energies

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Finite Physical Dimensions Thermodynamics Analysis and Design of Closed Irreversible Cycles

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Abstract: This paper develops simplifying entropic models of irreversible closed cycles. The entropic models involve the irreversible connections between external and internal main operational parameters with finite physical dimensions. The external parameters are the mean temperatures of external heat reservoirs, the heat transfers thermal conductance, and the heat transfer mean log temperatures differences. The internal involved parameters are the reference entropy of the cycle and the internal irreversibility number. The cycle's design might use four possible operational constraints in order to find out the reference entropy. The internal irreversibility number allows the evaluation of the reversible heat output function of the reversible heat input. Thus the cycle entropy balance equation to design the trigeneration cycles only through external operational parameters might be involved. In designing trigeneration systems, they must know the requirements of all consumers of the useful energies delivered by the trigeneration system. The conclusions emphasize the complexity in designing and/or optimizing the irreversible trigeneration systems.

Keywords: closed irreversible cycles; number of internal irreversibility; reference entropy; operational constraints; irreversible energy efficiency; trigeneration

Citation: Dumitrașcu, G.; Feidt, M.; Grigorean, Ş. Finite Physical Dimensions Thermodynamics Analysis and Design of Closed Irreversible Cycles. *Energies* **2021**, *14*, 3416. https://doi.org/10.3390/en14123416

Academic Editor: Antonio Calvo Hernández

Received: 22 April 2021
Accepted: 2 June 2021
Published: 9 June 2021

1. Introduction

Since energy needs are rising continuously, energy systems remain as decisive research items. The design of these systems is focused on the energy client's requirements and it is following the optimized efficiency. This specific design ignores the energy systems connections, and requirements of different clients, and the global impact on the environment. The planning and management of complex energy systems might generate extra restrictive constraints, e.g., variable energy requirements. The national or international interconnected electricity grids safely ensure the variable energy needs of any customer. The variable heating and refrigeration—e.g., heating and conditioning systems—are designed using scenarios with variable operation shapes. The best energy solutions are obtained for the steady state operation. The non-steady state energy processes generates energy/exergy losses generated by irreversibility. Actually, specific studies are related to design, optimization, management, and planning of new applications. Reference [1] developed a very specific model with two complementary constraints, minimization of operating costs and reducing the carbon footprint for a local grid of customers with different needs of power, cooling and heating; [2] simulated the non-steady state operation of a trigeneration system uniting a gas turbine engine with a helium based reverse cycle, working both in the refrigeration mode and heat pump mode, coupled to a heat storage systems and to a cooling storage system delivering the needed heating and cooling; [3] assessed, on the basis of the levelized cost of electricity, the energy and exergy efficiency and the environmental effects of a biomass gasification based trigeneration system comprising a solid oxide fuel cell, a closed Brayton engine, an absorption refrigeration machine and a hot water boiler;

reference [4] compared the performances of two solar based organic Rankine cycles and using two kinds of solar concentrators, one parabolic through and the other one linear Fresnel reflectors, and emphasized the operational differences; reference [5] made an optimization of a combined storage of thermal energy and of electrical one for a net-zero-energy district, and considering optimal strategy of energy generators location and of energy distribution network; reference [6] performed a modeling of a biofueled small local trigeneration system delivering useful energies, cooling, heating, and power by means of specific storage system for cooling and heating and electrical power, and compared the convenient ratio heat/electricity for two engines, reciprocating and Stirling; reference [7] made an actual review regarding the solar based polygeneration systems—furnishing cooling, heating, power, fresh water, and hydrogen—and was comparing three types of solar concentrators, parabolic through, solar photovoltaic thermal, and solar tower; reference [8] analyzed a hybrid system including photovoltaic thermal operating as heat source or heat sink for a reversible heat pump, and having different sizes and storage tank volumes, and working in different climate conditions; reference [9] analyzed the integration of a high temperature heat pump inside a trigeneration system including an absorption chiller which provided the heat pump input heat from the chiller condenser; reference [10] presents a life cycle assessment for a natural gas based local small trigeneration system, and used environmental data to perform exergoenvironmental assessments; reference [11] performed a simulation regarding the integration of an adsorption unit inside a combined cycle and evaluated the operational impacts; reference [12] judges the performances of a small-scale CCHP—(combined cooling heat and power) system comprising a biogas externally fired microturbine, an absorption refrigeration unit and multiple heat exchangers for supplying energies demanded by a Bolivian small dairy farms. All these very specific studies are developing methods and models for planning and management of CCHP systems, see for instance [13], where Mixed Integer Linear Program solvers were developed in order to minimize the operating costs of a very complex energy grid including different CCHP systems, and auxiliary peak boilers and heat storage units.

Improvement of general design models for trigeneration systems is completed by the following logical levels, presented below, see also [14].

1. Stating the complete reversible trigeneration cycles for the fundamental energy schemes of providing imposed useful cooling, heating and power. This design level is very easily well done by considering ideal Carnot engine and refrigeration cycles. The ideal trigeneration built with completely ideal Carnot cycles gives the specific maximum maximorum energy efficiency, not depending on the working fluids' nature, depending only on the external heat reservoirs temperatures related to the ideal Carnot engine cycle and to the ideal Carnot refrigeration cycle. These maximum maximorum energy efficiencies could be used for all kinds of assessments regarding the lost energy/exergy/irreversible entropy minimization.
2. Defining the general endoreversible trigeneration thermodynamic models for all possible patterns of providing useful energies. This design level might be well completed through FPDT (finite physical dimensions thermodynamics) mathematical models based on the endoreversible Carnot cycles, see [14]. The limits of endoreversible Carnot cycles are surpassed through the mean thermodynamic temperature concept. However, it is obvious that the actual mean thermodynamic temperature is depending on the working fluids nature—i.e., on the thermodynamic properties—and on the type of the specific non adiabatic process. The FPDT (finite physical dimensions thermodynamics) mathematical models are general ones. The applications have to choose the working fluids and therefore the optimization might find the best working/convenient fluid by imposing simple or complex optimization criteria. The best actual FPDT endoreversible trigeneration becomes the reference case for all irreversible trigeneration cycles having the same working pattern.
3. Defining the reference models assessing the irreversibility influence. The classical irreversibility analysis might be completed through thorough sensitivity analyses and

specific optimization methods implying mean thermodynamic temperatures, specific lost exergy and irreversible entropy generation concepts. The FPDT assessments delineate a single concept evaluating priori the accumulated internal irreversibility, see for instance the short communications [15,16]. This evaluation of internal accumulated irreversibility by a single parameter is directly connecting the internal irreversibility to the external energy interactions through entropy balance equation. Therefore, through this single irreversibility dimensionless parameter the irreversible trigeneration assessments might be completed without knowing the working fluid nature and the type of thermal system. Although before each FPDT work they must be defined the operational possible domain range of this single parameter depending on the working fluid nature and on the thermal system type. They must also mention that the generalizing FPDT models of irreversible trigeneration have to adopt a new mean temperature of an external heat reservoir [14]. This new mean temperature is defined on the basis of the mean thermodynamic temperatures of the working fluid during the heat transfer processes and of the mean log temperature differences related to the linear heat transfer law. This new mean temperature can unify the first law and the linear heat transfer law without errors.

4. Defining the design optimization approaches of reference reversible and irreversible cycles. The optimization procedures consider either pure thermodynamic criteria, or CAPEX criteria, or operational costs criteria, or environmental criteria. The more elaborated methods combine different criteria, e.g., multi-objective optimization.
5. Defining the management optimization methods for possible interconnected trigeneration grids. They might be involved adaptive/intelligent management systems or trained predictive ones—e.g., training through fuzzy algorithms, management optimization methods such as MILP models, see reference [13].

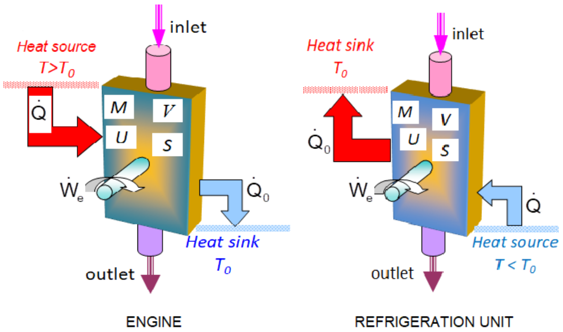
This paper is a work inside stage three and develops FPDT generalizing entropic approaches of irreversible closed cycles depending on a single dimensionless parameter characterizing all internal irreversibility without specifying the working fluid nature and the type of the thermal system. The mathematical models involve the irreversible connections between external and internal main operational parameters with finite physical dimensions. The external parameters are the mean temperatures of external heat reservoirs, the heat transfers thermal conductance and the mean log temperatures differences. The internal involved parameters are the reference entropy of the cycle and the internal irreversibility number. The cycle's design might use four possible operational constraints in order to find out the reference entropy. The number of internal irreversibility allows the evaluation of the reversible heat output function of the reversible heat input. Thus the cycle entropy balance equation to design the trigeneration cycles might be involved only through external operational parameters. In designing trigeneration systems they must know the requirements of all consumers of the useful energies delivered by the trigeneration system. The final conclusions emphasize the complexity in designing and/or optimizing the irreversible trigeneration systems.

2. The Irreversible Closed Cycles—The Irreversible Energy Efficiency—The Reference Entropy—The Number of Internal Irreversibility

Let us suppose the general basic irreversible thermal systems interacting with the 'environment' by heat transfers, mass transfers and power transfers, see Figure 1. These thermal systems can be analyzed taking into account either the whole irreversibility, internal and external, or only the internal irreversibility. Three types of thermal systems would be analyzed:

- the enlarged open thermal system comprising three interconnected parts and completely isolated from the universe, i.e., the proper open thermal system deformable under the external pressure which is joined with the external heat transfer reservoirs having known mean temperatures and heat capacities and joined with the deformation work and mass transfer reservoirs having known parameters, mass composition and specific energies (enthalpies, kinetic, and potential energies);

- the enlarged non-deformable closed thermal system that has two coupled parts isolated from the universe, the proper closed thermal system joined only with the external heat transfer reservoirs having known mean temperatures and specific heat capacities, and
- the closed thermal system/cycle considered alone but connected to external heat reservoirs with unknown parameters.



M: mass of the working fluid surrounded by the inner walls at a certain operational time.

V: working fluid volume defined by the inner walls at a certain operational time, supposed deformable under the external pressure.

U: internal energy of the working fluid surrounded by the inner walls at a certain operational time.

S: entropy of the working fluid surrounded by the inner walls at a certain operational time.

Figure 1. Basic irreversible thermal systems.

2.1. Assumptions for the First Case, the Enlarged Thermal Systems

- They will be analyzed non steady-state enlarged basic open thermodynamic systems, including both the thermal system, and the external heat reservoirs controlling the heat transfers, and the environment allowing the mass transfers and the deformation work transfer under the external pressure, see Figure 1;
- The working fluid is a mixture of different chemical species, the inlet and outlet compositions might be different because of chemical reactions that can appear during the flow through the thermal system, e.g., combustion;
- The inner boundary of the flow path through the thermal system is deformable under the environmental pressure;

Correlating the first law Equation (1) with the second law Equation (2), they can obtain the most general equation of the irreversible power (3) connected to the complete reversible cycle.

$$\frac{\partial U}{\partial t} = \left(\dot{Q} - \left|\dot{Q}_0\right|\right) - \dot{W}_e - p_e\frac{\partial V}{\partial t} + \sum_{inlet}\dot{m}\left(h + \frac{c^2}{2} + gZ\right) - \sum_{outlet}\dot{m}\left(h + \frac{c^2}{2} + gZ\right) \quad (1)$$

$$\dot{S}_{gen}^{irrev} = \frac{\partial S}{\partial t} - \left(\frac{\dot{Q}}{T} - \frac{\left|\dot{Q}_0\right|}{T_0}\right) - \sum_{inlet}\dot{m}s + \sum_{outlet}\dot{m}s \geq 0 \quad (2)$$

$$\begin{gathered}\dot{W}_e = \dot{W}_e^{rev} + \dot{W}_{lost}^{irrev} = \dot{Q}\left(1 - \tfrac{T_0}{T}\right) + \sum_{inlet}\dot{m}\left((h - T_0 s) + \tfrac{c^2}{2} + gZ\right) \\ - \sum_{outlet}\dot{m}\left((h - T_0 s) + \tfrac{c^2}{2} + gZ\right) - \tfrac{\partial}{\partial t}(U + p_e V - T_0 S) - T_0\dot{S}_{gen}^{irrev}\end{gathered} \quad (3)$$

where:

- $\dot{Q}, \dot{Q}_0, \dot{W}_e, \dot{W}_e^{rev}, \dot{W}_{lost}^{irrev}$ are the heat transfer rates from the heat source and to the heat sink, the real irreversible power, the complete reversible power and the lost power through irreversibility;
- $p_e\frac{\partial V}{\partial t}$ is the deformation work transfer under the external pressure, p_e is the external pressure and V is the deformable volume of the thermal system;

– $\dot{m}$, h, s are the mass flow rates, the specific enthalpy inclosing both the chemical and physical parts and the specific entropy inclosing both the chemical and physical parts, compulsory to obey to the first law of thermodynamics and considering all possible internal chemical processes, e.g., combustion;
– $\frac{c^2}{2}$, gZ are the specific kinetic and potential energies;
– T, T_0 are the mean temperatures of the heat source and of the heat sink;
– $\dot{S}_{gen}^{irrev}$ is the entropy rate generated through whole irreversibility.

The Equation (3) allows to define the complete reversible power including three components

$$\dot{W}_{e}^{rev} = \dot{W}_{e,Q}^{rev} + \dot{W}_{e,flow}^{rev} + \dot{W}_{e,storage}^{rev} \tag{4}$$

respectively

– the reversible power transfer related to the reversible heat transfers, ideal Carnot cycle

$$\dot{W}_{e,Q}^{rev} = \dot{Q}\left(1 - \frac{T_0}{T}\right) \tag{5}$$

– the reversible power transfer caused by the reversible flow from the inlet states to outlet ones

$$\dot{W}_{e,flow}^{rev} = \sum_{inlet} \dot{m}(h^* - T_0 s) - \sum_{outlet} \dot{m}(h^* - T_0 s) \tag{6}$$

– the reversible power transfer related to the system reversible 'energy inertia' caused by non-steady state processes of the working fluid surrounded by the inner walls at a certain operational time

$$\dot{W}_{e,storage}^{rev} - \frac{\partial}{\partial t}(U + p_e V - T_0 S) \tag{7}$$

Therefore, the lost power through whole irreversibility has the general equation

$$\dot{W}_{lost}^{irrev} = -T_0 \dot{S}_{gen}^{irrev} \tag{8}$$

The thermal energy consumed to produce power has two components, the heat transfer rate get from the heat source $\dot{Q}$, and the generalized enthalpy rate/variation associated to the flow $\sum_{inlet} \dot{m}h^* - \sum_{outlet} \dot{m}h^*$, where $h^* = h + \frac{c^2}{2} + gZ$ is the so called methalpy ('generalized' enthalpy), see [17,18].

2.2. Assumptions Considering Closed Thermal Systems

Let us suppose the general basic closed irreversible thermal systems, see Figure 2. The assumptions are

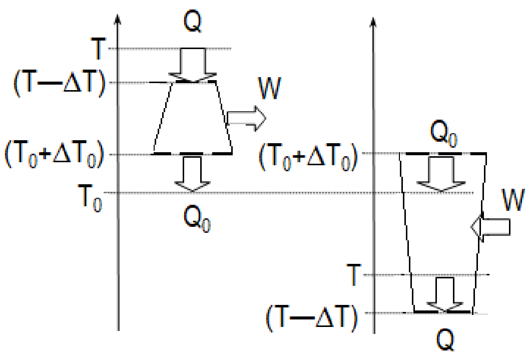


Figure 2. Scheme of the irreversible heat transfer interactions.

– no mass transfers

$$\sum_{inlet} \dot{m}h^* - \sum_{outlet} \dot{m}h^* = 0 \text{ and } - \sum_{inlet} \dot{m}s + \sum_{outlet} \dot{m}s = 0 \quad (9)$$

– non deformable boundary walls, and

$$-p_e \frac{\partial V}{\partial t} = 0 \quad (10)$$

– steady state operation

$$\frac{\partial U}{\partial t} = 0 \text{ and } \frac{\partial S}{\partial t} = 0 \quad (11)$$

The associated entropy balance equation for the enlarged thermal system is, see Figure 2

$$-\,\mathrm{Irr}\frac{\dot{Q}}{T} + \frac{\left|\dot{Q}_0\right|}{T_0} = 0 \quad (12)$$

The associated entropy balance equation only for the alone thermal system is, see Figure 2

$$-\,N_{irr}\frac{\dot{Q}}{T-\Delta T} + \frac{\left|\dot{Q}_0\right|}{T_0+\Delta T_0} = 0 \quad (13)$$

The relation between Irr and N_{irr} is obtained from entropy balance Equations (12) and (13)

$$\mathrm{Irr} = N_{irr}\frac{T}{T_0}\frac{T_0+\Delta T_0}{T-\Delta T} = N_{irr}\theta_{HR}\theta_{mtt} \quad (14)$$

where:

– $\dot{Q}, \dot{Q}_0$ are the heat transfer rates;
– $T,\ T_0,\ \Delta T,\ \Delta T_0$ are the mean temperatures of the heat source, of the heat sink and the corresponding mean log temperature differences controlling the heat transfers; they have to state that $(T - \Delta T)$ and $(T_0 + \Delta T_0)$ are the mean thermodynamic temperatures of the working fluid for the reversible non adiabatic processes of the cycle, i.e., the reversible heating and cooling processes;
– Irr is the comprehensive dimensionless irreversibility function linking the heat transfers through the entropy balance equation for the enlarged thermal system, it includes both the external irreversibility and the internal one;
– N_{irr} is the internal dimensionless irreversibility function linking the heat transfers through entropy balance equation only for the thermal system, it includes only the internal irreversibility; in this paper N_{irr} is called as the number of internal irreversibility;
– θ_{HR}, θ_{mtt} are the ratios of mean temperatures of external heat reservoirs and of mean thermodynamic temperatures of cycle's non-adiabatic processes.

2.3. Irreversible Energy Efficiency of Enlarged Closed Thermal System

They will be demonstrated the comprehensive irreversible energy efficiency related to the complete reversible Carnot cycle, i.e., for the enlarged thermal system, see Figure 2.

- Engine

The delivered power of the enlarged engine cycle

$$\dot{W}_e = \dot{W}_e^{rev} + \dot{W}_{lost}^{irrev} = \dot{Q}_{rev}\left(1 - \frac{T_0}{T}\right) - T_0\dot{S}_{gen}^{irrev} \quad (15)$$

The irreversible energy efficiency, $EE^{irrev}_{engines}$

$$EE^{irrev}_{engines} = \frac{\dot{W}_e}{\dot{Q}_{rev}} = \frac{\dot{W}^{rev}_e + \dot{W}^{irrev}_{lost}}{\dot{Q}_{rev}} = \frac{\dot{W}^{rev}_e}{\dot{Q}_{rev}} + \frac{\dot{W}^{irrev}_{lost}}{\dot{Q}_{rev}} = 1 - \frac{T_0}{T} - \frac{T_0\dot{S}^{irrev}_{gen}}{\dot{m}(T-\Delta T)\Delta s_q}$$
$$= 1 - \frac{T_0}{T}\left(1 + \frac{\theta_{SLT}\dot{S}^{irrev}_{gen}}{\dot{m}\Delta s_q}\right) = 1 - \frac{T_0}{T}Irr < EE_{Carnot} = 1 - \frac{T_0}{T} \quad (16)$$

where:

$\theta_{SLT} = \frac{T}{T-\Delta T}$ is a dimensionless temperature ratio related to the second law of thermodynamics

$\dot{m}\Delta s_q$ is the reversible entropy variation rate of the working fluid during the reversible heat input, $\dot{Q}_{rev}$, and $\left(1 + \frac{\theta_{SLT}\dot{S}^{irrev}_{gen}}{\dot{m}\Delta s_q}\right) = Irr$ is the primary form of the overall irreversibility dimensionless function.

- Refrigeration unit

The consumed power of the enlarged refrigeration unit

$$\left|\dot{W}_e\right| = -\dot{W}^{rev}_e - \dot{W}^{irrev}_{lost} = -\dot{Q}_{rev}\left(1 - \frac{T_0}{T}\right) + T_0\dot{S}^{irrev}_{gen} = \dot{Q}_{rev}\left(\frac{T_0}{T} - 1\right) + T_0\dot{S}^{irrev}_{gen} \quad (17)$$

The irreversible energy efficiency, $EE^{irrev}_{refrigeration}$

$$EE^{irrev}_{refrigeration} = \frac{\dot{Q}_{rev}}{\left|\dot{W}_e\right|} = -\frac{\dot{Q}_{rev}}{\dot{W}^{rev}_e + \dot{W}^{irrev}_{lost}} = -\frac{1}{\frac{\dot{W}^{rev}_e}{\dot{Q}_{rev}} + \frac{\dot{W}^{irrev}_{lost}}{\dot{Q}_{rev}}} = \frac{1}{\frac{T_0}{T} - 1 + \frac{T_0\dot{S}^{irrev}_{gen}}{\dot{m}(T-\Delta T)\Delta s_q}}$$
$$= \frac{1}{\frac{T_0}{T}\left(1 + \frac{\theta_{SLT}\dot{S}^{irrev}_{gen}}{\dot{m}\Delta s_q}\right) - 1} = \frac{1}{\frac{T_0}{T}Irr - 1} < COP_{Carnot} = \frac{1}{\frac{T_0}{T} - 1} \quad (18)$$

where:

$\theta_{SLT} = \frac{T}{T-\Delta T}$ is a dimensionless temperature ratio related to the second law of thermodynamics

$\dot{m}\Delta s_q$ is the working fluid entropy variation rate during the reversible heat input $\dot{Q}_{rev}$, and $\left(1 + \frac{\theta_{SLT}\dot{S}^{irrev}_{gen}}{\dot{m}\Delta s_q}\right) = Irr$ is the primary form of the overall irreversibility dimensionless function.

2.4. Irreversible Energy Efficiency Only for the Closed Thermal System

They will be demonstrated the comprehensive irreversible energy efficiency related to the endoreversible Carnot cycle, see Figure 2.

- Engine

The delivered power of the alone closed engine cycle

$$\dot{W}_e = \dot{W}^{rev}_e + \dot{W}^{irrev}_{lost,cycle} = \dot{Q}_{rev}\left(1 - \frac{T_0 + \Delta T_0}{T - \Delta T}\right) - (T_0 + \Delta T_0)\dot{S}^{irrev}_{gen,cycle} \quad (19)$$

The irreversible energy efficiency, $EE^{irrev}_{engines}$

$$EE^{irrev}_{engines} = \frac{\dot{W}_e}{\dot{Q}_{rev}} = \frac{\dot{W}^{rev}_e + \dot{W}^{irrev}_{lost,\,cycle}}{\dot{Q}_{rev}} = \frac{\dot{W}^{rev}_e}{\dot{Q}_{rev}} + \frac{\dot{W}^{irrev}_{lost,cycle}}{\dot{Q}_{rev}} = 1 - \frac{T_0+\Delta T_0}{T-\Delta T} - \frac{(T_0+\Delta T_0)\dot{S}^{irrev}_{gen,cycle}}{\dot{m}(T-\Delta T)\Delta s_q}$$
$$= 1 - \frac{T_0+\Delta T_0}{T-\Delta T}\left(1 + \frac{\dot{S}^{irrev}_{gen,cycle}}{\dot{m}\Delta s_q}\right) = 1 - \frac{T_0+\Delta T_0}{T-\Delta T}N_{irr} < EE_{Carnot} = 1 - \frac{T_0+\Delta T_0}{T-\Delta T} \quad (20)$$

where:

$\dot{m}\Delta s_q$ is the working fluid entropy variation rate during the reversible heat input $\dot{Q}_{rev}$, and

$\left(1 + \frac{\dot{S}^{irrev}_{gen,cycle}}{\dot{m}\Delta s_q}\right) = N_{irr}$ is the primary form of the internal irreversibility dimensionless function.

- Refrigeration unit

The consumed power of the alone closed refrigeration unit

$$\begin{aligned} \left|\dot{W}_e\right| = -\dot{W}_e^{rev} - \dot{W}^{irrev}_{lost,cycle} &= -\dot{Q}_{rev}\left(1 - \tfrac{T_0+\Delta T_0}{T-\Delta T}\right) + (T_0 + \Delta T_0)\dot{S}^{irrev}_{gen,cycle} \\ &= \dot{Q}_{rev}\left(\tfrac{T_0+\Delta T_0}{T-\Delta T} - 1\right) + (T_0 + \Delta T_0)\dot{S}^{irrev}_{gen,cycle} \end{aligned} \quad (21)$$

The irreversible energy efficiency, $EE^{irrev}_{refrigeration}$

$$\begin{aligned} EE^{irrev}_{refrigeration} &= \frac{\dot{Q}_{rev}}{\left|\dot{W}_e\right|} = -\frac{\dot{Q}_{rev}}{\dot{W}_e^{rev} + \dot{W}^{irrev}_{lost,cycle}} = -\frac{1}{\frac{\dot{W}_e^{rev}}{\dot{Q}_{rev}} + \frac{\dot{W}^{irrev}_{lost,cycle}}{\dot{Q}_{rev}}} = \frac{1}{\frac{T_0+\Delta T_0}{T-\Delta T} - 1 + \frac{(T_0+\Delta T_0)\dot{S}^{irrev}_{gen,cycle}}{\dot{m}(T-\Delta T)\Delta s_q}} \\ &= \frac{1}{\frac{T_0+\Delta T_0}{T-\Delta T}\left(1 + \frac{\dot{S}^{irrev}_{gen,cycle}}{\dot{m}\Delta s_q}\right) - 1} = \frac{1}{\frac{T_0+\Delta T_0}{T-\Delta T}N_{irr} - 1} < COP_{Carnot} = \frac{1}{\frac{T_0+\Delta T_0}{T-\Delta T} - 1} \end{aligned} \quad (22)$$

where:

$\dot{m}\Delta s_q$ is the working fluid entropy variation rate during the reversible heat input $\dot{Q}_{rev}$, and

$\left(1 + \frac{\dot{S}^{irrev}_{gen,cycle}}{\dot{m}\Delta s_q}\right) = N_{irr}$ is the primary form of the internal irreversibility dimensionless function.

Remark 1. *Dimensionless functions*, Irr *and* N_{irr}, *depend on the entropy variation rate,* $\Delta\dot{S} = \dot{m}\Delta s_q$, *and on the corresponding* $\dot{S}^{irrev}_{gen}$ *and* $\dot{S}^{irrev}_{gen,cycle}$. *At their turn, both parameters,* $\Delta\dot{S} = \dot{m}\Delta s_q$ *and* $\dot{S}^{irrev}_{gen}$ and $\dot{S}^{irrev}_{gen,cycle}$, *will be strongly shaped through the working fluids nature and their thermodynamic properties. On the basis of equations defining* Irr *an* Nirr, *the reference entropy becomes always* $\Delta\dot{S} = \dot{m}\Delta s_q > 0$, *i.e., the working fluid entropy variation rate during the reversible heat input on the cycle. The references [15,16] includes some demonstrated equations of both* N_{irr} *and irreversible energy efficiency, e.g., for some closed engine cycle, and for closed refrigeration cycle.*

Remark 2. *Heat rates exchanged with external heat reservoirs are the reversible heat rates for constant pressure processes where the irreversibility is defined by pressure drops equivalent to a constant enthalpy process (equivalent throttling), see Figures 3 and 4.*

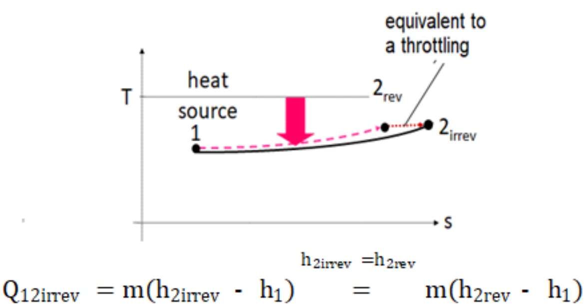
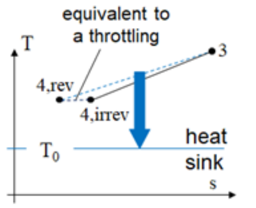


Figure 3. Irreversible heat input is equalizing the reversible one, constant pressure heating.



$$Q_{34irrev} = m(h_{4irrev} - h_3) \overset{h_{4irrev} = h_{4rev}}{=} m(h_{4rev} - h_3)$$

Figure 4. Irreversible heat output is equalizing the reversible one, constant pressure cooling.

When we have different irreversible non adiabatic processes, e.g., constant temperature, polytropic, constant volume we have to define the irreversibility either through adequate and known pressure drops caused by friction or through irreversible lost work alike it is defined the isentropic efficiency of an adiabatic process.

Remark 3. *Reversible entropy variation during the heat output can be evaluated through the number of internal irreversibility,* N_{irr}*, i.e.,* $\Delta\dot{S}_0 = \dot{m}\Delta s_{q0} = -N_{irr}\Delta\dot{S} = -N_{irr}\dot{m}\Delta s_q$*. Knowing the reversible cyclic heat input and heat output they can be assessed the irreversible power and the irreversible energy efficiency.*

3. Design Imposed Operational Conditions

The first design of irreversible cycles can be performed using four imposed conditions see for instance [19]. These operational imposed conditions might be also combined, for instance constant specific power and constant energy efficiency for engines and constant heat input and constant energy efficiency for refrigeration units.

The analysis and design of irreversible cycles has always two directions. The first one is to analyze the cycle ignoring the energy interactions with the environment, see Section 3.1 below. The second one uses the energy interactions as main control functions and takes into consideration only the number of internal irreversibility as a general internal function quantifying the all internal irreversibility and linking the external heat transfers with external heat reservoirs, see the following Section 4.

3.1. FPDT Internal Design through Imposed Operational Conditions

They have to define the internal main finite physical dimension parameters and after that to establish the dependence functions characterizing the performances of the irreversible cycle. For instance, [19] applied the four imposed operational conditions to evaluate the performances of a Joule-Brayton cycle working with two ideal gases, air and CO_2. The main finite physical dimension parameter was the classical compression ratio, π_C, and the dependence functions characterizing the performances of the irreversible cycle were:

- the maximum temperature on the cycle, T_{3irr} [K], see Figure 5,
- the specific power, w, [J/kg], imposed $w = 500$ kJ/kg,
- the energy efficiency, EE_{irr}, see Figure 6, and
- the number of internal irreversibility, N_{irr}, see Figure 7.

The internal irreversible entropy generation was known through isentropic efficiencies of compressor, η_{sC}, and of gas turbine, η_{sT}, and through the pressure drops inside exchangers, r_p.

The admissible maximum temperature on the cycle, (1100 K to 1400 K), imposes its own limits for the maximum compression ratio. These limitations establish actual compulsory limitations for the specific power, for the energy efficiency, and consequently for the maximum admissible number of internal irreversibility. The all limitations would be controlled by the working fluids nature and by the magnitude of irreversibility.

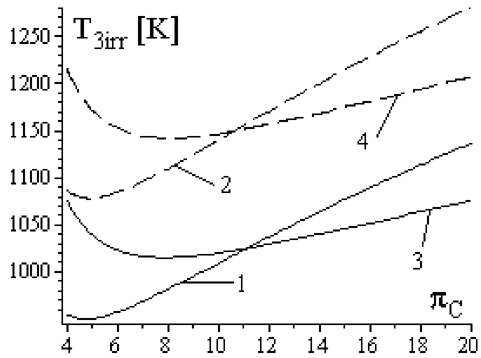


Figure 5. Dependences $T_{3irr} = f_T(\pi_C)$.

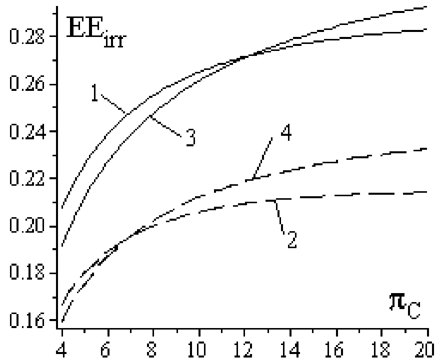


Figure 6. Dependences $EE_{irr} = f_E(\pi_C)$.

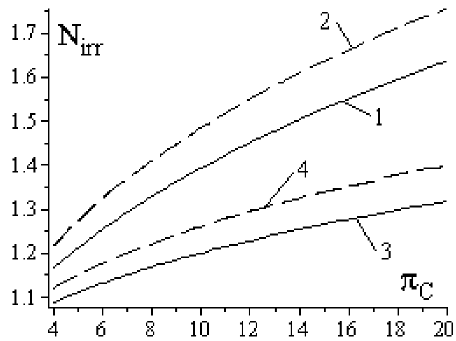


Figure 7. Dependences $N_{irr} = f_N(\pi_C)$.

Below are selected numerical results [19], see Figures 5–7, for imposed constant power w = 500 kJ/kg and imposed irreversibility for graphs 1, 2, 3 and 4:
1: air, $\eta_{sC} = 0.85$, $\eta_{sT} = 0.9$, $r_p = 0.975$; **2**: air, $\eta_{sC} = 0.8$ and $\eta_{sT} = 0.85$, $r_p = 0.95$
3: CO_2, $\eta_{sC} = 0.85$, $\eta_{sT} = 0.9$, $r_p = 0.975$; **4**: CO_2, $\eta_{sC} = 0.8$ and $\eta_{sT} = 0.85$, $r_p = 0.95$

4. Irreversible Trigeneration Cycles External Design Based on FPDT

The [14] presented the analysis of endoreversible trigeneration cycles design based on FPDT. This section is extending the mathematical models to four irreversible closed trigeneration cycles:

a. engine cycle working in power mode and the reverse cycle working in refrigeration mode, the summer season;
b. engine cycle working in cogeneration mode and the reverse cycle working in refrigeration mode, the winter season;

c. engine cycle working in power mode and the reverse cycle working both in refrigeration mode and heat pump mode, the winter season; and
d. engine cycle working in cogeneration mode and the reverse cycle working both in refrigeration mode and heat pump mode, the winter season.

For each cycle, engine or refrigeration unit, they were adopted the related four external and two internal control parameters with finite dimensions. The external control parameters are pertaining to external heat transfers and the internal control parameters are the generalizing reference entropy and the number of internal irreversibility.

4.1. Basic Mathematical Model

The mathematical model joins the first law and the linear heat transfer law with the second law. The useful energies are the power, the refrigeration and the heating rates. The useful thermal energies must be known through the ratio of refrigeration rate to power (x) and the ratio of heating rate to power (y).

4.1.1. Engine Irreversible Cycle

The reference entropy variation rate is:

$$\Delta\dot{S}_E = \dot{m}\Delta s_q \tag{23}$$

The finite physical dimension control parameters are:

- Mean log temperature differences ΔTH [K] at the hot side and ΔTC [K] at the cold side.
- Thermal conductance $(UA)_H$ [kW/K] allocated to the hot side, and thermal conductance $(UA)_C$ [kW/K] allocated to the cold side.
- Thermal conductance inventory:

$$G_{TE} = G_H + G_C = (UA)_H + (UA)_C \left[kW\cdot K^{-1}\right] \tag{24}$$

$$g_H = \frac{G_H}{G_{TE}},\ g_C = \frac{G_C}{G_{TE}},\ g_H + g_C,\ g_C = 1 - g_H \tag{25}$$

where U [$kW\cdot m^{-2}\cdot K^{-1}$] is the overall heat transfer coefficient and A [m^2] is the heat transfer area.

First Law Equations

$$\dot{Q}_H = g_H G_{TE}\Delta T_H = T_H\Delta\dot{S}_E = (\theta_{HS}T_{CS} - \Delta T_H)\Delta\dot{S}_E \text{ at the hot side} \tag{26}$$

$$\overset{(26)}{\widehat{\Rightarrow}} G_{TE} = \frac{(\theta_{HS}T_{CS} - \Delta T_H)\Delta\dot{S}_E}{g_H\Delta T_H} \tag{27}$$

$$\dot{Q}_C = -(T_{CS} + \Delta T_C)\Delta\dot{S}_E N_{irr,E} = -(1 - g_H)G_{TE}\Delta T_C \text{ at the cold side} \tag{28}$$

$$\overset{(27,28)}{\widehat{\Rightarrow}} \Delta T_C = \frac{g_H\Delta T_H N_{irr,E}}{\theta_{HS}\left\{1 - g_H - \frac{\Delta T_H\left[1+g_H\left(N_{irr,E}-1\right)\right]}{\theta_{HS}T_{CS}}\right\}} \tag{29}$$

$$\begin{aligned}\dot{W}_E = \dot{Q}_H + \dot{Q}_C = (\theta_{HS}T_{CS} - \Delta T_H)\Delta\dot{S}_E \\ - \left(T_{CS} + \frac{g_H\Delta T_H N_{irr,E}}{\theta_{HS}\left\{1-g_H-\frac{\Delta T_H\left[1+g_H\left(N_{irr,E}-1\right)\right]}{\theta_{HS}T_{CS}}\right\}}\right)\Delta\dot{S}_E N_{irr,E}\end{aligned} \tag{30}$$

$$EE_{irr,E} = \frac{\dot{W}_E}{\dot{Q}_H} = 1 - \frac{\left(T_{CS} + \frac{g_H \Delta T_H N_{irr,E}}{\theta_{HS}\left\{1 - g_H - \frac{\Delta T_H\left[1+g_H\left(N_{irr,E}-1\right)\right]}{\theta_{HS}T_{CS}}\right\}}\right) N_{irr,E}}{(\theta_{HS}T_{CS} - \Delta T_H)} \tag{31}$$

where

- $\dot{m}$ [kg·s^{-1}] is the mass flow rate through engine;
- $\dot{Q}_H$ [kW] is the reversible heat input rate;
- $\dot{Q}_C$ [kW] is the reversible heat output rate;
- $\dot{W}_E$ [kW] is the power;
- $EE_{irr,E}$ is the irreversible energy efficiency;
- T_H [K] is the mean thermodynamic temperature, cycle hot side;
- $T_{HS} = T_H + \Delta T_H$ [K] is the heat source proper mean temperature;
- T_C [K] is the mean thermodynamic temperature, cycle cold side;
- $T_{CS} = T_C - \Delta T_C$ [K] is the heat sink proper mean temperature;

The performance functions get explicit forms if they are replacing the reference entropy through one imposed operational condition. Additionally imposing the energy efficiency, we can simplify the computational procedures. The main difficulty is to correctly evaluate the possible imposed energy efficiency by a sensitivity analysis and to define the domain range of N_{irr}. The proof results are correlating the internal and external FPDT evaluations.

As a very rapid computational example, they were imposed mixed operational conditions, constant power and constant energy efficiency: $\dot{W}$ = 100 kW, θ_{HS} = 4, T_{CS} = 323 K, and $EE_{irr,E}$ = 0.35, and some numbers of internal irreversibility, see Figures 8–10. The imposed energy efficiency allowed to find the relationship $\Delta T_H = \varphi(g_H, N_{irr,E})$:

$$\Delta T_H = 795.0769(1 - g_H) \text{ with } N_{irr,E} = 1.00$$

$$\Delta T_H = \frac{670.846(1 - g_H)}{1 + 0.25 g_H} \text{ with } N_{irr,E} = 1.25$$

$$\Delta T_H = \frac{546.615(1 - g_H)}{1 + 0.5 g_H} \text{ with } N_{irr,E} = 1.50$$

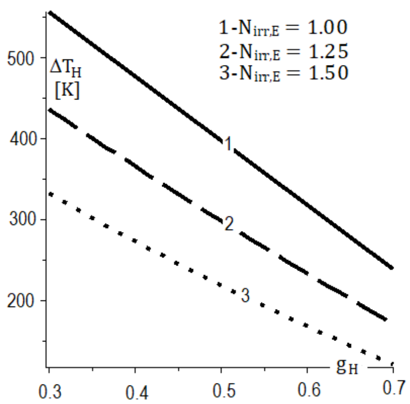


Figure 8. Dependence between the mean log temperature difference at the hot side and the dimensionless thermal conductance at the hot side, $\Delta T_H = f(g_H)$.

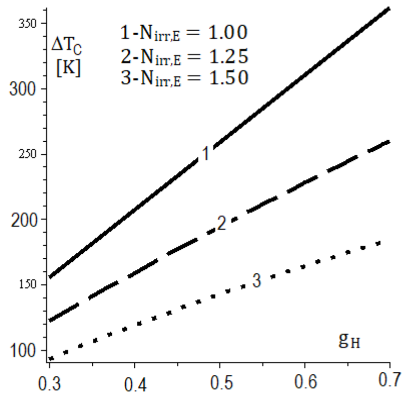


Figure 9. Dependence between the mean log temperature difference at the cold side and the dimensionless thermal conductance at the hot $\Delta T_C = f(g_H)$.

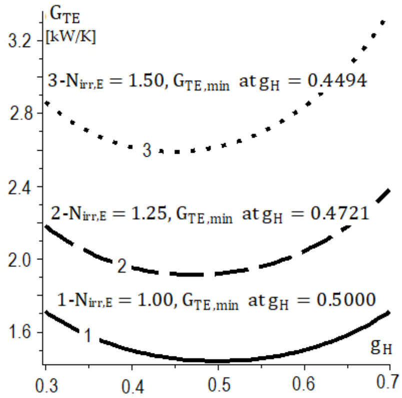


Figure 10. Dependence between the thermal conductance inventory and the dimensionless thermal conductance at the hot side $G_{TE} = f(g_H)$.

The numerical results emphasized that as larger the number of internal irreversibility as larger the thermal conductance inventory and smaller the mean log temperature differences. Respectively, as higher the internal irreversibility as lower the external irreversibility in order to maintain constant energy efficiency.

4.1.2. Refrigeration Irreversible Cycle

- The reference entropy variation rate is:

$$\Delta \dot{S}_R = \dot{m}\Delta s_q \tag{32}$$

- The finite physical dimension control parameters are: mean log temperature differences ΔT_R [K] and ΔT_0 [K], inside of heat exchangers at the heat source and at the heat sink;
- Thermal conductances $(UA)_R$ inside the heat exchanger at the heat source, and $(UA)_0$ inside the heat exchanger at the heat sink:

$$G_{TR} = G_R + G_0 = (UA)_R + (UA)_0 \left[\text{kW}\cdot\text{K}^{-1}\right] \tag{33}$$

$$g_R = \frac{G_R}{G_{TR}},\ g_0 = \frac{0}{G_{TR}},\ g_R + g_0 = 1,\ g_o = 1 - g_R \tag{34}$$

where U [$\text{kW}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$] is the overall heat transfer coefficient and A [m^2] is the heat transfer area.

- First law balance equations:

$$\dot{Q}_R = g_R G_{TR} \Delta T_R = T_R \Delta \dot{S}_R = \left(\frac{T_{0S}}{\theta_{RS}} - \Delta T_R \right) \Delta \dot{S}_R \quad (35)$$

$$\overset{(34)}{\widehat{\Rightarrow}} G_{TR} = \frac{\left(\frac{T_{0S}}{\theta_{RS}} - \Delta T_R \right) \Delta \dot{S}_R}{G_{TR} \Delta T_R} \quad (36)$$

$$\dot{Q}_0 = -(1 - g_R) G_{TR} \Delta T_0 = -(T_{0S} + \Delta T_0) \Delta \dot{S}_R N_{irr,R} \quad (37)$$

$$\overset{(35,36)}{\widehat{\Rightarrow}} \Delta T_0 = \frac{g_R \theta_{RS} \Delta T_R N_{irr,R}}{1 - g_R - \frac{\theta_{RS} \Delta T_R}{T_{0S}} (1 + g_R (N_{irr,R} - 1))} \quad (38)$$

$$\dot{W}_R = \dot{Q}_R + \dot{Q}_0 = \left[\left(\frac{T_{0S}}{\theta_{RS}} - \Delta T_R \right) - \left(T_{0S} + \frac{g_R \theta_{RS} \Delta T_R N_{irr,R}}{1 - g_R - \frac{\theta_{RS} \Delta T_R}{T_{0S}} (1 + g_R (N_{irr,R} - 1))} \right) N_{irr,R} \right] \Delta \dot{S}_R \quad (39)$$

$$\dot{EE}_{irr,R} = \frac{\dot{Q}_R}{\left| \dot{W}_R \right|} = \frac{\frac{T_{0S}}{\theta_{RS}} - \Delta T_R}{\left(T_{0S} + \frac{g_R \theta_{RS} \Delta T_R N_{irr,R}}{1 - g_R - \frac{\theta_{RS} \Delta T_R}{T_{0S}} (1 + g_R (N_{irr,R} - 1))} \right) N_{irr,R} - \left(\frac{T_{0S}}{\theta_{RS}} - \Delta T_R \right)} \quad (40)$$

where

- $\dot{m}$ [$kg \cdot s^{-1}$] is the mass flow rate of the working fluid through the refrigeration machine;
- $\dot{Q}_R$ [kW] is the refrigeration heat rate;
- $\dot{Q}_0$ [kW] is the heat rate at the heat sink;
- $\dot{W}_R$ [kW] is the consumed power
- $T_R =$ [K] is the mean thermodynamic temperature at the cycle cold side;
- $T_{RS} = \frac{T_{0S}}{\theta_{RS}} = T_R + \Delta T_R$ [K] is the mean temperature of the heat source;
- T_0 [K] is mean thermodynamic temperature at the cycle hot part;
- $T_{0S} = T_0 - \Delta T_0$ is the mean temperature of the heat sink.

As an example, they were imposed mixed operational conditions, constant heat input and constant energy efficiency: $\dot{Q}_R = 0.1\dot{W}_E = 10$ kW, T_{RS} = 263 K, T_{0S} = 323 K, $EE_{irr,R}$ = COP = 2, see Figures 11–13. The extra imposed energy efficiency allowed to find the first explicit operational function $\Delta T_R = f(g_R, N_{irr,R})$.

$$\Delta T_R = \frac{143(1 - g_H)}{3} \text{ with } N_{irr,E} = 1.00$$

$$\Delta T_R = \frac{78.4(1 - g_H)}{3 + 0.3 g_H} \text{ with } N_{irr,E} = 1.10$$

$$\Delta T_R = \frac{13.8(1 - g_H)}{3 + 0.6 g_H} \text{ with } N_{irr,E} = 1.20$$

The numerical results emphasized the similarity with those obtained for the irreversible engine cycle, respectively as larger the number of internal irreversibility as larger the thermal conductance inventory and smaller the mean log temperature differences. Respectively as higher the internal irreversibility as lower the external irreversibility in order to maintain constant energy efficiency. The influences of internal irreversibility number are more sensitive as in the case of engine cycle.

The Equations (23)–(40) allow the external evaluation of closed irreversible engine cycles. This evaluation has to be correlated with internal evaluation in order to obtain the best design variant. Probably, a multi-objective evaluation might be a good mathematical way. The design optimization must correlate and agree the computational results obtained through both internal cycle design (see Section 3.1) and external design (see Section 4), especially when we use mixed operational constraints.

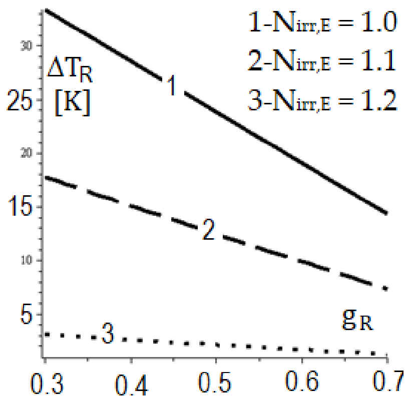


Figure 11. Dependence between the mean log temperature difference at the cold side and the dimensionless thermal conductance at the cold side, $\Delta T_R = f(g_R)$.

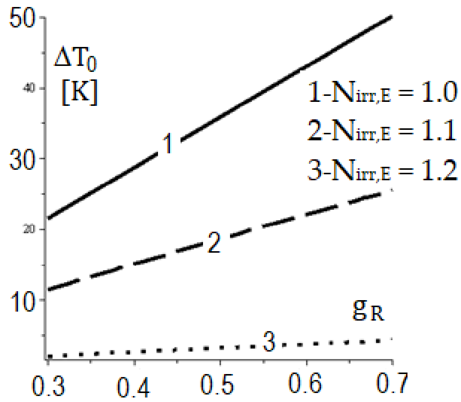


Figure 12. Dependence between the mean log temperature difference at the hot side and the dimensionless thermal conductance at the cold side, $\Delta T_0 = f(g_R)$.

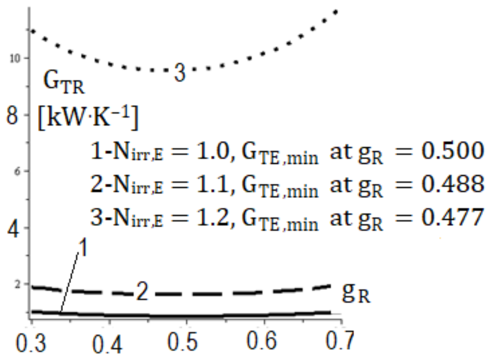


Figure 13. Dependence between the thermal conductance inventory and the dimensionless thermal conductance at the cold side ide, $G_{TR} = f(g_R)$.

4.2. Irreversible Trigeneration System

The design and/or optimization of trigeneration systems define first the clients/customers/end users energy needs and after that choose the proper type of trigeneration system, see the beginning of Section 4. It follows a preliminary selection of closed irreversible cycle and working fluids, a preliminary evaluation of the domain range of internal number of irreversibility, and a preliminary design of trigeneration cycles based on (23) to (40) equations and on the imposed operational conditions. Knowing the mean log temperatures differences of all heat exchangers and the thermal conductance inventories we verify the proper mean temperatures of external heat reservoirs involving in parallel the internal cycle assessments. In order to simplify, we have to define the energy connections defined through useful power and the ratios of refrigeration rate to power (x) and of heating rate to power (y).

The general equation of energy efficiency (EE) is the ratio of useful energy to the consumed one. The energy efficiency has the below equations, specific to type of trigeneration operational pattern, see the beginning of Section 4, cases a, b, c, and d, see also [14], but for all cases the consumed energy is the input reversible heat for the engine cycle.

- Case "a"—energy efficiency:

$$EE_a = \frac{\dot{W}_E - \left|\dot{W}_R\right| + \dot{Q}_R}{\dot{Q}_H} = EE_E\left(1 + x\frac{COP - 1}{COP}\right) \tag{41}$$

- Case "b"—energy efficiency:

$$EE_b = \frac{\dot{W}_E - \left|\dot{W}_R\right| + \dot{Q}_R + \left|\dot{Q}_C\right|^*}{\dot{Q}_H} = EE_{cog} + EE_E x\frac{COP - 1}{COP} \tag{42}$$

- Case "c"—energy efficiency:

$$EE_c = \frac{\dot{W}_E - \left|\dot{W}_R\right| + \dot{Q}_R + \left|\dot{Q}_0\right|}{\dot{Q}_H} = EE_E(1 + 2x) \tag{43}$$

- Case "d"—energy efficiency:

$$EE_b = \frac{\dot{W}_E - \left|\dot{W}_R\right| + \dot{Q}_R + \left|\dot{Q}_C\right|^* + \left|\dot{Q}_0\right|}{\dot{Q}_H} = EE_{cog} + 2EE_E x \tag{44}$$

In above equations, the useful power is $\dot{W}_u = \dot{W}_E - \left|\dot{W}_R\right|$, where $\dot{W}_E$ is the engine power and $\left|\dot{W}_R\right|$ is power consumed by the refrigeration unit. The useful thermal energies are the refrigeration rate $\dot{Q}_R$, the heat rate produced by cogeneration $\left|\dot{Q}_C\right|^* \leq \left|\dot{Q}_C\right|$ and the heat rate delivered by the reverse cycle working also in heat pump mode $\left|\dot{Q}_0\right|$. The consumed energy is always the engine heat input rate $\dot{Q}_H$. x is the ratio of the refrigeration rate to engine power $x = \dot{Q}_R/\dot{W}_E$ and $EE_{cog} = \frac{\dot{W}_E + \left|\dot{Q}_C\right|^*}{\dot{Q}_H}$ is the energy efficiency of cogeneration and $EE_E = \frac{\dot{W}_E}{\dot{Q}_H}$ is the energy efficiency of the engine working in power mode.

They must emphasize that Equations (41)–(44) are identical for ideal reversible, endoreversible and irreversible trigeneration systems. They have to know the real energy efficiencies of system components—i.e., EE_{cog}, $EE_{E,real}$, and COP_{real}—and ratio x.

At the European level the engine cogeneration energy efficiency must be $EE_{cog} \geq 0.85$. Also, for the specified heating system type they might be accounted the heat losses along the delivering path of useful thermal energies (heating) for all cases.

For all cases, the minimum useful power compels the maximum x ratio

$$\dot{W}_u = \dot{W}_E - \left|\dot{W}_R\right| = \dot{W}_E\left(1 - \frac{x}{COP}\right) \geq \dot{W}_{u,min} \Rightarrow x \leq COP\left(1 - \frac{\dot{W}_{u,min}}{\dot{W}_E}\right) \tag{45}$$

where $\dot{W}_{u,min}$ is the minimum admissible useful power, required by the power end users.

Equations (23)–(44) provide the general mathematical model to design/optimize the irreversible trigeneration systems.

The Tables 1 and 2 include main external parameters for engine and refrigeration units possibly to be coupled in a trigeneration systems, cases a, b, c, and d. They were adopted the dimensionless thermal conductance corresponding to the minimum thermal conductance inventory, g_H and g_R.

Table 1. External parameters for the engine, $\dot{W}_E = 100$ kW, EE_E = 0.35 (imposed), θ_{HS} = 4 (imposed).

Trigeneration	N_{irr}	g_H	T_{CS} (K)	ΔT_H (K)	ΔT_C (K)	G_{TE} ($kW \cdot K^{-1}$)
	1.00	0.5000	308	379	246	1.507
(a)	1.25	0.4721	308	302	176	2.003
	1.50	0.4494	308	234	124	2.713
	1.00	0.5000	343	422	274	1.354
(b)	1.25	0.4721	343	336	196	1.799
	1.50	0.4494	343	261	138	2.436
	1.00	0.5000	273	336	218	1.701
(c)	1.25	0.4721	273	268	156	2.261
	1.50	0.4494	273	208	110	3.061
	1.00	0.5000	343	422	274	1.354
(d)	1.25	0.4721	343	336	196	1.799
	1.50	0.4494	343	261	138	2.436

Table 2. External parameters for the refrigeration unit, $\dot{Q}_R = 10$ kW.

Trigeneration	N_{irr}	g_R	T_{0S} (K)	T_{RS} (K)	ΔT_R (K)	ΔT_0 (K)	G_{TR} ($kW \cdot K^{-1}$)	COP
	1.0	0.500	308	253	23.83	35.75	0.839	2
(a)	1.1	0.488	308	253	13.24	18.83	1.547	2
	1.2	0.477	308	253	3.15	4.31	6.503	2
	1.0	0.500	273	253	24.13	32.17	0.829	3
(b)	1.1	0.488	273	253	13.56	17.13	1.511	3
	1.2	0.477	273	253	3.49	4.24	5.88	3
	1.0	0.500	343	253	23.60	39.33	0.875	1.5
(c)	1.1	0.488	343	253	13.00	20.53	1.577	1.5
	1.2	0.477	343	253	2.88	4.38	7.106	1.5
	1.0	0.500	343	253	23.60	39.33	0.875	1.5
(d)	1.1	0.488	343	253	13.00	20.53	1.577	1.5
	1.2	0.477	343	253	2.88	4.38	7.106	1.5

The trigeneration system might have different external operational features depending on the adopted case (a, b, c, d), on the number of internal irreversibility of engine and of refrigeration unit, on the ratios of refrigeration and delivered ratio to power.

The comparison of different kind of trigeneration systems might be assessed only if they have similar operational features, see for instance Figure 14a with: $EE_{irr,E}$ = 0.35, and COP = 2, and EE_{cog} = 0.85 and the minimum useful power 50% from engine power, i.e., x_{max} = 1.

In Figure 14b are compared the ideal reversible energy efficiency for ideal trigeneration cycle built with ideal Carnot cycles, with EE_E = 0.75 for θ_{HS} = 4 as in Table 1, and COP = $T_{RS}/(T_{0S} - T_{RS})$ with temperatures from Table 2, and EE_{cog} = 1 and the minimum useful power 50% from engine power, i.e., x_{max} = 1.

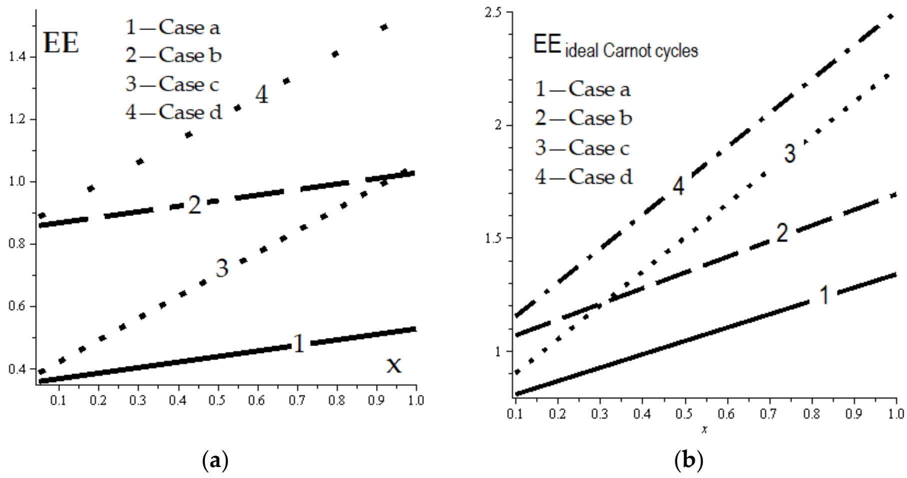


Figure 14. (**a**) The irreversible energy efficiency of trigeneration systems. (**b**) The ideal reversible energy efficiency of trigeneration systems.

5. Conclusions

The paper suggests an original and generalizing FPDT mathematical model to design irreversible trigeneration cycles. The mathematical model minimizes the finite physical dimensions control parameters, and operational corresponding dependence functions of engine and refrigeration cycles included in a trigeneration system.

There are two kind of control parameters, four external and two internal. The four external control parameters are pertaining to external heat transfer—i.e., two mean log temperature differences and two dimensionless thermal conductance inventories. The internal ones are the reference entropy and the number of internal irreversibility which delineate a single dimensionless concept a priori evaluating the accumulated internal irreversibility.

The specific dependence of the reference entropy function of the working fluid nature and of the thermal system type is replaced through the operational adopted condition—i.e., the reference entropy particular value is expressed either through the imposed power, or through the imposed heat input as in this paper, or through the imposed energy efficiency or through the imposed reference entropy.

The number of internal irreversibility is a dimensionless parameter generalizing the evaluation of accumulated irreversible entropy generated along the cycle. Therefore, the irreversible trigeneration assessments might be completed without knowing the working fluid nature and the type of thermal systems.

Although before each FPDT work they must be defined the operational possible domain range of the number of internal irreversibility depending on the working fluid nature and on the thermal system type.

The evaluated specific numerical results for the found specific minimum thermal conductance inventory, see Figures 10 and 13, emphasized that as internal irreversibility increased, so too did the conductance inventory, while the mean log temperature differences decreased. Respectively as higher the internal irreversibility as lower the external irreversibility in order to maintain constant energy efficiency.

The Equations (41)–(44) are universal, can be applied for ideal reversible trigeneration cycle, endoreversible, or irreversible ones, see for instance Figure 14a,b. The comparison reversible–endoreversible–irreversible has to use the operational similarity and thus they can be completed various analyses and optimizing assessments.

Author Contributions: Conceptualization: G.D., M.F.; Methodology: G.D.; Software: Ş.G.; Formal analysis: G.D.; M.F.; Resources: G.D.; Writing—original draft: G.D. All authors have read and agreed to the published version of the manuscript.

Funding: Research grant PCCDI 32/2018.

Institutional Review Board Statement: The study was conducted according to the guidelines of the Declaration of Helsinki, and approved by the Institutional Review Board of "Gheorghe Asachi" Technical University of Iaşi, Romania.

Informed Consent Statement: Informed consent was obtained from all subjects involved in the study.

Data Availability Statement: MDPI Research Data Policies at https://www.mdpi.com/ethics (accessed on 20 March 2021).

Acknowledgments: This work was supported by UEFISCDI Romania research grant PCCDI 32/2018 (implicit endorsement for any dissemination).

Conflicts of Interest: The authors declare no conflict of interest.

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