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Influence of Coil Current and Oil Film Thickness on Hopf Bifurcation of MLDSB

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Abstract: Magnetic-liquid double-suspension bearing (MLDSB) is composed of an electromagnetic supporting system and a hydrostatic supporting system. Due to its greater supporting capacity and stiffness, it is appropriate for middle-speed applications, overloading, and frequent starting. However, because it contains two sets of systems, its structure and rotor support system are more complex. It contains strong nonlinear links. When the parameters of the system change, the bearing rotor may feature Hopf bifurcation, resulting in system flutter and reducing the operational stability of the magnetic fluid double-suspension bearing rotor, which has become one of the key problems restricting its development and application. As key parameters of MLDSB, the coil current and oil film thickness exert a major impact on Hopf bifurcation. Therefore, the mathematical model of MLDSB is established in this paper, and the border and direction of Hopf bifurcation, period, and amplitude of limit cycle are analyzed. The calculation, simulation, and experimental results show that when the coil current and oil film thickness of the bearing system are greater than the boundary value of the Hopf bifurcation, Hopf bifurcation will occur, resulting in the vibration of the bearing rotor and affecting the stability of the system. In addition, when analyzing the combined effects of coil current and oil film thickness on the Hopf bifurcation of the system, it was found that the boundary value of Hopf bifurcation in the system is reduced compared with when it is affected solely due to the coupling of the two parameters. The period, amplitude and vibration speed of limit cycle increase with increases in the coil current and oil film thickness. Hopf bifurcation experiment was conducted on MLDSB testing system. The results show that Hopf bifurcation does not occur when $i_0 < 0.5$ A, the bearing rotor operates stably in the balanced position, $i_0 > 1.0$ A, Hopf bifurcation occurs in the system, and the bearing rotor vibrates with equal amplitude, which reduces the stability of operation. The research in this paper can provide a theoretical reference for the Hopf bifurcation analysis of MLDSB.

Keywords: MLDSB; nonlinear supporting system; Hopf bifurcation; limit cycle; phase trajectories; x-t diagram



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1. Introduction

As a novel supporting system, MLDSB is dominated by electromagnetic suspension and supplemented by hydrostatic support. It offers plenty of advantages, such as a lack of mechanical contact, strong carrying capacity, high stiffness, and so on, which make it appropriate for deep-sea exploration, hydroelectric power generation, and other fields, especially for working in conditions of middle speed, overloading, and frequent starting [1].

MLDSB includes a bracket, a variable speed motor, coupling, a stepped shaft, journal bearing, axial bearing, an axial loading motor, and a journal motor, as shown in Figure 1.

The radial unit of MLDSB is composed of a step shaft, a magnetic guide sleeve, a supporting cavity, a magnetic pole, an inlet pipe, a shell, a coil, and an outlet, as shown in Figure 2.

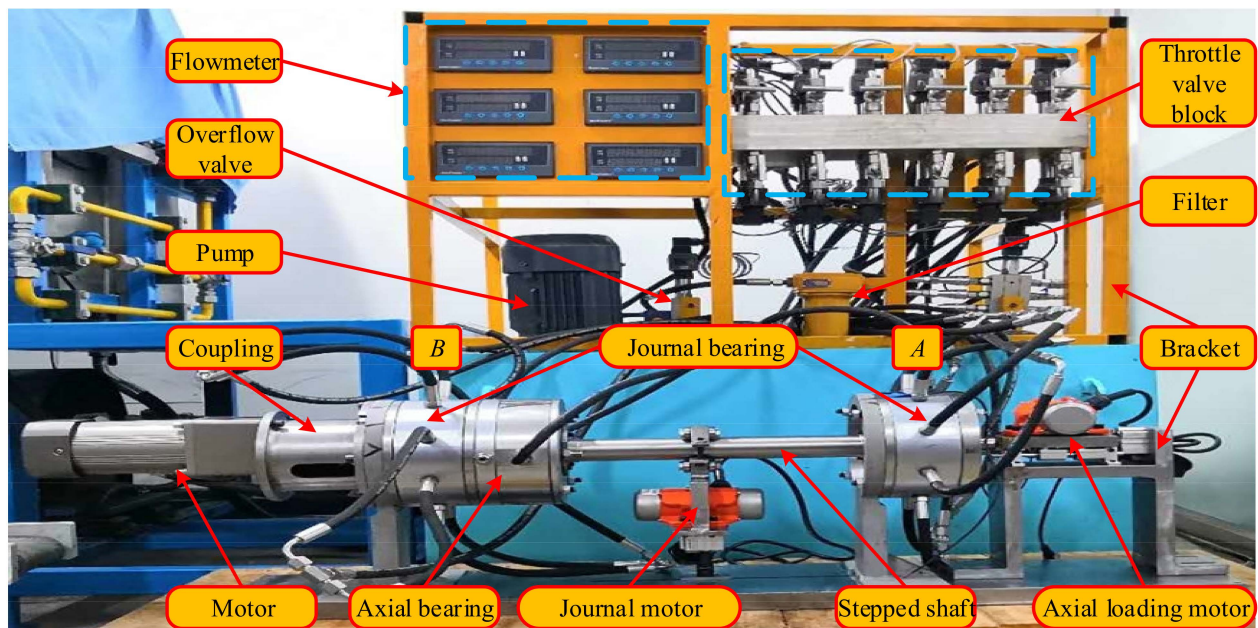


Figure 1. Overall drawing of MLDSB test bench.

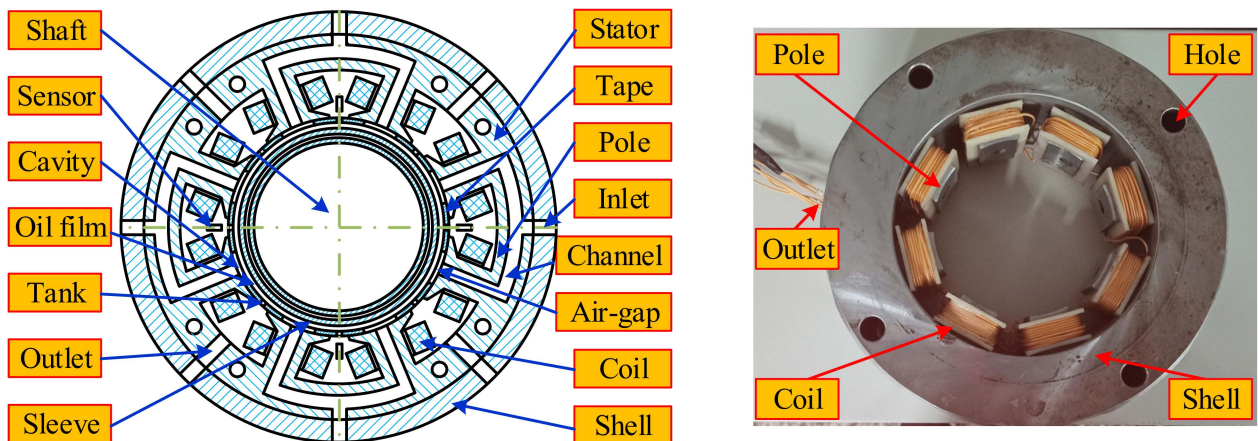


Figure 2. Structure of radial unit of MLDSB.

The regulation principle of MLDSB is shown as Figure 3 [2]. Due to its strong nonlinear characteristics and complicated damping characteristics, it features mutual coupling between its two supporting systems, and Hopf bifurcation phenomenon occurs easily; consequently, the reliability and operational stability of MLDSB are reduced.

In recent years, many scholars have studied the Hopf bifurcation behavior of electromagnetic bearings and achieved fruitful results. Hopf bifurcation is a relatively simple and important bifurcation problem in nonlinear dynamic systems. It belongs to local dynamic bifurcation. Specifically, it refers to the phenomenon whereby the system suddenly bifurcates from the equilibrium point to the limit cycle at the non-hyperbolic equilibrium point in response to changes in the bifurcation parameters. It is widely used in the study of the dynamic characteristics of various complex nonlinear dynamic systems. The inherent nonlinear dynamic essence of the system is revealed through Hopf bifurcation, and a corresponding control strategy and optimization method are proposed.

Due to the introduction of hydrostatic force, the rotor force of MLDSB is more complex than that of traditional electromagnetic bearing, so it is difficult to analyze Hopf bifurcation behavior.

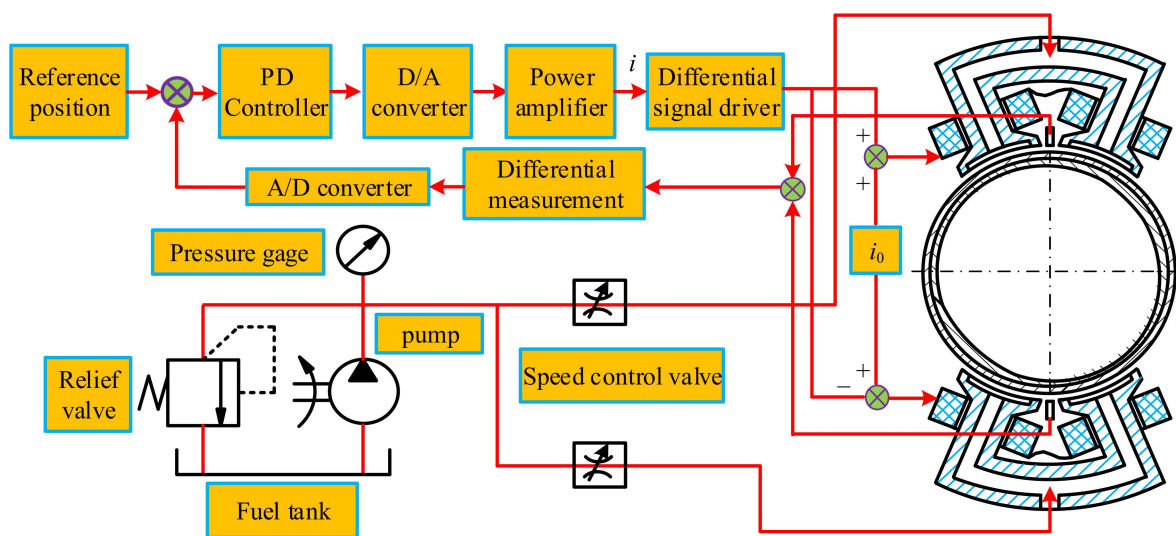


Figure 3. Single-DOF supporting system for MLDSB.

As the key parameters in the supporting characteristics of MLDSB, oil film thickness and coil current exert a major impact on Hopf bifurcation behavior. Therefore, this paper intends to establish a mathematical model of the magneto-hydraulic coupling force of MLDSB and to explore the internal influence laws governing the influence oil film thickness and coil current on the Hopf bifurcation boundary, limit cycle period, and phase trajectory of a single-DOF supporting system, so as to provide a theoretical basis for the design and stable operation of MLDSB.

2. Dynamics Model of Single-DOF Supporting System

The dynamics equation for a single-DOF supporting system in MLDSB is as follows [2]:

$$\begin{cases} \dot{x} = y \\ y = -\frac{1}{m}f(x, \dot{x}) - \frac{1}{m}g(x, \dot{x}) \end{cases} \quad (1)$$

where $f(x, \dot{x}) = F_{l0}\dot{x}\left[\frac{1}{(1+\bar{x}\cos\theta)^3} - \frac{1}{(1-\bar{x}\cos\theta)^3}\right]$;

$$g(x, \dot{x}) = F_{e0}\left[\left(\frac{1-\bar{i}_c}{1+\bar{x}\cos\theta}\right)^2 - \left(\frac{1+\bar{i}_c}{1-\bar{x}\cos\theta}\right)^2\right] - F_{l0}\left[\frac{1}{(1+\bar{x}\cos\theta)^3} - \frac{1}{(1-\bar{x}\cos\theta)^3}\right];$$

$\bar{i}_c = i_c/i_0$; $\bar{x} = x/h_0$; x is the displacement, y is the velocity, g is the acceleration of gravity, i_c is the regulation current, and θ is the included angle of magnetic poles.

The design parameters of MLDSB are shown in Table 1 [3]. The range of coil current i_0 is (0.5, 1.2) A. $\bar{x}$ is the displacement after dimensionless treatment, F_{e0} is the initial electromagnetic force, and F_{l0} is the initial hydraulic pressure.

Table 1. Design parameters of MLDSB.

Symbol	Variable	Value
A	Length of Supporting Cavity	0.1 m
A_1	Magnetic Area	1000 mm ²
A_b	Extrusion area	1056 mm ²
A_e	Supporting Area	1504 mm ²
B	Width of Supporting Cavity	0.02 m
$\bar{B}$	Flow coefficient	4.3611
K_d	Differential Coefficient	0.05
K_p	Scale Coefficient	−70

Table 1. Cont.

Symbol	Variable	Value
N	Turns of Coil	60
a	Length of Sealing Tape	0.006 m
h_0	Oil Film Thickness	30 μm
i_0	Initial current	2 A
k	Electromagnetic coefficient	2.38×10^{-5}
m	Rotor Mass	100 kg
q_0	Supporting Cavity Flow	3.2622 L/min
μ	Viscosity	1.307×10^{-3} Pa.s
b	Width of Sealing Tape	0.004 m
μ_0	Air Permeability	$4\pi \times 10^{-7}$ H/m

3. Influence of Coil Current and Oil Film Thickness on Hopf Bifurcation of MLDSB

3.1. Influence of Coil Current on Hopf Bifurcation of MLDSB

3.1.1. Hopf Bifurcation Boundary

According to Hopf bifurcation theory [4–6], the Jacobian matrix of Equation (1) is obtained as follows:

$$A = \begin{pmatrix} 0 & 1 \\ a_{21} & a_{22} \end{pmatrix} \quad (2)$$

$$\lambda^2 - a_{22}\lambda - a_{21} = 0 \quad (3)$$

$$\lambda_{1,2} = \frac{\alpha_1}{2} \pm \frac{1}{2}\sqrt{\alpha_1^2 + 4\beta_1} \quad (4)$$

$$\text{where } a_{21} = \frac{8i_0^2 k \cos^2 \theta}{mh_0^3} - \frac{6\delta_1 \cos \theta}{mh_0^4} + \frac{1200K_p k i_0 \cos \theta}{mh_0^2};$$

$$a_{22} = \frac{-2\delta_2}{mh_0^3} + \frac{1200K_d i_0 k \cos \theta}{mh_0^2}; \delta_1 = \frac{2\mu q_0 A_e \cos \theta}{B}; \delta_2 = \frac{2\mu A_e A_b \cos^2 \theta}{B};$$

$$\alpha_1 = 693.31i_0 - 597.17; \beta_1 = 2.83 \times 10^6 i_0^2 - 9.71 \times 10^5 i_0 - 3.07 \times 10^6.$$

According to Equations (2)–(4), the Hopf bifurcation boundary of a single-DOF supporting system is $i_0 = 0.86$ A. When $i_0 > 0.86$ A, Hopf bifurcation occurs in the bearing system, resulting in the vibration of the bearing rotor and affecting the stability of operation.

3.1.2. Hopf Bifurcation Direction

Take the partial derivative of both sides of Equation (3) with respect to i_0 .

$$\frac{d\lambda}{di_0} = \frac{\varepsilon_1}{\varphi_1} \quad (5)$$

$$\text{where } \varepsilon_1 = \frac{16k \cos \theta i_0}{mh_0^3} + \frac{1200k \cos \theta}{mh_0^2} (K_p + K_d \lambda);$$

$$\varphi_1 = 2\lambda + \frac{4\mu A_e A_b \cos^2 \theta}{mBh_0^3} - \frac{1200K_d i_0 k \cos \theta}{mh_0^2} \quad \lambda_{1,2} = -0.46 \pm 1.35 \times 10^3 i \text{ when } i_0 = 0.86 \text{ A, and}$$

the cross-sectional coefficient can be obtained as follows.

$$\sigma'(i_0) = \text{Re} \left(\frac{d\lambda}{di_0} \bigg|_{i_0 = 0.86} \right) = 346.67 \quad (6)$$

According to Equation (6), supercritical Hopf bifurcation occurs when the cross-sectional coefficient is greater than zero. In other words, when $i_0 < 0.86$ A, the single-DOF supporting system achieves a stable balance state without Hopf bifurcation. When $i_0 > 0.86$ A, the phase trajectory is a stable limit cycle and the Hopf bifurcation phenomenon occurs.

3.1.3. Influence of Coil Current on Limit Cycle Period

According to Hopf bifurcation theory, the period of the limit cycle is shown as follows.

$$T = \frac{4\pi}{\sqrt{-(\eta_1^2 + 4\gamma_1)}} \quad (7)$$

where $\eta_1 = 693.31i_0 - 597.17$; $\gamma_1 = 2.83 \times 10^6 i_0^2 - 9.71 \times 10^5 i_0 - 3.07 \times 10^6$.

Substituting the data in Table 1 into Equation (7), the period T of limit cycle can be obtained as shown in Figure 4.

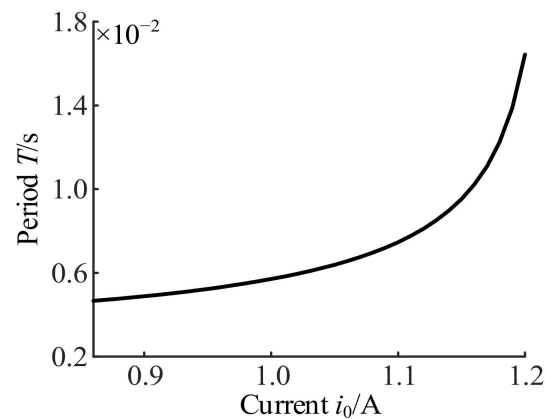


Figure 4. Curve of period T and current i_0 .

According to Figure 4, the period of the limit cycle increases with the increase in current i_0 . When $i_0 < 1.1$ A, it creates little impact on the period of the limit cycle. When $i_0 > 1.1$ A, the limit cycle period is greatly affected.

3.2. Influence of Oil Film Thickness on Hopf Bifurcation of MLDSB

3.2.1. Hopf Bifurcation Boundary

Set $i_0 = 1.2$ A, the root of characteristic equation of Jacobian matrix can be obtained as follows:

$$\frac{\alpha_2}{2} \pm \frac{1}{2} \sqrt{\alpha_2^2 + 4\beta_2} \quad (8)$$

where $\alpha_2 = \frac{7.49 \times 10^{-7}}{h_0^2} - \frac{1.61 \times 10^{-11}}{h_0^3}$; $\beta_2 = \frac{1.10 \times 10^{-7}}{h_0^3} - \frac{2.49 \times 10^{-12}}{h_0^4} - \frac{1.05 \times 10^{-3}}{h_0^2}$.

According to Equation (8), the bifurcation boundary is $h_0 = 2.5 \times 10^{-5}$ m.

3.2.2. Hopf Bifurcation Direction

The derivative of bifurcation parameter h_0 on both sides of Equation (3) is obtained as follows.

$$\frac{d\lambda}{dh_0} = \frac{\varepsilon_{2,1} + \varepsilon_{2,2}}{\varphi_2} \quad (9)$$

where $\varepsilon_{2,1} = \frac{\cos^2 \theta}{m} \left(\frac{-24ki_0^2}{h_0^4} + \frac{48\mu q_0 A_e}{Bh_0^5} - \frac{2400K_p k i_0}{h_0^3 \cos \theta} \right)$; $\lambda_{1,2} = 84 \pm 1003.7i$.

$\varepsilon_{2,2} = \left(\frac{12\mu A_e A_b \cos^2 \theta}{mBh_0^4} - \frac{2400K_d k i_0 \cos \theta}{mh_0^3} \right) \lambda$; $\varphi_2 = 2\lambda + \frac{4\mu A_e A_b \cos^2 \theta}{mBh_0^3} - \frac{1200K_d i_0 k \cos \theta}{mh_0^2}$.

When $h_0 = 2.5 \times 10^{-5}$ m, the cross-sectional coefficient can be obtained as follows.

$$\sigma'(h_0) = \text{Re} \left(\frac{d\lambda}{dh_0} \Big|_{h_0=2.5 \times 10^{-5}} \right) = 1.34 \times 10^7 \quad (10)$$

According to Equation (10), supercritical Hopf bifurcation occurs when the cross-cutting coefficient is greater than zero. In other words, when $h_0 < 2.5 \times 10^{-5}$ m, the single-

DOF supporting system achieves a stable balance state without Hopf bifurcation. When $h_0 > 2.5 \times 10^{-5}$ m, the phase trajectory is the stable limit cycle and Hopf bifurcation occurs.

3.2.3. Effect of Oil Film Thickness on Limit Cycle Period

According to Hopf bifurcation theory, the limit cycle period is obtained as follows:

$$T = \frac{4\pi}{\sqrt{-(\eta_2^2 + 4\gamma_2)}} \quad (11)$$

where $\eta_2 = \frac{7.49 \times 10^{-7}}{h_0^2} - \frac{1.61 \times 10^{-11}}{h_0^3}$; $\gamma_2 = \frac{1.10 \times 10^{-7}}{h_0^3} - \frac{1.05 \times 10^{-3}}{h_0^2} - \frac{2.49 \times 10^{-12}}{h_0^4}$.

Substituting the data in Table 1 into Equation (11), the period T of limit cycle is obtained, as shown as Figure 5.

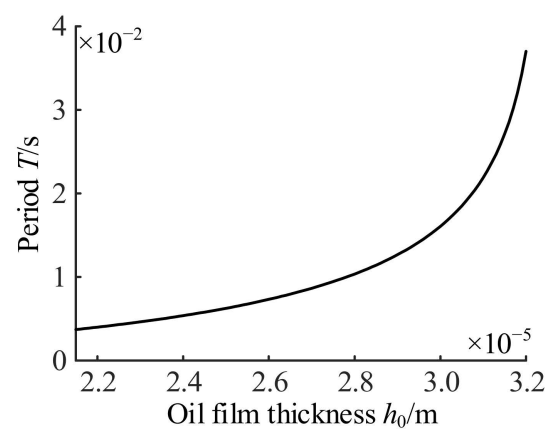


Figure 5. Curve of period T and oil film thickness h_0 .

According to Figure 5, the period of limit cycle increases with the increase in oil film thickness. When $h_0 < 2.8 \times 10^{-5}$ m, the oil film thickness exerts little effect on the limit cycle period. When $h_0 > 2.8 \times 10^{-5}$ m, the limit cycle period is greatly affected.

3.3. Compound Effect of Coil Current and Oil Film Thickness on Hopf Bifurcation of MLDSB

3.3.1. Hopf Bifurcation Boundary

Setting i_0 and h_0 as the variables, the roots of characteristic equation of Jacobian matrix can be obtained as follows.

$$\frac{\alpha_3}{2} \pm \frac{1}{2} \sqrt{\alpha_3^2 + 4\beta_3} \quad (12)$$

where $\alpha_3 = \frac{6.24 \times 10^{-7} i_0}{h_0^2} - \frac{1.61 \times 10^{-11}}{h_0^3}$;

$$\beta_3 = \frac{7.65 \times 10^{-8} i_0^2}{h_0^3} - \frac{2.49 \times 10^{-12}}{h_0^4} - \frac{8.74 \times 10^{-4} i_0}{h_0^2}.$$

According to Equation (12), the bifurcation boundary curve is obtained as follows.

$$i_0 = \frac{h_0^3 + 1.61 \times 10^{-4}}{6.24 h_0} \quad (13)$$

3.3.2. Hopf Bifurcation Direction

According to Equations (6) and (10), the cross-sectional coefficient is greater than zero; consequently, the Hopf bifurcation direction can be inferred as shown in Figure 6.

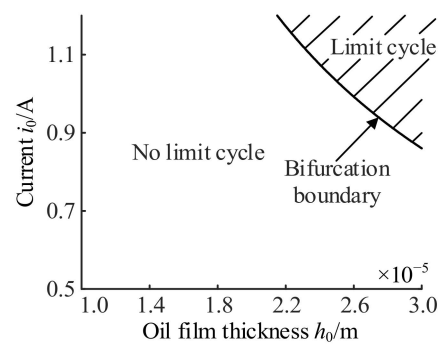


Figure 6. Bifurcation region.

According to Figure 6, when the parameters are on the left side of the boundary line, the single-DOF supporting system achieves stable balance without Hopf bifurcation. When the parameters are on the right side (the shaded region), the phase trajectory is the stable limit cycle and Hopf bifurcation phenomenon occurs. When one parameter is small, the stability region of another parameter can be effectively expanded.

3.3.3. Limit Cycle Period

According to Hopf bifurcation theory, the limit cycle period is obtained.

$$T = \frac{4\pi}{\sqrt{-(\eta_3^2 + 4\gamma_3)}} \quad (14)$$

$$\text{where } \eta_3 = \frac{6.24 \times 10^{-7} i_0}{h_0^2} - \frac{1.61 \times 10^{-11}}{h_0^3};$$

$$\gamma_3 = \frac{7.65 \times 10^{-8} i_0^2}{h_0^3} - \frac{2.49 \times 10^{-12}}{h_0^4} - \frac{8.74 \times 10^{-4} i_0}{h_0^2}.$$

Substituting the data in Table 1 into Equation (14), the period T of limit cycle is obtained as follows.

According to Figure 7, the period of limit cycle increases with the increase in coil current and oil film thickness. The oil film thickness exerts little effect on the limit cycle period when the coil current and oil film thickness are small, and the limit cycle period is greatly affected while larger.

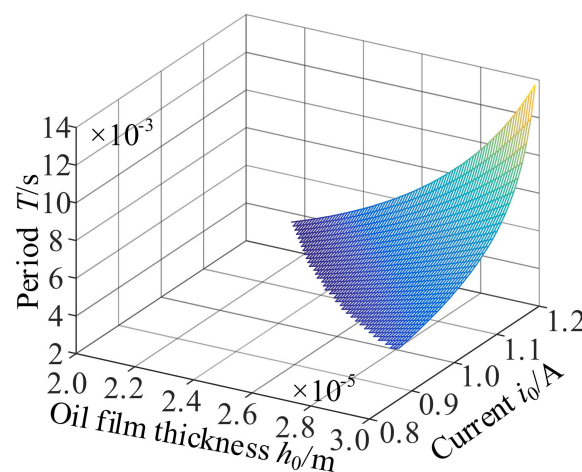


Figure 7. Surface diagram of relation between T and i_0 and h_0 .

4. Simulation and Experiment

When the phase trajectory becomes a limit cycle, Hopf bifurcation occurs; subsequently, the oscillation degree can be reflected by the amplitude and maximum vibration

velocity of the limit cycle. Therefore, the phase trajectory and x-t diagram under different circumstances were simulated.

4.1. Simulation of Phase Trajectories and X-T Diagrams

4.1.1. Phase Trajectories and X-T Diagrams under The Influence of Coil Current

The initial displacement x_0 and velocity v_0 were selected, and the phase trajectory and x-t diagram were simulated, as shown in Figure 8.

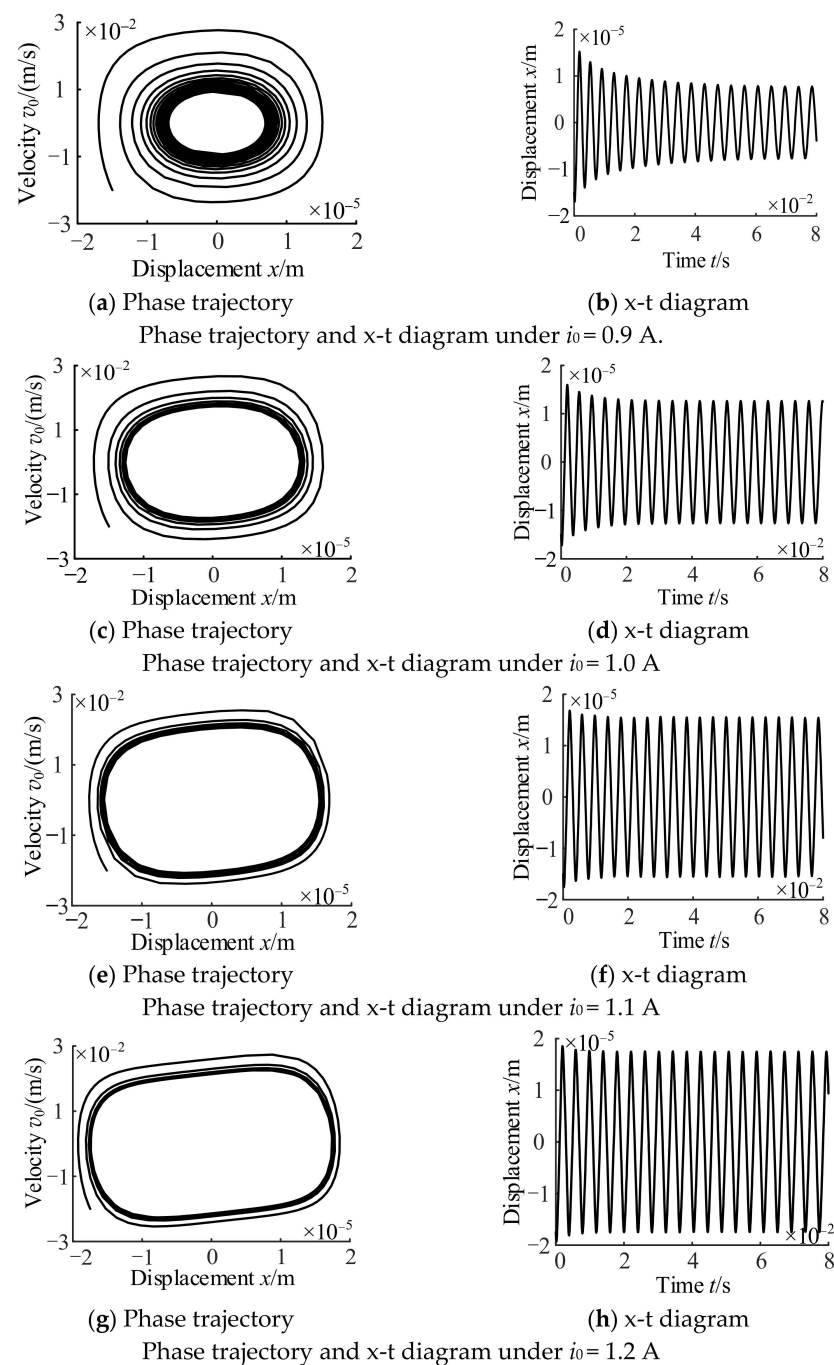
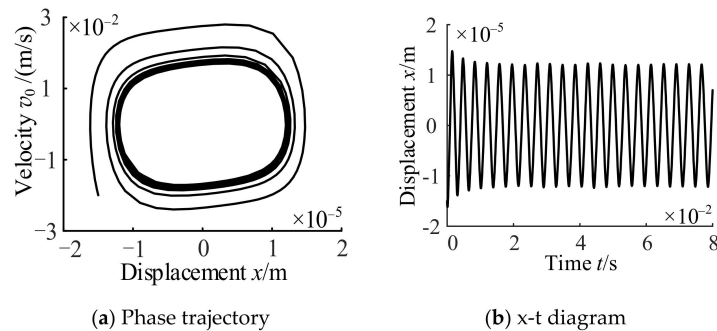


Figure 8. Phase trajectory and x-t diagram.

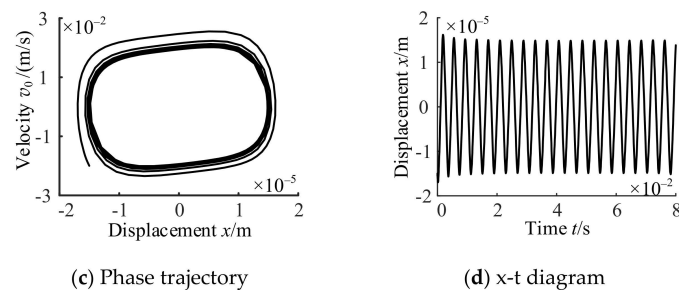
According to Figure 8, when i_0 increases from 0.9 A to 1.2 A, the vibration amplitude increases from 8.0×10^{-6} m to 1.75×10^{-5} m, and the maximum vibration velocity increases from 0.011 m/s to 0.023 m/s. The larger i_0 is, the more serious the constant-amplitude oscillation phenomenon will be and the weaker the stability will be.

4.1.2. Phase Trajectories and X-T Diagrams under The Influence of Oil Film Thickness

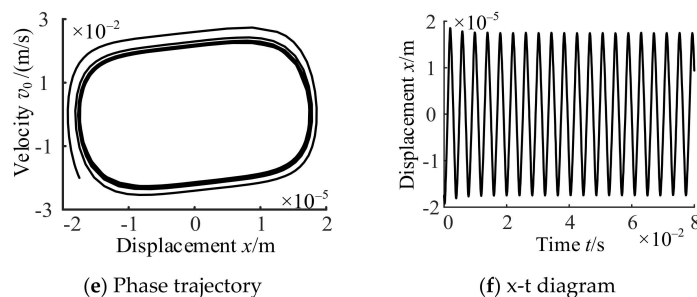
The initial displacement x_0 and velocity v_0 were selected, and the phase trajectory and x-t diagram were simulated, as shown in Figure 9.



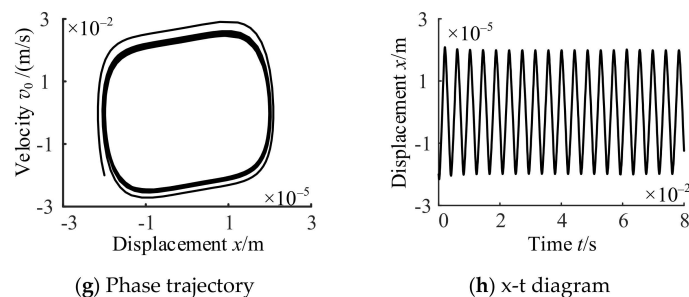
Phase trajectory and x-t diagram under $h_0 = 2.6 \times 10^{-5}$ m



Phase trajectory and x-t diagram under $h_0 = 2.8 \times 10^{-5}$ m



Phase trajectory and x-t diagram under $h_0 = 3.0 \times 10^{-5}$ m



Phase trajectory and x-t diagram under $h_0 = 3.2 \times 10^{-5}$ m

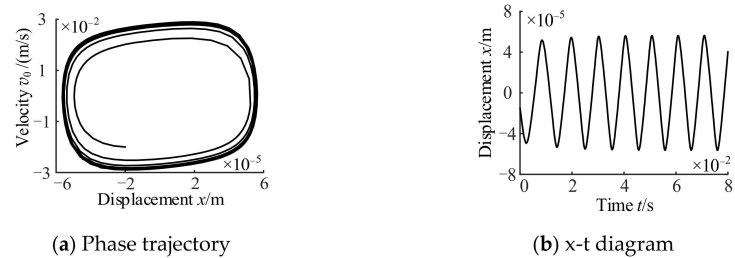
Figure 9. Phase trajectory and x-t diagram.

According to Figure 9, when h_0 increases from 2.6×10^{-5} m to 3.2×10^{-5} m, vibration amplitude increases from 1.2×10^{-5} m to 2.0×10^{-5} m and the maximum vibration velocity

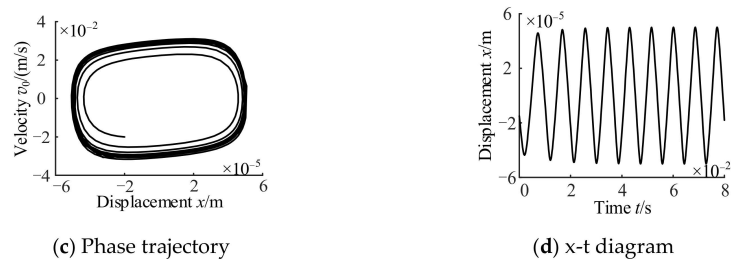
increases from 0.017 m/s to 0.025 m/s. The larger h_0 is, the more serious constant-amplitude oscillation and the weaker the stability will be.

4.1.3. Phase Trajectory and X-T Diagram under The Combined Influence of Coil Current and Oil Film Thickness

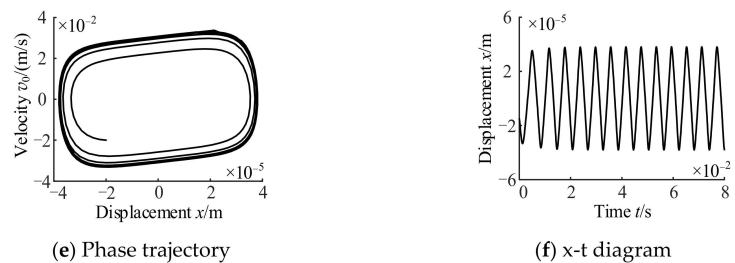
The initial displacement x_0 and velocity v_0 were randomly selected, and the phase trajectory and x-t diagram were simulated as shown as Figure 10.



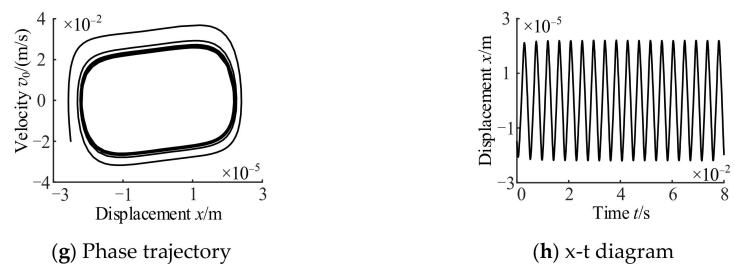
Phase trajectory and x-t diagram when $i_0 = 0.88$ A and $h_0 = 7.4 \times 10^{-5}$ m



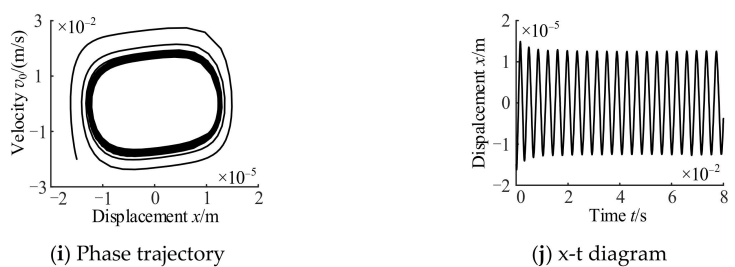
Phase trajectory and x-t diagram when $i_0 = 0.92$ A and $h_0 = 6.6 \times 10^{-5}$ m



Phase trajectory and x-t diagram when $i_0 = 1.10$ A and $h_0 = 5.0 \times 10^{-5}$ m



Phase trajectory and x-t diagram when $i_0 = 1.18$ A and $h_0 = 3.4 \times 10^{-5}$ m



Phase trajectory and x-t diagram when $i_0 = 1.22$ A and $h_0 = 2.6 \times 10^{-5}$ m

Figure 10. Phase trajectory and x-t diagram.

According to Figure 10, when i_0 increases from 0.88 A to 1.22 A and h_0 decreases from 7.4×10^{-5} m to 2.6×10^{-5} m, the vibration amplitude decrease from 5.6×10^{-5} m to 1.2×10^{-5} m. The maximum vibration velocity increases from 0.029 m/s to 0.033 m/s and then decreases to 0.018 m/s. Compared with i_0 , the influence of h_0 on vibration amplitude is more obvious; accordingly, the maximum vibration velocity increases first and then decreases.

4.2. Experimental Result of Decoupling

4.2.1. Brief Introduction to MLDSB Testing System

The MLDSB testing system is composed of an electronic control system, a hydrostatic system, and a bearing body, as shown in Figure 11.

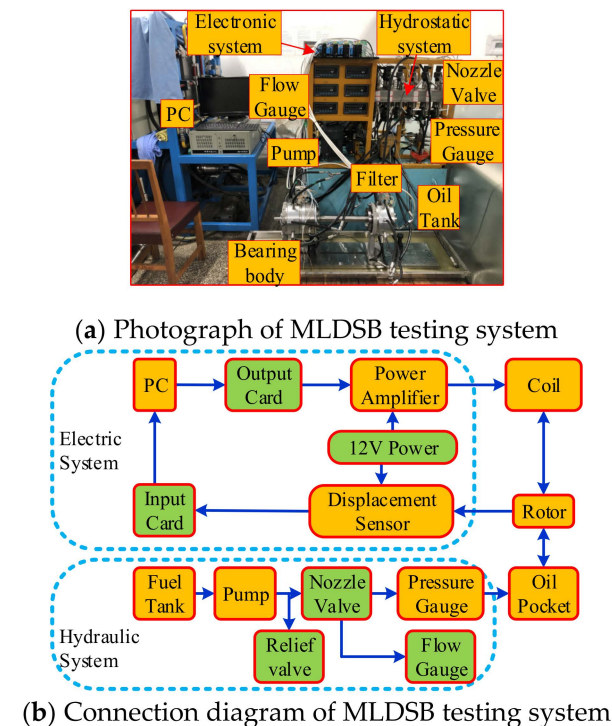


Figure 11. MLDSB testing system.

A hydrostatic system is a constant-pressure supporting model, and its flow is adjusted by needle valve. An electronic control system is a closed-loop position control system, and its current is adjusted by PD controller.

The parameters of the MLDSB testing system are as follows.

- (1) Hydraulic pump, model TGPVL4-200SH, pressure 14 MPa, flow 16 L/min, rotate speed 1450 r/min.
- (2) Relief valve, model DBD-H-6-P-10-B-NG10, pressure 10 MPa, size 6 mm.
- (3) Nozzle valve, model A7-2-KL2-0KL20-PTFE, size 6 mm.
- (4) Flow gauge, model LWGY-S, pressure 2 MPa, flow 120–2400 mL/min, and accuracy 2%.
- (5) Pressure gauge, model HSTL-802, pressure 10 MPa, accuracy 0.25%.
- (6) Displacement gauge, model VB-Z9900, range for 4 mm, accuracy 1.5%.
- (7) Coil, materials for Cu, diameter 1.0 mm, electrical resistivity $0.02240 \Omega/\text{m}$, length 25 m.
- (8) PC, model IPC-610L, mainboard AIMB-705BG, CPU I5-6500.
- (9) Output card, model NI6723, 8 channel output.
- (10) Input card, model PCI1716, 16 channel input.
- (11) 12V Power, model S1500-12, voltage 12V, current 125 A, power 1500 W.
- (12) Power amplifier, model AQMD3620NS-A2, power 400 w, input voltage 36 V, current 16 A.

4.2.2. Analysis of Experimental Results

The displacement curves when $i_0 = 0.5$ A, 1.0 A, and 1.2 A are shown in Figures 12–14.

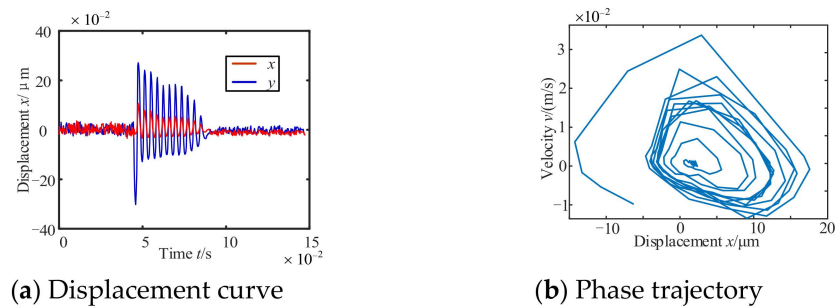


Figure 12. Displacement curve and phase trajectory under $i_0 = 0.5$ A.

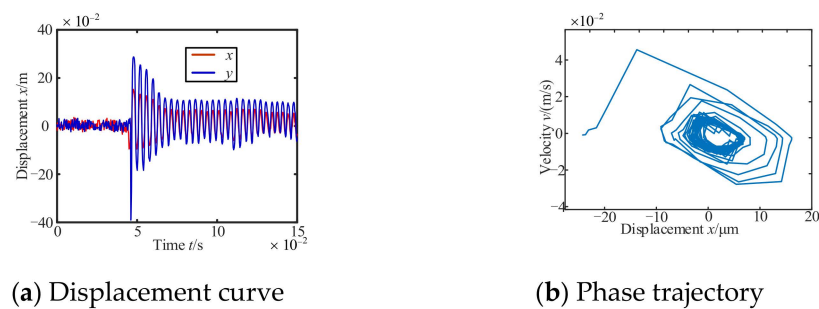


Figure 13. Displacement curve and phase trajectory under $i_0 = 1.0$ A.

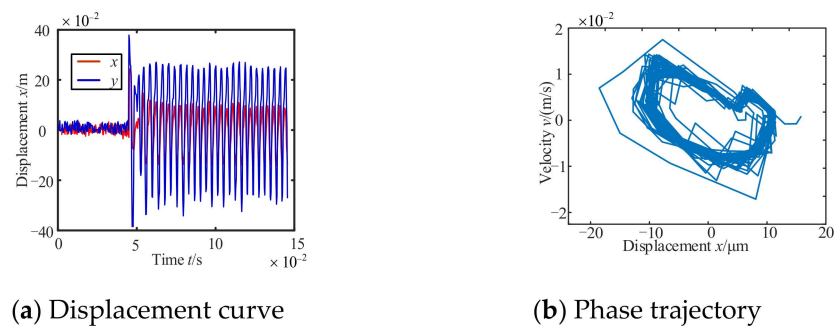


Figure 14. Displacement curve and phase trajectory under $i_0 = 1.2$ A.

It can be seen from the experimental results that the rotor can return to the original position and Hopf bifurcation does not occur at this time after the external interference disappears, when i_0 is 0.5 A. When i_0 is 1.0 A, the rotor gradually approaches equal amplitude oscillation and Hopf bifurcation occurs. When i_0 is 1.2 A, the rotor also gradually approaches equal amplitude oscillation. Compared with the case in which i_0 is 1.0 A, the oscillation amplitude increases, the adjustment time decreases, the maximum vibration speed increases, the oscillation phenomenon is more serious, and the stability of MLDSB becomes weaker.

5. Conclusions

- (1) The stable limit cycle and Hopf bifurcation occur when the coil current and oil film thickness are greater than the threshold; at this point, the bearing rotor features constant amplitude vibration and its motion loses stability. Therefore, in order to ensure the stable operation of the bearing rotor, the values of the coil current and oil film thickness should be set below the threshold, or the threshold should be avoided by adjusting other control parameters.

- (2) Under the combined influence of the coil current and oil film thickness, the boundary value of the Hopf bifurcation of the bearing system decreases, and the amplitude of the Hopf bifurcation's out-of-limit cycle increases with the increase in the coil current and oil film thickness. In addition, compared with the coil current, the oil film thickness creates a greater impact on the vibration amplitude of the limit cycle. Therefore, in order to ensure the stable operation of the bearing rotor, we should try our best to ensure that the values of the coil current and oil film thickness are less than the boundary value of the Hopf bifurcation.
- (3) The Hopf bifurcation results obtained using the MLDSB Testing System show that Hopf bifurcation does not occur when $i_0 < 0.5\text{A}$, although it occurs when $i_0 > 1.0\text{A}$. Therefore, in order to ensure the stable operation of the bearing system, the coil current should be controlled below 1A as far as possible.

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