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Sliding Mode Observer-Based Fault Detection in Continuous Time Linear Switched Systems

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Abstract: This paper studies the problem of fault detection for continuous time linear switched systems in the presence of disturbance. For this purpose, a fault detection sliding mode observer approach is designed to generate the residual signal. To minimize the effect of disturbance from the residual, the problem is formulated into H∞ filtering technique to increase more robustness. To deal with the issue of the switched systems stability, the Lyapunov-Krasovskii functional method is utilized along with average dwell time, and linear matrix inequalities are formulated to derive the sufficient conditions. The residual signal is evaluated, and an adaptive threshold is computed for both modes of the switched system. Finally, a simulation example for a case study of boost converter and a numerical example with both abrupt and incipient faults are illustrated to prove the efficacy of the proposed method.

Keywords: fault detection; H∞ control; linear matrix inequalities; sliding mode observer; switched systems

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Citation: Ali, S.; Raza, M.T.; Abbas, G.; Ullah, N.; Al Otaibi, S.; Luo, H. Sliding Mode Observer-Based Fault Detection in Continuous Time Linear Switched Systems. *Energies* **2022**, *15*, 1090. https://doi.org/10.3390/en15031090

Academic Editor: Jose Luis Calvo-Rolle

Received: 26 December 2021
Accepted: 25 January 2022
Published: 1 February 2022

1. Introduction

Fault diagnosis has a significant demand to achieve high performance, safety, and reliability in practical engineering systems. For complex systems in industry, the possibility of occurring faults increases because of the huge presence of actuators, sensors, and process components. Therefore, Fault Detection (FD) has become an active research area in the field of control systems. Among various fault diagnosis techniques, the model-based FD method [1,2] is the most popular analytical scheme because a model is run in parallel with the original system in a computer, instead of a redundant hardware component being installed. Observer-based fault diagnosis is a special type of model-based approach in which, with the availability of the system model in Reference [3–5], an observer is designed to estimate error between the actual system and the observer. Thus, observer-based fault diagnosis is developed in the context of entrenched theory of control and observer. Various methods have been suggested for fault diagnosis purpose, such as unknown input observer method, reduced order observer method, adaptive observer method, Sliding Mode Observer (SMO) method, high gain observer method, etc. [2–4,6].

Due to increase in robustness to unknown inputs, disturbances and uncertainties, sliding mode techniques have emerged as a hot research area in control community. Ref. [7] proposed an adaptive Sliding Mode Controller (SMC) for tracking accuracy of a DC-DC buck converter with time varying disturbances and uncertainties. The SMC problem is solved in Reference [8] for discrete time switched systems by means of an event triggered strategy. A second order SMC is designed and implemented in Reference [9] for a buck converter under external disturbances and model uncertainties. However, it should noted that the SMO-based fault diagnosis method [10,11] has been one of the most prominent methods because of its faster response, and the convergence rate of observer and system states are asymptotically stable. In addition, estimated output error converges to zero finitely. The SMO design method provides a nonlinear switching term to feedback output estimation error in nonlinear fashion [11], which is used to eliminate disturbances and faults. Another important property of SMO is the sliding surface, also called the sliding patch. It is that section where sliding motion ensures, and any deviation from this sliding area results in an observer failure to converge. A large proportion of literature associated with the SMO method has been studied for several systems, such as switching systems [12,13], linear systems [14], and markovian jump systems [15].

On the other hand, fault diagnosis for switched systems is an additional demand and attractive choice [16]. Switched systems are normally used when there is the need to model a system which undergoes certain abrupt changes. Switched systems are common in academic study and engineering applications, such as process control, communication systems, aircrafts, and power systems. Fruitful results on switched systems associated with observer design, stability and stabilization, and controller design have been produced [17,18]. In Reference [19], the authors reported an FD filter design for continuous time switched linear systems. An observer is designed in Reference [20] based on unknown switching while investigating switched systems. Robust Fault Estimation (FE) for switched systems in the presence of unknown inputs based on switched Lyapunov function and Average Dwell Time (ADT) approach is studied in Reference [21]. A hybrid observer-based Fault Detection and Isolation (FDI), in Reference [22], is developed for discrete time switched systems. Various practical examples for switched systems fault diagnosis are studied in the literature, including fault diagnosis based on SMO in Reference [12,23] that uses a practical example of a DC-DC boost converter which acts as a switched system model. FD for multi cellular and boost converter is illustrated in Reference [24] by using reduced order SMO. In addition, in Reference [25], a DC-DC buck boost converter is used as an application of switched systems that detect and estimate sensor faults. In Reference [26], a sensor-less Induction motor drive is used as a real time application for SMO-based fault diagnosis purposes. SMO for large scale systems in Reference [27] is used for robust Fault Detection and Estimation (FDE). In Reference [13], two types of observers are designed for switched systems that investigate their state and FE problems. A robust SMO for FD sensitive to both disturbance and fault is estimated in Reference [14,28]. An adaptive fuzzy finite-time control problem is designed in Reference [29] for nonlinear switched systems with prescribed performance. In addition, the control problem in Reference [30] is studied uncertain under actuated nonlinear switched systems with actuator faults using adaptive fuzzy hierarchical SMC. An observer-based Takagi-Sugeno (T-S) fuzzy model is considered in Reference [31], to formulate the quantized output for discrete-time nonlinear systems. A non-fragile $H\infty$ filtering problem is considered for continuous time T-S fuzzy systems in Reference [32]. One pioneering work is described in Reference [33] for discrete time polynomial fuzzy systems. The fault detection filter problem is combined with H_{-}/H_{∞} optimization such that it achieves the best robustness and sensitivity to disturbance and fault, respectively. This motivated us further to investigate the robust fault detection problem for switched systems by using SMO. More recently, one of the challenging problems for model-based hybrid system was studied in Reference [34], for discrete component prognosis with intermittent faults.

Although there exist several achievements in the area of SMO-based fault diagnosis for switched systems, a few results are available in Reference [12,23,25]. However, it still necessary to pay more attention to explore further in this area. Our major focus in this work is the SMO-based FD, taking the switched system in continuous time in the presence of disturbances and faults. This paper has the following main contributions. First, in order to make the residual signal robust regarding disturbance effects, we transform the switched system and residual error dynamics into an augmented form. Second, the problem is formulated into H∞ performance index to minimize disturbance effect from residual, and a Lyapunov function is also used for switched system stability, while considering Average Dwell Time (ADT) constraints. Hence, the dwell time restriction is relaxed for each mode of the switched system by means of ADT. Third, the evaluated residual with adaptive threshold is derived in the design technique for perfect fault detection. Finally, two simulation examples depict the performance of the prospective method.

The remaining part in this paper is arranged into the following sections: The switched system model description and SMO design is illustrated in Section 2. The main results are described in Section 3. Residual evaluation and adaptive threshold computation steps are derived in Section 4. Simulation results are demonstrated with two examples in Section 5, and Section 6 presents the conclusion.

2. Description of the System

2.1. Switched System Design

Switched system in continuous time case represents the following class as:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + E_{d\sigma(t)}d(t) + E_{f\sigma(t)}f(t) \\ y(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}u(t) + F_{d\sigma(t)}d(t) + F_{f\sigma(t)}f(t) \end{cases}, \tag{1}$$

where $x(t) \in R^n$ and $u(t) \in R^m$ are state and input vectors. $y(t) \in R^p$, $d(t) \in R^r$, and $f(t) \in R^s$ are output, disturbance, and fault vectors, respectively. $A_{\sigma(t)}$, $B_{\sigma(t)}$, $C_{\sigma(t)}$, $D_{\sigma(t)}$, $E_{d\sigma(t)}$, $F_{d\sigma(t)}$, $E_{f\sigma(t)}$, and $F_{f\sigma(t)}$ are matrices of suitable dimensions for each modes. We denote $\sigma(t) = i$, which is known as the switching signal of the subsystem, as $A_{\sigma(t)}$=A_i.

2.2. Sliding Mode Observer Design

The SMO design for the switched system model is given to construct residual signal for effective fault detection as

$$\begin{cases} \dot{\hat{x}}(t) = A_{\hat{\sigma}_{(t)}}\hat{x}(t) + B_{\hat{\sigma}_{(t)}}u(t) + Gl\hat{\sigma}_{(t)}(e_y(t)) + \nu(t) \\ \hat{y}(t) = C_{\hat{\sigma}_{(t)}}\hat{x}(t) + D_{\hat{\sigma}_{(t)}}u(t) \\ r(t) = y(t) - \hat{y}(t) \end{cases}. \tag{2}$$

Here, $\hat{\sigma}(t) = i$ is the observer switching signal. As both subsystem and observer are switching in a synchronous manner, the phenomenon of synchronous switching arises between them. Thus, we assume two modes of the switched system, in which Mode 1 is operated for $\sigma(t) = 1$, and Mode 2 is also active when $\sigma(t) = 0$. G_l is the traditional linear Luenberger observer gain which may be used to expand sliding area, and $\nu(t)$ is the nonlinear injection term involved in SMO. The output error to be estimated is $e_y(t)$, and $e(t)$ is state error estimation. The nonlinear switching term is given below, where P_i is a positive definite matrix, and ρ a positive scalar.

$$\nu(t) = \rho P_i^{-1} \frac{e(t)}{\| e(t) \|}.$$

Remark 1. *The nonlinear injection term $\nu(t)$ is going to be discontinuous, and any mismatch between what the plant is doing and what the observer is doing will come through on that signal. This is a very important property for fault detection and monitoring of the system. The observer is also associated with nonlinear switching and has no problems for high gains because of a model run in a computer, which switches as much as possible.*

The error dynamics and generated residual signal are given as:

$$\begin{cases} \dot{e}(t) = (A_i - G_{li}C_i)e(t) + (E_{di} - G_{li}F_{di})d(t) \\ \qquad +(E_{fi} - G_{li}F_{fi})f(t) - \nu(t) \\ r(t) = C_ie(t) + F_{di}d(t) + F_{fi}f(t) \end{cases}. \tag{3}$$

Definition 1. *See Reference [35]. ADT is defined as the switching signal between consecutive switching at least τ_{av}. This technique is less conservative than that of arbitrary switching because of slow switching.*

Let $N_\sigma(t_a, t_b)$ be the number of switching in switching signal $\sigma(t)$ during the interval (t_a, t_b), if the following condition holds

$$N_\sigma(t_a, t_b) \leq N_o + \frac{t_b - t_a}{\tau_{av}},$$

where $N_o > 0$ and $\tau_{av} > 0$ are chattering bound and ADT, respectively.

Lemma 1. *See Reference [19]. There exists a Lyapunov function $V_i(x(t))$ for the switched system, along with ADT, which is said to be globally asymptotically stable, satisfying the performance of H∞ with an index no greater than $\gamma = max(\gamma_i)$ as*

$$\dot{V}_i(x(t)) \leq -\beta V_i(x(t)) - r^T(t)r(t) + \gamma_i^2 u^T(t)u(t).$$

ADT is defined as

$$\tau_a > \tau_a^* = \frac{ln\mu}{\varsigma}.$$

3. Main Results

In this section, we provide a solution to the FD problem based on SMO with H∞ filtering. The Lyapunov-Krasovskii functional method is used for switched system stability, and Linear Matrix Inequalities (LMI's) are derived to provide sufficient conditions. The schematic diagram in Figure 1 illustrates a clear description of the complete design procedure of system, with the observer to produce the residual. This difference in plant and observer output is evaluated, along with varying thresholds for perfect fault detection, as shown in separate blocks of figure. Input to the system is fault and disturbance. Our assumption in switching between the observer and the subsystem is synchronous, i.e., they are switching at the same time without any delay in identifying the observer.

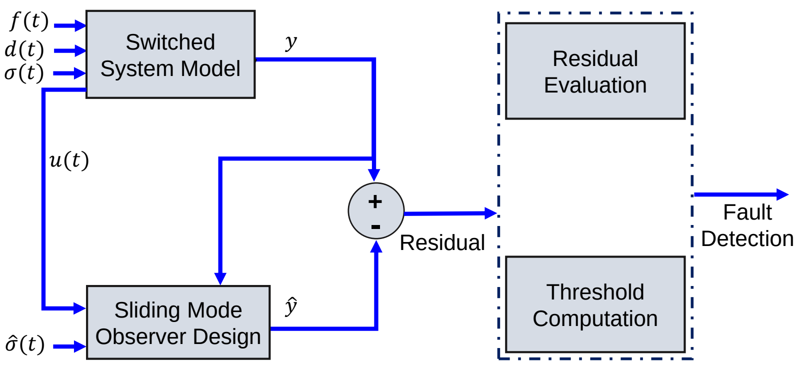

Figure 1. Design scheme of fault detection.

Here, our aim is to reduce disturbance effect from residual signal, so (3) is modified as

$$\begin{cases} \dot{e}(t) = \bar{A}_i e(t) + \bar{B}_{di} d(t) - \nu(t) \\ r(t) = \bar{C}_i e(t) + \bar{D}_{di} d(t). \end{cases} \tag{4}$$

Remark 2. *We transform the error and residual generator dynamics of the switched system into an augmented form as in (4), so as to reduce disturbance effect from the residual signal.*

Theorem 1. *Given the constants $\mu > 0$, $\beta > 0$, $\rho > 0$, if there exists a positive definite matrix $P_i > 0$, then the following inequality becomes true, and system (4) is asymptotically stable and achieves robust performance (9).*

$$\begin{bmatrix} \bar{A}_i^T P_i + P_i \bar{A}_i + \beta P_i & P_i \bar{B}_{di} & \bar{C}_i^T \\ * & -\gamma_i^2 I & \bar{D}_{di}^T \\ * & * & -I \end{bmatrix} < 0. \tag{5}$$

Proof of Theorem 1. Define the common Lyapunov function for system (4) as

$$V_i(e(t)) = e^T(t) P_i e(t). \tag{6}$$

Taking derivative of (6) gives

$$\dot{V}_i(e(t)) = \dot{e}^T(t) P_i e(t) + e^T(t) P_i \dot{e}(t). \tag{7}$$

The robust performance of the residual signal to disturbance is as follows:

$$\frac{\| r(t) \|_2}{\| d(t) \|_2} < \gamma_i, \tag{8}$$

where (8) can be rewritten as

$$r^T(t) r(t) < \gamma_i^2 d^T(t) d(t).$$

To make residual signal attenuating the disturbance effect, the following H∞ performance index holds for the FD observer as

$$\int_0^\infty r^T(t) r(t) dt < \gamma_i^2 \int_0^\infty d^T(t) d(t) dt. \tag{9}$$

□

Remark 3. *Note that, in order to make the residual signal robust regarding disturbance, the optimization algorithm of H∞ performance is combined with our switched system. Thus, the residual signal minimizes the effect of disturbance and achieve more robustness.*

Therefore, by considering nonzero d(t) $\in l_2[0, \infty)$, and under zero initial conditions, the performance index is modified by using Lemma 1.

$$r^T(t) r(t) - \gamma_i^2 d^T(t) d(t) + \dot{V}_i(e(t)) + \beta V_i(e(t)) \le 0. \tag{10}$$

By substituting the value of $\dot{e}(t)$ from (4) into (7), we get

$$\dot{V}_i(e(t)) = \begin{array}{l} [\bar{A}_i e(t) + \bar{B}_{di} d(t) - \nu(t)]^T P_i e(t) \\ + e^T(t) P_i [\bar{A}_i e(t) + \bar{B}_{di} d(t) - \nu(t)] \end{array}. \tag{11}$$

Expand (11) by substituting the value of $\nu(t)$ derived as

$$\dot{V}_i(e(t)) = \begin{array}{l} e^T(t) \bar{A}_i^T P_i e(t) + e^T(t) P_i \bar{A}_i e(t) + d^T(t) \bar{B}_{di}^T P_i e(t) \\ + e^T(t) P_i \bar{B}_{di} d(t) - 2\rho \| e(t) \| \end{array}. \tag{12}$$

Substitute $r(t)$, $V_i(e(t))$, and $\dot{V}_i(e(t))$, from (4), (6), and (12) in (10), evaluates the inequality as

$$\begin{array}{c} [\bar{C}_i e(t) + \bar{D}_{di} d(t)]^T \times [\bar{C}_i e(t) + \bar{D}_{di} d(t)] - \gamma_i^2 d^T(t) d(t) + e^T(t) \bar{A}_i^T P_i e(t) \\ + e^T(t) P_i \bar{A}_i e(t) + d^T(t) \bar{B}_{di}^T P_i e(t) + e^T(t) P_i \bar{B}_{di} d(t) \\ + \beta e^T(t) P_i e(t) - 2\rho \| e(t) \| \le 0 \end{array}. \tag{13}$$

In the above equation, ρ is a positive constant, and the second norm of $\| e(t) \|$ is also positive. Therefore, $-2\rho\| e(t) \| < 0$. These inequalities above can be written as

$$\begin{bmatrix} e(t) \\ d(t) \end{bmatrix}^T \sqcap \begin{bmatrix} e(t) \\ d(t) \end{bmatrix} \leq 0; \tag{14}$$

here, $\sqcap < 0$ becomes

$$\sqcap = \begin{bmatrix} \triangle_a & P_i\bar{B}_{di} + \bar{C}_i^T\bar{D}_{di} \\ \bar{B}_{di}^T P_i + \bar{D}_{di}^T\bar{C}_i & -\gamma_i^2 I + \bar{D}_{di}^T\bar{D}_{di} \end{bmatrix} \tag{15}$$

$$\triangle_a = \bar{A}_i^T P_i + P_i\bar{A}_i + \bar{C}_i^T\bar{C}_i + \beta P_i.$$

From the definition of Schur complement [36], we know that the following conditions hold:

$$\begin{array}{c} \text{a) } S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} < 0 \\ \text{b) } S_{22} < 0, \quad S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0 \end{array}. \tag{16}$$

Hence, (15) is expanded into the following form

$$\begin{bmatrix} \triangle_b & P_i\bar{B}_{di} \\ \bar{B}_{di}^T P_i & -\gamma_i^2 I \end{bmatrix} + \begin{bmatrix} \bar{C}_i^T\bar{C}_i & \bar{C}_i^T\bar{D}_{di} \\ \bar{D}_{di}^T\bar{C}_i & \bar{D}_{di}^T\bar{D}_{di} \end{bmatrix} \leq 0 \tag{17}$$

$$\triangle_b = \bar{A}_i^T P_i + P_i\bar{A}_i + \beta P_i.$$

By using (16), we get the inequality as

$$\begin{bmatrix} \bar{A}_i^T P_i + P_i\bar{A}_i + \beta P_i & P_i\bar{B}_{di} & \bar{C}_i^T \\ * & -\gamma_i^2 I & \bar{D}_{di}^T \\ * & * & -I \end{bmatrix} < 0. \tag{18}$$

To transform the above nonlinear matrix inequality into LMI, we first substitute the values of $\bar{A}_i$, $\bar{B}_{di}$, $\bar{C}_i$, $\bar{D}_{di}$, and taking the change of variables as $R_i = P_iG_{li}$ and $R_i^T = G_{li}^T P_i^T$ gives the following LMIs

$$\bar{A}_i = A_i - G_{li}C_i, \quad \bar{B}_{di} = E_{di} - G_{li}F_{di}, \quad \bar{C}_i = C_i, \quad \bar{D}_{di} = F_{di}$$

$$\wedge_i = \begin{bmatrix} \wedge_{11} & \wedge_{12} & \wedge_{13} \\ * & \wedge_{22} & \wedge_{23} \\ * & * & \wedge_{33} \end{bmatrix}$$

$$\begin{array}{l} \wedge_{11} = A_i^T P_i - C_i^T R_i^T + P_iA_i - R_iC_i + \beta P_i \\ \wedge_{12} = P_iE_{di} - R_iF_{di}, \quad \wedge_{13} = C_i^T \\ \wedge_{22} = -\gamma_i^2 I, \quad \wedge_{23} = F_{di}^T \\ \wedge_{33} = -I \end{array}.$$

The linear observer gain G_{li} for both the modes of the switched system is determined by solving the matrix variables P_i and R_i from the given LMIs as

$$G_{li} = P_i^{-1}R_i. \tag{19}$$

□

4. Residual Evaluation and Adaptive Threshold Computation

Generally, residual signal is corrupted with unknown inputs and disturbances, even when there is no fault in the system. So, based on available residual signal, the residual is evaluated for successful fault detection. A threshold setup over a zero fault case with bounded disturbance exists. Thus, comparison of the generated residual with that of the threshold accurately detects the occurrence of fault. Several functions of evaluation are discussed in the literature, e.g., for switched systems in Reference [37]. We choose the residual evaluation function as

$$\| R(t) \|_{2,\tau} = \| R_d(t) \|_{2,\tau} + \| R_u(t) \|_{2,\tau},$$

where $R_d(t)$ and $R_u(t)$ can be defined as

$$R_d(t) = R(t)_{f=0,u=0}, \quad R_u = R(t)_{f=0,d=0}.$$

The threshold can be determined by $J_{th} = J_{thd} + J_{thu}$, which has two parts, consisting of the constant and input dependent varying threshold, as in Reference [38].

$$J_{th} = J_{thd} + \gamma_v \sqrt{\int_0^{t+\tau} u(t)^T u(t) d(t)}, \tag{20}$$

where the starting time of evaluation is 0, and the evaluation window is τ.

$$\gamma_v = sup \| R_u(t) \|_2 \, / \, \| u(t) \|_2,$$
$$J_{thd} = \sup_{f=0,d\in l_2} \| R_d(t) \|_{2,\tau} \,. \tag{21}$$

Remark 4. *The threshold in (20) is adaptive because any change in the system input results in a change of the variable threshold part.*

For the decision of fault to occur, the following decision rules are employed:
(1) $\| R(t) \|_{2,\tau} > J_{\text{th}}$ Fault is detected.
(2) $\| R(t) \|_{2,\tau} < J_{\text{th}}$ No fault is detected.

5. Simulation Examples

In this section, we give two examples, including a case study of boost converter and a numerical example with both incipient and abrupt faults, to illustrate usefulness of the proposed mechanism.

5.1. Example 1: A Case Study of Boost Converter

A boost converter is a specific type of power converters which acts as a switch system, as shown in Figure 2. We consider the boost converter circuit in Reference [39] driven by pulse width modulation technique. The dynamic model of the switching system is given as:

$$A_1 = \begin{bmatrix} -\frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}, \quad A_2 = \begin{bmatrix} -\frac{1}{RC} & 0 \\ 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}.$$

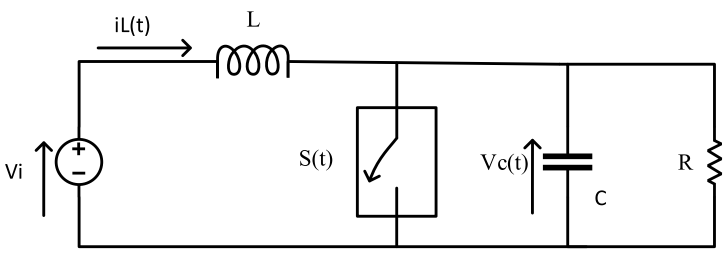

Figure 2. Boost converter circuit.

Let $x(t) = [v_c(t) \;\; i_L(t)]^T$ be the state vector, where $v_c(t)$ is the capacitor voltage, and $i_L(t)$ is the inductor current. By using the same normalization technique as in Reference [39,40], the design matrices for both the modes of switch systems are given as:

$$
\begin{aligned}
&A_1 = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.6 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
&E_{d1} = \begin{bmatrix} 0.5 & 0.2 \\ 0.01 & 0.3 \end{bmatrix}, \quad F_{d1} = \begin{bmatrix} 0.1 & 0.02 \\ 0.02 & 0.2 \end{bmatrix}, \quad E_{f1} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad F_{f1} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix} \\
&A_2 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
&E_{d2} = \begin{bmatrix} 0.2 & 0.01 \\ 0.01 & 0.1 \end{bmatrix}, \quad F_{d2} = \begin{bmatrix} 0.1 & 0.3 \\ 0.02 & 0.1 \end{bmatrix}, \quad E_{f2} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad F_{f2} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}
\end{aligned}.
$$

The positive definite matrix variables and the observer gain matrices for both the modes of the switched systems can be determined from the LMIs.

$$
\begin{aligned}
&P_1 = \begin{bmatrix} 1.695 & -0.740 \\ -0.740 & 2.347 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.671 & -0.007 \\ -0.007 & 12.30 \end{bmatrix}, \quad G_{l1} = \begin{bmatrix} 6.464 & -0.050 \\ 1.553 & 1.424 \end{bmatrix} \\
&G_{l2} = \begin{bmatrix} 0.009 & 2.702 \\ -0.146 & 1.403 \end{bmatrix}
\end{aligned}.
$$

The fault signal with a unit magnitude of a pulse generator is simulated between 40 and 60 s in MATLAB/Simulink. The simulation time is set as T = 100 s and takes the L2 norm bounded disturbance signal as $d \leq 0.5$. Thus, (5) provides a feasible solution by choosing $\gamma = 0.12$, for a good disturbance attenuation level. For the switching signal, choose the parameter value as $\mu = 1.5$ and $\varsigma = 0.35$, to calculate average dwell time as 1.1584. Hence, the switching interval from one mode to the other is greater than 1.1584. Switching signals of subsystem, observer, and fault signal are shown in Figure 3.

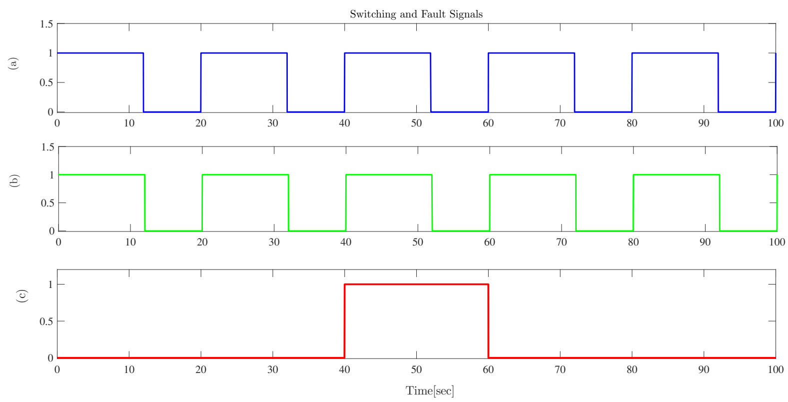

Figure 3. (**a**) Subsystem switching signal. (**b**) Observer switching signal. (**c**) Fault signal.

The residual signal generated with and without the presence of fault is illustrated in Figure 4. Here, the nonzero residual signals, even in the absence of faults, need to be evaluated.

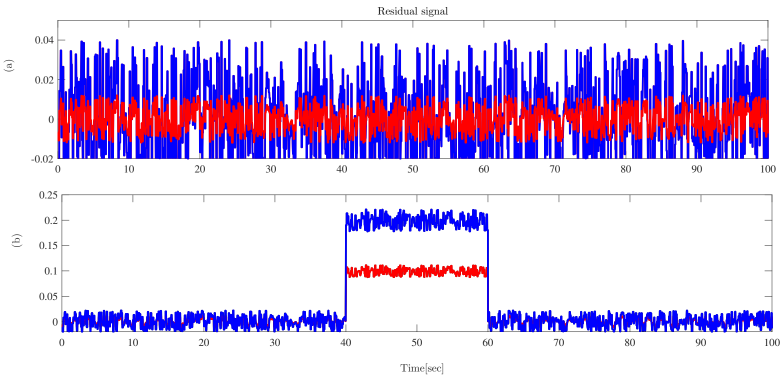

Figure 4. (**a**) Residual signals without fault. (**b**) Residual signals with fault.

Figure 5 shows the evaluated residual along with adaptive threshold and state trajectory of the system, which maintains its stability, even when the fault is applied. The adaptive threshold is computed by using (20), in which the constant threshold is calculated offline as $j_{thd} = 0.0563$ for both of the modes. In addition, the adaptive threshold is computed at $\tau = 47.694$ as $j_{thadpt} = 0.6244$. Thus, fault is detected when the evaluated residual is greater than its threshold, i.e., $r = 0.6246 > 0.6244$.

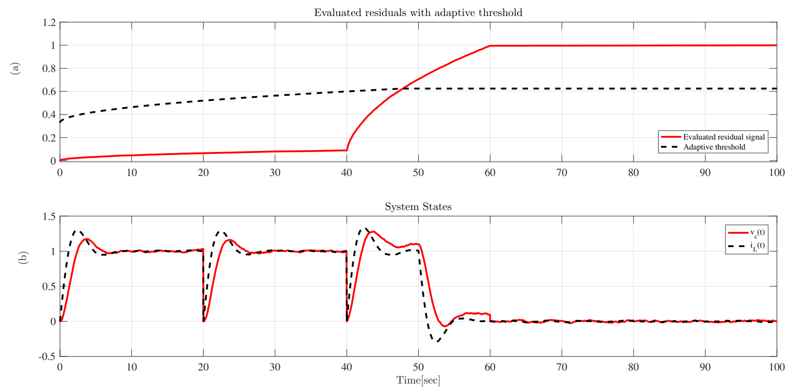

Figure 5. (**a**) Evaluated residual with adaptive threshold. (**b**) Switched system states.

5.2. Example 2: A Numerical Example

Consider the general system from (1), with two modes in state space form, which is:

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + E_{di} d(t) + E_{fi} f(t) \\ y(t) = C_i x(t) + D_i u(t) + F_{di} d(t) + F_{fi} f(t) \end{cases}.$$

For $i \in [1, 2]$, we consider two modes of the switched system. Mode 1 is in operation when subsystem 1 of the switched system and subsystem 1 of the observer are active. Similarly, for Mode 2, subsystem 2 of the switched system and observer are activated. We also assume a known switching sequence as $\sigma_{(t)} = 1$, $\sigma_{(t)} = 0$, for activation of both the modes, respectively. Choose the system matrices as:

System matrices for Mode 1:

$$A_1 = \begin{bmatrix} -6 & 3 & 1 \\ -0.3 & -4 & 1 \\ -2 & -0.6 & -1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0.6 & 0 \end{bmatrix}^T, C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0 & 0.5 & 0 \end{bmatrix}^T, \quad E_{f1} = \begin{bmatrix} 0.2 \\ 0.6 \\ 0.7 \end{bmatrix}, \quad F_{f1} = \begin{bmatrix} 0.3 \\ 0.4 \\ 0.8 \end{bmatrix}.$$

$$E_{d1} = \begin{bmatrix} -0.3 & 0.2 & 0.04 \\ 0.2 & -0.3 & 0.2 \\ 0.2 & 0.04 & 0.1 \end{bmatrix}, \quad F_{d1} = \begin{bmatrix} 0.1 & 0.02 & 0.14 \\ 0.03 & -0.3 & 0.2 \\ -0.06 & 0.03 & 0.4 \end{bmatrix}$$

System matrices for Mode 2:

$$A_2 = \begin{bmatrix} -8 & 4 & 4 \\ -2 & -4 & -1 \\ -0.8 & -1 & -4 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0.9 & 0 \end{bmatrix}^T, \quad C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 0 & 0.4 & 0 \end{bmatrix}^T, \quad E_{f2} = \begin{bmatrix} 0.2 \\ 0.9 \\ 0.6 \end{bmatrix}, \quad F_{f2} = \begin{bmatrix} 0.7 \\ 0.1 \\ 0.4 \end{bmatrix}.$$

$$E_{d2} = \begin{bmatrix} -0.19 & 0.06 & 0.4 \\ 0.3 & -0.6 & 0.2 \\ 0.2 & 0.06 & 0.02 \end{bmatrix}, \quad F_{d2} = \begin{bmatrix} 0.2 & 0.08 & 0.3 \\ 0.2 & -0.4 & 0.2 \\ -0.09 & 0.06 & 0.5 \end{bmatrix}$$

The positive definite matrices P_1 and P_2 are determined from the LMIs:

$$P_1 = \begin{bmatrix} 0.2867 & 0.1277 & -0.2422 \\ 0.1277 & 0.4485 & -0.1742 \\ -0.2422 & -0.1742 & 1.0202 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.1516 & -0.0039 & 0.0397 \\ -0.0039 & 0.3355 & -0.0758 \\ 0.0397 & -0.0758 & 0.3816 \end{bmatrix}.$$

In addition, the linear observer gain matrices are calculated from (19).

$$G_{l1} = \begin{bmatrix} -1.14 & -1.73 & 3.72 \\ 0.25 & 3.47 & -0.44 \\ 0.39 & 0.18 & 1.54 \end{bmatrix}, \quad G_{l2} = \begin{bmatrix} 5.44 & -0.16 & 0.69 \\ 0.0002 & 4.29 & 0.25 \\ -0.06 & 0.65 & 2.15 \end{bmatrix}.$$

The simulation time is set as T = 30sec. The disturbance attenuation level for both modes are set as $\gamma_1 = 0.6734$ and $\gamma_2 = 0.5467$. By choosing the parameters values $\mu = 1.4$, $\varsigma = 0.2$, ADT is set as 1.6823, which means that interval of switching among two subsystems exceeds this value. The unknown input or disturbance is some type of random number and is taken as $d \in [-0.5, 0.5]$. Due to importance in real physical systems, two types of faults are considered in this paper, as in Reference [38], including abrupt faults in which destructive system failure is prohibited by detecting the faults early or configuring the systems earlier. On the other hand, for slowly developing or incipient faults, the time of fault detection may be larger and cannot be easily detected. Hence, we apply these faults to both the modes of the switched systems with bounded disturbance. Figure 6 shows the switching signals of the subsystem and observer. Duration of Mode 1 is from time 0 to 5 s, and that of Mode 2 from 5 to 10 s.

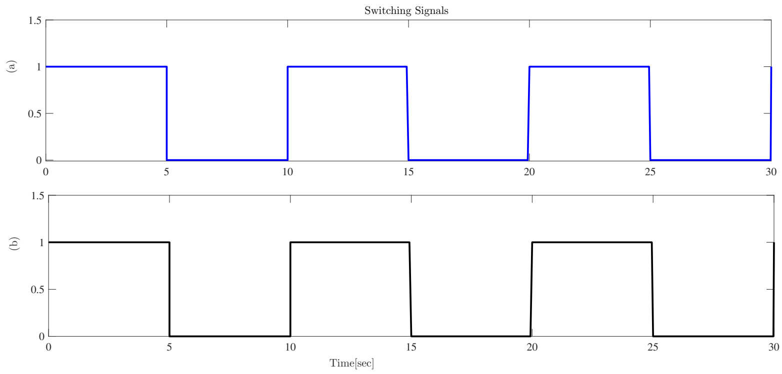

Figure 6. (**a**) Subsystem switching signal. (**b**) Observer switching signal.

5.2.1. Abrupt Fault Case

Figure 7 illustrates two sudden faults of pulse generator signals at a phase delay of 3 and 8 s, respectively. It means the first fault is detected at Mode 1, and the second fault is detected at Mode 2, alternatively.

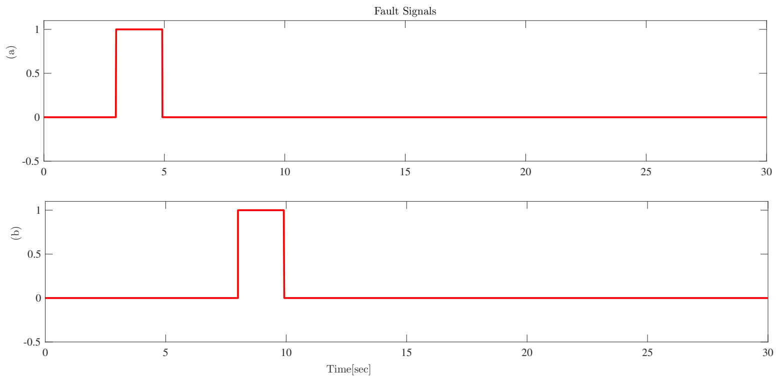

Figure 7. (**a**) Fault signal for Mode 1. (**b**) Fault signal for Mode 2.

The three types of residual signals generated for each mode are depicted in Figure 8. In Figure 9, the evaluated residual signals are represented by solid lines, with adaptive threshold represented by dashed lines. By computing, mathematically, the threshold is set by using (20), where the constant threshold for both the modes is calculated offline as $j_{thd_1} = 0.1451$ and $j_{thd_2} = 0.2365$. Thus, the adaptive threshold for Mode 1 is calculated at $\tau = 3.7862$ as $J_{th1} = 0.8456$, and $J_{th2} = 0.6462$ at $\tau = 8.563$ for Mode 2. A fault is detected when the residual evaluated is greater than its threshold, such as $r_1 = 0.8458 > 0.8456$ and $r_2 = 0.6465 > 0.6462$, where r_1 and r_2 are residuals evaluated for both the modes.

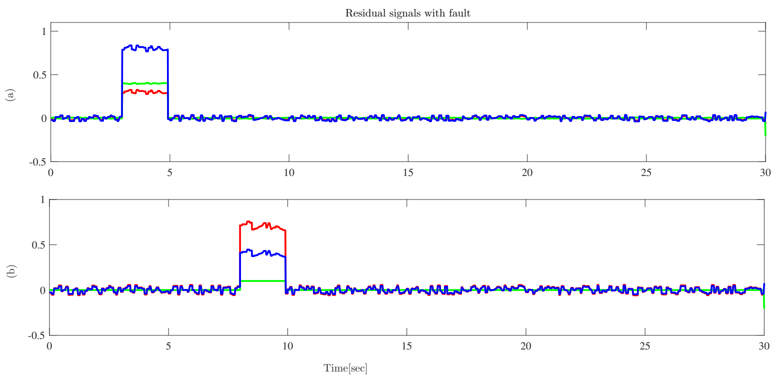

Figure 8. (**a**) Residual signals for Mode 1 (**b**) Residual signals for Mode 2.

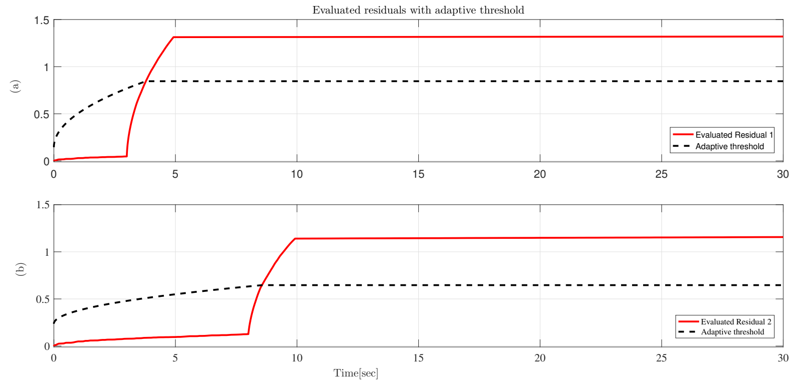

Figure 9. (**a**) Evaluated residual with adaptive threshold for Mode 1. (**b**) Evaluated residual with adaptive threshold for Mode 2.

5.2.2. Incipient Fault Case

Slowly developing faults using ramp signal are applied between time 3 to 5 s for first mode, and between time 5 to 10 s for the second mode as in Figure 10. The generated residual signals in Figure 11 show the fault occurrence with disturbance. The evaluated residual signals and their threshold are given in Figure 12. Thus, setting of the threshold gives $J_{th1} = 0.807$ at $\tau = 3.38$, and $J_{th2} = 0.6202$ for $\tau = 7.51$. Hence, the fault occurs when the evaluated residuals exceed their threshold, as $r_1 = 0.809 > 0.807$ and $r_2 = 0.6204 > 0.6202$.

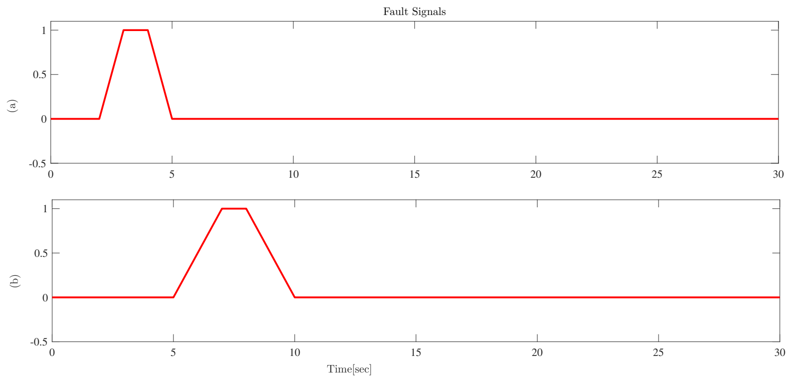

Figure 10. (**a**) Fault signal for Mode 1. (**b**) Fault signal for Mode 2.

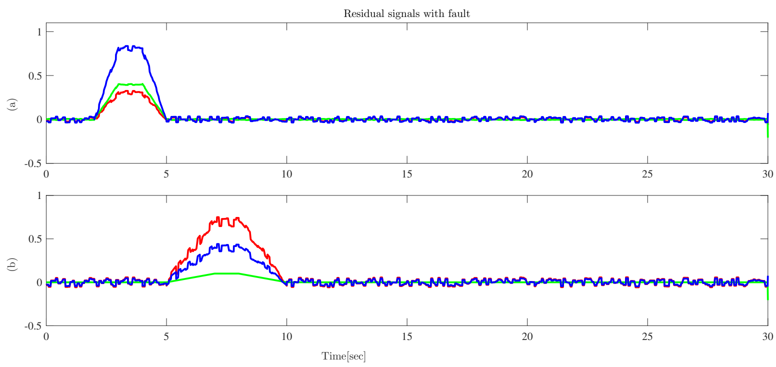

Figure 11. (**a**) Residual signals for Mode 1. (**b**) Residuals signal for Mode 2.

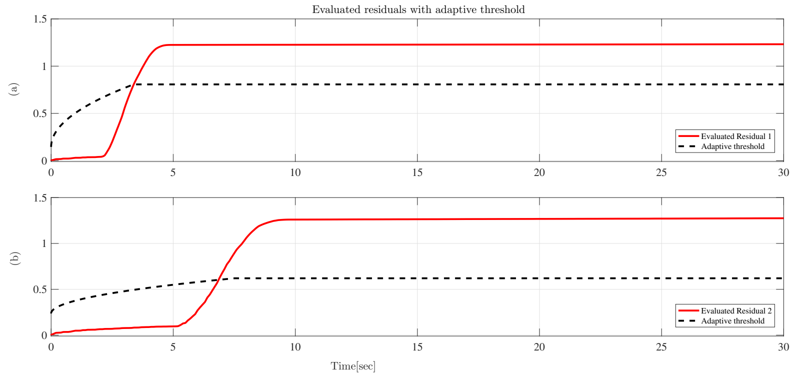

Figure 12. (**a**) Evaluated residual with adaptive threshold for Mode 1. (**b**) Evaluated residual with adaptive threshold for Mode 2.

6. Conclusions

In this paper, we have discussed FD problem of switched linear systems based on the SMO approach. H∞ criteria is implemented to achieve robustness of residual against disturbances. The system stability issues are dealt with Lyapunov function, while taking the ADT switching into account. For efficient detection of a fault, an adaptive threshold is set up for both the modes. At the end, simulation results are illustrated, for efficacy of our suggested approach. Thus, the developed method has the advantage of not only detecting the fault perfectly but also providing robustness against process disturbances. This has been verified in the design simulation examples. Recently, some useful results have been presented on SMO-based robust fault diagnosis for switched systems using H∞ performance. Moreover, this work can further be extended to fault diagnosis for complex asynchronous switching systems, while using SMO. In addition, fault coupling with disturbance and uncertainties using less conservative techniques, while investigating switched systems, will be our further future work.

Author Contributions: Conceptualization, M.T.R. and S.A.; methodology, S.A. and M.T.R.; software, S.A.; validation, H.L. and M.T.R.; formal analysis, S.A.; investigation, S.A.and M.T.R.; resources, H.L.; data curation, S.A., G.A., and M.T.R.; writing—original draft preparation, S.A.; writing—review and editing, H.L. and G.A.; visualization, S.A.; supervision, H.L. and M.T.R.; project administration, S.A.O. and N.U.; funding acquisition, S.A.O., N.U., and G.A. All authors have read and agreed to the published version of the manuscript.

Funding: This research work was supported by Taif University Researchers Supporting Project number (TURSP-2020/228), Taif University, Taif, Saudi Arabia.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors acknowledge the financial support from Taif University Researchers Supporting Project Number (TURSP-2020/228), Taif University, Saudi Arabia. The authors are also thankful to the Department of Control Science and Engineering, Harbin Institute of Technology, Harbin, China for providing the facilities to conduct this research.

Conflicts of Interest: The authors declare no conflict of interest.

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