energies

MDPI

Article

# Optimum Thickness of Thermal Insulation with Both Economic and Ecological Costs of Heating and Cooling

**Robert Dylewski [1,*] and Janusz Adamczyk [2]**

[1] Institute of Mathematics, Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra, Licealna 9, 65-417 Zielona Góra, Poland
[2] Institute of Management and Quality, Faculty of Economics and Management, University of Zielona Góra, Licealna 9, 65-417 Zielona Góra, Poland; J.Adamczyk@wez.uz.zgora.pl
* Correspondence: R.Dylewski@wmie.uz.zgora.pl; Tel.: +48-6-8328-2821

**Abstract:** The energy efficiency of the construction sector should be determined by the cleanliness of the environment and, thus, the health of society. The scientific aim of this article was to develop a methodology for determining the optimum thickness of thermal insulation, taking into account both economic and ecological aspects and considering both heating and cooling costs. The method takes into account the number of degree days of the heating period, as well as the number of degree days of the cooling period. Variants in terms of different types of thermal insulation, various types of construction materials for building walls, climatic zones and heat sources, were taken into consideration. In order to find the optimum thicknesses of thermal insulation, both in economic and ecological terms, a metacriterion was used. The optimum thicknesses of thermal insulation with the use of the metacriterion were obtained in the range of 0.11–0.55 m. It was observed that the values of the optimum heat transfer coefficients for economic and ecological reasons do not depend on the type of construction materials used for vertical walls. The type of applied heat source is of the greatest importance for the size of the economic and ecological benefits. The proposed mathematical model for determining the optimum thickness of thermal insulation with the use of a metacriterion is a kind of generalization of earlier models from the literature.

**Keywords:** mathematical model of thermal insulation; optimum thickness of thermal insulation; heating degree days; cooling degree days; heating and cooling costs; life cycle assessment; economic and ecological benefits; two-criteria analysis

check for updates

**Citation:** Dylewski, R.; Adamczyk, J. Optimum Thickness of Thermal Insulation with Both Economic and Ecological Costs of Heating and Cooling. *Energies* **2021**, *14*, 3835. https://doi.org/10.3390/en14133835

Academic Editor: Marc Medrano

Received: 27 May 2021
Accepted: 23 June 2021
Published: 25 June 2021



## 1. Introduction

The use of conventional energy carriers to ensure the fulfillment of an appropriate level of human needs is the cause of the deterioration of the environment's condition. Technical progress and the increase in the quality of life in society generate a large number of positive and negative effects. Currently, the priority task of scientists and decision makers is to minimize the negative effects of the development of global society, including the reduction of energy consumption.

Primary energy use in the European Union remains high [1], which is related to the level of socioeconomic development and demographics [2]. Energy savings are perceived in the improvement of the energy efficiency of devices, installations and buildings [3].

The European Union's activities aimed at improving energy efficiency date back to 2006, when the Energy End-use Efficiency and Energy Services Directive (ESD) was adopted. It was the first directive on energy efficiency in the EU, which committed to achieving the overall national target by 2016 for energy savings of 9% compared to the average final energy consumption in 2001–2005 [4,5]. In December 2012, the Energy Efficiency Directive (EED) was adopted, which replaced the previous Directive (ESD). The Directive (EED) sets a much higher target for European Union countries in terms of energy efficiency of 20%, which they should have achieved by 2020. In absolute terms, it means that the total energy consumption in the EU should not exceed 1483 million tons of oil equivalent (Mtoe), primary energy or 1086 Mtoe of final energy [5,6]. As part of the "Clean Energy for All Europeans" package, in 2018, a new Directive on Energy Efficiency (EED) [7] was agreed. The headline target for energy efficiency was set for 2030, which is at least 32.5%. This target was set in relation to the 2007 modeling projections. In absolute terms, this means that the EU's energy consumption in 2030 should not exceed 1273 Mtoe of primary energy and/or not more than 956 Mtoe of final energy.

The highest primary energy consumption in Europe is attributed to the transport sector, 30.5%; followed by households, 26.1%; industry, 25.8%; services, 14.2%; and agriculture, forestry and others, 0.5% [8].

Buildings in the EU are responsible for the emission of approx. 30% of greenhouse gases. The greatest energy losses occur in buildings constructed in the years 1945–1980 [3]. The high rates of energy consumption and greenhouse gas emissions in Europe were the reason why Directive 2002/91/EC of the European Parliament and of the Council of 16 December 2002 on the energy performance of buildings was established in 2002. This Directive was the first so-called Energy Performance Buildings Directive (EPBD). The main goal of this EPBD Directive was to build awareness among building owners and users about energy-efficient buildings [9–11]. This Directive introduced important legal instruments that made it possible to provide relevant knowledge to interested persons about the energy efficiency of the building in which they lived, wanted to rent or buy.

Another Directive in this area appeared in the legal regulations of the European Union as early as 2010. It introduced the concept of the so-called nearly zero-energy building (nZEB). Thus, Member States were obliged to achieve nearly zero-energy consumption by 31 December 2020, for all new buildings and after 31 December 2018, for all new buildings occupied by public authorities [12–14].

On 19 June 2018, the amendment to Directive 2010/31/EU EPBD was published in the Official Journal of the EU. The new Directive (EU) 2018/844 of the European Parliament and of the Council of 30 May 2018 amending Directive 2010/31/EU on the energy performance of buildings and Directive 2012/27/EU on energy efficiency upholds the previous assumption of converting existing buildings into nearly zero-energy buildings [15].

The pace of changes in legal regulations in the field of energy efficiency of buildings may indicate great opportunities to improve the energy efficiency of this sector. In line with the provisions of the Directive, Member States should strive for a cost-effective balance between the decarbonization of energy supplies and the reduction of final energy consumption (Directive 2018/844). The scientific aim of this article is to develop a methodology for determining the optimum thickness of thermal insulation, taking into account both economic and environmental aspects. As indicated above, this is in accordance with the provisions of the Directive 2018/844.

## 2. Literature Review

The issue of energy efficiency in residential buildings has become a research problem for scientists from around the world. It is possible to encounter a lot of studies on the optimum thickness of thermal insulation. Nematchoua et al. [16,17] developed an economic model covering the cost of the insulation material and the present value of the energy consumption and the cost over the lifetime of the building, which is 22 years. This model was used to find the optimum insulation thickness for external walls. In the study, polystyrene was chosen as an insulation material, and then the variants of two typical wall constructions (HCB-concrete block) and walls made of compressed stabilized earth blocks (CSEB) were used. The annual loads of cooling transmission, depending on the orientation of the walls and the percentage of blocked radiation, were calculated by the method of apparent finite differences under set periodic conditions. The value of the optimum insulation thickness oscillated between 0.09 m [16] and from 0.08 m to 0.11 m [17]. The optimum thickness of the insulation, taking into account the energy and economic aspect, was also made in the studies of De'Rossi et al. [18], Sahu et al. [19] and Aktemur and Atikol [20].

Orzechowski and Orzechowski [21] presented a method for calculating the optimum thickness of thermal insulation related to the fuel prices and quality of energy supply. They assumed that the total cost after renovation is the sum of heating costs and thermal modernization costs. They showed that when calculating the optimum thickness of the insulation, it is necessary to take into account the thermal resistance of the external walls, the characteristics of the heat source and local weather conditions. Two heat sources were analyzed. The authors found that with thermal modernization with the use of polystyrene of optimum thickness, the total investment costs are almost the same, regardless of the thermal properties of the polystyrene.

Dylewski [22] proposed a method for determining the optimum thickness of thermal insulation for external walls, taking into account economic and ecological heating costs. The analysis did not include the factor related to the cooling of the building. In the study, attention was drawn to the fact that it was preferable to use higher thicknesses of thermal insulation than optimum for economic reasons. Then, greater ecological benefits are obtained from the investment in thermal insulation, with a slight reduction in economic benefits. The studies (Çetintaş, Yilmaz [23]; Yildiz [24]), apart from the economic aspect, also include the ecological aspect.

Barrau et al. [25] determined the optimum thickness of thermal insulation in terms of economy, energy and environment. Four thermal insulation materials were tested. The thickness of the insulation depends to a large extent on the unit costs associated with the production of the materials. The analysis did not take into account the costs related to the cooling of the building.

In his study, Kon [26] used a large number of variants in the selection of heat sources (four) and cooling (one), thermal insulation materials (three) and construction materials (two). The study looked at the heating and cooling period in a variable location in Turkey. The analysis of the optimum thickness of the insulation was made on the basis of the economic aspect. The author noticed that the optimum insulation thickness would be much greater in the case of construction materials where the difference between the thermal conductivity value and the density was higher.

Hernandez and Kenny [27] proposed the concept of "net energy" in the built environment, which they introduced and applied. They also developed a methodology that takes into account the built-in energy of building components along with the energy consumption during operation [28].

This article proposes an innovative method for determining the optimum thickness of thermal insulation, taking into account both economic and ecological costs. The method includes the number of degree days for heating and cooling, so it can be used in different types of climate.

## 3. Optimum Insulation Thicknesses

In order to find the optimum thickness of thermal insulation for economic or ecological reasons, insulation of the external walls of the building can be treated as an investment. The basic tool for evaluating the investment is the net present value (NPV). In the case of thermal insulation, negative flows (costs) will be associated with the implementation of thermal insulation, while positive flows (profits) will reduce the building's energy demand for heating and cooling.

### *3.1. Thermal Insulation Thickness, Optimum for Economic Reasons*

Taking into account the economic aspects, it is necessary to establish the annual economic cost of heating, referred to 1 m$^2$ of external wall area:

$$G_{OH} = K_{OH} \cdot c \cdot HDD \quad [(\mathrm{PLN} \cdot \mathrm{K})/(\mathrm{W} \cdot \mathrm{year})] \tag{1}$$

and annual economic cost of cooling, referred to 1 $m^2$ of the surface of exterior wall:

$$G_{OC} = K_{OC} \cdot c \cdot CDD \quad [(\text{PLN} \cdot \text{K})/(\text{W} \cdot \text{year})] \tag{2}$$

where:

$K_{OH}$—cost of heat generation for a given heat source (PLN/Wh);
$K_{OC}$—cost of producing coolness (PLN/Wh);
$c$—24 (h/day);
$HDD$—number of degree days of the heating period (K·day/year);
$CDD$—number of degree days of the cooling period (K·day/year);

Based on the above costs, the economic net present value can be determined depending on the thickness of the thermal insulation:

$$NPV\ (d) = -(K_m \cdot d + K_w) + S_N \cdot (G_{OH} + G_{OC}) \cdot (U_0 - \lambda/(d + \lambda/U_0)) \quad (\text{PLN/m}^2) \tag{3}$$

where:

$d$—thickness of the thermal insulation layer (m);
$K_m$—cost of 1 $m^3$ of the thermal insulation material (PLN/$m^3$);
$K_w$—costs of performing thermal insulation of the 1 $m^2$ building wall surface (PLN/$m^2$);
$S_N = \sum_{j=1}^{N} \frac{(1+s)^j}{(1+r)^j}$ —cumulative discount factor;
$N$—number of years of thermal insulation use;
$r$—real annual interest rate;
$s$—real annual growth (in percentage) of heating costs;
$\lambda$—thermal conductivity of the thermal insulation material (W/mK);
$U_0$—heat transfer coefficient of the wall without the thermal insulation layer (W/$m^2$K).

Let us observe that $U\ (d) = \lambda/(d + \lambda/U_0)$ is the heat transfer coefficient of the wall, taking into account the thermal insulation layer with thickness $d$.

Formula (3) is a generalization of Formula (1) to $NPV$ from Reference [29], in which only heating was considered; that is, when the building does not use air conditioning, then $G_{OC} = 0$, and Formula (3) is simplified to Formula (1) from [29].

Based on Formula (3), it is possible to find the optimum thickness of insulation for economic reasons. Because $NPV$ with respect to $d$ is strictly concave and bounded from the above function, at the point where the derivative with respect to $d$ is equal to 0, the $NPV$ function reaches its maximum value. The optimum thickness of thermal insulation for economic reasons is therefore:

$$d_{opt} = \text{sqrt}\ (\lambda \cdot (G_{OH} + G_{OC}) \cdot S_N/K_m) - \lambda/U_0 \quad (\text{m}), \tag{4}$$

while the optimum heat transfer coefficient of the wall with a thermal insulation layer:

$$U_{opt} = U\ (d_{opt}) = \lambda/(d_{opt} + \lambda/U_0) \quad (\text{W/m}^2\text{K}) \tag{5}$$

Substituting $d_{opt}$ (from Formula (4)) into Formula (5) for $U_{opt}$, the term "$\lambda/U_0$" will be reduced. As a result, $U_{opt}$ does not depend on $U_0$.

### *3.2. Optimum Thickness of Thermal Insulation for Ecological Reasons*

When assessing thermal insulation for environmental reasons, it is crucial to determine the annual ecological cost of heating, referred to as 1 $m^2$ of the external wall area:

$$G_{EH} = K_{EH} \cdot c \cdot HDD \quad ((\text{Pt} \cdot \text{K})/(\text{W} \cdot \text{year})) \tag{6}$$

and annual ecological cost of cooling, referred to as 1 $m^2$ of the surface of exterior wall:

$$G_{EC} = K_{EC} \cdot c \cdot CDD \quad ((\text{Pt} \cdot \text{K})/(\text{W} \cdot \text{year})) \tag{7}$$

where:

$K_{EH}$—LCA analysis result of obtaining 1 Wh of useful thermal energy for a given heat source (Pt/Wh);
$K_{EC}$—LCA analysis result of obtaining 1 Wh of useful cooling energy (Pt/Wh);
other—as defined earlier.

Based on the ecological costs (6) and (7), the ecological net present value can be determined depending on the thickness of the insulation:

$$NPV_E\ (d) = -(K_I \cdot d) + N \cdot (G_{EH} + G_{EC}) \cdot (U_0 - \lambda/(d + \lambda/U_0)) \quad (\mathrm{Pt/m^2}) \tag{8}$$

where:

$K_I$—LCA analysis result for 1 $m^3$ of the thermal insulation material (Pt/$m^3$);
other—as defined earlier.

Formula (8) is a generalization of Formula (3) for $NPV_E$ from Reference [29], where only heating was considered. For a building where no air conditioning is used, $G_{EC} = 0$, and Formula (8) is simplified to Formula (3) from [29].

As in the case of the economic analysis, on the basis of Formula (8), it is possible to determine the optimum thickness of the insulation for ecological reasons. Due to $d$, the $NPV_E$ function is also strictly concave and bounded from above. Thus, at the point where the derivative with respect to $d$ is equal to 0, $NPV_E$ is at its maximum. For ecological reasons, the optimum thickness of the thermal insulation is therefore:

$$d_{Eopt} = \mathrm{sqrt}\ (\lambda \cdot (G_{EH} + G_{EC}) \cdot N/K_I) - \lambda/U_0 \quad (\mathrm{m}), \tag{9}$$

while the optimum for ecological reasons of the heat transfer coefficient of the wall with a thermal insulation layer:

$$U_{Eopt} = U\ (d_{Eopt}) = \lambda/(d_{Eopt} + \lambda/U_0) \quad (\mathrm{W/m^2K}) \tag{10}$$

As in the case $U_{opt}$ also $U_{Eopt}$ does not depend on $U_0$.

### 3.3. Optimum Thickness of Thermal Insulation Taking into Account Costs Both Economically and Ecologically

In a previous study [29], it turned out that the optimum thickness of insulation for ecological reasons may significantly differ (may be much greater) from the optimum thickness for economic reasons. Below we propose a method to find the optimum thickness, including both economic and ecological costs.

First, on the basis of Formulas (3) and (4), as well as (8) and (9), the following should be determined:

$$NPV_{max} = NPV\ (d_{opt}) \quad \text{and} \quad NPV_{Emax} = NPV_E\ (d_{Eopt}) \tag{11}$$

Based on these values, it is possible to define the so-called metacriterion $MK$, taking into account the level of satisfaction of the I type:

$$MK\ (d) = w_1 \cdot NPV\ (d)/NPV_{max} + w_2 \cdot NPV_E\ (d)/NPV_{Emax} \tag{12}$$

where $w_1 + w_2 = 1$, $w_1, w_2 \geq 0$. For $NPV\ (d) \geq 0$ and $NPV_E\ (d) \geq 0$, $MK\ (d)$ is a unitless quantity and takes values between 0 and 1. The metacriterion defined in (12) is a convex combination of two strictly concave functions and those bounded from above. Thus, it is also a strictly concave function, bounded from above. It reaches its maximum for thickness:

$$d_{MKopt} = \mathrm{sqrt}\ (\lambda \cdot c_E/c_T) - \lambda/U_0 \quad (\mathrm{m}), \tag{13}$$

and the heat transfer coefficient of the wall with a thermal insulation layer corresponding to this thickness is:

$$U_{MKopt} = U\,(d_{MKopt}) = (U_0 - \lambda/(d_{MKopt} + \lambda/U_0)) \quad (\mathrm{W/m^2K}), \tag{14}$$

where:

$$c_E = (w_1/NPV_{max})\cdot \mathrm{S_N}\cdot(G_{OH} + G_{OC}) + (w_2/NPV_{Emax})\cdot N\cdot(G_{EH} + G_{EC}), \tag{15}$$

$$c_T = (w_1/NPV_{max})\cdot K_m + (w_2/NPV_{Emax})\cdot K_I \tag{16}$$

The derivation of Formula (13) is included in Appendix A.

### 3.4. The Use of LCA for Ecological Assessment

LCA analysis is a technique more and more often used for environmental assessment of products (Bras [30], Dylewski and Adamczyk [31], Dzikuć [32], Dzikuć and Dzikuć [33]). In this article, the LCA technique was used to determine $K_I$, $K_{EH}$ and $K_{EC}$ (see Section 3.2). The entire LCA analysis was performed in accordance with the international standards ISO 14040 [34] and ISO 14044 [35].

The LCA methodology is supported by a large number of commercial computer programs. The article uses the Sima Pro version 8.2 computer program and the ReCiPe endpoint method, egalitarian version (the result includes the environmental impact of all impact categories that are included in the method). The ReCiPe method is a commonly used method in scientific research (Dekker et al. [36]; Wolfova et al. [37]). Using the ReCiPe method and weighing procedure, the final result of the analysis can be obtained, expressed in the unit (Pt). The data library implemented in the SimaPro program: Ecoinvent 3 and European Life Cycle Database v3.1 (ELCD) was used. Due to the requirements of the standards describing the LCA analysis, the system boundaries and functional units of the analyzed products are presented. The system boundaries cover the phases of the product life cycle from raw materials to the factory gate (“from the cradle to the factory gate”). The functional unit for thermal insulation materials is 1 $m^3$ and, for heat sources, the production of 1 kWh of useful thermal energy.

## 4. Economic and Ecological Analysis for the Climatic Conditions in Poland

This section presents an economic and ecological analysis for a thermal insulation investment, using the methodology in Section 3. Various variants were taken into account due to the significant impact of initial conditions. It is important to consider what climatic zone the building is in, the parameters of the exterior walls without thermal insulation and the heat source in the building. Various thermal insulation materials were also considered in order to assess which type of thermal insulation is most favorable for the particular initial conditions.

### 4.1. Description of Variants Accepted for Analysis

There are five climatic zones [38] in Poland, which differ significantly. Table 1 presents data on the degree days of the heating period *HDD* and the degree days of the cooling period *CDD* determined on the basis of data from Eurostat [39]. The base temperature for the number of degree days of heating is 15 °C, and the base temperature for the number of cooling degree days is 24 °C. Average values from 16 years (2003–2018) for selected regions in Poland (each in a different zone) and for the whole of Poland are presented. Let us notice that the lowest number of *HDD* degree days occurs in the Lubuskie region (Zone II) and the highest in the Podlaskie region (Zone IV), by over 21% more. The number of *CDD* degree days in Poland is less than 1% of the number of *HDD* degree days.

**Table 1.** Average values of degree days of the heating and cooling period for the whole of Poland and selected regions in Poland.

| | *HDD* (K·Day/Year) | *CDD* (K·Day/Year) |
|---|---|---|
| Poland | 3386.9 | 21.5 |
| Zachodniopomorskie (Zone I) | 3272.0 | 12.3 |
| Lubuskie (Zone II) | 3074.8 | 30.6 |
| Mazowieckie (Zone III) | 3448.0 | 23.4 |
| Podlaskie (Zone IV) | 3734.1 | 12.8 |

There are three variants of exterior walls without thermal insulation. Table 2 and Figure 1 summarize the data on walls needed in the analysis. As seen, they differ significantly in the $U_0$ heat transfer coefficient.

**Table 2.** Construction materials.

| Construction Material | Cellular Concrete 600 (C1) | Lime and Sand Blocks SILKA E (C2) | Ceramic Hollow Blocks Max (C3) |
|---|---|---|---|
| Thickness of walls (m) | 0.24 | 0.24 | 0.29 |
| Thermal conductivity $\lambda_c$ (W/mK) | 0.16 | 0.55 | 0.29 |
| Heat transfer coefficient $U_0$ (W/m$^2$K) | 0.60 | 1.65 | 0.86 |

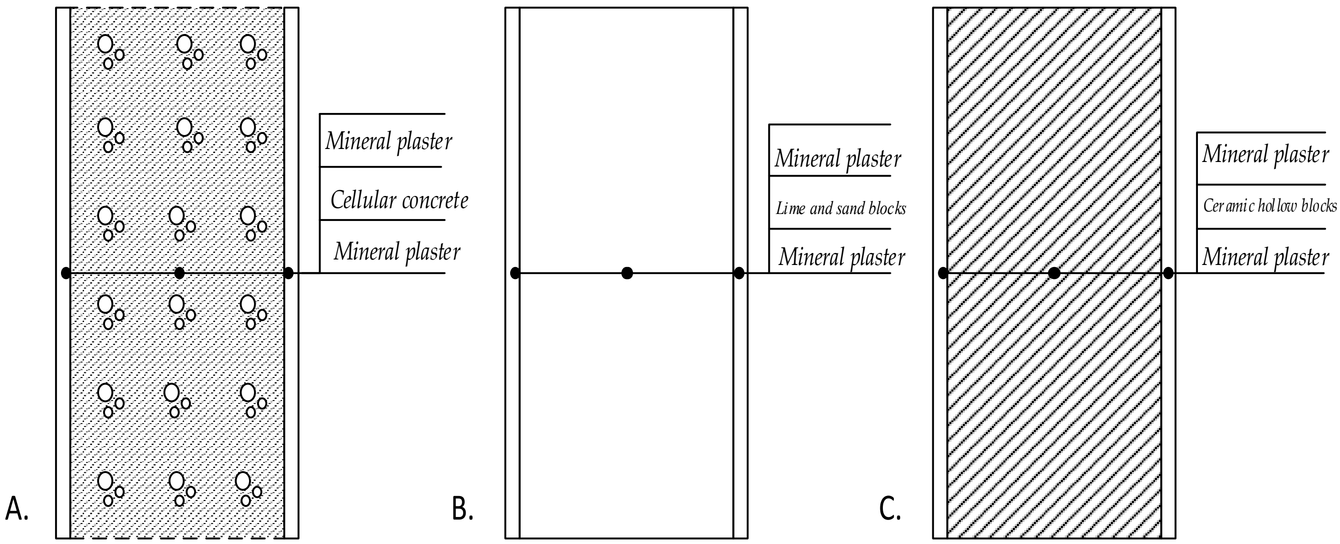


**Figure 1.** Cross-sections of the analyzed partitions made of: (**A**) cellular concrete; (**B**) lime and sand blocks; (**C**) ceramic hollow blocks.

Three types of heat sources were considered. The necessary information is presented in Table 3. The highest economic costs (EUR 1 ≈ PLN 4.50) are related to heating with the use of an electric boiler (S3). The highest ecological costs are for heating with a coal-fired boiler (S1). $N$ = 25 years was adopted for the analysis, while the rates $r$ = 5% and $s$ = 2%, in line with the guidelines for comparing investment projects in Poland.

**Table 3.** Heat sources.

| Source of Heat | Coal Boiler (S1) | Condensing Gas Boiler (S2) | Electricity Boiler (S3) |
|---|---|---|---|
| Efficiency | 82% | 94% | 99% |
| $K_{OH}$ (PLN/Wh) | $0.162 \times 10^{-3}$ | $0.245 \times 10^{-3}$ | $0.707 \times 10^{-3}$ |
| $K_{EH}$ (Pt/Wh) | $0.124 \times 10^{-3}$ | $0.027 \times 10^{-3}$ | $0.107 \times 10^{-3}$ |

One type of air conditioning was considered (see Table 4). A seasonal energy efficiency ratio (SEER) for air conditioning was assumed in the range for the energy efficiency class A ($5.10 \leq \text{SEER} \leq 5.60$, in accordance with the ELD 2010/30/EU Directive).

**Table 4.** Data for cooling.

| Coolness Source | Air Conditioner (SC) |
|---|---|
| Seasonal energy efficiency ratio (SEER) | 5.30 |
| $K_{OC}$ (PLN/Wh) | $0.132 \times 10^{-3}$ |
| $K_{EC}$ (Pt/Wh) | $0.020 \times 10^{-3}$ |

Three different thermal insulation materials were included in the research. A summary of the necessary data is presented in Table 5. These materials differ slightly in thermal conductivity (from 0.032 for I3 to 0.040 for I2) but also in economic and ecological costs. Both the differences in economic costs (from 205.00 PLN/m$^3$ for I2 to 420.00 PLN/m$^3$ for I3) and ecological costs (from 6.77 Pt/m$^3$ for I2 to 31.90 Pt/m$^3$ for I3) are very large.

**Table 5.** Thermal insulation materials.

| Thermal Insulation Mat. | Mineral Wool (I1) | Polystyrene EPS (I2) | Polystyrene XPS (I3) |
|---|---|---|---|
| Density $\rho$ (kg/m$^3$) | 90.0 | 12.5 | 40.0 |
| Thermal conductivity $\lambda$ (W/mK) | 0.038 | 0.040 | 0.032 |
| $K_m$ (PLN/m$^3$) | 233.00 | 205.00 | 420.00 |
| $K_w$ (PLN/m$^2$) | 45.00 | 40.00 | 40.00 |
| $K_l$ (Pt/m$^3$) | 19.10 | 6.77 | 31.90 |

### *4.2. Optimum Thickness of Thermal Insulation for Economic Reasons*

Using the methodology presented in Section 3.1, the optimum values of the $d_{opt}$ insulation thicknesses were determined (see Formula (4)) for economic reasons. Table 6 shows the results for the two selected regions: the region with the lowest number of *HDD* degree days (II) and the region with the highest number of *HDD* degree days (IV). The same source of cooling was assumed in all variants (see Table 4). As seen, the results vary widely, depending on all four factors taken into account. The difference in optimum thickness between Regions II and IV can be up to several cm. Due to the heat source, the greatest thicknesses were obtained for the S3 variant due to the highest heating costs for the electric boiler (see Table 3). Taking into account thermal insulation materials, polystyrene EPS (I2) is the cheapest of the considered thermal insulation materials, hence the highest optimum thicknesses are for this material. Due to the construction material of the wall, the greatest optimum thickness is obtained for the C2 wall, which has the worst (largest) heat transfer coefficient $U_0$.

**Table 6.** Thermal insulation optimum thicknesses for economic reasons $d_{opt}$ (m).

| Constr. Mat. | Region | Heat Source and Thermal Insulation Mat. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **S1-I1** | **S1-I2** | **S1-I3** | **S2-I1** | **S2-I2** | **S2-I3** | **S3-I1** | **S3-I2** | **S3-I3** |
| C1 | II | 0.122 | 0.136 | 0.074 | 0.165 | 0.183 | 0.102 | 0.323 | 0.356 | 0.211 |
| | IV | 0.141 | 0.156 | 0.086 | 0.187 | 0.208 | 0.118 | 0.362 | 0.399 | 0.238 |
| C2 | II | 0.163 | 0.179 | 0.107 | 0.205 | 0.225 | 0.136 | 0.364 | 0.399 | 0.245 |
| | IV | 0.181 | 0.199 | 0.120 | 0.228 | 0.250 | 0.152 | 0.403 | 0.441 | 0.272 |
| C3 | II | 0.141 | 0.157 | 0.090 | 0.184 | 0.203 | 0.119 | 0.342 | 0.376 | 0.227 |
| | IV | 0.160 | 0.177 | 0.102 | 0.207 | 0.228 | 0.134 | 0.382 | 0.419 | 0.254 |

Additionally, the $U_{opt}$ heat transfer coefficients (from Formula (5)) corresponding to the optimum insulation thicknesses for economic reasons (see Table 7) were determined. It can be noticed that the $U_{opt}$ coefficient value does not depend on the construction material of the wall. For instance, for Region II and Variants C1-S1-I1, C2-S1-I1 and C3-S1-I1, it has the same value ($U_{opt}$ = 0.205 W/m$^2$K), of course obtained with other optimum thicknesses of thermal insulation $d_{opt}$.

**Table 7.** Optimum heat transfer coefficients of the wall with a thermal insulation layer $U_{opt}$ (W/m$^2$K).

| Constr. Mat. | Region | Heat Source and Thermal Insulation Mat. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **S1-I1** | **S1-I2** | **S1-I3** | **S2-I1** | **S2-I2** | **S2-I3** | **S3-I1** | **S3-I2** | **S3-I3** |
| C1 | II | 0.205 | 0.197 | 0.252 | 0.167 | 0.160 | 0.205 | 0.098 | 0.095 | 0.121 |
| | IV | 0.186 | 0.179 | 0.230 | 0.152 | 0.146 | 0.187 | 0.089 | 0.086 | 0.110 |
| C2 | II | 0.205 | 0.197 | 0.252 | 0.167 | 0.160 | 0.205 | 0.098 | 0.095 | 0.121 |
| | IV | 0.186 | 0.179 | 0.230 | 0.152 | 0.146 | 0.187 | 0.089 | 0.086 | 0.110 |
| C3 | II | 0.205 | 0.197 | 0.252 | 0.167 | 0.160 | 0.205 | 0.098 | 0.095 | 0.121 |
| | IV | 0.186 | 0.179 | 0.230 | 0.152 | 0.146 | 0.187 | 0.089 | 0.086 | 0.110 |

### *4.3. Optimum Thickness of Thermal Insulation for Ecological Reasons*

Based on Section 3.2, the optimum thickness $d_{Eopt}$ of the thermal insulation for ecological reasons was determined from Formula (9). The results are summarized in Table 8, as for the economic analysis, for two selected regions (II and IV). In the ecological analysis, the thicknesses significantly depend on the region in which the building is located. It can be observed that for each variant, the optimum thickness for ecological reasons is much greater than the optimum thickness for economic reasons (see Tables 6 and 8). Due to the heat source, the greatest thicknesses were obtained for the S1 variant due to the highest ecological heating costs with the use of a coal-fired boiler (see Table 3). Taking into account thermal insulation materials, the environmental costs $K_I$ are the lowest for EPS (I2) polystyrene (see Table 5), hence the highest optimum thicknesses are for this material. As in the case of the economic analysis, the highest optimum thickness is obtained for the C2 wall, which has the worst (highest) heat transfer coefficient $U_0$.

**Table 8.** Thermal insulation optimum thicknesses for ecological reasons $d_{Eopt}$ (m).

| Constr. Mat. | Region | Heat Source and Thermal Insulation Mat. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **S1-I1** | **S1-I2** | **S1-I3** | **S2-I1** | **S2-I2** | **S2-I3** | **S3-I1** | **S3-I2** | **S3-I3** |
| C1 | II | 0.612 | 1.097 | 0.426 | 0.253 | 0.478 | 0.171 | 0.564 | 1.014 | 0.392 |
| | IV | 0.680 | 1.215 | 0.475 | 0.284 | 0.532 | 0.193 | 0.628 | 1.124 | 0.437 |
| C2 | II | 0.652 | 1.139 | 0.460 | 0.293 | 0.520 | 0.205 | 0.604 | 1.057 | 0.426 |
| | IV | 0.721 | 1.257 | 0.509 | 0.324 | 0.574 | 0.227 | 0.668 | 1.166 | 0.471 |
| C3 | II | 0.631 | 1.117 | 0.442 | 0.272 | 0.498 | 0.187 | 0.583 | 1.034 | 0.408 |
| | IV | 0.699 | 1.235 | 0.491 | 0.303 | 0.552 | 0.209 | 0.647 | 1.144 | 0.453 |

The heat transfer coefficients $U_{Eopt}$ (from Formula (10)) were also determined, corresponding to the optimum thicknesses of thermal insulation for ecological reasons (see Table 9). As in the case of the economic analysis, the value of the $U_{Eopt}$ coefficient does not depend on the construction material of the wall.

**Table 9.** Optimum heat transfer coefficients of the wall with a thermal insulation layer $U_{Eopt}$ (W/m$^2$K).

| Constr. Mat. | Region | Heat Source and Thermal Insulation Mat. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **S1-I1** | **S1-I2** | **S1-I3** | **S2-I1** | **S2-I2** | **S2-I3** | **S3-I1** | **S3-I2** | **S3-I3** |
| C1 | II | 0.056 | 0.034 | 0.067 | 0.120 | 0.073 | 0.143 | 0.061 | 0.037 | 0.072 |
| | IV | 0.051 | 0.031 | 0.061 | 0.109 | 0.067 | 0.130 | 0.055 | 0.034 | 0.065 |
| C2 | II | 0.056 | 0.034 | 0.067 | 0.120 | 0.073 | 0.143 | 0.061 | 0.037 | 0.072 |
| | IV | 0.051 | 0.031 | 0.061 | 0.109 | 0.067 | 0.130 | 0.055 | 0.034 | 0.065 |
| C3 | II | 0.056 | 0.034 | 0.067 | 0.120 | 0.073 | 0.143 | 0.061 | 0.037 | 0.072 |
| | IV | 0.051 | 0.031 | 0.061 | 0.109 | 0.067 | 0.130 | 0.055 | 0.034 | 0.065 |

In Poland, since 2021, the maximum value for the heat transfer coefficient of a vertical exterior wall has been $U_N$ = 0.20 W/m$^2$K (see [40]). In the variants considered in the article, $U_{opt} \leq U_N$ was not obtained in every case (see Table 7, Variant S1-I3). However, taking into account ecological considerations, in each case, $U_{Eopt} \leq U_N$ (see Table 9). Therefore, a method of determining the optimum thickness of the thermal insulation $d_{MKopt}$ was proposed (see Section 3.3), including both economic and ecological costs. Consequently, the optimum thickness of the insulation $d_{MKopt}$ is between $d_{opt}$ and $d_{Eopt}$. The same applies to the heat transfer coefficient $U_{Eopt} \leq U_{MKopt} \leq U_{opt}$.

### *4.4. Optimum Thickness of Thermal Insulation Taking into Account Costs Both Economically and Ecologically*

Figure 2 shows an example situation for the P1-S2-I3-II variant and Figure 3 for the P2-S3-I1-II variant. Additional markings were introduced:

$$NPV^{\sim}(d) = NPV(d)/NPV_{max} \text{ and } NPV_E^{\sim}(d) = NPV_E(d)/NPV_{max}.$$

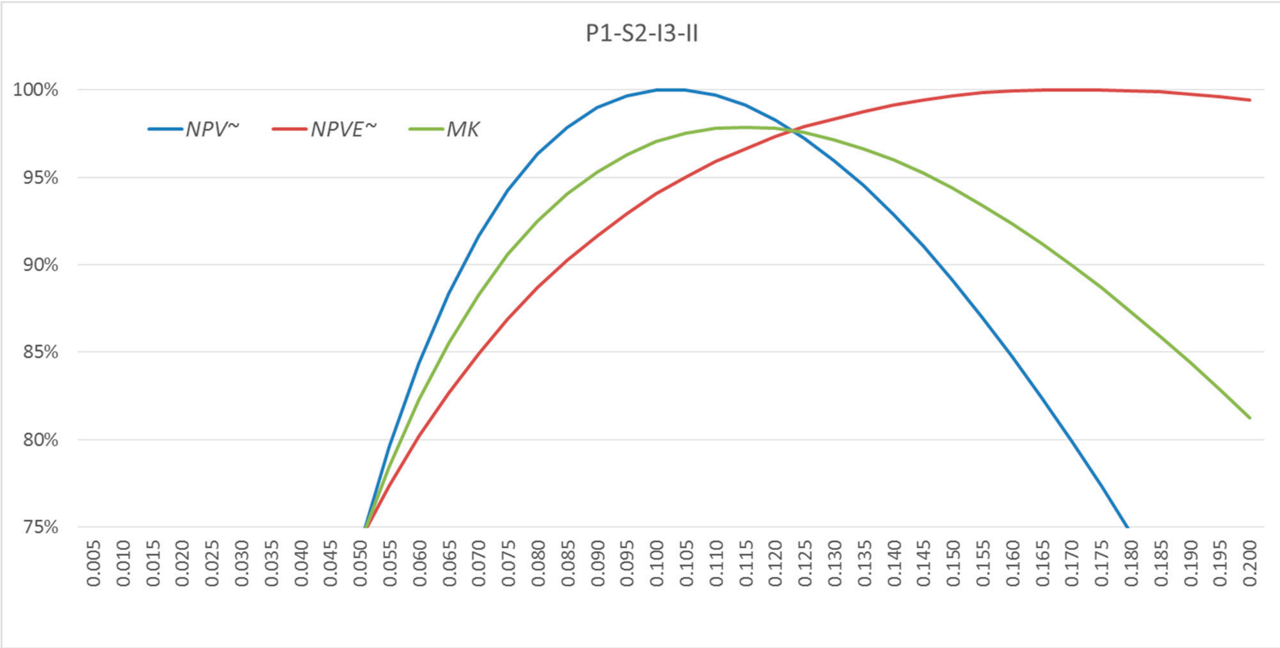


**Figure 2.** $NPV^{\sim}$, $NPV_E{}^{\sim}$ and $MK$ values depending on the thickness $d$ for Variant P1-S2-I3-II.

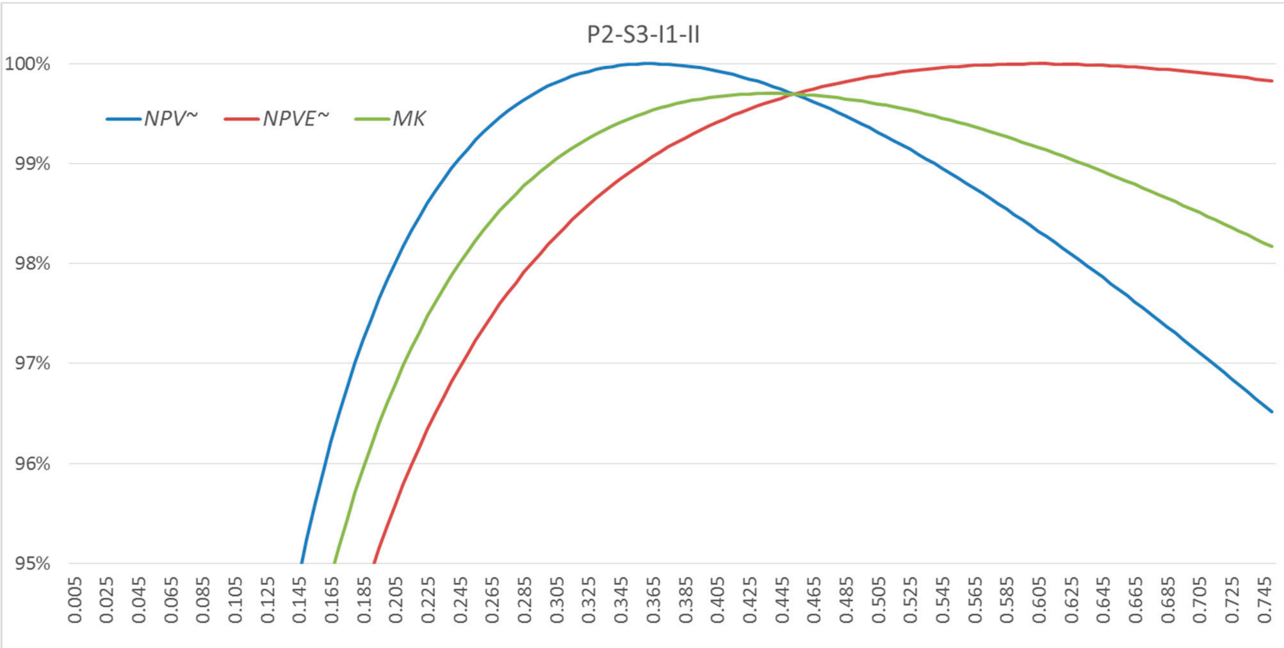


**Figure 3.** $NPV^{\sim}$, $NPV_E{}^{\sim}$ and $MK$ values depending on the thickness $d$ for Variant P2-S3-I1-II.

The metacriterion $MK$ was determined according to Formula (12), with the weights assumed $w_1 = w_2 = \frac{1}{2}$ (it was assumed that both criteria have the same weight). Both $NPV^{\sim}(d)$, as well as $NPV_E{}^{\sim}(d)$, take values $\leq 1$, with $NPV^{\sim}(d) = 1$ (100%) for $d = d_{opt}$, a $NPV_E{}^{\sim}(d) = 1$ (100%) for $d = d_{Eopt}$. The metacriterion $MK$ takes the maximum value for the thickness $d = d_{MKopt}$ between $d_{opt}$ and $d_{Eopt}$. Because the graph of $NPV_E{}^{\sim}(d)$ is flatter around the maximum than the graph of $NPV^{\sim}(d)$, the optimum thickness of $d_{MKopt}$ is however much closer to the $d_{opt}$ than $d_{Eopt}$. The situation is similar for all other variants.

Table 10 summarizes the determined $d_{MKopt}$ on the basis of Formula (13) for the considered variants. Let us consider that the optimum thicknesses due to the metacriterion $MK$ are already feasible. For Variants S1-I2 and S3-I2, the optimum $d_{Eopt}$ thicknesses for ecological reasons were greater than 1 m (see Table 8). The optimum thicknesses for the *MK* for these variants are already much smaller.

**Table 10.** Optimum insulation thicknesses $d_{MKopt}$ (m).

| Constr. Mat. | Region | Heat Source and Thermal Insulation Mat. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **S1-I1** | **S1-I2** | **S1-I3** | **S2-I1** | **S2-I2** | **S2-I3** | **S3-I1** | **S3-I2** | **S3-I3** |
| C1 | II | 0.130 | 0.151 | 0.075 | 0.184 | 0.219 | 0.115 | 0.389 | 0.460 | 0.257 |
| | IV | 0.160 | 0.182 | 0.095 | 0.212 | 0.253 | 0.135 | 0.437 | 0.518 | 0.290 |
| C2 | II | 0.212 | 0.237 | 0.140 | 0.235 | 0.286 | 0.159 | 0.438 | 0.522 | 0.298 |
| | IV | 0.238 | 0.266 | 0.158 | 0.261 | 0.318 | 0.177 | 0.485 | 0.579 | 0.331 |
| C3 | II | 0.171 | 0.193 | 0.107 | 0.209 | 0.252 | 0.137 | 0.412 | 0.490 | 0.277 |
| | IV | 0.198 | 0.223 | 0.126 | 0.236 | 0.285 | 0.156 | 0.460 | 0.547 | 0.310 |

Table 11 presents $U_{MKopt}$ determined on the basis of Formula (14) for the considered variants. Due to the fact that the thicknesses $d_{MKopt} \geq d_{opt}$, the obtained heat transfer coefficients $U_{MKopt} \leq U_{opt}$. Only in a few cases (C1-S1-I3 and C3-S1-I3) was $U_{MKopt} \geq U_N$ = 0.20 W/m$^2$K not meeting the requirements.

**Table 11.** Optimum heat transfer coefficient $U_{MKopt}$ (W/m$^2$K).

| Constr. Mat. | Region | Heat Source and Thermal Insulation Mat. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **S1-I1** | **S1-I2** | **S1-I3** | **S2-I1** | **S2-I2** | **S2-I3** | **S3-I1** | **S3-I2** | **S3-I3** |
| C1 | II | 0.196 | 0.184 | 0.249 | 0.154 | 0.140 | 0.190 | 0.084 | 0.076 | 0.103 |
| | IV | 0.170 | 0.161 | 0.215 | 0.138 | 0.125 | 0.170 | 0.076 | 0.068 | 0.093 |
| C2 | II | 0.162 | 0.153 | 0.201 | 0.147 | 0.129 | 0.180 | 0.082 | 0.073 | 0.101 |
| | IV | 0.146 | 0.138 | 0.180 | 0.134 | 0.117 | 0.163 | 0.075 | 0.066 | 0.091 |
| C3 | II | 0.177 | 0.167 | 0.221 | 0.150 | 0.134 | 0.184 | 0.083 | 0.075 | 0.102 |
| | IV | 0.157 | 0.148 | 0.196 | 0.136 | 0.121 | 0.166 | 0.075 | 0.067 | 0.092 |

## 5. Discussion

Taking into consideration the results obtained in the previous paragraph, the values of *NPV* and $NPV_E$ for a few selected variants were compiled for the optimum thickness $d_{MKopt}$ with regard to the *MK* metacriterion for the considered thermal insulation materials. On this basis, it is possible to check which thermal insulation material will bring the greatest economic benefits and which ecological. Table 12 presents a summary for Variant P1-S2-II and P1-S2-IV, Table 13 for Variant P3-S1-II and P3-S1-IV and Table 14 for Variant P2-S3-II and P2-S3-IV. For each variant, the highest value of *NPV* and $NPV_E$ in terms of thermal insulation material is marked in bold. Let us observe that in each case the highest *NPV* and $NPV_E$ values are obtained for the thermal insulation material I2 (polystyrene EPS). This material has the highest value of the λ coefficient but the lowest economic and ecological costs (see Table 5). For this material, the lowest value of the heat transfer coefficient of the wall with insulation $U_{MKopt}$ is obtained with the optimum thickness of the insulation $d_{MKopt}$. The situation is similar for other variants. Moreover, in each case, the level of satisfaction measured by the *MK* function for the optimum thickness $d_{MKopt}$ is over 90%.

**Table 12.** Summary of results for Variant P1-S2-II I P1-S2-IV.

| Variant | Insulation Mat. | $d_{MKopt}$ | $U_{MKopt}$ | $NPV$ ($d_{MKopt}$) | $NPV_E$ ($d_{MKopt}$) | $MK$ ($d_{MKopt}$) |
|---|---|---|---|---|---|---|
| P1-S2-II | I1 | 0.184 | 0.154 | 54.34 | 18.88 | 98.7% |
| | I2 | 0.220 | 0.140 | **61.60** | **21.62** | 95.8% |
| | I3 | 0.115 | 0.190 | 42.29 | 16.90 | 97.8% |
| P1-S2-IV | I1 | 0.212 | 0.138 | 83.73 | 23.97 | 99.0% |
| | I2 | 0.253 | 0.125 | **91.23** | **27.09** | 96.4% |
| | I3 | 0.135 | 0.170 | 69.13 | 21.78 | 98.3% |

**Table 13.** Summary of results for Variant P3-S1-II I P3-S1-IV.

| Variant | Insulation Mat. | $d_{MKopt}$ | $U_{MKopt}$ | $NPV$ ($d_{MKopt}$) | $NPV_E$ ($d_{MKopt}$) | $MK$ ($d_{MKopt}$) |
|---|---|---|---|---|---|---|
| P3-S1-II | I1 | 0.171 | 0.177 | 59.52 | 153.32 | 93.8% |
| | I2 | 0.193 | 0.167 | **66.82** | **157.48** | 92.5% |
| | I3 | 0.107 | 0.222 | 49.85 | 142.80 | 91.7% |
| P3-S1-IV | I1 | 0.198 | 0.157 | 88.28 | 191.66 | 94.5% |
| | I2 | 0.223 | 0.148 | **95.87** | **196.29** | 93.4% |
| | I3 | 0.126 | 0.196 | 76.50 | 180.53 | 92.8% |

**Table 14.** Summary of results for Variant P2-S3-II I P2-S3-IV.

| Variant | Insulation Mat. | $d_{MKopt}$ | $U_{MKopt}$ | $NPV$ ($d_{MKopt}$) | $NPV_E$ ($d_{MKopt}$) | $MK$ ($d_{MKopt}$) |
|---|---|---|---|---|---|---|
| P2-S3-II | I1 | 0.438 | 0.082 | 1289.13 | 301.65 | 99.7% |
| | I2 | 0.522 | 0.073 | **1297.60** | **308.30** | 99.2% |
| | I3 | 0.298 | 0.101 | 1254.17 | 296.87 | 99.6% |
| P2-S3-IV | I1 | 0.485 | 0.075 | 1592.48 | 368.60 | 99.7% |
| | I2 | 0.579 | 0.066 | **1601.23** | **375.98** | 99.3% |
| | I3 | 0.331 | 0.091 | 1553.10 | 363.34 | 99.6% |

The highest values of $NPV(d_{MKopt})$ and $NPV_E(d_{MKopt})$ occur, obviously, in the colder climatic zone Zone IV due to the greater number of heating degree days (see Tables 12–14). According to the earlier observations that the values of the $U_{opt}$ and $U_{Eopt}$ coefficients do not depend on the construction materials of vertical walls, the type of heat source is the most important for the $NPV(d_{MKopt})$ and $NPV_E(d_{MKopt})$ values. In Poland, the source of heat, an electricity boiler (S3), is one of the most expensive solutions when it comes to generating thermal energy. The cost of generating thermal energy (1 kWh) in an electricity boiler (S3) is almost three times higher than in a natural gas boiler (S2) and over four times higher than in a coal boiler (S1). Therefore, for (S3), the optimum $d_{MKopt}$ insulation thickness values due to the metacriterion are the highest.

According to the authors' knowledge, the method proposed in the article for determining the optimum thickness of thermal insulation is the only one that allows taking into account both economic and ecological aspects and includes both heating and cooling costs. On the other hand, the proposed model is a kind of generalization of earlier models from the literature. It is so universal that if, due to climatic aspects, a cooling system is not used in the building, it is enough to assume $CDD = 0$ in the model, and if there is no suitable heating system, it is enough to take $HDD = 0$.

## 6. Conclusions

The energy efficiency of existing buildings is an area that scientists have been focusing on for many years. One of the methods that can have a significant impact on the maximization of economic and ecological profits is to determine the optimum thickness of insulation taking into account these two aspects. The method proposed in the article allows determining the optimum solution, which is a good compromise between economically optimum and ecologically optimum.

The economically and ecologically optimum values of insulation thickness determined in the above article are not constant values due to high fluctuations in the prices of energy carriers, prices of thermal insulation materials and construction works. With regard to ecological aspects, it is necessary to understand the changing environmental impact of the methods of obtaining thermal energy, in particular of obtaining electricity, as well as the production methods and types of applied insulation materials. The study clearly shows that these are not the only variables determining the optimum values of insulation thickness. The variability of climatic conditions also significantly influences the optimum thickness of the insulation, but the differences are relatively small. The method proposed in the article takes into account both the costs of heating and cooling the building. In Poland, heating is much more important than cooling a building.

It is also worth noticing that the article introduces consciously the variability of climatic conditions in order to visualize the universality of the application of the proposed methodology.

It is obvious that in the economic analysis, the costs of thermal insulation materials and thermal energy have a decisive impact on the optimum thickness of thermal insulation. A responsible policy of decision makers in the use of economic instruments of the ecological policy can significantly increase the economic benefits resulting from thermal modernization. In Poland, in the field of thermal modernization, the following are used: direct payments to replace the boiler with an "ecological" one, reimbursement of part of the costs in the thermal modernization investment in the form of a personal tax refund and subsidies for photovoltaic installations. This part of the costs, as mentioned above, is influenced by decision makers and, of course, by market rules. However, the ecological benefits or losses in the above analysis are the result of the raw materials used for the production of thermal insulation materials and the type of fuels used for the production of thermal energy. Minimizing the impact on the environment is also on the part of decision makers by using economic instruments encouraging users to replace heat sources with more "ecological" ones. Other groups responsible for reducing the environmental impact of the construction sector are designers, manufacturers and innovators who have the potential to develop new "green" thermal insulation materials or design with lower environmental impact insulation materials.

In the European Union countries, newly built buildings, due to legal regulations, should be almost zero energy. Buildings already built in the years when the normative values of heat transfer coefficients were high are a problem. Currently, these buildings require high economic and ecological outlays, which, as a consequence, can effectively influence the minimization of thermal energy consumption. It should also be noticed that the life cycle of a building varies from several dozen to even several hundred years. In the above article, the shorter 25-year period of thermal insulation use was analyzed, where, nevertheless, relatively high values of $NPV(d_{MKopt})$ economic benefits and $NPV_E(d_{MKopt})$ ecological benefits were obtained.

The proposed methodology for determining the optimum thicknesses of thermal insulation is dedicated to designers, users and building managers. In further research, it is planned to include a social aspect to determine the optimum thicknesses of thermal insulation with the use of a metacriterion.

**Author Contributions:** Conceptualization, R.D. and J.A.; methodology, R.D.; software, R.D. and J.A.; validation and formal analysis, R.D. and J.A.; investigation, R.D. and J.A; resources and data curation, J.A. and R.D.; writing—original draft preparation, review and editing, R.D. and J.A.; visualization, R.D.; supervision, R.D. and J.A. Both authors have read and agreed to the published version of the manuscript.

**Funding:** This research received no external funding.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Data is contained within the article.

**Conflicts of Interest:** The authors declare no conflict of interest.

## Appendix A

The $MK$ metacriterion depending on the thickness $d$ was defined in (12) as follows:

$$MK\ (d) = w_1 \cdot NPV\ (d)/NPV_{max} + w_2 \cdot NPV_E\ (d)/NPV_{Emax}.$$

Substituting into the formula $NPV\ (d)$ according to (3) and $NPV_E\ (d)$ according to (8) we obtain:

$$MK\ (d) = (w_1/NPV_{max}) \cdot (-\ (K_m \cdot d + K_w) + S_N \cdot (G_{OH} + G_{OC}) \cdot (U_0 - \lambda/(d + \lambda/U_0))) +$$

$$+\ (w_2/NPV_{Emax}) \cdot (-\ (K_I \cdot d) + N \cdot (G_{EH} + G_{EC}) \cdot (U_0 - \lambda/(d + \lambda/U_0))) =$$

$$= -\ ((w_1/NPV_{max}) \cdot K_m + (w_2/NPV_{Emax}) \cdot K_I) \cdot d - (w_1/NPV_{max}) \cdot K_w +$$

$$+\ ((w_1/NPV_{max}) \cdot S_N \cdot (G_{OH} + G_{OC}) + (w_2/NPV_{Emax}) \cdot N \cdot (G_{EH} + G_{EC})) \cdot (U_0 - \lambda/(d + \lambda/U_0)).$$

Accepting

$$c_E = (w_1/NPV_{max}) \cdot S_N \cdot (G_{OH} + G_{OC}) + (w_2/NPV_{Emax}) \cdot N \cdot (G_{EH} + G_{EC}),$$

$$c_T = (w_1/NPV_{max}) \cdot K_m + (w_2/NPV_{Emax}) \cdot K_I,$$

we obtain:

$$MK\ (d) = -c_T \cdot d - (w_1/NPV_{max}) \cdot K_w + c_E \cdot (U_0 - \lambda/(d + \lambda/U_0)).$$

Thus, the function $MK$ with regard to $d$ is a strictly concave function and bounded from above. It reaches its maximum for thickness $d_{MKopt}$, at which the derivative $MK$ with respect to $d$ is equal to 0, i.e.,

$$MK'\ (d) = -c_T + c_E \cdot \lambda/(d + \lambda/U_0)^{\ 2} = 0.$$

Hence, we obtain:

$$d_{MKopt} = \text{sqrt}\ (\lambda \cdot c_E/c_T) - \lambda/U_0.$$

## References

1. Energy Consumption in 2018, Primary and Final Energy Consumption Still 5% and 3% away from 2020 Targets. Available online: https://ec.europa.eu/eurostat/documents/2995521/10341545/8-04022020-BP-EN.pdf/39dcc365-bdaa-e6f6-046d-1b4d241392ad (accessed on 23 February 2021).
2. Beretta, G.P. World energy consumption and resources: An outlook for the rest of the century. *Int. J. Environ. Technol. Manag.* **2007**, *7*, 99–112. [CrossRef]
3. Good Practice in Energy Efficiency. For a Sustainable, Safer and More Competitive Europe. Available online: https://ec.europa.eu/energy/sites/ener/files/documents/good_practice_in_ee_-web.pdf (accessed on 23 February 2021).
4. Directive 2006/32/EC of the European Parliament and of the Council of 5 April 2006 on Energy End-Use Efficiency and Energy Services and Repealing Council Directive 93/76/EEC. OJ L 114, 27.4.2006. pp. 64–85. Available online: https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32006L0032&from=EN (accessed on 23 February 2021).
5. Tsemekidi Tzeiranaki, S.; Bertoldi, P.; Diluiso, F.; Castellazzi, L.; Economidou, M.; Labanca, N.; Ribeiro Serrenho, T.; Zangheri, P. Analysis of the EU Residential Energy Consumption: Trends and Determinants. *Energies* **2019**, *12*, 1065. [CrossRef]

6. Directive 2012/27/EU of the European Parliament and of the Council of 25 October 2012 on Energy Efficiency, Amending Directives 2009/125/EC and 2010/30/EU and Repealing Directives 2004/8/EC and 2006/32/EC Text with EEA Relevance. Available online: https://eur-lex.europa.eu/legal-content/EN/TXT/?qid=1399375464230&uri=CELEX:32012L0027 (accessed on 23 February 2021).
7. Directive (EU) 2018/2002 of the European Parliament and of the Council of 11 December 2018 Amending Directive 2012/27/EU on Energy Efficiency. Available online: https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=uriserv%3AOJ.L_.2018.328.01.0210.01.ENG (accessed on 23 February 2021).
8. Energy Statistics—An Overview. Available online: https://ec.europa.eu/eurostat/statistics-explained/index.php/Energy_statistics_-_an_overview#Final_energy_consumption (accessed on 23 February 2021).
9. Directive 2002/91/EC of the European Parliament and of the Council of 16 December 2002 on the Energy Performance of Buildings. Available online: https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32010L0031&from=PL (accessed on 3 March 2021).
10. Fabbri, K.; Tronchin, L.; Tarabusi, V. Real Estate market, energy rating and cost. Reflections about an Italian case study. International Conference on Green Buildings and Sustainable Cities. *Procedia Eng.* **2011**, *21*, 303–310. [CrossRef]
11. Tronchin, L.; Fabbri, K. Energy Performance Certificate of building and confidence interval in assessment: An Italian case study. *Energy Policy* **2012**, *48*, 176–184. [CrossRef]
12. Directive 2010/31/EU of the European Parliament and of the Council of 19 May 2010 on the Energy Performance of Buildings. Available online: https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=celex%3A32010L0031 (accessed on 3 March 2021).
13. Dascalaki, E.G.; Balaras, C.A.; Gaglia, A.G.; Droutsa, K.G.; Kontoyiannidis, S. Energy performance of buildings—EPBD in Greece. *Energy Policy* **2012**, *45*, 469–477. [CrossRef]
14. Gaitani, N.; Casesb, L.; Mastrapostolia, E.; Eliopoulou, E. Paving the way to nearly zero energy schools in Mediterranean region-ZEMedS project. 6th International Building Physics Conference. *Energy Procedia* **2015**, *78*, 3348–3353. [CrossRef]
15. Directive (EU) 2018/844 of the European Parliament and of the Council of 30 May 2018 Amending Directive 2010/31/EU on the Energy Performance of Buildings and Directive 2012/27/EU on Energy Efficiency. Available online: https://eur-lex.europa.eu/legal-content/EN/TXT/HTML/?uri=CELEX:32018L0844&from=pl (accessed on 3 March 2021).
16. Nematchoua, M.K.; Raminosoa, C.R.R.; Mamiharijaona, R.; René, T.; Orosa, J.A.; Elvis, W.; Meukam, P. Study of the economical and optimum thermal insulation thickness for buildings in a wet and hot tropical climate: Case of Cameroon. *Renew. Sustain. Energy Rev.* **2015**, *50*, 1192–1202. [CrossRef]
17. Nematchouaa, M.K.; Ricciardia, P.; Reiterb, S.; Yvon, A. A comparative study on optimum insulation thickness of walls and energy savings in equatorial and tropical climate. *Int. J. Sustain. Built Environ.* **2017**, *6*, 170–182. [CrossRef]
18. De' Rossi, F.; Marigliano, M.; Marino, C.; Minichiello, F. A technical and economic analysis on optimal thermal insulation thickness for existing office building in Mediterranean climates. *Int. J. Heat Technol.* **2016**, *34*, S561–S568. [CrossRef]
19. Sahu, D.K.; Sen, P.K.; Sahu, G.; Sharma, R.; Bohidar, S. A Review on Thermal Insulation and Its Optimum Thickness to Reduce Heat Loss. *Int. J. Innov. Res. Sci. Technol.* **2015**, *2*, 1–6. Available online: http://www.ijirst.org/articles/IJIRSTV2I6001.pdf (accessed on 23 February 2021).
20. Aktemur, C.; Atikol, U. Optimum Insulation Thickness for the Exterior Walls of Buildings in Turkey Based on Different Materials, Energy Sources and Climate Regions. *Int. J. Eng. Technol.* **2017**, *3*, 72–82. [CrossRef]
21. Orzechowski, T.; Orzechowski, M. Optimal thickness of various insulation materials for different temperature conditions and heat sources in terms of economic aspect. *J. Build. Phys.* **2018**, *41*, 377–393. [CrossRef]
22. Dylewski, R. Optimal Thermal Insulation Thicknesses of External Walls Based on Economic and Ecological Heating Cost. *Energies* **2019**, *12*, 3415. [CrossRef]
23. Çetintaş, K.F.; Yilmaz, Z. Optimization of thermal insulation material and thickness for building energy efficiency in Mediterranean climates based on life cycle perspective. *ITU A/Z* **2017**, *14*, 99–112. [CrossRef]
24. Yildiz, A.; Gürlek, G.; Erkek, M.; Özbalta, N. Economical and environmental analyses of thermal insulation thickness in buildings. *J. Therm. Sci. Technol.* **2008**, *28*, 25–34.
25. Barrau, J.; Ibanez, M.; Badia, F. Impact of the insulation materials' features on the determination of optimum insulation thickness. *Int. J. Energy Environ. Eng.* **2014**, *5*, 79. [CrossRef]
26. Kon, O. Determination of optimum insulation thicknesses using economical analyse for exterior walls of buildings with different masses. *Int. J. Optim. Control Theor. Appl.* **2017**, *7*, 149–157. [CrossRef]
27. Hernandez, P.; Kenny, P. From net energy to zero energy buildings: Defining life cycle zero energy buildings (LC-ZEB). *Energy Build.* **2010**, *42*, 815–821. [CrossRef]
28. Hernandez, P.; Kenny, P. Development of a methodology for life cycle building energy ratings. *Energy Policy* **2011**, *39*, 3779–3788. [CrossRef]
29. Dylewski, R.; Adamczyk, J. Impact of the Degree Days of the Heating Period on Economically and Ecologically Optimal Thermal Insulation Thickness. *Energies* **2021**, *14*, 97. [CrossRef]
30. Brás, A.; Gomes, V. LCA implementation in the selection of thermal enhanced mortars for energetic rehabilitation of school buildings. *Energy Build.* **2015**, *92*, 1–9. [CrossRef]

31. Dylewski, R.; Adamczyk, J. Life cycle assessment (LCA) of building thermal insulation materials. In *Eco-Efficient Construction and Building Materials: LCA, Eco-Labelling and Case Studies*; Pacheco-Torgal, F., Cabeza, L.F., Labrincha, J., de Magalhaes, A., Eds.; WoodHead Publishing Ltd.: Cambridge, UK, 2014; pp. 267–286.
32. Dzikuć, M. Environmental management with the use of LCA in the Polish energy system. *Management* **2015**, *19*, 89–97. [CrossRef]
33. Dzikuć, M.; Dzikuć, M. The analysis of opportunities to reduce low emissions in the Middle Odra using LCA techniques. In *Społeczne i Ekologiczne Pogranicza: Monografia*; Wawrzyniak, W., Zaborowski, T., Eds.; Instytut Badań i Ekspertyz Naukowych, Politechnika Poznańska: Poznań, Poland, 2018; pp. 150–162.
34. *ISO 14040 Environmental Management-Life Cycle Assessment-Principles and Framework*; European Committee for Standardisation: Brussels, Belgium, 2006.
35. *ISO 14044 Environmental Management-Life Cycle Assessment-Requirements and Guidelines*; European Committee for Standardisation: Brussels, Belgium, 2006.
36. Dekker, E.; Zijp, M.C.; van de Kamp, M.E.; Temme, E.H.M.; van Zelm, R. A taste of the new ReCiPe for life cycle assessment: Consequences of the updated impact assessment method on food product LCAs. *Int. J. Life Cycle Assess* **2020**, *25*, 2315–2324. [CrossRef]
37. Wolfova, M.; Estokova, A.; Ondova, M.; Monokova, A. Comparing of the external bearing wall using three cultural perspectives in the life cycle impact assessment. *IOP Conf. Ser. Mater. Sci. Eng.* **2018**, *385*, 012064. [CrossRef]
38. Strefy Klimatyczne Polski. Available online: http://www.nieruchomosci.egospodarka.pl/art/galeria/55914,Kolektor-sloneczny-na-dachu-budynku,3,79,1.html (accessed on 3 March 2021).
39. Eurostat. Available online: http://appsso.eurostat.ec.europa.eu/nui/show.do?dataset=nrg_chdd_a&lang=en (accessed on 15 March 2021).
40. Rozporządzenie ministra transportu, budownictwa i gospodarki morskiej z dnia 5 lipca 2013 r. zmieniające rozporządzenie w sprawie warunków technicznych, jakim powinny odpowiadać budynki i ich usytuowanie, Dz. U. 2013, nr 0, poz. 926. Regulation of the minister of transport, construction and maritime economy of 5 July 2013 amending the regulation on the technical conditions to be met by buildings and their location. *J. Laws* **2013**, *0*, 926. Available online: http://isap.sejm.gov.pl/isap.nsf/DocDetails.xsp?id=WDU20130000926 (accessed on 3 March 2021). (In Polish).