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Research on Air Gap Magnetic Field Characteristics of Trapezoidal Halbach Permanent Magnet Linear Synchronous Motor Based on Improved Equivalent Surface Current Method

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Abstract: Accurate analysis of the air gap magnetic field is the focus of research in the field of precision permanent magnet linear synchronous motors. In this paper, the two-dimensional air gap magnetic field of a secondary trapezoidal Halbach permanent magnet array coreless permanent magnet linear synchronous motor (PMLSM) was taken as our research object. On the basis of the equivalent surface current method, we proposed an improved equivalent analytical algorithm with a trapezoidal side length unit. The equivalent analytical model of the magnetic induction vector of the two-dimensional air gap was established, and the air gap magnetic field of the trapezoidal Halbach array coreless PMLSM was calculated. At the same time, we analyzed the influence of the bottom angle α of a trapezoidal permanent magnet equivalent width coefficient α_w , pole height coefficient α_h , and air gap height coefficient α_g on the amplitude (B_{peak}) and total harmonic distortion (THD_B) of the central magnetic field in the air gap. The results show that α and α_w have a significant influence on the B_{peak} and THD_B of the central magnetic field air gap. With the synergy of α and α_w , we identified the “flux convergence” effect, which makes the maximum range of B_{peak} $\alpha > 90^\circ$ and $\alpha_w < 0.5$. We also found the “equilateral” effect, which causes the minimum region of THD_B to change linearly. The calculation results of the improved equivalent surface current analytical model established in this paper agree with those verified by the finite element method. The calculation is convenient, and the accuracy of the result is high. This research provides a new method for analyzing the air gap magnetic field of a permanent magnet with a nonrectangular cross-section and lays a theoretical foundation for optimizing the PMLSM pole model.

Keywords: permanent magnet linear synchronous motor (PMLSM); equivalent surface current method; analysis; air gap magnetic field; total harmonic distortion (THD_B)



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1. Introduction

PMLSMs have the advantages of a simple structure, large thrust density ratio, high efficiency, and accurate positioning, among others. It is increasingly used in precision and ultraprecision servo drive systems [1–3]. The coreless PMLSM has zero slot effect, zero positioning force, and smooth motion because it does not affect the slot and end effect. Because of the unique structure of coreless PMLSMs, the motor thrust is low. The currently established method for increasing the motor thrust uses a Halbach array. However, the shape and arrangement of permanent magnets have a greater impact on the air gap magnetic field of coreless PMLSMs. The distribution of the air gap magnetic field plays a decisive role in the strength of the motor's thrust fluctuation [4,5]. Therefore, accurate analysis of the air gap magnetic field of coreless PMLSMs and the study of the B_{peak} change and THD_B of the air gap central magnetic field with Halbach array permanent magnet structure are of great interest in the PMLSM research.

At present, the numerical method represented by the finite element method is most frequently used to calculate complex boundaries, multiple media, and nonlinear problems.

However, the preprocessing and calculation of the finite element method is time-consuming and generally used to verify electromagnetic performance after determining various size parameters. Some scholars have studied the air gap magnetic field using the equivalent magnetization, magnetic circuit, magnetic network, and surface current methods. Krop et al. calculated the no-load air gap magnetic field of a PMLSM using an equivalent magnetization method. By optimizing the shape and size of the permanent magnet, the sinusoidal distribution of the no-load air gap magnetic field of the motor was improved [6–8]. This method applies only to the solution of the electromagnetic field of a regularly shaped magnet whose boundary is parallel to the coordinate axis; the medium must be uniform, and the constraint condition that the magnetization direction is completely parallel to the direction of the coordinate system must be satisfied. Shei et al. used the equivalent magnetic circuit method to divide the magnetic field to be solved into several independent elements and calculate the magnetic conductivity of each element. They connected the nodes to form a magnetic network model in order to calculate the magnetic circuit and compare the calculation results in the finite element method [9,10]. The method is challenging when dealing with small structures. For example, when modeling the motor's magnetic field, it is necessary to consider the small changes in the structure of the magnetic network caused by the changes in the primary and secondary relative positions. Liu et al. divided the motor into several independent units—magnetic fields—with uniform medium and regular geometry to calculate the equivalent magnetic conductivity. Because of the similarities between the magnetic and the electrical networks, the magnetic network is calculated by the node method, and the distribution of air gap magnetic density is obtained [11,12]. The method is problematic in solving the magnetic conductivity of adjacent nodes, as the amount of data calculated before and after the nodes move is large, and the calculation model lacks universality.

The equivalent surface current method is effective for calculating the magnetic field of a permanent magnet. The method treats the interior of the permanent magnet as a vacuum, and the magnetic field generated by the permanent magnet is equivalent to the magnetic field generated by its surface current layer. The method does not consider the complex calculation inside the magnet but converts the complex-shaped magnet into the current layer magnetic field calculation on its corresponding surface, effectively improving the calculation accuracy. Lee used the equivalent surface current method to analyze and calculate the PMLSM air gap magnetic field of the trapezoidal Halbach permanent magnet array and obtained the trapezoidal bottom angle that maximizes the motor thrust [13]. Xue et al. combined the equivalent surface current method with the subregion model and the magnetic field superposition principle and divided the permanent magnet motor into two subregion models for the analytical calculation of its magnetic field [14]. However, the calculation required to conduct the permanent magnet's equivalent processing is extensive, resulting in the low analytical accuracy of the air gap magnetic field. Sun et al. also used the equivalent surface current method to treat the permanent magnet equivalently [15–17].

Some scholars have studied a permanent magnet linear synchronous motor with a rectangular permanent magnet structure. Boduroglu proposed three new coreless PMLSMs with a rectangular permanent magnet arrangement; analyzed the back EMF, air gap magnetic induction strength, and THD_B of the motor; and experimentally verified the motor thrust and thrust fluctuation [18]. Ma proposed a single-layer concentric armature winding structure and analyzed and verified the secondary magnetic field composed of rectangular permanent magnets in the motor [19]. It can be seen from the above literature review that most research is focused on rectangular permanent magnets. The design scheme of trapezoidal permanent magnets changes the right-angle structure of traditional rectangular permanent magnets, resulting in the need to reconsider the influence of trapezoidal bottom angles in calculating the air gap magnetic field. Research on the influence of trapezoidal bottom angles on the B_{peak} and THD_B of magnetic fields in the air gap center of permanent magnet arrays whose bottom angle is not equal to 90° still needs to be completed. In addition, the analytical calculation method of the air gap magnetic field is greatly influenced

by the geometric shape of the permanent magnet. When its shape is irregular and the magnetization direction is complex, the difficulty of using the traditional equivalent surface current method to calculate the air gap magnetic field is particularly evident; the method cannot reflect the internal characteristics of the real air gap magnetic field.

To sum up, in this paper, we establish an improved equivalent analysis algorithm based on the principle of the equivalent surface current method, with the trapezoidal side-length unit, in order to accurately calculate the air gap magnetic field of the trapezoidal Halbach permanent magnet linear synchronous motor and reveal the influence of the trapezoidal bottom angle α on the B_{peak} change and THD_B of the central air gap magnetic field. We study the bottom angle α of a trapezoidal permanent magnet, the equivalent width coefficient α_w , the pole height coefficient α_h , the influence of the air gap height coefficient α_g on the B_{peak} of the air gap's center magnetic field, and the THD_B .

2. Model of Improved Equivalent Analytical Algorithm

The trapezoidal Halbach array coreless PMLSM model studied in this paper is shown in Figure 1. In Figure 1a, the primary is the armature winding, and the secondary is the trapezoidal permanent magnet array. The trapezoidal permanent magnet array is mounted on the back yoke. The secondary structures on both sides of the motor are symmetrical and orderly. The difference between the magnetization directions of adjacent permanent magnets is 90° . Figure 1b is a two-dimensional structure diagram of the trapezoidal Halbach array coreless PMLSM model. The symmetrical Halbach array period is shown in the red box area in Figure 1b. The trapezoidal Halbach permanent magnet is symmetrical along the center of the vertically magnetized permanent magnet in a period. It is assumed that the vertically magnetized trapezoidal permanent magnet is the main magnetic pole, and the parallel magnetized trapezoidal permanent magnet is the auxiliary magnetic pole. In order to facilitate the establishment of the model, the derivation shown below sets the absolute coordinate system xoy based on the geometric center of the main magnetic pole.

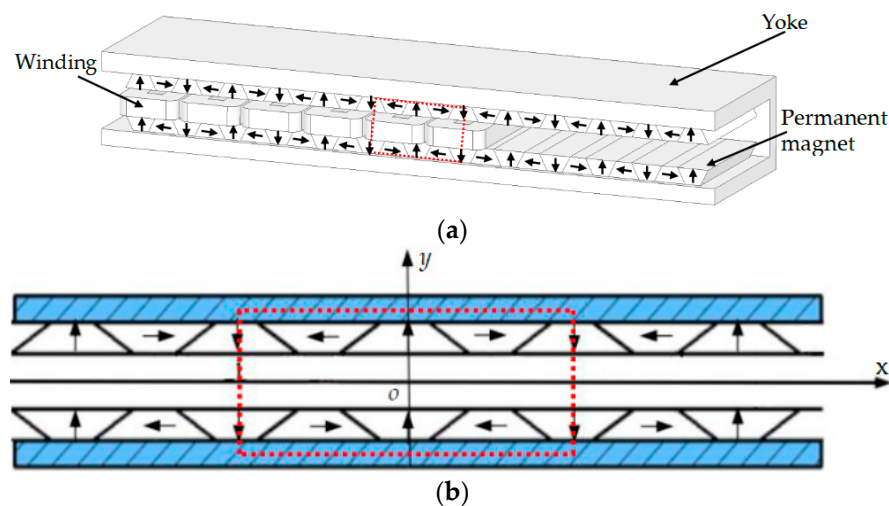


Figure 1. Topology of coreless PMLSM for trapezoidal Halbach permanent magnet array: (a) topology diagram of motor structure, and (b) schematic diagram of the two-dimensional structure of trapezoidal Halbach permanent magnet array.

3. Improved Equivalent Analytical Algorithm Modeling and Calculation Results

Assume the two-dimensional magnetic field of the air gap of the motor is as follows:

1. The secondary array of the motor is infinite along the z -axis;
2. The secondary magnetization of the motor is uniform;
3. The motor yoke is made of aluminum. In the calculation, the air, aluminum, and magnet permeability are taken as vacuum permeability.

3.1. Improved Equivalent Surface Current Method

By Biot–Savart’s Law:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \vec{e}_r}{r^2} \quad (1)$$

where $\vec{e}_r$ is the unit vector, $\mu_0 = 4\pi \times 10^{-7}$ is the permeability in a vacuum, and r is the vector diameter from the source point (i.e., any point on I) to the $p(x, y)$.

On the basis of the principal coordinate system established in Figure 1, take the horizontal left magnetized magnetic pole as an example and set a relative coordinate system $x_t o_t y_t$ of each side of the left magnetized magnetic pole in the Halbach permanent magnet array period, as shown in Figure 2a. The center of the side length is o_t , y_t coincides with the side length, the included angle with M is an acute angle α_{vi} , and x_t is perpendicular to the side length, as shown in Figure 2b. Since the magnetizing direction of the magnetic pole is horizontal to the left, its surface current is composed of four sides (I, II, III, IV). Side IV is the equivalent of the current inflow, and the other three sides are the equivalent of the current outflow. Figure 2c shows the included angle α_{v1} between side length I and the magnetizing direction.

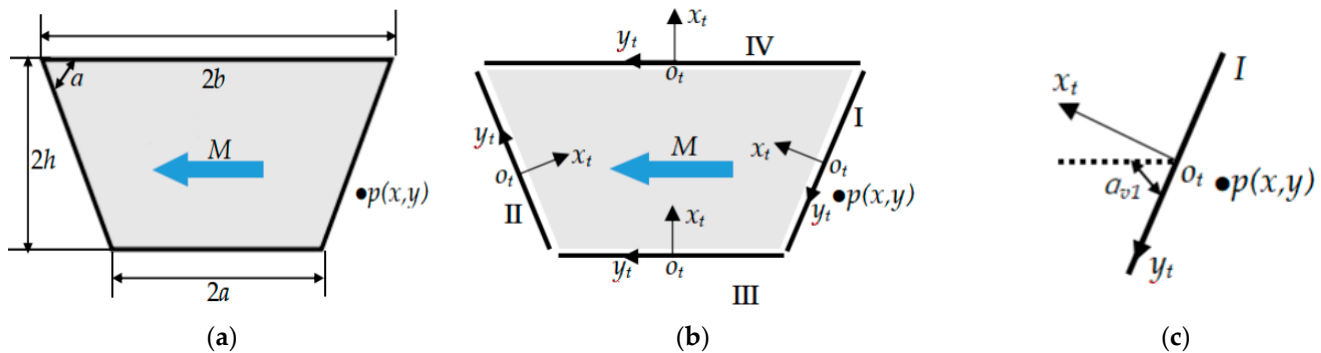


Figure 2. Schematic diagram of improved equivalent analytic algorithm modeling with trapezoidal side length as a unit: (a) horizontal left magnetization, (b) relative coordinate system established by side length, and (c) angle α_{v1} between side I and the magnetizing direction.

After the relative coordinate system $x_t o_t y_t$ of any edge in Figure 2b is established, the magnetic field generated by the surface current edge I in the air gap magnetic field to any point $p(x_t, y_t)$ in the air gap magnetic field can be obtained from Equation (1) and Figure 2c, as shown in Equation (2):

$$\begin{aligned} B_{ix_t}(x_t, y_t, \alpha_{vi}, L) &= \text{sign}(I) \frac{M\mu_0 \cos \alpha_{vi}}{4\pi} \ln \frac{(y_t + L_i/2)^2 + x_t^2}{(y_t - L_i/2)^2 + x_t^2} \\ B_{iy_t}(x_t, y_t, \alpha_{vi}, L) &= \text{sign}(I) \frac{M\mu_0 \cos \alpha_{vi}}{2\pi} \left[\arctan \left(\frac{y_t - L_i/2}{x_t} \right) - \arctan \left(\frac{y_t + h/2}{x_t} \right) \right] \end{aligned} \quad (2)$$

$\text{Sign}(I)$ is a symbolic function, and the surface current I that flows out of the paper is +1; otherwise, it is −1. I is the serial number of the current side.

According to coordinate transformation, in the absolute coordinate system xoy , there are:

$$\begin{cases} x_t = (x - x_{0t}) \cos \alpha + (y - y_{0t}) \sin \alpha \\ y_t = -(x - x_{0t}) \sin \alpha + (y - y_{0t}) \cos \alpha \end{cases} \quad (3)$$

Subsuming Equation (3) into Equation (2) to obtain the basic mathematical model of magnetic field calculation in an absolute coordinate system with side length as the unit is shown in Equation (4):

$$\begin{aligned}
& B_{ix_i}(x, y, x_{0t}, y_{0t}, \alpha_{vi}, L_i, \alpha_i) \\
&= \text{sign}(I) \frac{M\mu_0 \cos \alpha_{vi}}{4\pi} \ln \frac{(-(x-x_{0t}) \sin \alpha_i + (y-y_{0t}) \cos \alpha_i + L_i/2)^2 + ((x-x_{0t}) \cos \alpha_i + (y-y_{0t}) \sin \alpha_i)^2}{(-(x-x_{0t}) \sin \alpha_i + (y-y_{0t}) \cos \alpha_i - L_i/2)^2 + ((x-x_{0t}) \cos \alpha_i + (y-y_{0t}) \sin \alpha_i)^2} \\
& B_{iy_i}(x, y, x_{0t}, y_{0t}, \alpha_{vi}, L_i, \alpha_i) \\
&= \text{sign}(I) \frac{M\mu_0 \cos \alpha_{vi}}{2\pi} \left(\arctan \frac{(y-y_{0t}) \cos \alpha_i - L_i - (x-x_{0t}) \sin \alpha_i}{(x-x_{0t}) \cos \alpha_i + (y-y_{0t}) \sin \alpha_i} - \arctan \frac{(y-y_{0t}) \cos \alpha_i + L_i - (x-x_{0t}) \sin \alpha_i}{(x-x_{0t}) \cos \alpha_i + (y-y_{0t}) \sin \alpha_i} \right)
\end{aligned} \quad (4)$$

where α_i is the angle between the relative coordinate and the absolute coordinate, (x_{0t}, y_{0t}) is the origin coordinate of the migration coordinate system, and L_i is the side length of the calculated surface current.

When Equation (4) is introduced into Equation (3), the magnetic induction vector of the i_{th} equivalent current edge in the absolute coordinate system at $p(x, y)$ can be obtained, as shown in Equation (5):

$$\begin{cases} B_{ix}(x, y, x_i, y_i, \alpha_{vi}, L_i, \alpha_i) = B_{ix_i}(x, y, x_i, y_i, \alpha_{vi}, L_i, \alpha_i) \cos \alpha_i - B_{iy_i}(x, y, x_i, y_i, \alpha_{vi}, L_i, \alpha_i) \sin \alpha_i \\ B_{iy}(x, y, x_i, y_i, \alpha_{vi}, L_i, \alpha_i) = B_{ix_i}(x, y, x_i, y_i, \alpha_{vi}, L_i, \alpha_i) \sin \alpha_i + B_{iy_i}(x, y, x_i, y_i, \alpha_{vi}, L_i, \alpha_i) \cos \alpha_i \end{cases} \quad (5)$$

Equations (3)–(5) calculate only the magnetic field generated by a single side length of the magnetic pole.

Assume that the included angle between the side close to the air gap center and the trapezoidal waistline is the bottom angle. For the trapezoidal Halbach permanent magnet array, the trapezoidal bottom angle is divided into $0 \leq \alpha \leq 90^\circ$ and $90^\circ \leq \alpha \leq 180^\circ$, as shown in Figure 3a,b. In Figure 3, the y -axis is the symmetrical center of the main magnetic pole, the x -axis is the center line of the air gap, h is the height of the permanent magnet, g is the height of the air gap, τ is the length of the magnetic pole distance, and the equivalent width w is the width of the main magnetic pole waistline.

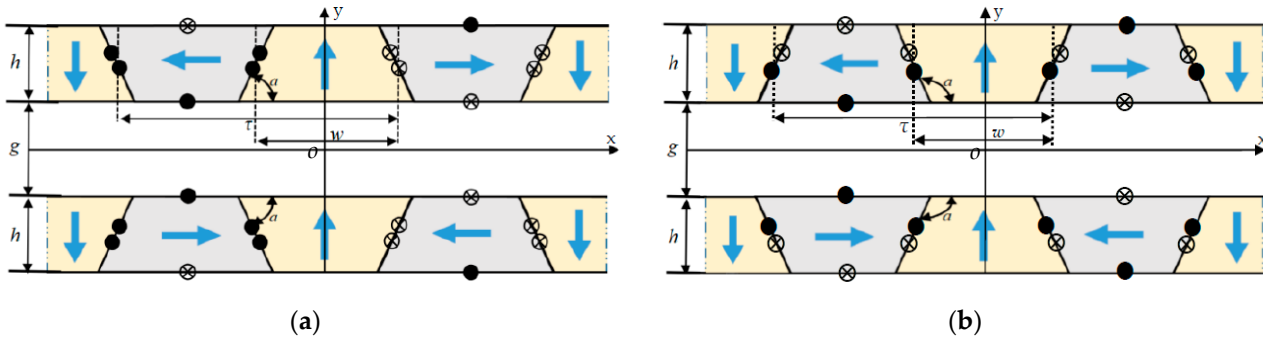


Figure 3. Analytical model of the modified equivalent surface current method for trapezoidal Halbach permanent magnet array: (a) $0 < \alpha < 90^\circ$ and (b) $90^\circ < \alpha < 180^\circ$.

For the universal research method, the model parameters are treated as dimensionless, and the feature length $\tau = 1$ is introduced. The following three dimensionless pole structure coefficients can be obtained: for the equivalent width coefficient of trapezoidal magnet α_w , take $\alpha_w = w/\tau$; for the height coefficient of trapezoidal magnet α_h , take $\alpha_h = h/\tau$; and for air gap height coefficient α_g , take $\alpha_g = g/\tau$.

Figure 3 shows that the bilateral Halbach array of a single period has 24 sides in total. Therefore, from Equation (5) and the relationship between the coordinates α_{vi} , L_i , α_i , and (x_i, y_i) of the midpoint of each current side, it can be shown that the magnetic induction vector generated by the trapezoidal Halbach permanent magnet array to $p(x, y)$ in the air gap magnetic field is:

$$\begin{aligned}
B_x(x, y) &= \sum_{i=1}^{24} B_{ix}(x, y, x_i, y_i, \alpha_{vi}, L_i, \alpha_i) \\
B_y(x, y) &= \sum_{i=1}^{24} B_{iy}(x, y, x_i, y_i, \alpha_{vi}, L_i, \alpha_i)
\end{aligned} \quad (6)$$

It can be seen from Figure 3 and Equation (6) that x_i , y_i , α_{vi} , L_i , and α_i can be replaced by the four basic parameters α , α_w , α_h , and α_g , and their expression is shown in Equation (7). For example, $\{\pm(\tau - 0.5\alpha_w), \pm\tau, \pm 0.5\alpha_w\}$ is a set of x_i , which takes α_w as the variable.

$$\begin{cases} x_i(\alpha_w) \in \left\{ \pm\left(\tau - \frac{1}{2}\alpha_w\right), \pm\tau, \pm\frac{1}{2}\alpha_w \right\}; \\ x_{i,j} = \{x_i \pm 2j\tau\}; \\ y_i(\alpha_g, \alpha_h) \in \left\{ \pm\frac{1}{2}\alpha_g, \pm\frac{1}{2}(\alpha_g + \alpha_h), \pm\left(\frac{1}{2}\alpha_g + \alpha_h\right) \right\}; \\ L_i(\alpha_w, \alpha_h, \alpha) \in \left\{ \tau - \alpha_w \pm \frac{\alpha_h}{\tan\alpha}, \alpha_h \sqrt{1 + (2/\tan\alpha)^2} \right\}; \\ \alpha_i(\alpha) \in \left\{ \pm\left(\frac{\pi}{2} + \alpha\right), \pm\left(\frac{\pi}{2} - \alpha\right), \pm\frac{\pi}{2} \right\}; (0 \leq \alpha \leq \frac{\pi}{2}) \\ \alpha_{vi}(\alpha) \in \{0, \alpha, \frac{\pi}{2} - \alpha\}; \\ i = 1, 2, \dots, 24 \quad j = 1, 2, \dots, \infty \end{cases} \quad (7)$$

To sum up, on the basis of the improved equivalent surface current method proposed by Equation (5), in this paper, first, the edges of each trapezoidal permanent magnet in the trapezoidal Halbach permanent magnet array are calculated equivalently. Then, the transformation relationship between the local and the global coordinate systems and the superposition principle is used to superimpose the magnetic fields generated by all the surface currents and finally obtain the complete and accurate air gap magnetic field distribution.

3.2. THD_B Calculation Method

In this paper, the total harmonic distortion THD_B [20] of the magnetic induction vector at the center of the air gap is taken as the characteristic quantity of the magnetic induction vector waveform sine at the center of the air gap:

$$THD_B = \frac{1}{B_1} \sqrt{\sum_{n=1}^{\infty} B_{2n+1}^2} \quad (8)$$

where B_{2n+1} is the odd harmonic amplitude of the air gap magnetic density and B_1 is the amplitude of the air gap magnetic density fundamental wave.

The amplitude of the magnetic induction vector at the center of the air gap in Equation (8) can be calculated by the Fourier transform of discrete periods.

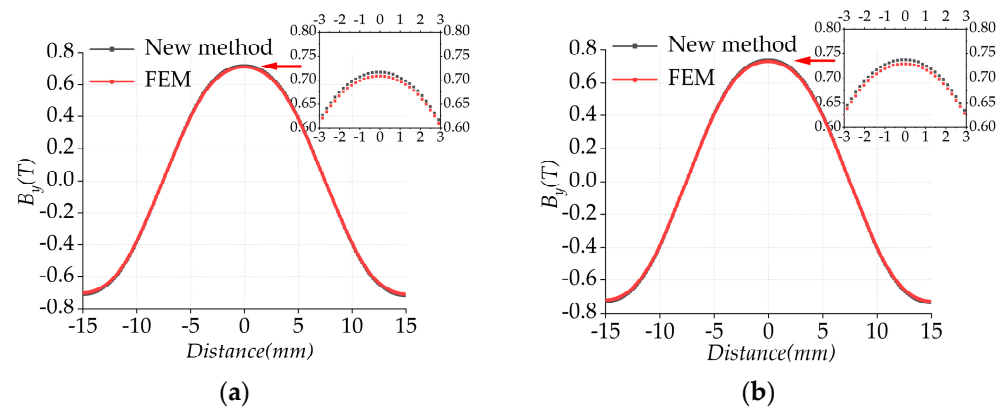
3.3. Calculation Results and Finite Element Verification

To verify the correctness of the improved equivalent surface current method, this paper takes the trapezoidal Halbach permanent magnet array model established in Figure 3 as an example and applies Equations (5)–(7) to analyze the B_y of the air gap's center in the trapezoidal Halbach permanent magnet array model, where the bottom angle α is equal to 70° and 110° , respectively. The calculation results were also validated by the finite element method. The parameters of the trapezoidal Halbach permanent magnet array are shown in Table 1, and the calculation results are shown in Figure 4.

In Figure 4, when $\alpha = 70^\circ$, the B_{peak} of B_y equals 0.7162 T. When $\alpha = 110^\circ$, the B_{peak} of B_y is equal to 0.7367 T. The improved equivalent surface current method established in this paper and the finite element method have the same calculation results for the air gap magnetic field. The waveforms of the air gap magnetic density all show sinusoidal characteristics. The calculation results of the two methods deviate only slightly in the local waveform. With the simulation results as the benchmark, the maximum relative error is 0.031%.

Table 1. Structural parameters of THPMA.

Symbol	Quantity	Value
w	the equivalent width of a trapezoidal permanent magnet	4.8 mm, 10.2 mm
h	height of trapezoidal permanent magnet	9 mm
τ	pole pitch	15 mm
g	air gap height	9 mm
M	magnetization	1.05×10^6 A/m
μ_0	permeability	$4\pi \times 10^{-7}$ T·m/A
α	bottom angle of trapezoidal	$70^\circ, 110^\circ$

**Figure 4.** Comparison of the improved equivalent surface current method and the finite element method results for the single-period air gap's center magnetic field: (a) $\alpha = 70^\circ$, and (b) $\alpha = 110^\circ$.

4. Analysis of the Influence of Trapezoidal Bottom Angles α and α_w , α_h , and α_g on the Air Gap Magnetic Field

On the basis of Figures 2 and 3 as well as Equations (5)–(7), this section analyzes the influence of the changes in the bottom angles α and α_w , α_h , and α_g on the air gap magnetic field in the ladder-type Halbach permanent magnet array model. The section explores the influence of various factors on the maximum value of the air gap central magnetic field amplitude B_{peak} and the minimum value of the total harmonic distortion THD_B . To ensure the universality of the calculation results, the following calculation parameters were selected: $\tau = 1$, $60^\circ \leq \alpha \leq 120^\circ$, $0.3 \leq \alpha_w \leq 0.7$, $0.3 \leq \alpha_h \leq 0.7$, and $0.5 \leq \alpha_g \leq 0.9$ (see Table 2 for detailed calculation and analysis scheme).

Table 2. Analysis scheme of the objective function and calculation parameters.

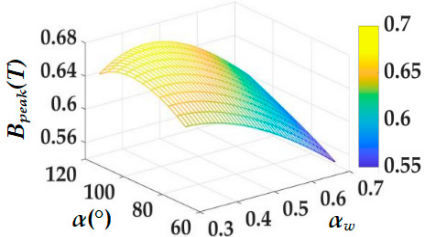
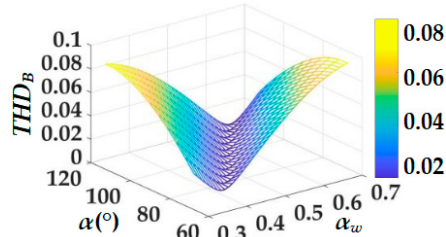
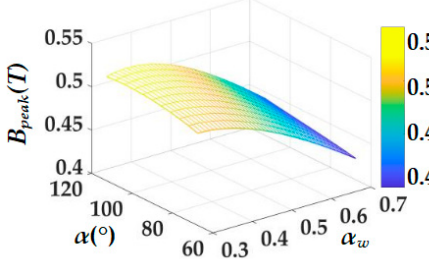
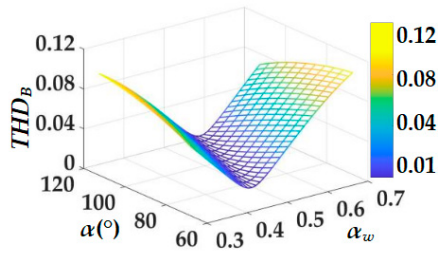
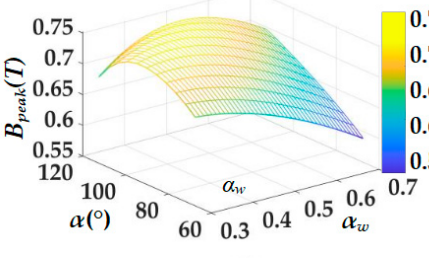
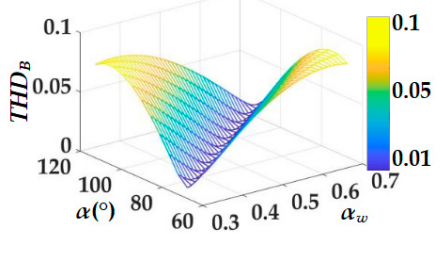
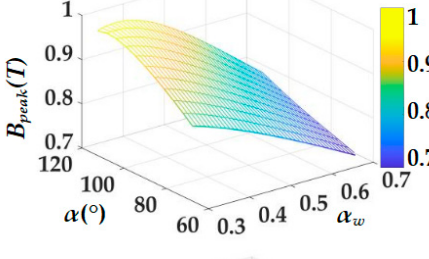
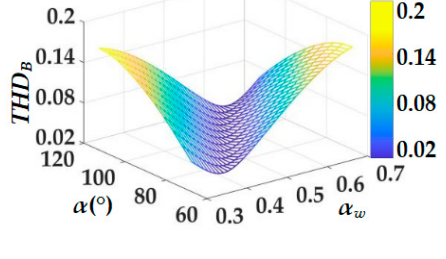
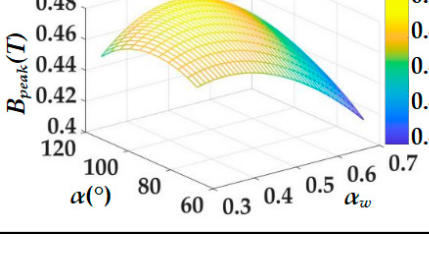
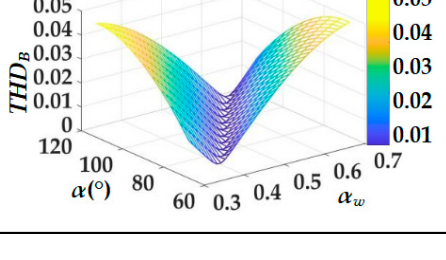
Objective Function	Calculation Parameters	
$B_{peak}(\alpha, \alpha_w)$ $THD_B(\alpha, \alpha_w)$	$\alpha_g = 0.5$	$\alpha_h = 0.5$
	$\alpha_g = 0.5$	$\alpha_h = 0.3$
	$\alpha_g = 0.5$	$\alpha_h = 0.7$
	$\alpha_h = 0.5$	$\alpha_g = 0.3$
		$\alpha_g = 0.7$

4.1. The Influence of α and α_w on the $B_{peak}(\alpha, \alpha_w)$ and $THD_B(\alpha, \alpha_w)$ of Air Gap Central Magnetic Field

Bottom angles α and α_w are variables with different α_h and α_g as parameters; the variation rules of the calculated air gap center magnetic field $B_{peak}(\alpha, \alpha_w)$ and $THD_B(\alpha, \alpha_w)$ are shown in Table 3. In the calculation area, $B_{peak}(\alpha, \alpha_w)$ has a maximum point in the area of $\alpha > 90^\circ$ and $\alpha_w < 0.5$, the relative change rate of $B_{peak}(\alpha, \alpha_w)$ is about 2%, and the change rate near the maximum is lower. $THD_B(\alpha, \alpha_w)$ have multiple minimum points, and the distribution of the minimum points changes linearly with the α and α_w . It can be seen from Table 3 that increasing or decreasing α_w has a significant influence on the distribution of the minimum of

$THD_B(\alpha, \alpha_w)$. The α_h value affects the trend of the distribution of the minimum of $THD_B(\alpha, \alpha_w)$ but does not change the shape of the distribution map of the minimum of $THD_B(\alpha, \alpha_w)$. The α_g value does not affect the trend of the distribution of the minimum of $THD_B(\alpha, \alpha_w)$ but changes the shape of the distribution map of the minimum of $THD_B(\alpha, \alpha_w)$.

Table 3. Analysis of the influence rule of α and α_w on $B_{peak}(\alpha, \alpha_w)$ and $THD_B(\alpha, \alpha_w)$ of the central magnetic field of the air gap.

Calculation Parameters	$B_{peak}(\alpha, \alpha_w)$	$THD_B(\alpha, \alpha_w)$
$\alpha_h = 0.5, \alpha_g = 0.5$ (Basic type, $B_{peak}(\alpha, \alpha_w)$ has a maximum value area, and the minimum value area of THD_B is linear.)		
$\alpha_h = 0.3, \alpha_g = 0.5$		
$\alpha_h = 0.7, \alpha_g = 0.5$ (The difference in α_h affects the B_{peak} and THD_B trends.)		
$\alpha_h = 0.5, \alpha_g = 0.3$		
$\alpha_h = 0.5, \alpha_g = 0.7$ (The difference in α_g affects the steepness of B_{peak} and THD_B .)		

In the PMLSM design, it is generally required that the thrust is large and the fluctuation is small; that is to say, the B_{peak} is maximum and the THD_B is minimum. It can be seen from Table 3 that under the coupling effect of the trapezoidal magnet bottom angle α and the

three pole structure parameters α_w , α_h , and α_g , α_w has the strongest influence on the B_{peak} and THD_B , while α_h and α_g have only monotonic influence on the B_{peak} and THD_B . The higher α_h is, the greater the air gap magnetic field strength is. However, when $\alpha_h > 0.75$, the enhancement of the air gap magnetic field strength is significantly slowed down. The smaller α_g is, the greater the air gap magnetic field strength is, but the reduction range of α_g is restricted by the thickness of the primary coil; α_h and α_g have little effect on the air gap central magnetic field THD_B . The accurate analysis of the influence mechanism of α and α_w on the B_{peak} and THD_B of the air gap central magnetic field is the basis for the correct selection of the trapezoidal magnet's structural parameters.

4.2. Analysis of Influence Mechanism of α and α_w on the B_{peak} and THD_B of Air Gap Central Magnetic Field

According to the analysis in Table 3, α_w is the most significant factor affecting the B_{peak} and THD_B among the four structure parameters. The maximum B_{peak} of the air gap center magnetic field is in the area where $\alpha > 90^\circ$ and $\alpha_w < 0.5$, and the minimum area of the THD_B changes linearly. This section was divided into three angles of $\alpha = 75^\circ$, 90° , and 105° taken as the calculation observation points with $\alpha_w = 0.4$, $\alpha_h = 0.55$, $\alpha_w = 0.5$, and $\tau = 15$ mm. The improved equivalent surface current method was used to calculate and analyze the magnetic field distribution in the air gap–magnet yoke area and analyze the α and α_w influence of on the B_{peak} and THD_B of the air gap center magnetic field. The calculation results are shown in Figures 5–7, respectively. Figures 5a, 6a and 7a show the distribution of magnetic induction vector $|B|$ in a period, and Figures 5b, 6b and 7b show the localized view of magnetic induction vector B .

It can be seen from Figures 5–7 that if the auxiliary magnetic pole is magnetized transversely, the magnetic induction vector acts on the side of the main magnetic pole, thus strengthening the magnetic field intensity in the main magnetic pole, resulting in the effect that the magnetic field intensity inside the main magnetic pole is greater than that inside the auxiliary magnetic pole. Comparing the magnetic induction vector inside the main magnetic pole and the auxiliary magnetic pole in Figures 5a, 6a and 7a, it is found that when $\alpha_w < 0.5$, the magnetic field intensity inside the main magnetic pole is significantly higher than that inside the auxiliary magnetic pole, which constitutes the “flux convergence” effect of the magnetic induction vector. When α_w is unchanged, $\alpha > 90^\circ$ is conducive to the formation of “flux convergence” effect on the geometric structure when the magnetic induction vector in the main magnetic pole enters the air gap magnetic field. Compared with the observation points shown in Figures 5b, 6b and 7b, it is evident that the magnetic induction vector of the main magnetic pole is further improved.

In Figures 5–7, the $|B|$ value on the center line of the air gap changes periodically with its adjacent magnetic poles. The maximum or minimum value of $|B|$ is obtained at the central position of the main magnetic pole, and $|B| = 0$ at the central position of the auxiliary magnetic pole. It can be seen from the periodic characteristics of harmonics that when the side lengths of the adjacent main and auxiliary magnetic poles at the air gap side are essentially the same, the THD_B is smaller (referred to as the “equilateral” effect), achieving the standard sinusoidal waveform. Table 3 shows that when the side-length ratio of the adjacent air gap between the main and the auxiliary magnetic poles changes by one because of the shift in α_h , it corresponds to the specific α_h . When THD_B reaches the minimum value, the side lengths of the main and the auxiliary magnetic poles in the adjacent air gap are not strictly equal.

The calculation results of the B_{peak} and THD_B of the center magnetic field of the air gap are shown in Table 4. When $\alpha = 105^\circ$, the magnetic induction vector in the main magnetic pole enters the air gap magnetic field to form a “flux convergence” effect in the geometric structure, and the magnetic induction vector of the main magnetic pole is the largest. When $\alpha = 75^\circ$, the side lengths of the main magnetic pole and the auxiliary magnetic pole near the air gap are approximately equal and the “equilateral” effect is noticeable, then the THD_B is minimal.

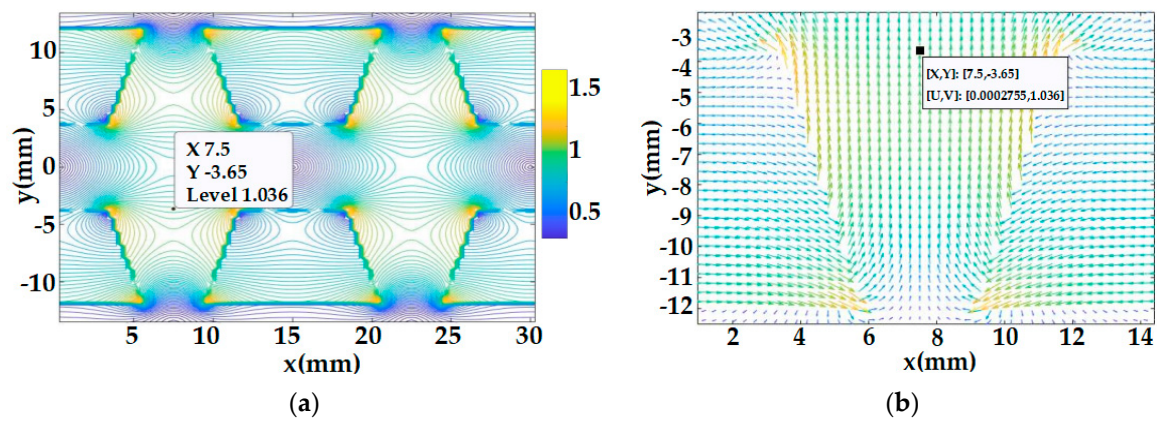


Figure 5. Distribution diagram of air gap magnetic field ($\alpha = 75^\circ$): (a) distribution cloud diagram of $|B|$ and (b) localized view of magnetic induction vector B .

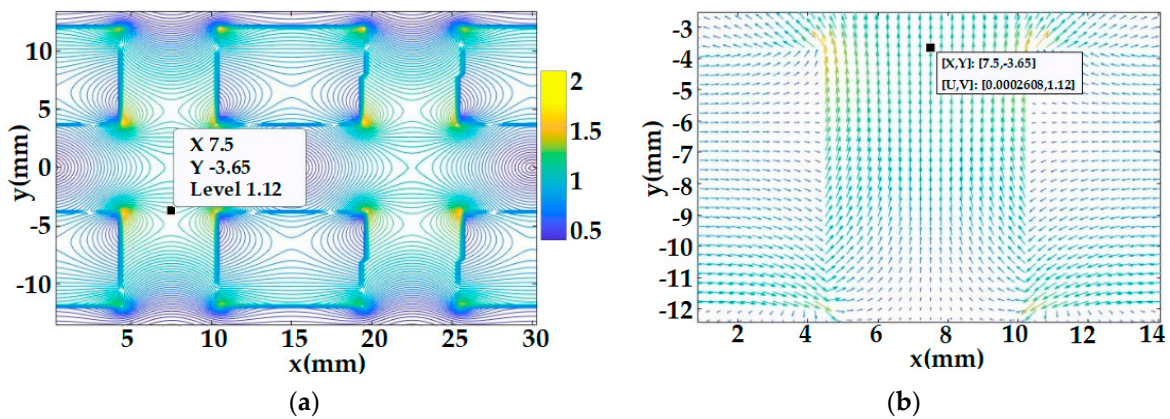


Figure 6. Distribution diagram of air gap magnetic field ($\alpha = 90^\circ$): (a) distribution cloud diagram of $|B|$ and (b) localized view of magnetic induction vector B .

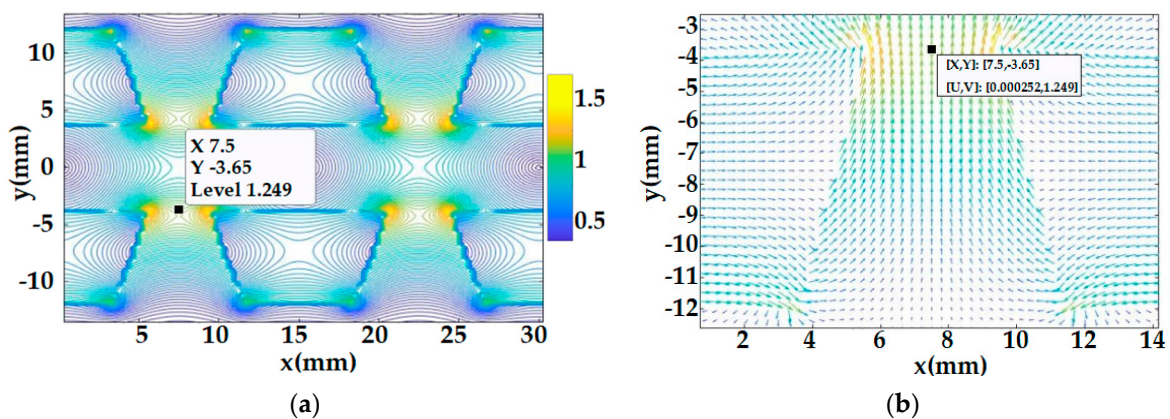


Figure 7. Distribution diagram of air gap magnetic field ($\alpha = 105^\circ$): (a) distribution cloud diagram of $|B|$ and (b) localized view of magnetic induction vector B .

Table 4. Calculation results of the B_{peak} and THD_B of the central magnetic field in the air gap.

α	B_{peak} (T)	THD_B
75°	0.8754	0.0136
90°	0.9027	0.0475
105°	0.9205	0.0687

5. Conclusions

In this paper, the characteristics of the air gap magnetic field of the coreless PMLSM of a trapezoidal Halbach array are studied on the basis of an improved equivalent surface current method. The main conclusions are as follows:

1. We present an improved equivalent surface current analytical algorithm for calculating the coreless PMLSM air gap magnetic field of a two-dimensional Halbach permanent magnet array, with the polygon side length as the element. This algorithm can fully reflect the internal characteristics of the real air gap magnetic field, and the calculation results are completely consistent with those of the finite element method, effectively proving the correctness of the new method;
2. Under the coupling effect of the bottom angle α of the trapezoidal magnet, α_w , α_h , and α_g , α_w is the most influential factor affecting the B_{peak} and THD_B among the three magnetic pole structure parameters, and α_h and α_g have only a monotonic effect on the B_{peak} and THD_B ;
3. The maximum B_{peak} value of the central magnetic field air gap is around $\alpha > 90^\circ$ and $\alpha_w < 0.5$, and the minimum value of the THD_B of the central magnetic field of the air gap changes linearly;
4. In the Halbach layout of the magnetic pole array, the “flux convergence” effect and the “equilateral” effect are the main factors influencing the area of the maximum B_{peak} and the area of the minimum THD_B .

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