the analysis results, the characteristic parameter values for the considered HT are specified as:

$$k_1 = 1.51, \; k_2 = 2.29, \; k_3 = 2.5 \tag{6}$$

Furthermore, the notations used in the analysis are defined as follows: T_{si} , W_{si} and T_{ri}, W_{ri} are the torques and angular velocities of the sun gear and ring gear of the ith PGT, respectively; and T_O, W_O, T_E, W_E, T_2, W_2 and T_1, W_1 are the torques and angular velocities of the OUT, EN, MG2 and MG1 components of the HT, respectively.

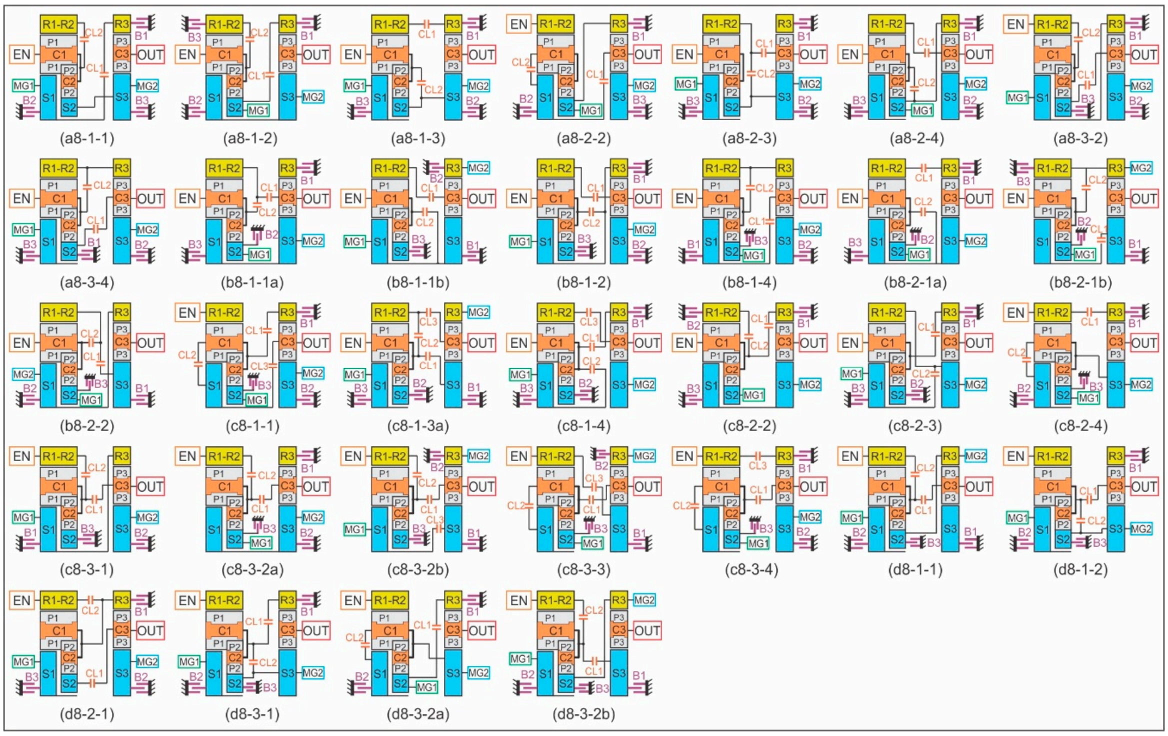
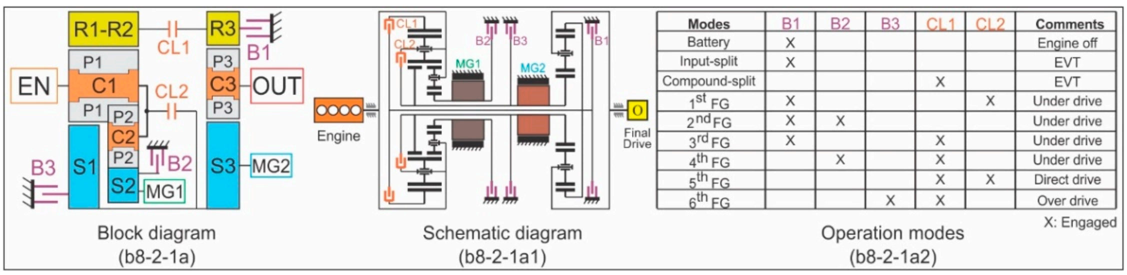


Figure 15. Atlas of new hybrid transmissions.



Modes	B1	B2	B3	CL1	CL2	Comments
Battery	X					Engine off
Input-split	X					EVT
Compound-split				X		EVT
1^{st} FG	X				X	Under drive
2^{nd} FG	X	X				Under drive
3^{rd} FG	X			X		Under drive
4^{th} FG		X		X		Under drive
5^{th} FG				X	X	Direct drive
6^{th} FG			X	X		Over drive

Figure 16. HT system and mode switching diagram considered in kinematic analysis.

10.1. Battery Mode

In the battery mode (see Figure 17), the EN is free and the vehicle is driven only by MG2. Moreover, only the SPGT works and the ring gear is fixed.

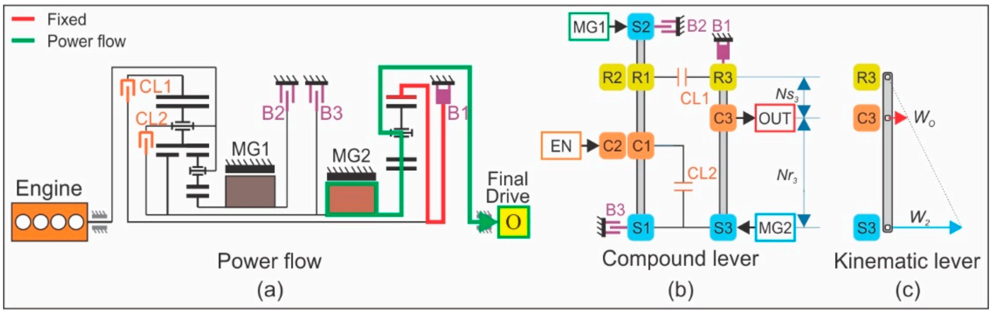


Figure 17. Battery mode.

From Equation (5), the speed and torque of OUT can be calculated as follows:

$$W_{s3}N_{s3} + W_{r3}N_{r3} = W_{c3}(N_{s3} + N_{r3}) \tag{7}$$

where $W_{s3} = W_2$, $W_{r3} = 0$, $W_{c3} = W_O$

Substituting Equations (2)–(4) and (6) into (7) gives:

$$W_O = W_2 \frac{1}{1+k_3} = 0.29\ W_2 \tag{8}$$

$$T_O = (1 + k_3)T_2 = 3.5\ T_2 \tag{9}$$

10.2. Input-Split Mode

The input-split mode (the first EVT mode) is used when the vehicle operates at low speed, and allows the vehicle to move with the motor alone, the EN alone, or with both the motor and the engine working together.

To facilitate this mode, the system requires a separation between the RGT and SPGT components. Furthermore, one component (the ring gear) needs to be fixed such that the power can be transmitted to OUT, as shown in Figure 18.

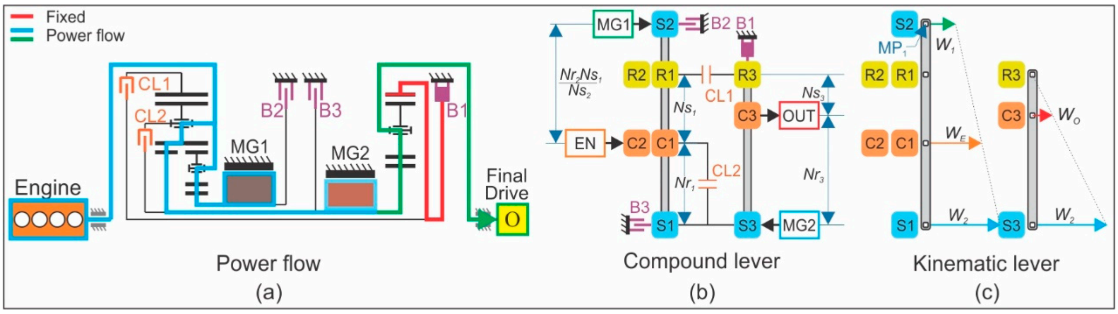


Figure 18. Input-split mode.

$$W_{s1}\frac{N_{r2}N_{s1}}{N_{s2}} + W_{s2}N_{r1} = W_{c1}\left(\frac{N_{r2}N_{s1}}{N_{s2}} + N_{r1}\right) \tag{10}$$

$$W_{s3}N_{s3} + W_{r3}N_{r3} = W_{c3}(N_{r3} + N_{s3}) \tag{11}$$

where $W_{s1} = W_{s3} = W_2$, $W_{s2} = W_1$, $W_{r3} = 0$, $W_{c1} = W_E$, $W_{c3} = W_O$

$$W_O = \frac{1}{k_2(1+k_3)}[(k_1 + k_2)W_E - k_1 W_1] = 0.47W_E - 0.19W_1 \tag{12}$$

$$T_{s3} = T_2 + T_{s1} \tag{13}$$

$$T_{s1} = \frac{k_2}{(k_1 + k_2)} T_E \tag{14}$$

$$T_{s3} = \frac{1}{1 + k_3} T_O \tag{15}$$

Substituting Equations (13) and (14) into (15) gives:

$$T_O = (1 + k_3)\left(\frac{k_2}{(k_1 + k_2)} T_E + T_2\right) = 2.1\ T_E + 3.5\ T_2 \tag{16}$$

10.3. Compound-Split Mode

The compound split mode (the second EVT mode) is used when the vehicle operates at high load and high speed and involves the use of all three main power components to drive the vehicle. Depending on the specific conditions, the controller unit divides the power contribution ratio of the components and controls the function of each part. For example, MG1 and MG2 sometimes work as generators and other times as motors. To accomplish such coordination, CL1 must be engaged (see Figure 19), and hence it follows that with $W_{s1} = W_{s3} = W_2$, $W_{s2} = W_1$, $W_{r1} = W_{r3}$, $W_{c1} = W_E$, $W_{c3} = W_O$:

$$\left[\frac{1}{k_1} - \frac{1 + k_1}{k_1(1 + k_3)}\right] W_1 + \left[\frac{1}{k_1}(k_2 - 1) + \frac{1 + k_1}{k_1(1 + k_3)}\right] W_E = \frac{k_2}{k_1} W_O \tag{17}$$

$$W_O = 0.12 W_1 + 0.87 W_E \tag{18}$$

$$\left[\frac{k_2 - 1}{k_1} + \frac{1 + k_1}{k_1(1 + k_3)}\right] T_O = \frac{k_2}{k_1} T_E + \frac{k_1 + k_2}{k_1} T_2 \tag{19}$$

$$T_O = 1.89\ T_2 + 1.14\ T_E \tag{20}$$

$$W_1 + W_2 \frac{k_2}{k_1} = W_E \left(1 + \frac{k_2}{k_1}\right) \tag{21}$$

$$W_2 = W_E \left(\frac{k_2 + k_1}{k_2}\right) - \frac{k_1}{k_2} W_1 = 1.66 W_E - 0.66\ W_1 \tag{22}$$

$$\left[k_3 \frac{1 + k_1}{1 + k_3}\right] T_2 + \left[1 - \frac{1 + k_1}{1 + k_3}\right] T_E = \left[k_2 - 1 + \frac{1 + k_1}{1 + k_3}\right] T_1 \tag{23}$$

$$T_1 = 0.14 T_E + 0.89 T_2 \tag{24}$$

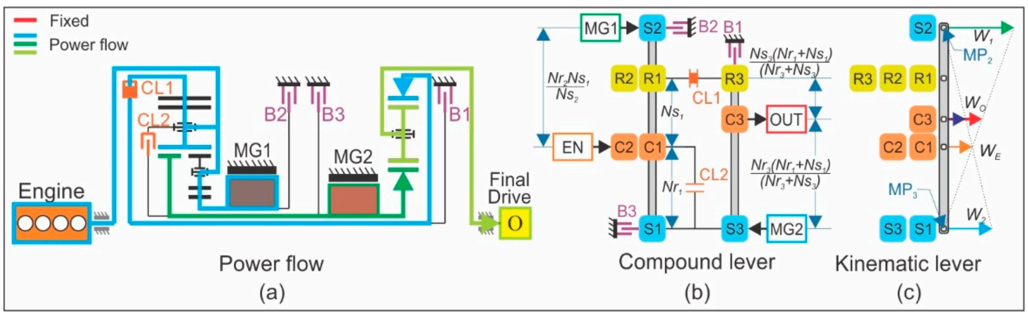


Figure 19. Compound-split mode.

10.4. Fixed Gear Modes

As described above, the novel HTs designed in the present study provide up to six gear ratios in the fixed gear mode, as shown in Figures 20–25. The speed and torque relations for each of these gear ratios are given as follows:

10.4.1. 1st Fixed Gear

With $W_{s1} = W_{s3} = W_2,\ W_{s2} = W_1\ , W_{r3} = 0$

$$W_O = \frac{1}{1+k_3} W_E = 0.28 W_E \tag{25}$$

$$T_O = (1+k_3)(T_1 + T_2 + T_E) = 3.5\ (T_1 + T_2 + T_E) \tag{26}$$

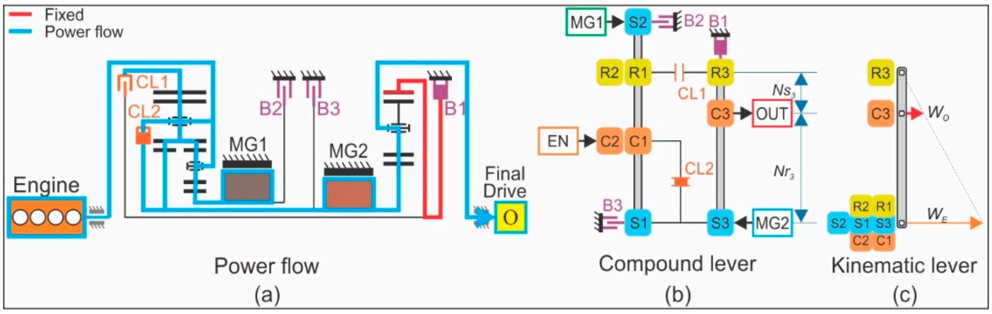


Figure 20. First fixed gear.

10.4.2. 2nd Fixed Gear

With $W_{s1} = W_{s3} = W_2,\ W_{s2} = W_1 = 0, W_{r3} = 0$

$$(1+k_3)k_2 W_O = (k_1 + k_2) W_E \tag{27}$$

$$W_O = 0.47 W_E \tag{28}$$

where $T_{s3} = T_{s1} + T_2$

$$T_{s1} = \frac{k_2}{k_1 + k_2} T_E \tag{29}$$

$$T_{s3} = \frac{1}{1+k_3} T_O \tag{30}$$

$$T_O = \frac{k_2(1+k_3)}{k_1 + k_2} T_E + (1+k_3) T_2 \tag{31}$$

$$T_O = 2.11 T_E + 3.5 T_2 \tag{32}$$

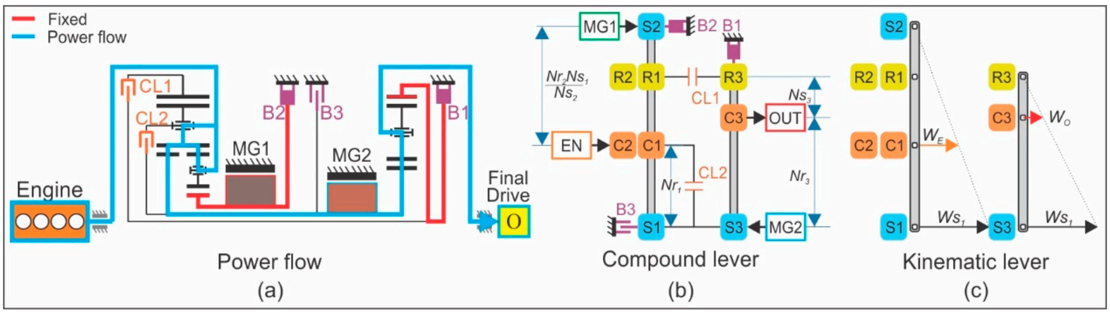


Figure 21. Second fixed gear.

10.4.3. 3rd Fixed Gear

With $W_{s1} = W_{s3} = W_2\ ,\ W_{r1} = W_{r3} = 0$

$$W_O = \frac{1+k_1}{1+k_3} W_E = 0.72 W_E \tag{33}$$

$$T_O = (1+k_3)\left[\frac{1}{1+k_1}T_E + T_2 - \frac{(k_2-1)}{1+k_1}T_1\right] = 1.39T_E + 3.5T_2 - 1.8T_1 \quad (34)$$

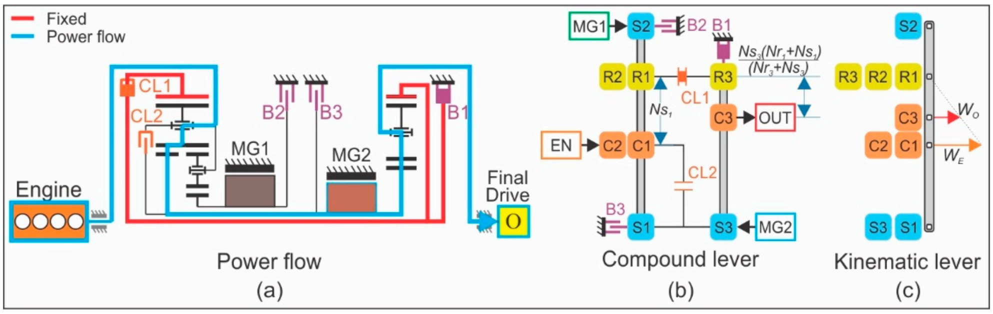


Figure 22. Third fixed gear.

10.4.4. 4th Fixed Gear

With $W_{s1} = W_{s3} = W_2$, $W_{r1} = W_{r3}$, $W_{s2} = W_1 = 0$,

$$W_O = \frac{(k_2-1)(1+k_3)+1+k_1}{k_2(1+k_3)}W_E = 0.88W_E \quad (35)$$

$$T_O = \frac{1+k_3}{(k_2-1)(1+k_3)+1+k_1}[k_2T_E + (k_2+k_1)T_2] = 1.14T_E + 1.89T_2 \quad (36)$$

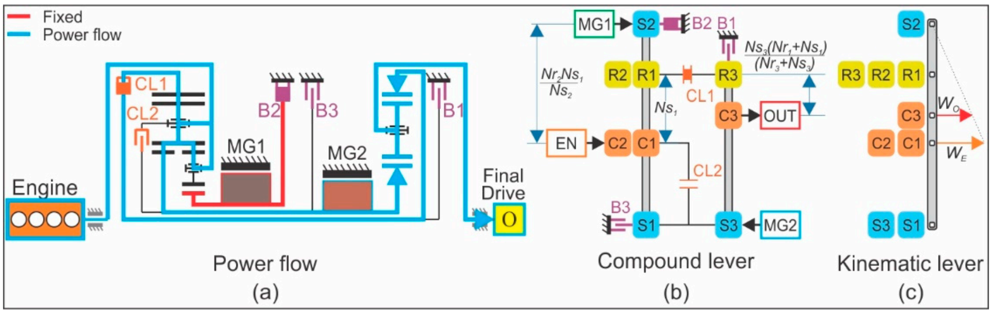


Figure 23. Fourth fixed gear.

10.4.5. 5th Fixed Gear

$$W_O = W_1 = W_2 = W_E \quad (37)$$

$$T_O = T_E + T_2 + T_1 \quad (38)$$

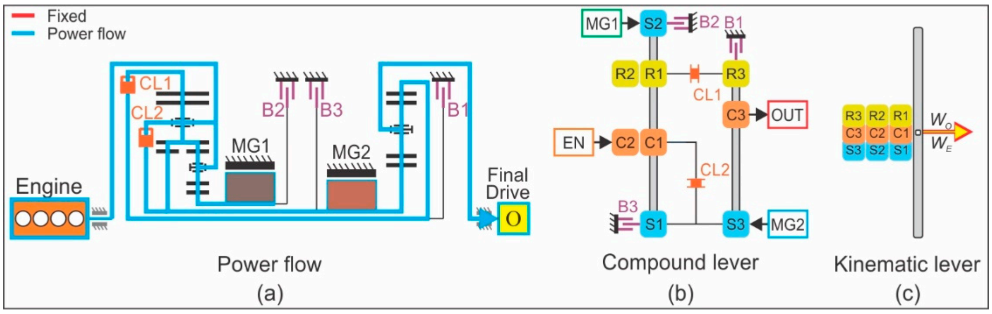


Figure 24. Fifth fixed gear.

10.4.6. 6th Fixed Gear

With $W_{s1} = W_{s3} = W_2 = 0$, $W_{r1} = W_{r3}$, $W_{s2} = W_1$,

$$W_O = \frac{k_3(1+k_1)}{k_1(1+k_3)} W_E = 1.18 W_E \tag{39}$$

$$T_O = \frac{1+k_3}{k_3(1+k_1)} [k_1 T_E + (k_2+k_1)T_1] = 0.84T_E + 2.12T_1 \tag{40}$$

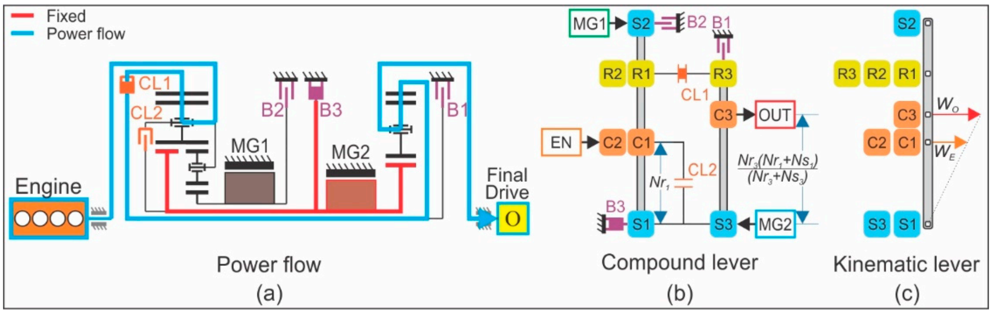


Figure 25. Sixth fixed gear.

11. Conclusions

This study has employed the creative design methodology proposed in [24] in conjunction with graph theory and the lever analogy method to synthesize feasible transmission systems for a hybrid electric vehicle. A total of 32 novel hybrid transmissions (HTs) have been proposed, where each design consists of one Ravigneaux gear train (RGT), one planetary gear train (PGT), one input port from the engine (EN), one output port (OUT) to the final drive, two electric motors (MG1 and MG2), two or three clutches (CLs) and three brakes (Bs). Each design supports a battery mode, an input-split mode, a compound-split mode and up to six fixed gear-ratio modes. The feasibility of the proposed HT designs has been demonstrated by performing the kinematic analysis of one of the designs. Overall, the results have shown that the proposed designs enable the vehicle to move smoothly with two EVT modes and six FG modes. In addition, to increase fuel economy because the engine can work in the fuel optimum regoins by regulating the speeds and torques of two eletric motors.

Overall, the design method employed in this study facilitates the design of complex HT systems comprising multiple gear trains (RGT and PGT) and a large number of members. Moreover, the design method is easily implemented using a straightforward step-by-step approach and is based mainly on the use of intuitive graphics at each stage, which minimizes the risk of calculation errors in the design process. Finally, the proposed method not only provides the means to synthesize all possible HT configurations for the same compound composition (e.g., one RGT and one SPGT in the present case), but also allows all possible correlation cases of the characteristic parameter values (k_1, k_2 and k_3) to be considered in the process of creating analogous lever.

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