

Review

Efficiency of Applying the Nyquist and V.M. Popov Criteria for Stability Analysis of Linearized Automatic Control Systems in Electromechanics and Power Engineering

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Abstract: It is well known that all real automatic control systems (ACS) are significantly nonlinear systems. Along with this, in engineering analysis the simplifications of their mathematic description, that is, linearization, are inevitable. These simplifications naturally introduce errors into engineering calculations. For the most complicated calculation of stability conditions, these errors can be very significant, since they lead to not only incorrect forecast of the ACS condition but also to the operating inability of the designed ACS. Most often, engineers simplify nonlinear structures in their calculations. In other words, they linearize these structures according to one of the numerous approaches which have been developed over 100 years. Calculation methods for linear ACS, in particular the Nyquist criterion, can be applied to the already linearized structure for the calculation of ACS stability condition. In this article, the authors suppose that for electromechanical and power ACS it would be more effective to widely use the absolute stability criteria or hyperstability which were developed when solving the “Aizerman problem” by V.M. Popov in the 1960s of the 20th century. This solution is not used as widely as the Nyquist criterion despite its outstanding qualities. In this article, the assumption was made that it is effective to apply the stability criterion of nonlinear ACS for the linearized models of nonlinear systems. Otherwise speaking, it is effective to use the criterion of V.M. Popov and its interpretation according to the logarithmic frequency characteristics of the ACS proposed earlier by the authors of the article. The authors provide simple and illustrative examples and give the results of the simulation.

Keywords: discreteness; frequency response; automatic control system; PID controller; variable structure system with sliding processes



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1. Introduction

Most of the real automatic control systems (ACS) in the electric power industry and electromechanics are significantly nonlinear systems. The nonlinear elements of the ACS solve very important problems, and it is precisely the nonlinearities that make it possible to solve them with high efficiency.

Examples of such high-tech nonlinear ACS elements are semiconductor power converters with pulse elements, AC motors; in particular, double-feed machines (DFM) in the electric power industry; high-precision mechanical structures with backlash selection devices, etc. Most often, it is impossible to accurately take into account all nonlinearities in engineering calculations.

The most important step in the analysis of a complex ACS is the assessment of its stability.

For several decades, the main method of stability analysis of a nonlinear ACS was the method called the method of harmonic balance in the USSR.

“The mathematical basis of this method . . . was described by N.M. Krylow and N.N. Bogolyubov . . . ” [1] (p. 274). Many authors [1–4] significantly simplified this method for

engineering calculations. The method was used to calculate specific technical systems and simple and illustrative models.

As applied to the frequency characteristics of ACS links, this method is reduced to the analysis of boundary stability according to the Nyquist criterion for a specific operating mode and to linearization of nonlinearity of this exact “operating point” [5]. This approach was applied to power systems since the operating modes of main electric generators were initially quite stable. The method of harmonic balance was cumbersome and gave large errors in the calculations of electromechanical ACS—tracking and stabilization systems. These errors became especially significant in the analysis of processes far from boundary stability, such as monotone processes. In addition, it is rather difficult to obtain the exact parameters of a real ACS. At the end of the 1940s, the Aizerman hypothesis, or “problem”, was formulated: the stability of a nonlinear ACS can be assessed according to the boundary values of nonlinearity using linear criteria [6]. Studies of this problem in different countries showed that this hypothesis was wrong, but the stability of nonlinear ACS with restrictions can be evaluated according to the special criterion called the criterion of absolute stability, or the V.M. Popov criterion [7]. The wording of the V.M. Popov criterion is given below, and it is important to note its two meanings:

- The criterion proves that stability of a nonlinear ACS can be assessed according to the boundary characteristics of nonlinearity;
- It is necessary to consider the frequency characteristic of an ACS linear part in the range from 0 to ∞ for the stability assessment in contradistinction to the Nyquist criterion, which considers frequency characteristics in the frequency range close to the cutoff frequency of ACS.

These features are very important both for theoretical understanding of the stability problem and for practical application of this theory.

This theory received a huge positive response in science, but the approaches based on the “linearization–linear criteria” consequence still prevail in engineering calculations. This leads to quite significant problems.

When calculating the accuracy or energy efficiency of ACS, the simplification of nonlinear dependencies leads to certain errors, which can be assessed in the conclusions. When assessing stability, simplifications of nonlinearities can lead to fundamentally incorrect solutions.

As is known from works on the theory of stability [1–4], the most significant influence on the stability of the ACS is exerted by ambiguous nonlinearities of the “link with hysteresis” type, dynamic nonlinearities, that is, links with varying transfer functions and linear “high-order” links with complex “right” or zero roots of the characteristic equations.

In this paper, the authors consider the influence of links with ambiguous characteristics and links with zero roots of characteristic equations, that is, integrators on the stability of the ACS.

In modern electromechanical ACS (Figure 1), the links with “right roots” are extremely rare, but integral ones are present in significant quantities. These are the links bonding mechanical torque and rotation speed of the motor shaft and operating aggregate, rotation speed, and torque on the output of elastic shafts. In the positioning or tracking ACS, there are links connecting rotation speed and movement. The regulators with integral channels, most often PID-regulators, only add to these links another zero root.

Many authors consider the links with ambiguous characteristics of the «hysteresis» type as a main problem of electromechanical ACS. This problem inhibits having a high accuracy and response, and these links are usually present in the ACS in the “detailed” analysis.

The most important stages in the analysis of stability of such an ACS is the justification of admissibility of the accepted simplifications of nonlinearities, which must be carried out for each specific calculation.

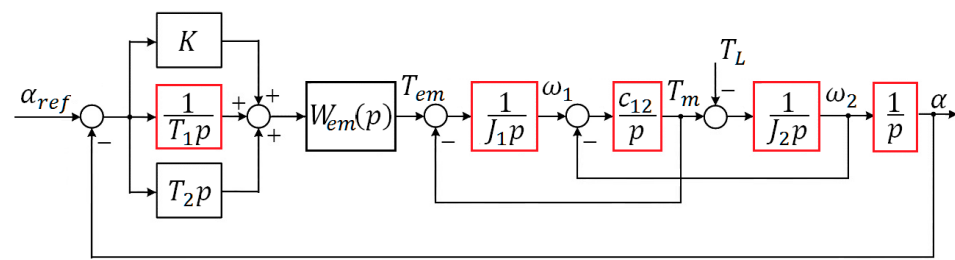


Figure 1. Structural diagram of electromechanical ACS, where T_L is a load torque, T_{em} is an electromagnetic torque, T_m is a mechanical torque, W_{em} is a transfer function of link of electromagnetic torque formation, T_1 and T_2 are time constants of integral and differential channels of the regulator, K is a regulator proportional gain; J_1 is a motor inertia moment; J_2 is an inertia moment of mechanism; ω_1 is a motor speed; ω_2 is a mechanism speed; c_{12} is a mechanism elasticity coefficient; α_{ref} is a reference angle; and α is a displacement angle.

2. Problem statement, State of the Issue

The most widely used criterion in the analysis of the stability of automatic control systems in recent decades is the Nyquist criterion for linear ACS and the V.M. Popov criterion [7] for nonlinear ACS.

The Nyquist stability condition is “non-coverage” by the hodograph of the frequency response of an open ACS $W(j\omega)$ on the complex plane of the “critical” point with coordinates $(-1/K, 0)$ (Figure 2).

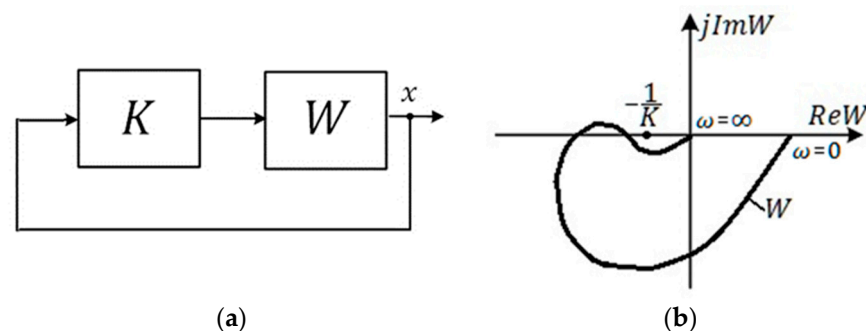


Figure 2. Scheme of the ACS (a) and Nyquist stability conditions for the frequency hodograph of the ACS open part (b).

The interpretation of the criterion according to the logarithmic frequency characteristics (LFC) of ACS is widely known (Figure 3). The Nyquist criterion limits the phase shift $\varphi(\omega_c)$ at the cutoff frequency ω_c to -180 degrees; the criterion is very clear and convenient for engineering calculations, but the presence of at least one nonlinearity makes its application incorrect.

Expressions (1)–(3) describe the Nyquist criterion, where $W(j\omega)$ is a transfer function in the exponential form, $A(\omega_c)$ is an amplitude on cutoff frequency ω_c , $\varphi(\omega_c)$ is a phase on this frequency, and φ_m is a phase margin:

$$W(j\omega) = A(\omega)e^{j\varphi(\omega)} \quad (1)$$

$$A(\omega_c) = 1; \quad (2)$$

$$\varphi(\omega_c) > -180^\circ; \quad (3)$$

$$\varphi_m = \varphi(\omega_c) + 180^\circ. \quad (4)$$

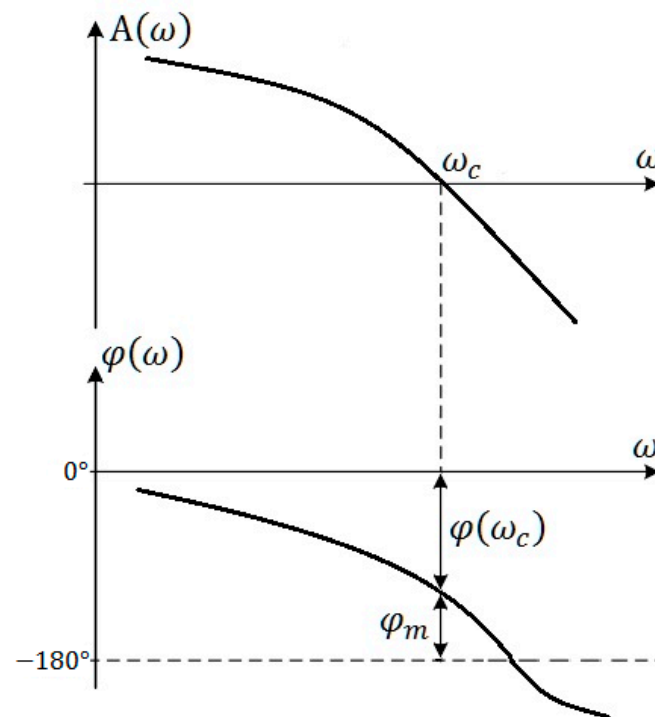


Figure 3. Stability criterion for LFC ACS.

At the same time, the fact of nonlinearity turns out to be more important than the actual values of the variation of any parameter. This is due to the fact that for the analysis of stability in such an ACS, instead of the Nyquist stability criterion one should use the V.M. Popov criterion, which requires the fulfillment of certain requirements for the frequency characteristics of the linear part in the entire frequency range—from 0 to ∞ , and not only in the cutoff frequency zone, as in the Nyquist criterion (Figure 4).

$$\Phi(x) \leq Kx \quad (5)$$

$$\operatorname{Re}(1 + j\omega q)W \geq -\frac{1}{K} \quad (6)$$

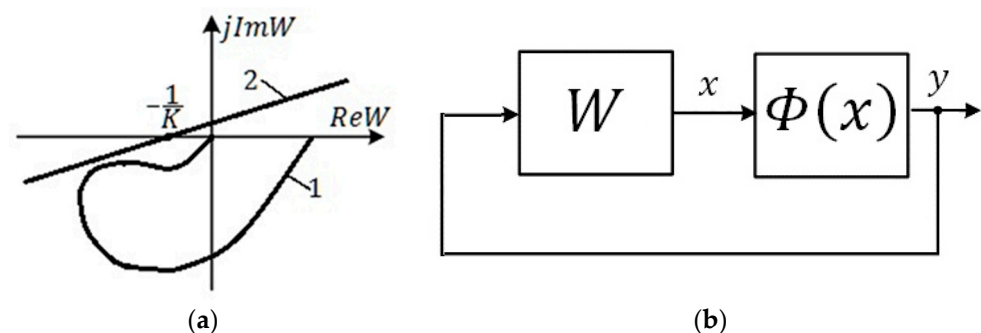


Figure 4. The V.M. Popov criterion, (5) and (6), demands the straight line “2” on frequency plane through critic point laid over hodograph “1” (a) of linear structure of ACS (b).

Many examples of calculations of ACS stability are considered in scientific papers according to the different methods, including the Nyquist criterion and the V.M. Popov criterion. Thus, the collection of “Automatic . . . control” conference materials at Cranfield (England, 1951) and the articles [8,9] provide examples of stability calculations, where the accuracy of the Nyquist criterion is noted and frequency hodographs are used. The method of harmonic balance is used in the same books to consider nonlinear systems.

Naumov's book [1] provides many examples of applying the V.M. Popov criterion. The linear parts and static nonlinearities with accurate description are accented (pp. 105–138); the frequency characteristics are used in the tracking systems when analyzing stability according to the method of harmonic balance (pp. 274–328).

Meerov [2] and other researchers [10–13] consider ACS with nonlinearities and use hodographs of linear parts in all cases. These hodographs are built on the complex plane. At the same time, when the linear ACS is calculated using the Nyquist criterion, the authors use logarithmic frequency characteristics, which are more convenient for engineers.

Unstable processes are most often evidence of emergency situations in power systems. The multidimensionality of the systems demands the greatest attention when analyzing the stability of complex electric power systems. The second method of Lyapunov is also applied, and this method is a basis of the V.M. Popov theory [14–18]. At the same time, frequency characteristics are applied quite rarely.

Electric generators are electromechanical systems with a hard-specified operating mode. The analysis methods based on averaging the parameters using the Krylov–Bogolyubov methods [5] are quite appropriate. Moreover, control of the generators becomes effective only if the condition of operating modes is observed, since the standard methods of controlling the synchronous and asynchronous electric machines are applied using the Park theory, which neglects nonlinearities. LFC and FC are not generally applied for the calculations.

Only wind turbines can switch to undesirable modes, as they operate in conditions of unstable wind load and parameters. They are switched off from the power grid, if it is impossible to correct their work. The standard algorithms are used in the control and inevitable nonlinearity is not taken into account. LFC and FC are not generally applied for the calculations.

It is important to pay attention to the calculation of stability of a nuclear reactor and other specific cases in the book of V.M. Popov [7]. The mathematical description of a nuclear reactor is of particular interest; the cycling of its operation and restrictions for nonlinear blocks are taken into account. The calculation of such a system according to V.M. Popov is most efficient [7] (pp. 302–307).

Obviously, the V.M. Popov criterion is the most logical in the sequence of stability analysis of nonlinear ACS, and it is difficult to use any “direct” approach, or sequence in other words, for this analysis.

- The linearization of a nonlinear element is the linear frequency method of stability analysis that requires an inaccurate stability margin on the nonlinearity. It is expedient to replace it by sequence.
- The linearization (probably harmonic) is the frequency condition of stability of nonlinear systems, which are based on nonlinearity restrictions. It is expedient to change the formulation of the criterion: instead of a complex hodograph, it is advisable to use convenient LFC of linear and linearized blocks of ACS.

In papers [19,20], the authors of this article proposed an interpretation of the V.M. Popov criterion for the LFC of the linear part of an ACS. This interpretation limits the phase shift of the combination of the linear part of the ACS, the boosting link, and the proportional link (Figure 5).

The criteria are very close in application technology, but the Nyquist criterion is most often used in engineering calculations. In these calculations, the structure of the ACS is simplified—the nonlinear functions of the links are replaced by linear ones (linearization of the nonlinearities is carried out) and the stability is estimated according to the engineering version of the Nyquist criterion. The presence of nonlinearities in the ACS is taken into account by a certain “phase stability margin” at the cutoff frequency of the open ACS. This margin should compensate for the action of linearized links and other factors not taken into account in the ACS analysis. When setting the ACS parameters to “average” dynamics indicators, this margin is usually sufficient for stability. Still, in automatic control systems, in which it is required to obtain “ultimate” accuracy and, especially, speed, the

“theoretical” stability conditions very often differ significantly from the real ones. There are a lot of reasons for these inconsistencies, and the nonlinearities of the ACS are not decisive among them.

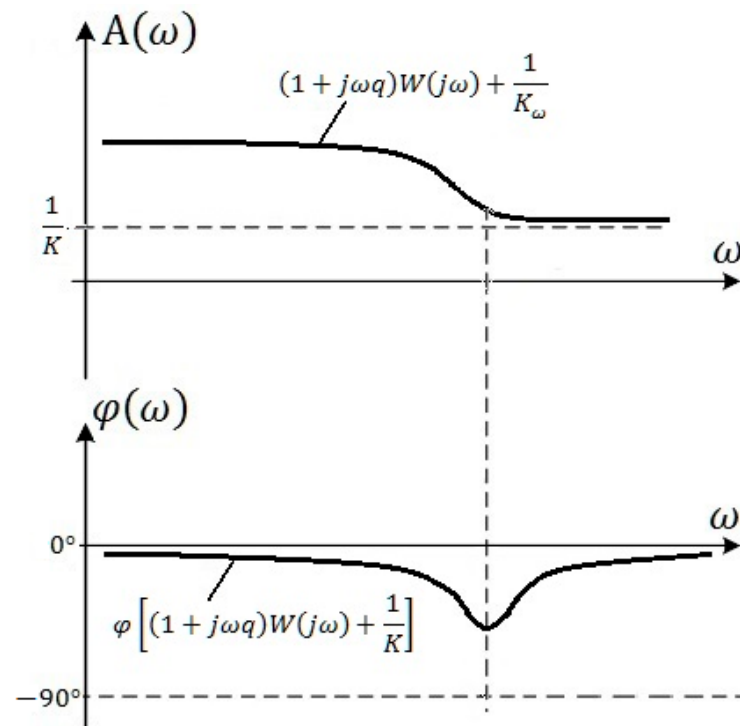


Figure 5. Stability condition according to V.M. Popov criterion is transferred on logarithmic frequency characteristics of ACS.

Proportional-integral-derivative regulators (PID) (Figure 6a) are most often applied in industrial electric drives and power systems. According to the provisions of Automatic Control Theory (ACT) [1–4], these regulators should ensure high efficiency of most complexes in industry and energy.

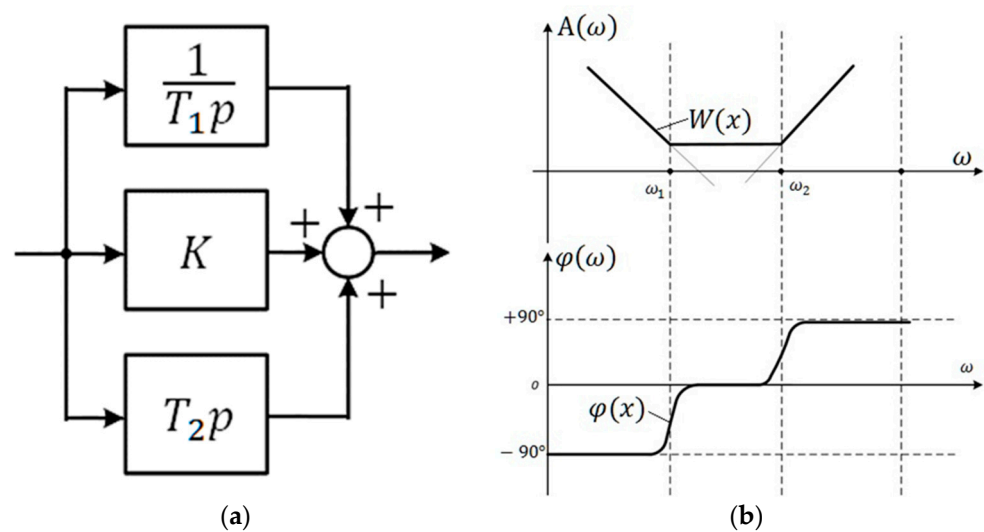


Figure 6. PID controller circuit (a) and its logarithmic frequency characteristic (b).

Expression (7) presents the transfer function W for PID controller, where K is a gain coefficient, T_1 —time constant of I-channel, T_2 —time constant of D-channel, p —Laplace operator.

$$W = K + T_1 p + \frac{1}{T_2 p} \quad (7)$$

According to the logarithmic frequency characteristics of regulator (Figure 6b), an integral (I) channel operates at low frequencies and provides high static accuracy; the proportional (P) channel operates at medium frequencies, at which the cutoff frequency of the ACS is formed. A derivative (D) channel improves the ACS stability at the same frequencies.

However, when optimizing real ACS, engineers quite often have to limit the operation of the integral channel or proportional gain to ensure the stability of the ACS, since the differential channel does not cope with this task. A detailed analysis of the mathematical features of nonlinear ACS has shown the possible reasons for the instability of such an ACS.

3. Theoretical “Origins” of the Problems

The modern ACS consists of an electric motor, electronic power amplifier, mechanical gearbox, and microprocessor regulating device (Figure 7).

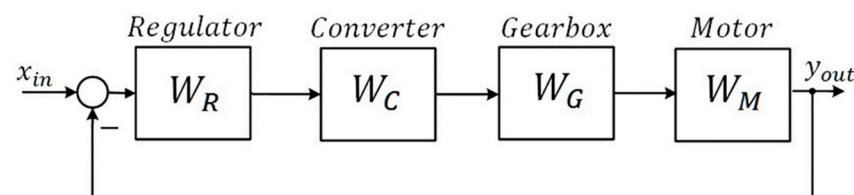


Figure 7. ACS functional diagram.

The dynamic characteristics of these links, as a rule, are significantly different, which most often allow to build stable closed-loop control systems.

Problems with stability arise when trying to obtain the maximum possible characteristics of ACS—accuracy and speed. The Nyquist criterion in the form generally accepted by engineers ceases to work effectively.

The most common formulation of the Nyquist criterion begins with the words: «If an open-loop system is stable . . . ». However, an open-loop control system with a PID-regulator does not fall, strictly speaking, to these systems, because an integral channel “adds” a “neutral” unstable pole to the open-loop part of the ACS.

The systems with the “slave” speed regulation of electric motor demand special attention. They implement two control loops—the speed and current of the armature of the electric motor (an analogue of the mechanical torque). Very often, both circuits are performed with integral channels of regulators—PI—current and speed regulators (Figure 8).

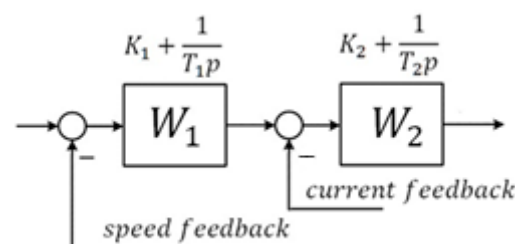


Figure 8. Structure of regulators with slave regulation.

Obviously, in the modes of simultaneous operation of the circuits with small mismatches in speed and variable load, the neutrality index of the open system becomes more

than two. The stability of such a system, evaluated by the engineering Nyquist criterion, is more than problematic.

It should be noted that most often the stability of an open-loop ACS is estimated by the overall LFC of the system, constructed in an approximate way. The same approximated LFCs have a number of errors, which are by no means always taken into account. Together with a not-too-strict adherence to the “letter” of sustainability criteria, the error in assessing sustainability can be significant.

According to the authors, a rigorous proof of the possibility of applying the specified formulation of the Nyquist criterion to linearized ACS is required, taking into account the errors of the approximated LFC of ACS links. The authors are not aware of any similar study that has been published in the last 15–20 years.

In addition to the inaccuracies of the stability criterion and the error in constructing the LFC of linear ACS, there is another serious methodological problem in assessing the stability of ACS.

These are significant nonlinearities of almost any electric drive, which should not be neglected in the “detailed” analysis. As indicated above, such a criterion is the criterion of V.M. Popov [4–7].

According to this criterion, the hodograph of the linear link must lie below the straight line passing through the critical point on the complex plane. For its implementation, it is not enough to correct the hodograph only near the critical point, which is easily achieved by the “stability margin” by the coefficient. It is necessary to impose more stringent requirements on the frequency response of the linear part of the ACS.

Let us consider two examples of linear automatic control systems, in which the introduction of one link with nonlinearity that is insignificant in terms of the actual value leads to the need, in accordance with the Popov criterion, to limit the coefficient more significantly than according to the Nyquist criterion with a stability margin, or to change the structure, since the automatic control system with a nonlinear link becomes a structurally unstable system, precisely according to the V.M. Popov criterion.

The link with ambiguousness (a hysteresis link) is offered to be used as a nonlinear link. There are two reasons for this:

1. All mechanical and electronic structures have this link: in gears as an interpretation of the mechanical gap, in transition algorithms between groups of electronic power devices, and in hysteresis characteristics of the magnetic structures of electric generators and motors. In other words, they are presented in all electromechanical and power structures. These links are particularly evident when changing speed of rotation and external torque, for example, in the wind turbines, and they influence their already low efficiency.
2. These links, among other nonlinearities, significantly reduce the stability of an ACS [14].

The method of harmonic linearization of this nonlinear link was tested in these calculations. During the simple experiment, the authors received very illustrative dependences in contradistinctions to the complex expressions and formulas. This method can be also used for other examples. The linearized part of ACS can be used instead of linear ACS part according to the V.M. Popov criterion.

In this study, the “Matlab Simulink” software was used for the simulation and ACS model building. The simulation time is 15 s.

Example 1. *There is a model of the second-order system. The analogue of the model is a real circuit of an integrated (-I)- motor current controller in an automatic control system with reversible thyristor converters.*

During the active introduction of thyristor converters in industrial electric drives in the 1970–1980s, the operation of I-current controllers in areas close to the dead zones of power converters and current sensors was an urgent problem.

In reversing converters, during the transitions from one group to another, a “current-less” pause was formed. The structure of the current circuit, taking into account the features of the operation of the TC, can be described by the model presented in Figure 9.

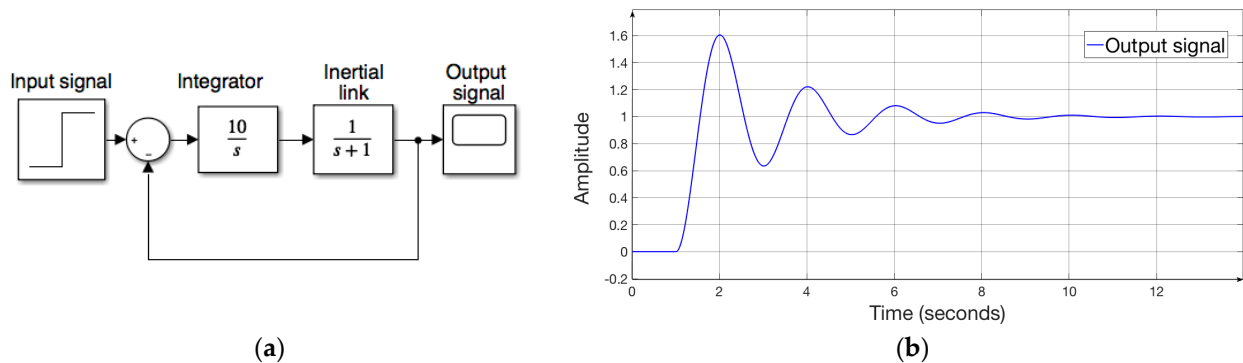


Figure 9. Scheme of a linear model (a) and processes in it (b).

The linear part, the integrator, and the inertial link, after being closed by a single negative connection, form a stable system (Figure 9) with the most “unsuccessful” parameters of the ACS links.

The introduction of a nonlinear link into the circuit violates the stability condition (Figure 10) (hereinafter in this article, undamped oscillations are considered an undesirable mode of operation and are taken as unstable).

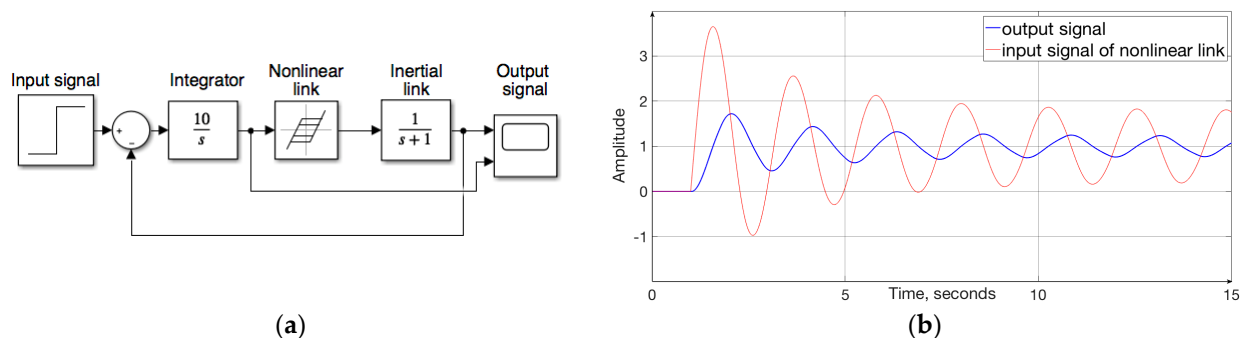


Figure 10. Scheme of the ACS model with nonlinearity (a) and processes in it (b).

If one linearizes a nonlinear link with any static coefficient, no violation should occur, since the original system of the second order is stable for any coefficient.

Studies of the nonlinear link model have shown that it should be interpreted not only by a static coefficient, but also by a dynamic link with a lagging phase, depending on the amplitude of the input signal. The simulation results are given in the Appendix A to the article. When the signal amplitude is at the level of the reference signal, the phase shift of the link is approximately 45–50 degrees. The frequency response of the linear part changes and an area of unstable values of the coefficient is formed. This area is determined both by the hodograph on the complex plane by the Nyquist criterion and by the Popov criterion. Yet this area is significantly smaller by the Nyquist criterion; that is, the allowable gains coefficients according to the criterion of M. Popov are significantly less than according to the Nyquist criterion. The simulation showed very clearly that V.M. Popov conditions are close to the experimental results, and the coefficients obtained by Nyquist are much larger than the real ones, which ensure stability in this simple ACS.

Reducing the coefficient made it possible to obtain a stable process (Figure 11).

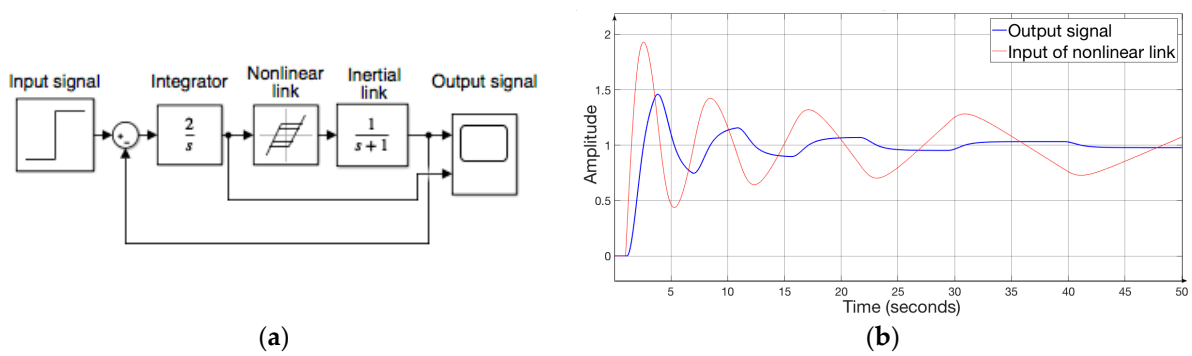


Figure 11. Diagram of a model with reduced gain nonlinearity(a) and simulation result (b).

At this point, the simulation confirmed that for ACS with a “simple” linear part (Figure 12a), one can use the Nyquist criterion, making a certain margin of stability (Figure 12b), but the value of this stock must be determined according to the criterion of V.M. Popov (Figure 12c).

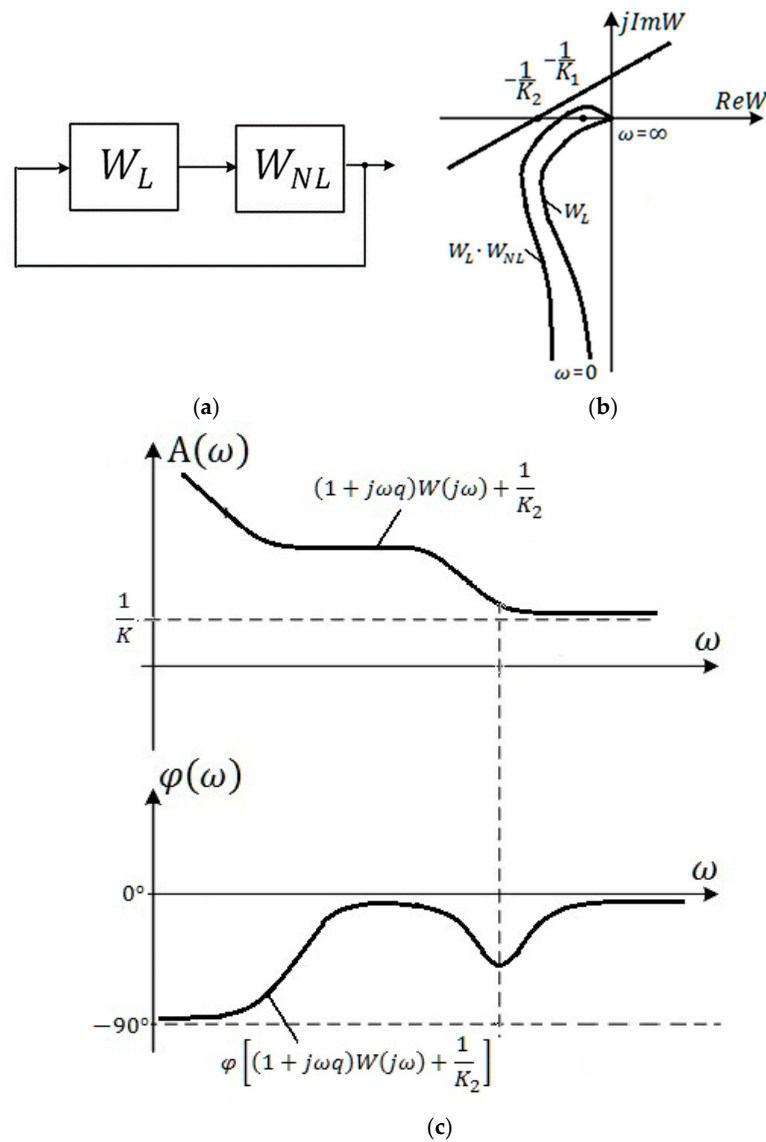


Figure 12. Structure of nonlinear ACS (a) and stability condition for it according to the criterion of V.M. Popov for frequency hodographs (b) and logarithmic frequency characteristics (c).

Expressions (8)–(14) describe the results of simulation in the context of mathematics. W_L is a transfer function of linear link of ACS, W_{NL} is a transfer function of nonlinear link of ACS in the exponential form, A_{NL} is an amplitude of nonlinear part of ACS, and φ_{NL} is a phase of the nonlinear part of ACS.

$$W_L = \frac{1}{p(p+1)} \quad (8)$$

$$W_{NL} = A_{NL}(\omega)e^{j\varphi_{NL}(\omega)} \quad (9)$$

$$A_{NL}(\omega) < A \quad (10)$$

$$W = W_L \cdot W_{NL} \quad (11)$$

$$\frac{1}{K_1} - \text{Nyquist stability} \quad (12)$$

$$\frac{1}{K_2} - \text{Popov stability} \quad (13)$$

$$K_1 > K_2 \quad (14)$$

conditions of the V.M. Popov criterion for Logarithmic frequency characteristics (LFC).

$$\varphi \left[(1 + j\omega q)W(j\omega) + \frac{1}{K_2} \right] > -90^\circ \quad (15)$$

Example 2. *There is a high-order automatic control system with mechanical nonlinearity.*

Figure 13 shows a simplified model of an electromechanical automatic control system, in which there are equivalent dynamic links of the regulator (integrator 1), transformation of speed into angular displacement (integrator 2), high-frequency link of the second order, describing the electromechanical links of the automatic control system and a nonlinear link with ambiguity, which interprets the nonlinearities that degrade the stability of the ACS. In structural analysis, this link is replaced by a dynamic nonlinear link.

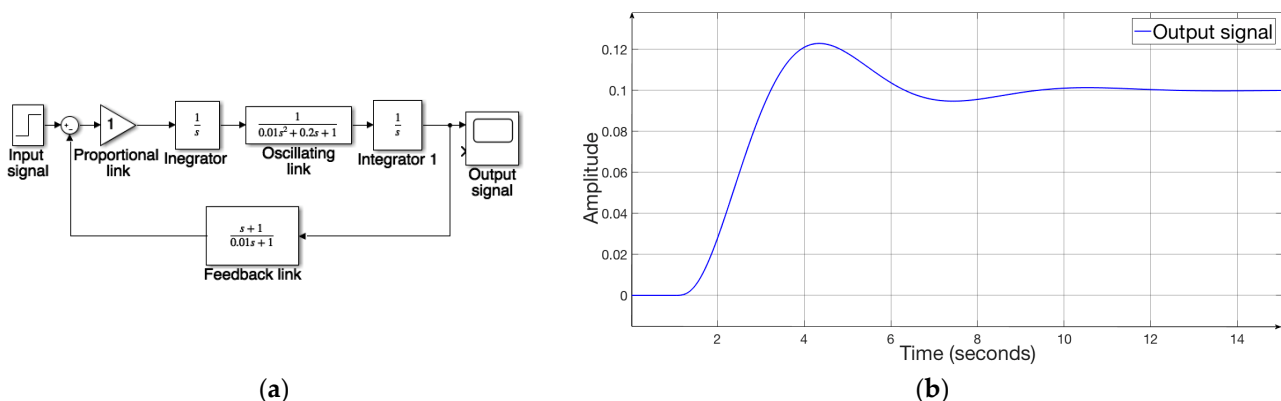


Figure 13. Scheme of a linear ACS of the 4th order (a) and simulation result (b).

The original linear structure (Figure 13) is stable due to the dynamic connection with the differential link. With the introduction of nonlinearity, the stability is violated. This is evidenced by the processes in the model (Figure 14), the hodographs on the complex plane, and the LFC of the system (Figure 15).

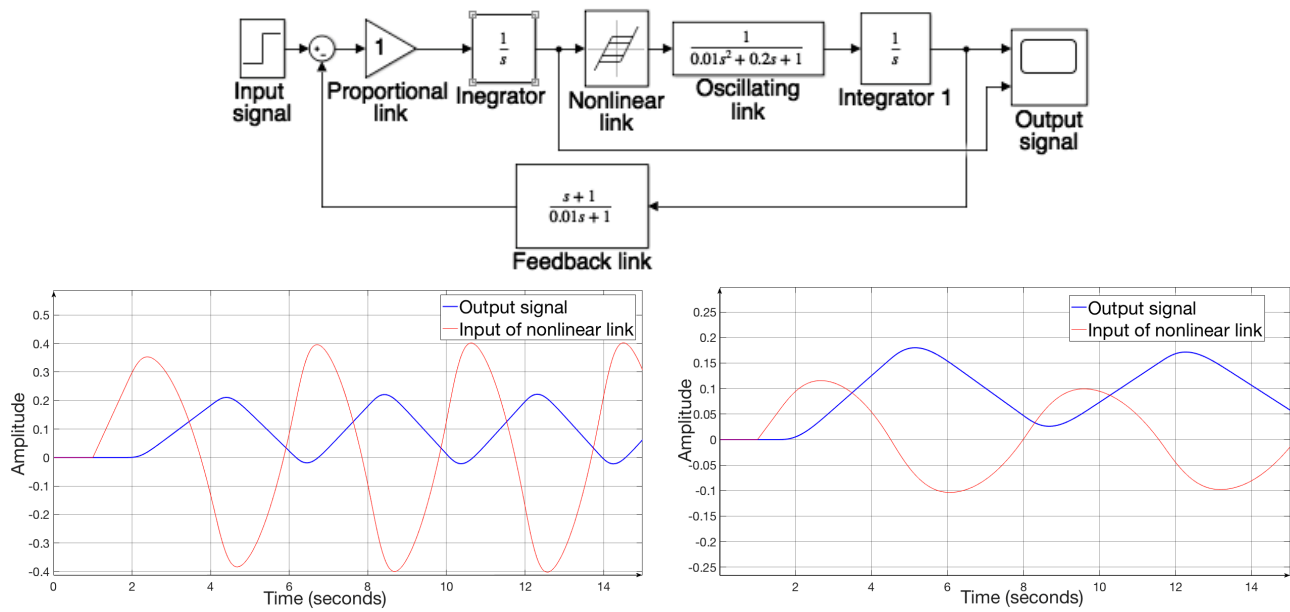


Figure 14. Scheme of an unstable nonlinear ACS and processes at different gain factors $K_1, 2$.

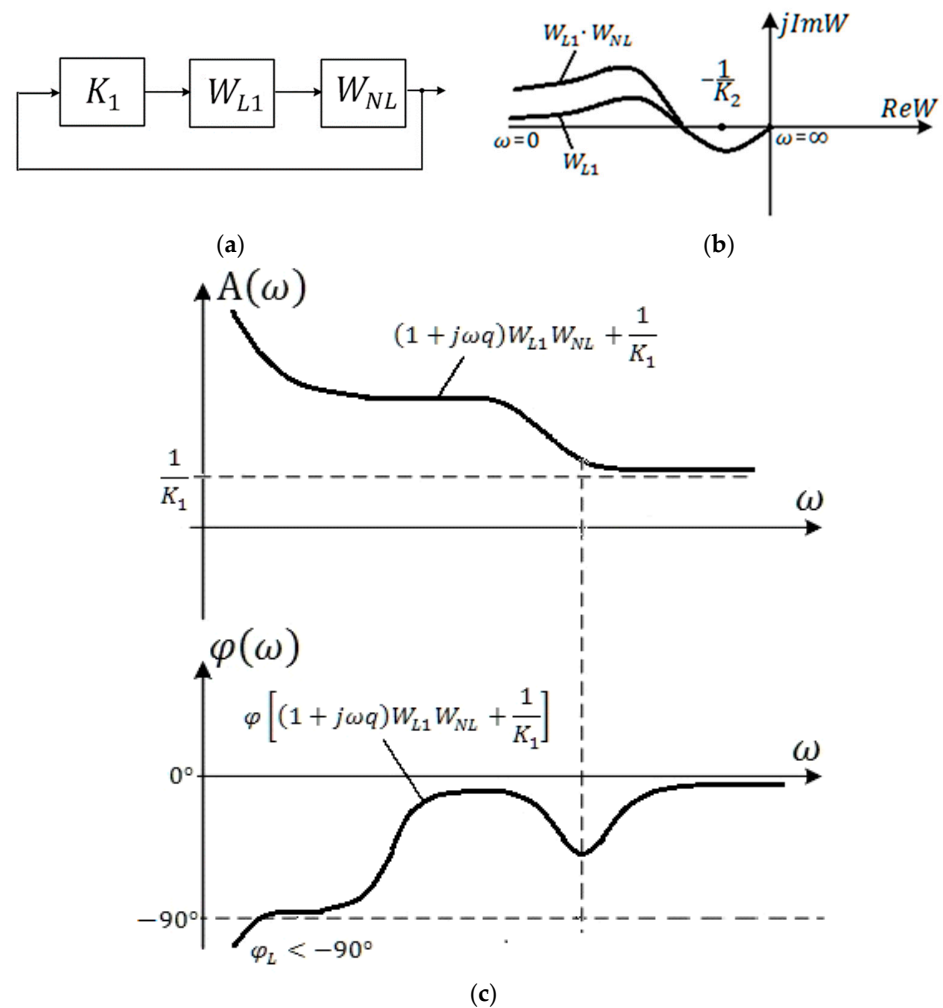


Figure 15. Structure of nonlinear ACS (a) and stability condition for it according to the criterion of V.M. Popov for frequency hodographs (b) and logarithmic frequency characteristics (c), which is not fulfilled for any K .

Expressions (16)–(19) illustrate the simulation results in the mathematical form. Herein W_{L1} is a transfer function of linear link, W_{NL} is a transfer function of nonlinear link in the exponential form, $\varphi(\omega)$ is a phase, and $A(\omega)$ is an amplitude.

$$W_{L1} = \frac{1}{p^2} \cdot \frac{1}{(0.01p^2 + 0.2p + 1)} \cdot \frac{p + 1}{(0.01p + 1)} \quad (16)$$

$$W_{NL} = A_{NL}(\omega)e^{j\varphi_{NL}(\omega)} \quad (17)$$

$$\varphi(\omega) < 0 \quad (18)$$

$$A(\omega) < A \quad (19)$$

$W_{L1} = \frac{1}{p^2}$. When passing from the Nyquist criterion to the criterion of V.M. Popov, the requirements for the linear part of the system change: non-covering of the critical point $(-1;0)$ by the hodograph of the linear part is replaced by the possibility of drawing a straight line through this critical point without touching the hodograph. That is, the Nyquist stability condition, performed for a linearized ACS, clearly does not coincide with the condition obtained by the criterion of V.M. Popov.

A stability margin obtained by the Nyquist criterion turns out to be insufficient for the stability of a system with nonlinearity and a simple decrease in the coefficient; as in the previous case, the problem of ACS stability cannot be solved. The system became structurally unstable.

Figure 13 shows stable processes in the initial linear ACS. Figures 14 and 15 show unstable processes in the ACS with a nonlinear link and reduced coefficients. That is, the structure became unstable. Changes in the amplitude and period of oscillations with decreasing coefficients confirm the changes in the linearized characteristic of the nonlinear element shown in the studies of the element (see Appendix A to the article). Reducing the amplitude of the input signal of a nonlinear link increases the phase shift of this link and contributes to the onset of oscillations with a lower frequency (Figure 15).

To solve this problem, it is necessary to correct the frequency response of the linear part—a double integrator with a second-order high-frequency link. To correct it for fulfilling the stability condition of M. Popov is to change it in the entire frequency range—from 0 to ∞ , so that a line can be drawn through the critical point in the complex plane, not touching the hodograph of the linear part.

There are many ways to solve a model problem. In this case, the task was not set to find any method of this correction. It is important to show the very possibility of such a solution. One of the possible ones is the correction of the linear part when replacing one of the integrators with an inertial link with a close inertia.

In Figure 16a, there is a scheme of ACS with an excluded integrator and an inertial link included in the circuit. Simulation result are presented in Figure 16b. The Nyquist conditions for both circuits (Figure 17) are unchanged: in the cutoff frequency zone, the inertia and the integrator act in the same way (Figure 18).

Expressions (20)–(26) illustrate the condition of V.M. Popov in the mathematical form. Herein, W_{L2} is a transfer function of linear link, W_{NL} is a transfer function of nonlinear link in the exponential form, $\varphi(\omega)$ is a phase, and $A(\omega)$ is an amplitude.

$$W_{L2} = \frac{1}{p} \cdot \frac{1}{p + 1} \cdot \frac{1}{(0.01p^2 + 0.2p + 1)} \cdot \frac{p + 1}{(0.01p + 1)} \quad (20)$$

$$W_{NL} = A_{NL}(\omega)e^{j\varphi_{NL}(\omega)} \quad (21)$$

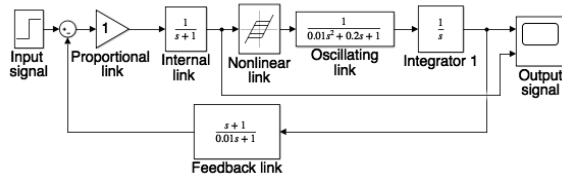
$$\varphi(\omega) < 0 \quad (22)$$

$$A(\omega) < A \quad (23)$$

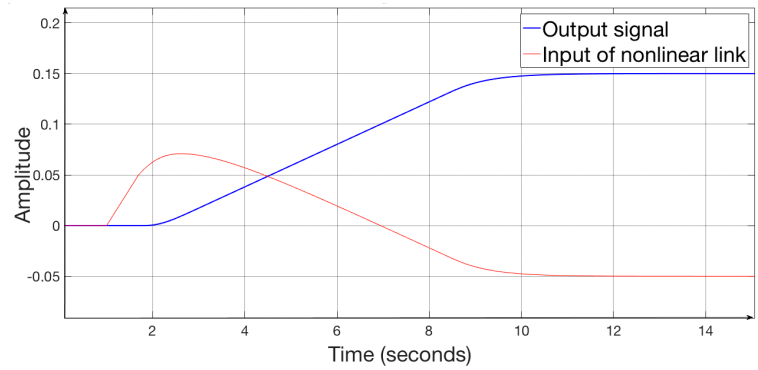
$$\operatorname{Re} \left[(1 + j\omega q)W_{L2}W_{NL} + \frac{1}{K_2} \right] > 0 \quad (24)$$

$$\varphi \left[(1 + j\omega q)W_{L2}W_{NL} + \frac{1}{K_2} \right] > -90^\circ \quad (25)$$

$$\Phi(x) \leq Kx \quad (26)$$



(a)



(b)

Figure 16. Stable nonlinear ACS (a) and simulation result (b).

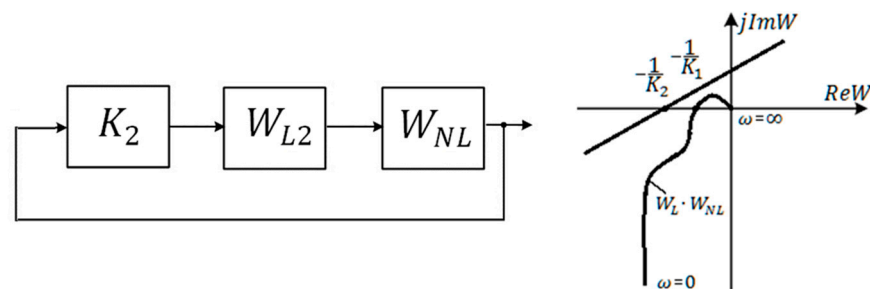


Figure 17. Structure and hodograph of a nonlinear stable ACS.

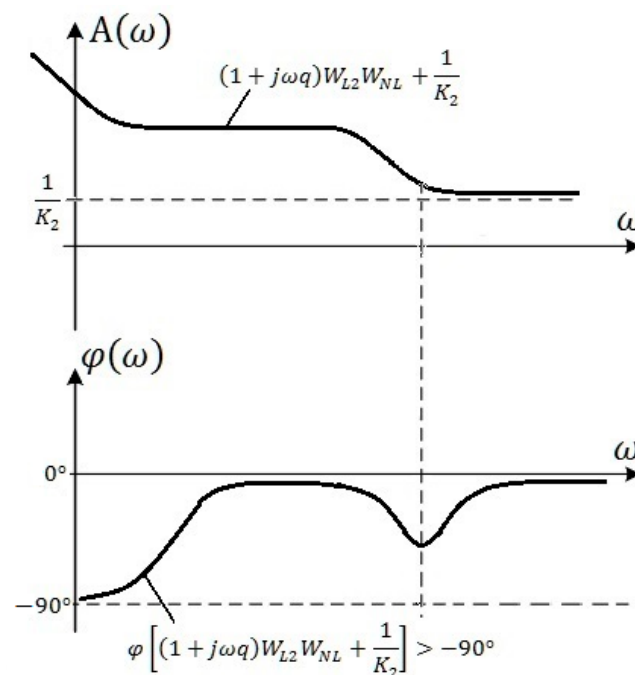


Figure 18. Condition of V.M. Popov fulfilled by the LFC of a nonlinear ACS.

The hodographs on the complex plane and LFC of the ACS (Figures 17 and 18) fully correspond to the simulation results. That is, a system with second-order astaticity and

ambiguous uncertainty in full accordance with the Popov criterion becomes structurally unstable. In an ACS with the same inertia, but with a lower level of astaticity, under certain conditions a straight line separating the hodograph from the critical point can be drawn, and the ACS becomes stable under such parameters.

4. Results and Discussion

Simulation Analysis

In the two considered models, the integrators form the low-frequency region on the complex plane and on the LFC. Oscillatory processes are also formed in the low-frequency region.

The high-frequency link in the second example does not form oscillatory processes (their frequency is low), but according to the criterion of V.M. Popov, it “interferes” with drawing a straight line above the hodograph of the linear part and, accordingly, with obtaining finite stability of ACS. Obviously, it is the high order of dynamic links that requires the transition to the Popov criterion for nonlinearities.

If there is any nonlinear link in a system with astaticism of the second and higher order, the condition for ACS stability should be formulated according to the criterion of V.M. Popov, since such a system can be structurally unstable. Nevertheless, the Nyquist criterion does not show this.

For ACS with a linear part not higher than the second order, the V.M. Popov condition can qualitatively coincide with the Nyquist condition with some “stability margin” if the critical point is shifted to the area to the left of the intersection of the hodograph and the axis of real values, that is, to the low coefficients area. The condition of drawing a straight line excludes some of the large coefficients without even specifying its nature; that is, an additional condition performed in the cutoff frequency zone ensures the fulfillment of Popov conditions for the entire frequency range.

In the course of this research, it was shown that the integrating links in the ACS create conditions for the occurrence of oscillatory or unstable processes. This is adequately reflected by the frequency stability criteria—the Nyquist criterion for linear systems and V.M. Popov criterion for nonlinear ones. It is hardly possible to accurately take into account the influence of these links, taking into account cross-links, and changing and nonlinear coefficients for engineering calculations. Frequency correction methods for ACS recommend creating a stability margin in phase or amplitude, but for systems with significant nonlinearities, this method is ineffective if the Nyquist criterion is used to assess stability.

The most important difference between the V.M. Popov criterion and the Nyquist criterion is the frequency response requirements that apply to the entire frequency range, and not to the cutoff frequency as in the Nyquist test.

Simulation has shown that in second-order systems, the transition to stability analysis with respect to M. Popov limits the coefficients, and in a higher-order ACS it takes into account structural instability and requires structure correction for stability. It is also shown that the previously proposed formulation of the V.M. Popov criterion on LFC is quite effective for engineering calculations.

5. Conclusions

1. The Nyquist and V.M. Popov criteria for real electromechanical and power systems (with decreasing characteristics) are quite close in terms of the mechanisms for obtaining the conditions.
2. The necessary stability margin for applying the Nyquist criterion can be determined quite accurately by the type of LFC of an open system when the V.M. Popov criterion is fulfilled.
3. Zero roots of the characteristic equations of the linear part can create structurally unstable nonlinear systems according to the criterion of V.M. Popov. The Nyquist criterion for linearized ACS will not show this structural instability.

4. For nonlinear automatic control systems with a complex linear part, the estimation method is more important than the accuracy of direct calculations. In complex systems with nonlinearities, when choosing integral channels in regulators, it is necessary to check the stability according to the V.M. Popov criterion.
5. The proposed interpretation of the Popov criterion for the LFC of the linear part of the ACS is fair and efficient for engineering analysis.
6. These experiments showed that the frequency characteristic of linearized ACS can be used in the conditions of stability according to the V.M. Popov criterion, along with the frequency characteristic of the linear part of the ACS.

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Data Availability Statement: In the research and the article, only materials from publicly available sources of information are used. The simulation results obtained in the Matlab software are presented.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. About the Nonlinear Element

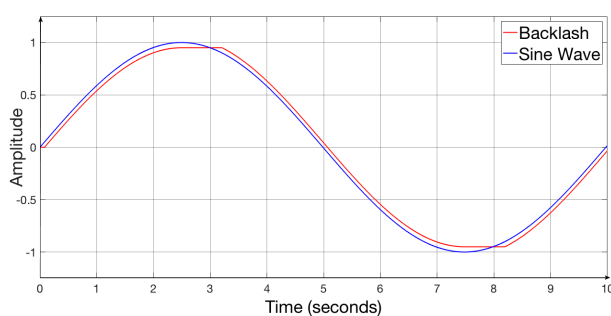
The nonlinear element with variable interchangeability in the analysis of stability is replaced by a combination of a link with a return stroke and a note on the value.

The dependence of phase shift and the link gain coefficient on frequency and the evaluation of the input signal are received as the result of direct simulation. The appendix contains the presented scheme, examples of registration of input and output signals, and desired dependencies which were used in the analysis of the stability of the ACS according to the criterion of V.M. Popov.

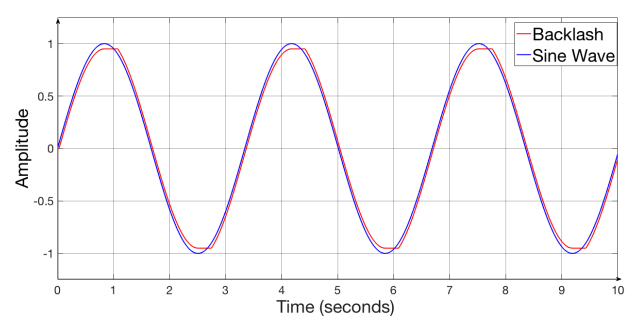
In the first experiment, the frequency of the input signal was changed at a constant amplitude (Table A1). Diagrams of input and output signals for cases 1 and 3 (Table A1) are presented in Figures A1a and A1b, respectively.

Table A1. Initial data.

Experiment Number	Amplitude of Input Signal	Frequency, Hz (rad/s)	Backlash
1	1	0.1 (0.63)	0.1
2	1	0.2 (1.26)	0.1
3	1	0.3 (1.88)	0.1
4	1	0.4 (2.51)	0.1
5	1	0.8 (5.03)	0.1



(a)



(b)

Figure A1. Simulation of cases 1 (a) and 3 (b). Amplitude is constant.

In the second experiment, the amplitude of the input signal was changed at a constant frequency of 0.1 Hz (Table A2). Diagrams of input and output signals for cases 3 and 4 (Table A2) are presented in Figures A2a and A2b, respectively.

Table A2. Initial data and data analysis of both experiments.

Experiment Number	Amplitude of Output Signal	Time Delay of Output Signal, s	Amplitude of Input Signal	Relationship of Amplitude of Output Signal to Amplitude of Input Signal
1	0.95	0.0794	1	0.95
2	0.75	0.9927	0.8	0.9375
3	0.45	0.0159	0.5	0.9
4	0.25	0.2658	0.3	0.8333
5	0.05	0.08255	0.1	0.5
6	0	0	0.05	0

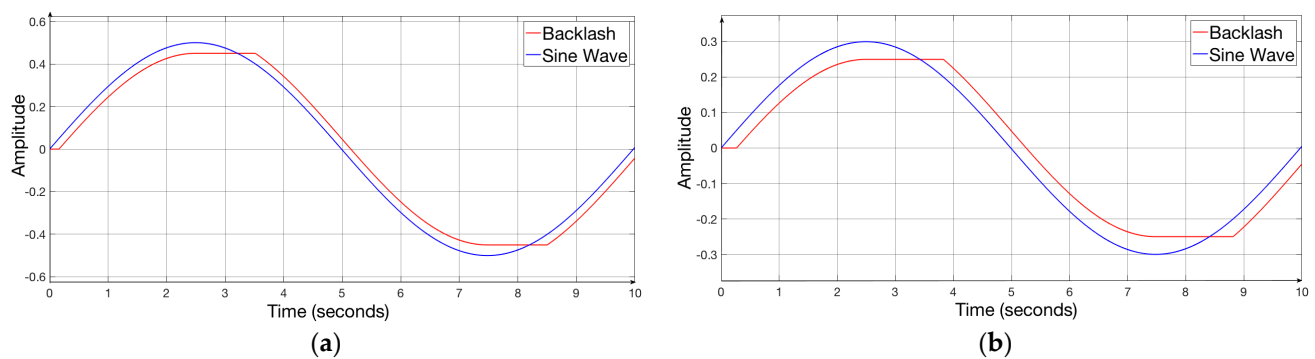


Figure A2. Simulation of cases 3 (a) and 4 (b). Amplitude is variable.

Graph of the output signal amplitude ratio to the input signal amplitude on the amplitude of the input signal is presented in Figure A3. This dependence is used in the analysis of stability.

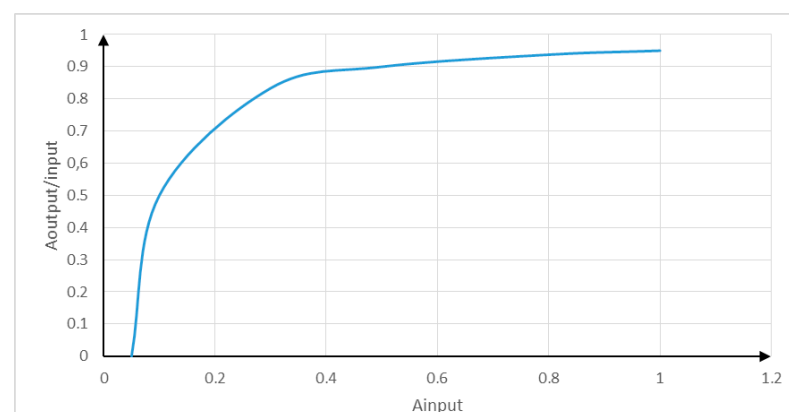


Figure A3. Graph of the output signal amplitude ratio to the input signal amplitude on the amplitude of the input signal.

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