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A Backwards Induction Framework for Quantifying the Option Value of Smart Charging of Electric Vehicles and the Risk of Stranded Assets under Uncertainty

Spyros Giannelos *, Stefan Borozan and Goran Strbac

Department of Electrical and Electronic Engineering, Imperial College London, London SW7 2AZ, UK; s.borozan@imperial.ac.uk (S.B.); g.strbac@imperial.ac.uk (G.S.)
* Correspondence: s.giannelos@imperial.ac.uk

Abstract: The anticipated electrification of the transport sector may lead to significant increase in the future peak electricity demand, resulting in potential violations of network constraints. As a result, a considerable amount of network reinforcement may be required in order to ensure that the expected additional demand from electric vehicles that are to be connected will be safely accommodated. In this paper we present the Backwards Induction Framework (BIF), which we use for identifying the optimal investment decisions, for calculating the option value of smart charging of EV and the cost of stranded assets; these concepts are crystallized through illustrative case studies. Sensitivity analyses depict how the option value of smart charging and the optimal solution are affected by key factors such as the social cost associated with not accommodating the full EV capacity, the flexibility of smart charging, and the scenario probabilities. Moreover, the BIF is compared with the Stochastic Optimization Framework and key insights are drawn.

Keywords: backwards induction framework; electric vehicles; option value; smart charging of EV; stochastic optimization

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1. Introduction

The integration of electric vehicles (EV) into electricity distribution networks is set to increase worldwide over the coming decades as part of the global decarbonization effort [1]. As a result, electricity distribution networks are expected to face considerable challenges associated with the resulting increased load peaks. Therefore, significant amount of network reinforcement may become necessary in order to facilitate the realization of this ongoing transition. Nonetheless, an important challenge associated with network reinforcement constitutes the increased amount of uncertainty that characterizes future EV deployment since it is not known a priori to the network planners how much EV capacity will eventually be connected to the system, thereby preventing fully informed network reinforcement decisions and creating the prospect of ending up with significantly underutilized, or stranded, network assets in the future. Hence, the presence of this uncertainty inadvertently creates the prospect of stranding risk, which necessitates departing away from the traditionally used deterministic modeling frameworks because they are not incorporating uncertainty into the decision-making process. On the other hand, it may be appropriate to develop and adopt new planning frameworks, such as the proposed Backwards Induction Framework (BIF).

During this ongoing transition, smart grid technologies such as the Smart Charging (SC) of EV may constitute viable alternatives to conventional network reinforcement, as the former technologies may enable network planners to accommodate new demand in a cost-effective way by taking advantage of the flexibility that these assets possess. The flexibility of Smart Charging resembles that of Demand Side Response, according to which the connected EV load can be optimally re-scheduled according to the most economic system operation [2], thereby leading to reduced peak demand. This flexibility can allow for deferral and/or displacement of investments in network assets, thereby generating cost-optimal investment strategies [3,4].

In this context, the economic value associated with this inherent flexibility that smart technologies possess, which enables network planners to optimally hedge against the risks associated with the underlying uncertainty, can be quantified through the concept of the Option Value [5]; this value constitutes the expected net economic benefit accrued by network planners from investing in a smart technology, under uncertainty. A positive option value is premised upon three criteria that must be fulfilled [6]. First, there must be temporal resolution of uncertainty as well as the possibility of learning [7] i.e., the state of knowledge about the system under study should increase over time. Second, there must be managerial flexibility i.e., the planner should be able to adopt different strategies according to the way by which uncertainty resolves. Finally, there must be irreversibility in the investment opportunities, i.e., inability to undo them, which is characteristic of conventional network reinforcements, since they have high capital costs and long lifetimes spanning several decades [8,9]. Therefore, investments in smart technologies, such as the Smart Charging of EV, qualify as the type of investments that can possess significant Option Value. This value can be captured through the Backwards Induction Framework (BIF) and the Stochastic Optimization Framework (SOF) since they can capture the presence of uncertainty in the system.

In this context, the structure of the paper is as follows. In Section 2 the broader context of network planning under uncertainty is discussed with a reference to various relevant methodologies. Section 3 presents the main operating principle of BIF. Section 4 demonstrates the application of BIF to an electricity distribution network and presents, in detail, the calculation of the option value, the optimal decisions and the stranding risk. Section 5 involves sensitivity analyses that provide insights into key factors that drive the Option Value of Smart Charging, such as the flexibility of smart charging, scenario probabilities, and social cost while Section 6 compares the BIF with the SOF framework extensively. Finally, Section 7 presents the conclusions and an outline of possible future research pathways.

2. Planning under Uncertainty

Electricity network investment planning has been an active research area with the majority of publications focusing on investment decision-making under perfect information (deterministic planning) such as in [10–12]. However, when there is significant amount of uncertainty present in the system, the selection of a deterministic approach introduces significant risk. For instance, there is the risk of proceeding to take investment decisions premised upon the assumption of one specific scenario realizing in the future, only to eventually realize that another scenario has occurred, thereby leading to invested network assets ending up being heavily underutilized or stranded [13].

In addition, deterministic approaches assume an a-priori certain evolution of the system parameters, thereby not enabling decision-makers to exert flexible decision-making [14,15]. The benefits arising from considering strategic flexibility through shifting from sequential long-term deterministic plans to adaptable strategies have been well documented in detail in [16].

There have been various modeling approaches toward obtaining a solution to the dynamic investment problem under uncertainty. For example, the Real Options Framework [17–20] allows employing the financial options theory for the evaluation of investment problems involving real i.e., tangible non-financial assets. However, given the significant differences between financial and real assets, this methodology includes significant amount of subjectivity, which may render its application particularly challenging [21]. Another approach to decision-making under uncertainty is the Minimax Regret Framework or Least-Worst Regret (LWR) [22], which aims at detecting the investment decisions that minimize the maximum economic cost, or regret, under the realization of the worst-case scenario.

A key characteristic of this approach is that it is not dependent on scenario probabilities, on the grounds that probabilities for future events are inherently subjective in nature. However, even though not incorporating probabilities may increase the robustness of the results, it may also result in an unrealistic treatment of events that are likely to possess low or high probability of occurrence.

A further modeling approach that can consider uncertainty is the Stochastic Optimization Framework (SOF), which examines all theoretically possible investment decisions, obtains their corresponding expected system costs (i.e., including investment and operational costs), and eventually selects as optimal the investment that results in the minimum expected (i.e., probability-weighted) system cost. This framework has been widely used in power systems to account for various sources of uncertainty such as generation costs, demand levels, or network outages [23–26].

An additional framework that can capture uncertainty is the Backwards Induction Framework (BIF). Its main characteristic is that instead of conducting an exhaustive search of all possible investment combinations in the grid, as SOF and LWR do, it is rather focused on specific pre-determined investment strategies, and through the backwards induction technique it leads to the quantification of the optimal solution.

In this context, uncertainty is defined as the lack of perfect information about the future value of a parameter, resulting in multiple scenarios describing the possible values of the uncertain parameter.

In view of this, the contributions of the present paper are as follows.

- Demonstration of the application of the BIF, for the first time in the context of power system investment planning under uncertainty.
- Quantification of the Option Value of Smart Charging of EV, for the first time, through the BIF.
- Quantification of the risk of stranded assets, for the first time through the BIF.
- Comparison of BIF and SOF, for the first time in the literature.
- Sensitivity analyses on key factors that are driving the Option Value of Smart Charging of EV, for the first time in the literature via the BIF.

3. The Backwards Induction Framework

In the previous two sections it was mentioned that the BIF can be utilized for the purpose of network planning under uncertainty. Although the BIF has not yet had an application in the context of power systems, traditionally it has found application in other scientific areas. Particularly, it has its roots in the scientific field of Game Theory, where it has been applied to sequential games [27–31], where the optimal solution is obtained by reasoning backwards in time, from the last stage of the problem to the initial stage. Below, the underlying principles of the framework are presented in detail.

The BIF makes use of a scenario tree with a structure as depicted in Figure 1 below. For simplicity, this tree is shown to consist of two stages (also known as epochs), but there is no restriction in the number of stages present. Every pair of stages, starts with a decision point, which is the time where the optimal investment decision is made by the network planner, and the second stage is the time when the uncertainty resolves.

This scenario tree structure illustrates the full domain of possible system costs C_{ij} under all possible investment decisions and scenario realizations. Specifically, the network planner's objective is to choose the optimal investment decision, which involves selecting one among a finite number N of candidate investment decisions $(D_1,\ D_2, \ldots,\ D_N)$; a number of M discrete scenarios, each with a corresponding probability of realization $p_k\ \forall\ k = 1, 2, \ldots, \mathrm{M}$, are used to characterize this uncertainty, with each scenario k resulting in a system cost C_{ki}, given the selection of investment strategy D_i. Notice that regardless of which investment decision has been made, the uncertainty resolves in an identical way in the second stage i.e., both the probabilities p_k as well as the number of scenarios remain the same across all uncertainty points.

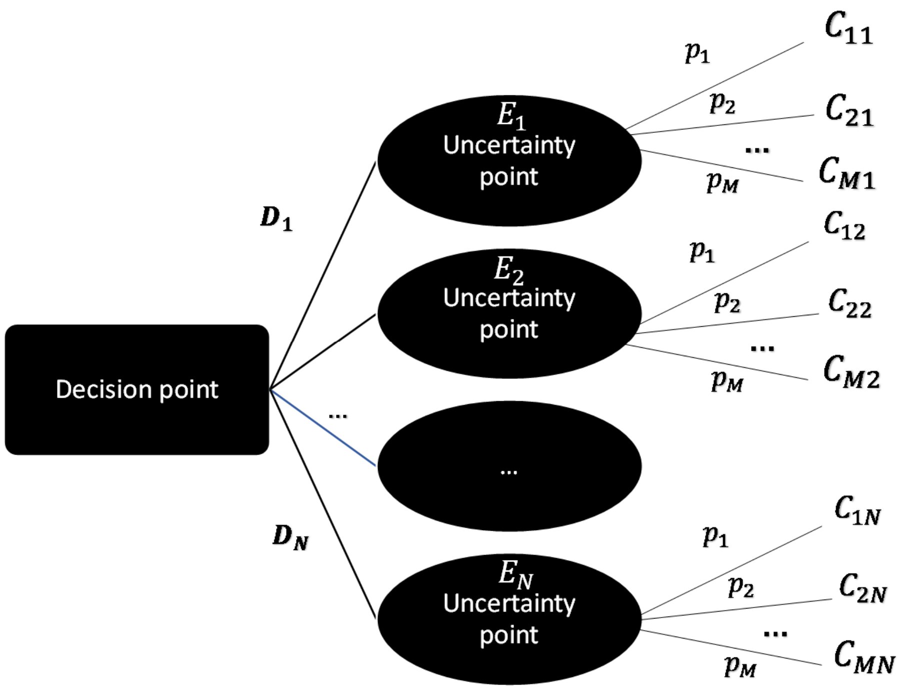

Figure 1. Scenario tree, describing the investment decision making process according to the BIF, consisting of M scenarios (each with a probability p_i) and N candidate investment decisions ($D_1, \ldots D_N$), one of which will be selected as the optimal.

In this context, the backwards induction technique begins at the very last epoch with the calculation of the system costs C_{ij}. These costs are calculated based on the specific investments that strategy D_i involves, under scenario j, as well as the corresponding operational costs. After obtaining the C_{ij} costs, for each decision D_i the corresponding expected cost E_i is calculated according to the formula $E_i = \sum_{k=1}^{M} p_k C_{ki}\ \forall i$. Ultimately, the optimal investment decision D_{i^*} is that whose expected system cost E_{i^*} is the smallest of all i.e., $E_{i^*} = \min_i E_i$.

Furthermore, it is important to mention the concept of *delay* (also known as build-time). Specifically, an investment may or may not become operational at the same epoch at which the decision to invest in it has been made. In the case of conventional reinforcements, this *delay* is typically in the order of years (e.g., two years for distribution networks and five years for transmission systems), which means that if the decision to invest in upgrading a distribution line is made at year t, then the investment will become operational after $t + 2$ years, primarily due to lengthy licensing procedures, necessary public works, and other factors (e.g., possible public opposition) [13]. On the other hand, investments in smart grid technologies are typically becoming operational in a shorter amount of time.

4. Case Study

This section describes a case study on the application of the BIF to electricity distribution network planning under uncertainty, as well as it presents the corresponding results.

4.1. Description

The present case study is about the investment decision-making problem of a network planner whose objective is to accommodate the increased future power flows caused by the integration of EV into the electricity distribution system. The key characteristic associated with this problem is the presence of uncertainty that makes the decision-making process non-straightforward and challenging; this uncertainty is captured through a two-stage scenario tree.

Specifically, the schematic diagram of Figure 2, below, shows the high-voltage (HV) electricity distribution network under study, which comprises three feeders consisting of a total number of 11 lines, each being of 1 km length, and 12 buses with bus 1 being the 33/11 kV primary substation through which the necessary energy is imported, from the higher voltage grid, to feed the loads. The peak demand is depicted in MW at each of these buses, with the total peak load summing up to 16 MW. Notice that Figure 2 illustrates the state of the system as it is in the first epoch (current time), where all power flows are fully accommodated (i.e., all loads are satisfied) and there have been no investments in the system and no smart technologies have been deployed. In addition, it is assumed that there is no EV demand in the first stage (i.e., that all is baseload demand). The analysis is conducted over two epochs (stages), and during the peak hourly period.

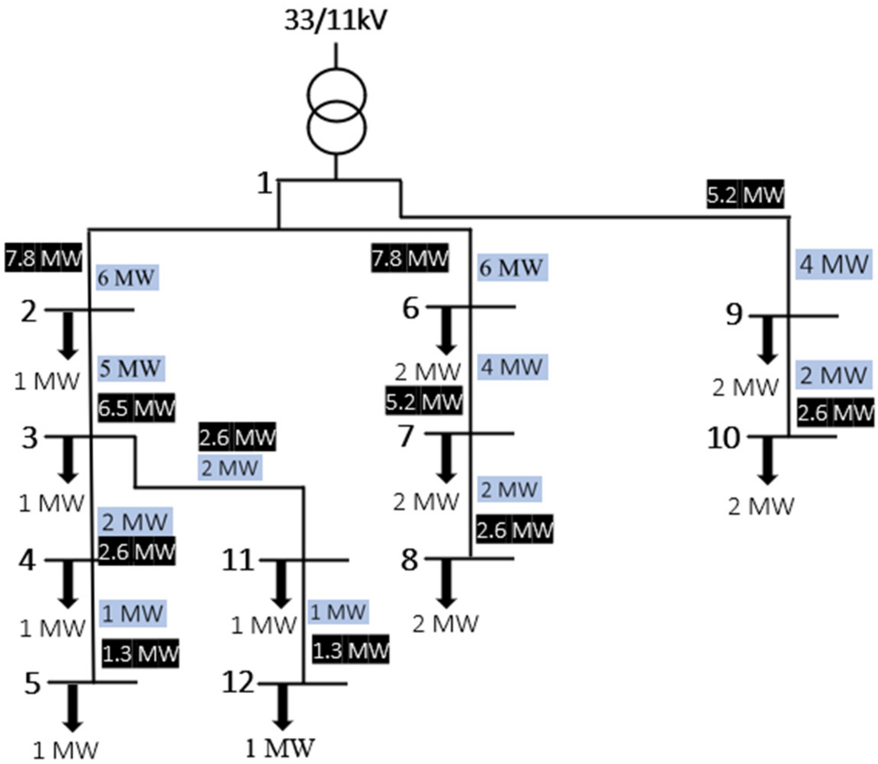

Figure 2. Schematic diagram of the HV electricity distribution network under study. All values shown correspond to the first stage of the problem; peak loads are shown under the black arrows, line thermal limits are shown in black, and the power flows are shown in blue. This schematic diagram applies to the first epoch, during the peak hourly period; it also applies to the second epoch only under scenario 3 realization (i.e., no EV integration, and baseload remains the same).

However, in the second stage, a number of EV may connect to the system, thereby driving the demand upwards and creating the need for investments. This demand growth can happen solely because of the electrification of the transport sector, which is expressed in the increase in the number of EV charging points connected to the system. This demand growth is uncertain, and this uncertainty is present across all buses in the network i.e., system wide. As such, the power flows across all distribution lines of the system, in the second epoch, may increase substantially.

According to Table 1 below, the uncertainty around EV penetration is described through three scenarios and a corresponding discrete probability distribution. In particular, these three scenarios characterize the second-stage peak demand growth.

Table 1. Description of the source of uncertainty in the problem through three scenarios for EV penetration.

Source of Uncertainty	Scenarios		Probabilities
Future EV penetration growth	S_1	High (i.e., 100% load growth per bus)	40%
	S_2	Medium (i.e., 50% load growth per bus)	35%
	S_3	No change	25%

The first scenario (i.e., scenario S_1) describes a situation where the total demand at each of the buses, in the second epoch, becomes twice as much as it is in the first epoch, due to additional electricity demand from the connected EV capacity, and this scenario is 40% likely to occur. The second scenario is related to a 50% growth in the total demand per bus, at 35% probability of occurrence. Finally, the last scenario (i.e., scenario S_3) corresponds to the case where there is no change in demand at all (i.e., the EV penetration remains at zero levels) with a probability of occurrence equal to 25%.

Notice that the increase in the second-stage demand is solely caused by EV penetration that corresponds to a number of connected EV. This number can be found by dividing this demand by the power of the EV charger, which we assume to be equal to 7 kW, and by the coincidence factor, which we assume to be 0.5. As an example, bus 10 (see Figure 2) accommodates 2 MW of baseload. If in the second stage a total of 2 MW of EV load gets connected, then this corresponds to $\frac{2000 \text{ kW}}{(0.5 \cdot 7 \text{ kW})} = 571$ electric vehicles.

The illustration of this discrete probability distribution, which reflects the described underlying uncertainty, can be made through the use of a scenario tree as displayed in Figure 3 below. The scenario tree consists of two stages (also known as epochs), where in the first stage the node is the decision point (i.e., where the network planner takes the optimal investment decision) with the branches emanating from this node representing the possible investment decisions $D_1 - D_4$, as described in the text below. In addition, the resolution of uncertainty takes place in the second stage, with three scenarios.

Regarding the possible investment decisions, they are described in Table 2 below. Particularly, in D_1 the conventional reinforcement is the only available technology for investment i.e., there is no availability of Smart Charging of EV. This decision involves conventional reinforcement of all distribution lines in the system, as explained in Section 4.2.1, with the required amount of MW so as to fully accommodate the increased load in the second epoch.

Regarding D_3, it involves Smart Charging as the sole investment technology i.e., there is no availability of conventional reinforcement. In this case, the planner deploys Smart Charging at all buses in the system as explained in Section 4.2.3.

Finally, decision D_4 involves both EV Smart Charging and conventional reinforcement technologies available to the planner for investment. This investment decision is explained in Section 4.2.4.

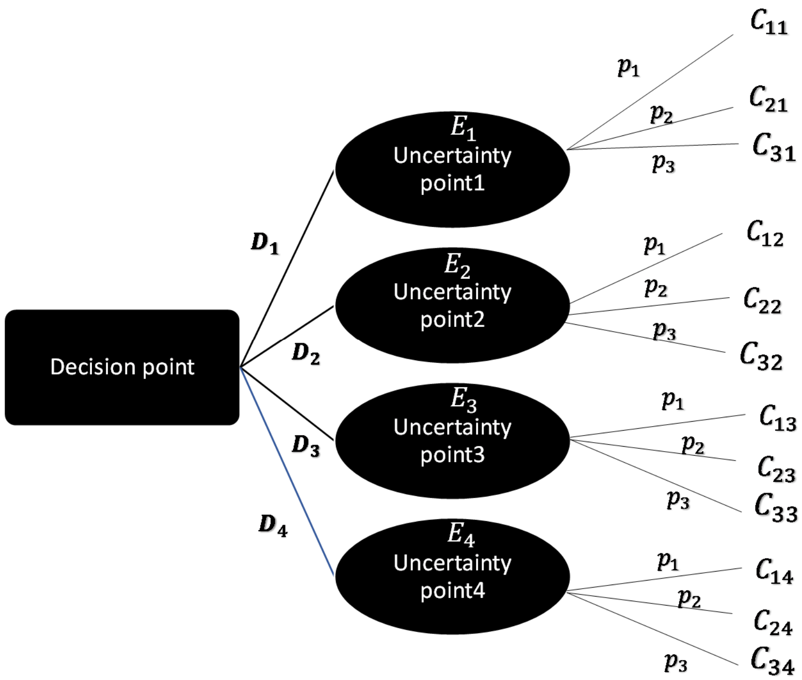

Figure 3. Illustration of the uncertainty through a scenario tree that consists of two stages, where the first stage is when the investment decision is taken ("decision point") and the second stage is when the uncertainty resolves (i.e., the planner learns which scenario realizes).

Table 2. Description of the availability of technologies for each of the candidate investment decisions.

D_1	Conventional network reinforcement is the only technology available to the planner.
D_2	The planner does not make any investments at all (Do-nothing approach).
D_3	Smart Charging of EV is the only technology available to the planner.
D_4	Both conventional network reinforcement and Smart Charging of EV are available to the planner.

Moreover, D_2 constitutes the do-nothing approach i.e., neither conventional nor Smart Charging technologies are available for investment and therefore the planner does not invest in the system at all.

Note that, as explained earlier, the commissioning of conventional reinforcements has a delay of one epoch. Therefore, any conventional reinforcement of lines must begin in the first epoch, i.e., as soon as the decision is made in the decision point. Conversely, smart investments carry zero construction delay, and as such, any decisions to invest in Smart Chargers can be realized in the second epoch after uncertainty resolution.

In terms of the techno-economical characteristics of these technologies, the conventional reinforcement of an existing distribution line includes a fixed cost equal to £15,000/km and a variable cost equal to £50/MW/km. Moreover, the installation of Smart Charging infrastructure across the system has an investment cost equal to £20/EV. In addition, the peak demand increase, which is caused by the realized EV penetration in the second stage, can be reduced by 40% through the implementation of the Smart Charging technology i.e., the flexibility of Smart Charging of EV is equal to 40%.

Note that flexibility equal to 40% is reflected in the reduction of the peak electricity demand, caused by EV, by 40% relative to its original value. For example, if the EV peak demand is equal to 1 kW, then the implementation of smart charging reduces it to $1\text{ kW} \times (100\% - 40\%) = 0.6\text{ kW}$.

Finally, the case study has been conducted assuming the possibility of a *social cost*, incurred by not being able to accommodate EV capacity. That is, we acknowledge that not being able to accommodate EV can have profound effects on the environment as well as on the society as a whole since electricity consumers will not be able to charge their vehicles [32]. The value of this cost has been selected to be equal to £50,000/MWh.

4.2. Results

The resulting costs C_{ij} are shown in Figure 4 below for every combination of scenario i and investment decision j. In total, there are three scenarios and four investment decisions $(D_1, D_2,\ D_3,\ D_4)$, resulting in 12 possible, discounted, system costs. In addition, the figure shows the corresponding expected values E_j, for every investment decision D_j and for every scenario realization i. The BIF has been modeled on Matlab R2021b.

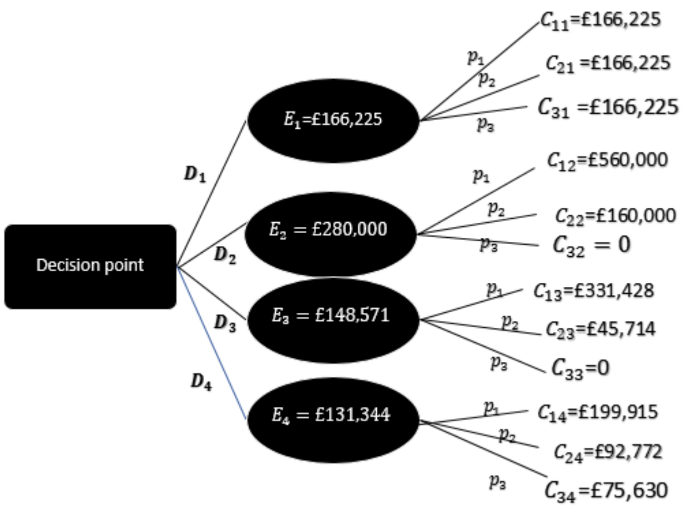

Figure 4. Two-stage scenario tree where stage1 includes the decision point (i.e., time when the investment decision is made) and stage 2 includes the uncertainty resolution, involving calculations on total system costs C_{ij} and the corresponding expected values E_j, where $i \in \{1,2,3\}$ is the scenario index, and $j \in \{1,2,3,4\}$ is the decision index, for each of the four investment decisions D_1, D_2, D_3, D_4.

It follows that the optimal investment decision is D_4 and the optimal total expected cost is £131,344. This is because this investment decision corresponds to the smallest expected cost of all candidate investment decisions.

In addition, the Option Value of Smart Charging is equal to the difference $E_1 - E_4 = £34,880$, where E_1 is the expected cost when decision D_1 has been selected (i.e., only conventional reinforcement is available and all lines have been reinforced) and E_4 is the expected cost for decision D_4 (i.e., conventional reinforcement and smart charging are available to the planner). The process for the derivation of the values shown in Figure 4 is explained in the paragraphs below.

4.2.1. Decision D_1

Regarding decision D_1, it corresponds to costs $C_{11},\ C_{21}, C_{31}$, depending on which scenario realizes and it involves conventional reinforcement being the only candidate technology for investment in the system. This type of investment involves delay/build-time of one epoch (also known as "stage"), i.e., it takes one epoch for these investments to become operational from the time the corresponding investment decision is made. That is, for the conventional reinforcements to become operational in the second stage the corresponding investment decision needs to be made in the first stage. For this reason, since the network planner aims to have all second-stage power flows fully accommodated, it is the first scenario that is considered for deciding how much to invest. This is because the first scenario (S_1) involves the maximum load growth. Otherwise, if for example the planner takes an investment decision, in the first stage, according to the assumption that there will be realization of S_2 or S_3 in the second stage, then if in fact S_1 ends up realizing in the second stage, then part of the second-stage EV load will not be met due to insufficient network capacity, thereby resulting in high social costs. Hence, decision D_1 involves investing according to the assumption that S_1 will realize in the second stage guarantees that the entire EV capacity in the second stage will be accommodated, regardless of which scenario ends up realizing. Therefore, D_1 involves investing in upgrading all lines in the first stage. By doing so, the second-stage costs, $C_{11},\ C_{21}, C_{31}$ involve zero social cost component (since all EV load is fully accommodated in the second stage) and therefore have investment component only. They are also equal to each other because the same investment decision is taken in the first stage, regardless of which scenario will be realized in the second stage; thus $C_{11} = C_{21} = C_{31}$.

Regarding cost C_{11} , which is the total system cost given that the planner has taken the decision D_1 and that scenario S_1 has been realized (see Figure 5), it is equal to £166, 225. This is found by taking into account the fact that scenario S_1 involves 100% load growth in the second stage across all buses, which leads to increased power flows across all 11 lines, thereby warrantying their reinforcement. Therefore, all 11 lines are upgraded, which involves £15,000/km fixed cost for each line (assuming every line having 1 km length) and a variable cost of £50/MW for each line. The total investment needed, summed across all lines, is equal to 24.5 MW, as explained further below. Thus, the resulting total fixed investment cost summed for all 11 lines is equal to $11 \cdot £15,000 = £165,000$ and the corresponding total variable investment cost is equal to 24.5 MW·£50/MW = £1225; hence, $C_{11} = £165,000 + £1225 = £166,225$. Moreover, $C_{11} = C_{21} = C_{31} = £166{,}225$; hence, the corresponding expected cost for this decision is $E_1 = p_1 C_{11} + p_2 C_{21} + p_3 C_{31} = £166{,}225$.

As mentioned, the planner will select to invest a total of 24.5 MW in upgrading all 11 lines. The decision to make this investment is taken in the first epoch and once the second stage arrives the uncertainty will resolve i.e., the planner will learn which of the three scenarios realizes. If S_1 happens then this investment will be fully utilized, and all 11 lines will be used to their full capacity. Thus, under S_1 the stranded cost will be £0.

However, if scenario S_2, or S_3 eventually realizes, then not all of this 24.5 MW invested capacity will be utilized, and the remaining unutilized capacity is known as a stranded asset, involving stranded cost. If S_3 realizes (i.e., zero second-stage EV penetration, see Figure 2) then this will mean that no EV will connect to the system in the second stage and the power flows in the second stage will be identical to those in the first stage. If the planner knew this in the first stage, then a decision to make no investments at all would have been made in the first stage. However, the planner does not know this, because of the presence of uncertainty, and therefore all 24.5 MW capacity will turn out to be stranded/underutilized and the corresponding costs of £166,225 will be stranded as well. Thus, under S_3 the stranded cost will be £166,225.

If S_2 realizes (see Figure 6), which means that the demand will grow by 50% in the second stage, a total of 7 MW of investment in line reinforcement will be needed in the second stage; all lines will require reinforcement, thereby incurring fixed cost of 11·£15,000 = £165,000. Since the planner assumed that S_1 would occur, this would mean that a total of 24.5 MW will have been invested, only to see that 7 MW was actually needed. This means that a total of $24.5 - 7$ = 17.5 MW will be stranded/underutilized. The investment of 17.5 MW is made across all 11 lines, thereby resulting in a fixed cost equal to 11·£15,000 = £165,000, which is not stranded because under S_2 all lines are upgraded as well. Whereas the variable cost is equal to 17.5 MW $\times$ £50 MW = £875, which is stranded since it corresponds to capacity (MW) not needed. Thus, under S_2 the stranded cost will be £875.

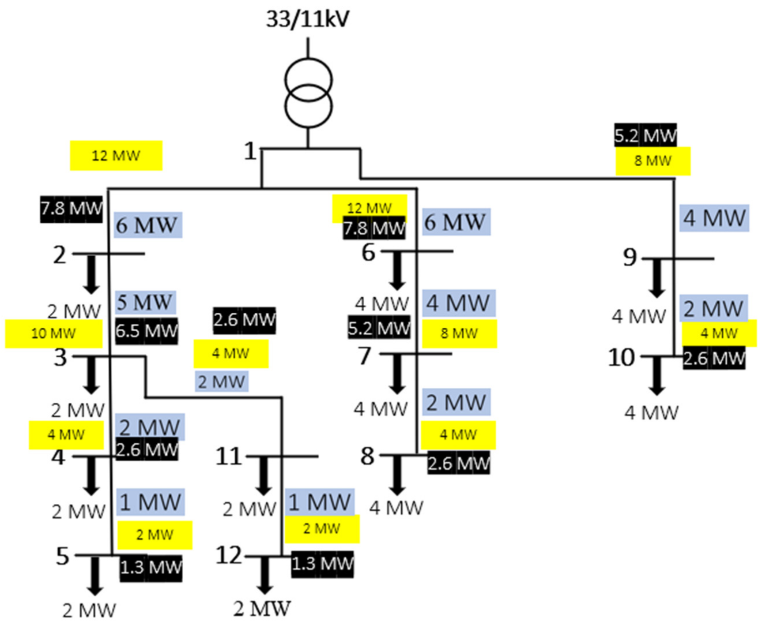

Figure 5. Schematic diagram of the HV electricity grid showing the first-stage power flows (in blue), the initial line thermal limits (in black), and the second-stage peak load (downward black arrows) assuming that in the second stage scenario S_1 has occurred (i.e., 100% load growth at every bus). Moreover, the second stage power flows are shown (in yellow) ignoring thermal line limits and investments (so that we can see the magnitude of the 2nd stage flows). Summing all second stage peak load yields 32 MW, consisting of 16 MW peak baseload (equal to that in the first stage i.e., no change in baseload) plus 16 MW peak EV load (all connected in the second stage). E.g., the total peak load at bus 5 is equal to 2 MW, where 1 MW is the peak baseload (same as in Figure 2) and the remaining 1 MW is the peak EV load (connected in the 2nd stage). The amount of capacity needed to be invested per line so that the second-stage flows are fully accommodated is equal to the difference between the values in yellow (power flows in the second stage) and those in black (first-stage thermal limits).

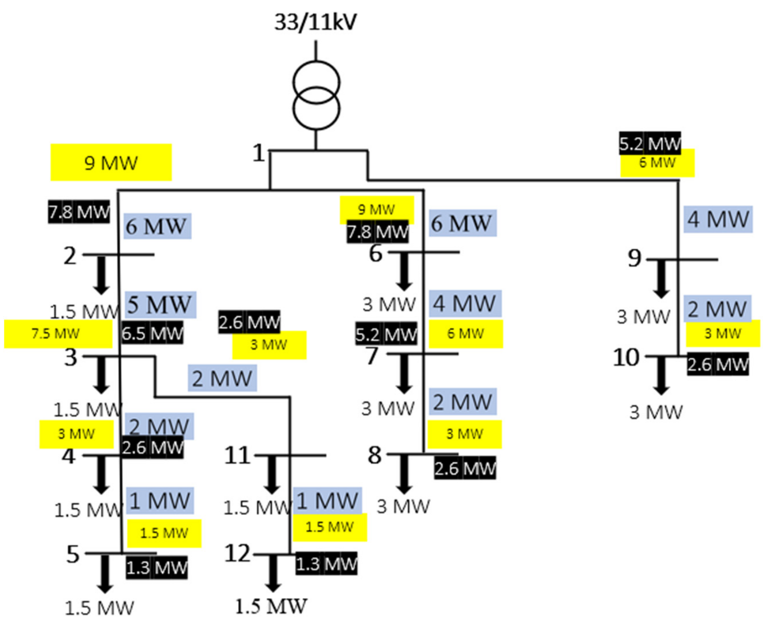

Figure 6. Schematic diagram of the HV electricity grid showing the first-stage power flows (in blue), the initial line thermal limits (in black), and the second-stage peak load (downward black arrows)

assuming that in the second stage scenario S_2 has occurred (i.e., 50% load growth at every bus). Moreover, the second stage power flows are shown (in yellow) ignoring thermal line limits and investments (so that we can see the magnitude of the 2nd stage flows). Summing all second stage peak load yields 24 MW, consisting of 16 MW peak baseload (equal to that in the first stage i.e., no change in baseload) plus 8 MW peak EV load (all connected in the second stage).

Thus, the expected stranded cost assuming that the planner has selected decision D_1 is equal to $p_1 \cdot £0 + p_2 \cdot £875 + p_3 \cdot £166,225 = 0.4 \cdot £0 + 0.35 \cdot £875 + 0.25 \cdot £166{,}225 = £306.25 + £41,556.25 = £41,862.5$.

4.2.2. Decision D_2

Regarding decision D_2, since it involves no investments at all, the investment cost is zero under all scenarios, but there is social cost due to the inability to accommodate EV due to insufficient network capacity.

Specifically, under S_3 (i.e., zero EV penetration) both the investment cost and the social cost are zero (all second stage load is fully accommodated) and so $C_{32} = 0$.

Regarding C_{12}, i.e., under the realization of S_1, without investments in the system the first-stage baseload of 16 MW in total, will all be met, and also some of the 16 MW EV load, that connects in the second stage, will be met as well because the first-stage thermal limits are higher than the first-stage peak baseload power flows, therefore some increase in the power flows can still be accommodated without investment. In the second stage, the total load that can be met is calculated by taking the sum of the first-stage thermal limits of the lines 1–2, 1–6, and 1–9, which is 7.8 + 7.8 + 5.2 = 20.8 MW; this is the total power that can flow to meet the system demand (see Figure 5) as losses are ignored. Out of this 20.8 MW of load that is met in the second stage, 16 MW are the baseload, which remains the same as in the first stage, and 4.8 MW are the EV peak load connecting in stage 2. Since the total second-stage load is equal to 32 MW under S_1, a total of 32–20.8 = 11.2 MW is not met, and all this is EV peak load; therefore, the corresponding social cost is equal to 11.2 MW$\times$1 h multiplied with £50,000/MWh, or $C_{12} = £560{,}000$.

Under the realization of S_2, the peak load that can be met in the second stage is equal to 7.8 + 7.8 + 5.2 = 20.8 MW as explained in the previous paragraph. Whereas the total load is equal to 24 MW (see Figure 6). Thus, the total load not met is 24 $-$ 20.8 = 3.2 MW, which is all EV peak load, thereby the corresponding social cost is C_{22} =3.2 MWh $\times$ £50,000 MWh = £160,000.

In this case, the corresponding expected cost for decision D_2 is $E_2 = p_1 C_{12} + p_2 C_{22} + p_3 C_{32} = £280{,}000$. Moreover, since no investments take place, the expected cost of stranded assets is zero.

4.2.3. Decision D_3

Regarding decision D_3, Smart Charging of EV, with flexibility equal to 40%, is the only technology available to the planner. Given that smart charging investments involve a zero-epoch build-time, the investment cost corresponding to D_3 depends on the scenario realization, because the decision to invest in Smart Charging is taken after the uncertainty has been resolved i.e., in the second stage.

Specifically, if scenario S_3 realizes, i.e., there is no EV penetration, then D_3 involves no investment in Smart Charging; therefore, it is $C_{33} = 0$.

Whereas, if S_1 realizes (see Figure 7), decision D_3 involves deploying Smart Charging of EV across the entire system, at every bus. For instance, at each of the buses 6–10, the first stage total peak load is 2 MW (see Figure 2), and under scenario S_1 a total of 2 MW extra load (due to EV penetration) connects; but since the presence of smart charging reduces the peak by 40%, the total load at each of these buses becomes 2 + 2 $\times$ 0.6 = 3.2 MW (see Figure 7). Similarly, the other buses' total first-stage peak load is 1 MW (see Figure 2), which in the second stage becomes equal to 1 + 1 $\times$ 0.6 = 1.6 MW. In this case, the total peak load in the second stage is equal to 25.6 MW, while the second-stage power flows (taking the line limits into consideration) are equal to the sum of the first-stage thermal limits of lines 1–2, 1–6, and 1–9 or 7.8 + 7.8 + 5.2 = 20.8 MW. Note that none of the lines has been upgraded since this is not considered in decision D_3. Hence, a total of 25.6 − 20.8 = 4.8 MW, of EV load, is not met, resulting in social cost equal to 4.8 MWh × £50,000/MWh = £240,000. Regarding the calculation of the investment cost of smart charging under S_1, a total of 16 MW of EV load connects in the second epoch. This is equivalent to 16,000/(7 kW)/0.5 electric vehicles, which corresponds to $\frac{£20}{EV} \times 16{,}000/$(7 kW)/0.5 = £91,428. Hence, C_{13} = £240,000 + £91,428 = £331,428.

On the other hand, if the planner has selected D_3, with smart charging of EV being deployed across the entire system, and S_2 realizes (i.e., 50% load growth, see Figure 8) then all EV will be accommodated (i.e., social cost is zero). For instance, the first-stage total peak load of each of the buses 6–10, which is 2 MW and it is entirely baseload, increases due to EV penetration and becomes 2 MW + 2 × (1 − 0.4) × 50% = 2.6 MW due to the flexibility of smart charging being 40%. Regarding the other buses that initially have 1 MW peak baseload, this becomes 1 + 1 × (1 − 0.4) × 50% =1.3 MW (see Figure 8). Hence, all power flows are safely accommodated i.e., all EV are accommodated and there is zero social cost. Regarding the calculation of the investment cost of smart charging under S_2, a total of 8 MW of EV load is connected to the system in the second epoch corresponding to 8000/7/0.5 electric vehicles with the corresponding investment cost of $8000/7/0.5 \times 20 = £45{,}714$.

The expected cost for decision D_3 is $E_3 = p_1 C_{13} + p_2 C_{23} + p_3 C_{33} = 0.4 \times £331,428 + 0.35 \times £45,714 + 0.25 \times £0 = £148,571$. Regarding the cost of stranded assets, this is zero since there are no stranded assets given the zero delay for smart charging.

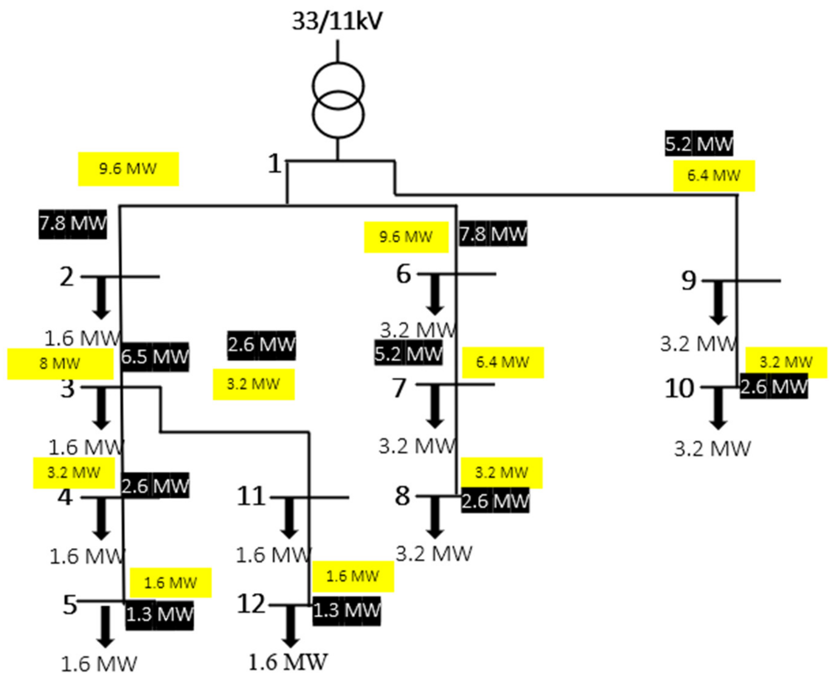

Figure 7. Schematic diagram of the HV electricity grid showing the initial line thermal limits (in black), and the second-stage peak load (downward black arrows) assuming that in the second stage scenario S_1 has occurred (i.e., 100% load growth at every bus) and that EV smart charging, with flexibility = 40%, has been deployed for all EVs in the system i.e., at every bus in the system. Moreover, the second stage power flows are shown (in yellow) ignoring thermal line limits.

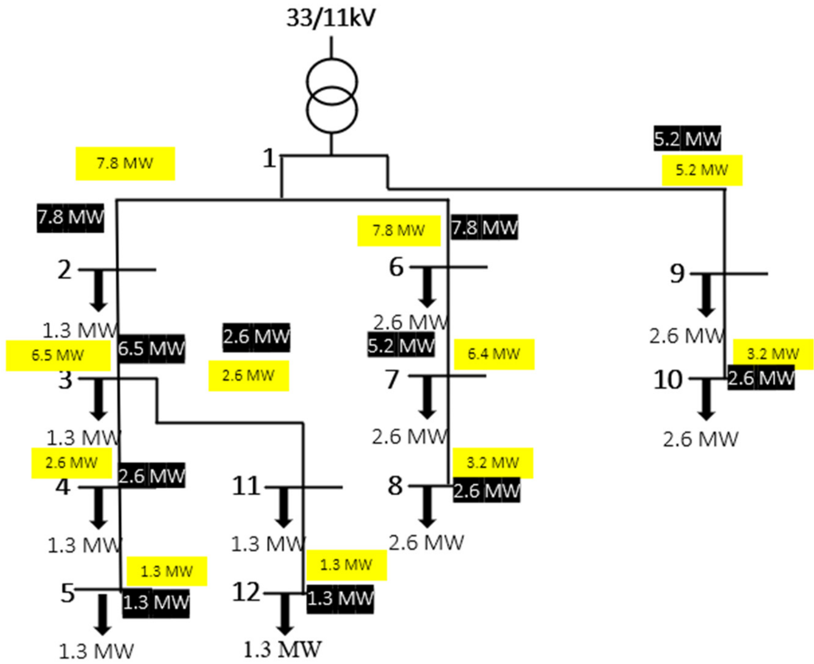

Figure 8. Schematic diagram of the HV electricity grid showing the initial line thermal limits (in black), and the second-stage peak load (downward black arrows) assuming that in the second stage scenario S_2 has occurred (i.e., 50% load growth at every bus) and that EV smart charging, with flexibility = 40%, has been deployed for all EVs in the system i.e., at every bus in the system. Moreover, the second stage power flows are shown (in yellow) ignoring thermal line limits.

4.2.4. Decision D_4

Regarding decision D_4, it involves a mix of conventional reinforcement and of Smart Charging investments. In this case, what is considered for D_4, is for the planner to conventionally upgrade the lines that supply the high-load buses of 2 MW (see Figure 2), i.e., feeders 1–8 and 1–10, i.e., in total 5 lines, and depending on the scenario realization, to then install Smart Charging infrastructure at the buses that are not supplied by these upgraded lines (buses 2–5, and buses 11–12).

Under scenario S_1, the conventional investment decision that has been made in the first stage, incurs a fixed investment cost equal to $5 \times £15,000 = £75,000$, and a variable investment cost of 12.6 MW×£50/MW = £630, where 12.6 MW is the investment required in the 5 lines in order to accommodate their second-stage power flows (see Figure 5 and feeders 1–8 and 1–10). Thus, the resulting total investment cost is £75,000 + £630 = £75,630. Moreover, under S_1 the total load at the buses 2–5 and 11, 12 is 9.6 MW (see Figure 7) but only 7.8 MW of this load can be met (see the thermal limit of line 1–2). Furthermore, the total load at all other buses is 20 MW (see Figure 5) and all is met since the lines are upgraded. Thus, the total load is 29.6 MW but 27.8 MW is met, leading to 1.8 MW not met, all of which is EV load, and corresponding to social cost of $1.8\ \text{MWh} \times £50,000/\text{MWh} = £90,000$. Moreover, a total of 6 MW of peak EV load gets connected in the second stage corresponding to an investment cost of smart charging equal to 6000/7/0.5×20 = £34,285. Thus, the total investment cost corresponding to D_4 under S_1 is £75,630 + £90,000 + £34,285 = £199,915 = C_{14}. Note that the stranded costs are zero since all investments are fully utilized, if this scenario realizes.

If scenario S_3 realizes (i.e., no load growth), then no investments in Smart Charging are made and, therefore, the only investment that is made is the conventional network reinforcement as described above; hence, it is $C_{34} = £\ 75,630$. In this case, all 12.6 MW of investment is stranded and the cost of £75,630 is stranded as well.

Under S_2, a total of 12.6 MW of investment is made in the aforementioned 5 lines as in S_1, resulting in total investment cost equal to £75,000 + £630 = £75,630. Moreover, there is zero social cost because all second stage load is accommodated. Finally, a total of 3 MW of peak EV load is connected with an investment cost of smart charging equal to 3000/7/0.5 × 20 = £17,142. Hence, C_{24} = £75,630 + £17,142 = £92,772. The stranded investment is 3.6 MW (see Figure 6, feeders 1–8, 1–10), which amounts to variable cost of 3.6 MW·£50/MW = £180.

In this case, the corresponding expected cost for decision D_4 is $E_4 = p_1 C_{14} + p_2 C_{24} + p_3 C_{34}$ = £131,344. The expected stranded cost, assuming that the planner has selected decision D_4 is equal to $p_1 \times$ £0 + $p_2 \times$ £180 + $p_3 \times$ £ 75,630 = $0.4 \times$ £0 + $0.35 \times$ £180 + $0.25 \times$ £ 75,630 = £18,970.5.

5. Sensitivity Analysis

The aforementioned case study was conducted by assuming that the conventional reinforcement of an existing distribution line involves a fixed cost equal to £15,000/km and a variable cost equal to £50/MW/km. In addition, the probabilities of each scenario occurring were selected to be $p_1 = 0.40$, $p_2 = 0.35$, $p_3 = 0.25$. Furthermore, the installation of Smart Charging infrastructure corresponded to an investment cost of £20/EV, assuming Smart Charging flexibility of 40% (i.e., its potential to reduce the peak demand). Finally, the social cost of consumers not being able to charge their EV due to insufficient network infrastructure stood at £50,000/MWh.

This section demonstrates the results of sensitivity analysis, conducted through the use of BIF, on smart charging flexibility (see Section 5.1), social cost (see Section 5.2), and scenario probabilities (see Section 5.3).

5.1. Sensitivity Analysis on Flexibility of Smart Charging

In this section, the flexibility of the Smart Charging technology takes on a series of values, between 10% until 100% as depicted in the first column of Table 3 below. For each case, the optimal value of the objective function is found (second column), the optimal investment decision (column 3) and the option value of Smart Charging (column 4). Figures 9 and 10 are the equivalent illustrations for this table. Note that all other input parameters remain at their initial values.

Table 3. Sensitivity analysis on the flexibility of Smart Charging of EV using BIF.

Flexibility of Smart Charging	Optimal Value (£) of Objective Function (i.e., min E_j)	Optimal Decision	Option Value of Smart Charging (£)
10%	166,225	1	0
20%	165,844	4	381
40%	131,344	4	34,881
60%	84,571	3	81,654
80%	52,571	3	113,654
100%	52,571	3	113,654

We can observe that as the flexibility of Smart Charging increases, the optimal value for the objective function (i.e., the corresponding optimal total expected system cost E_j) reduces. This happens because higher flexibility translates to smaller peak load in the second stage, resulting in smaller power flows and, as a result, in fewer conventional reinforcements, which are expensive, and, therefore, a higher option value of smart charging. Notice also, as shown in column 3, that for 10% flexibility, the optimal decision is D_1 (only conventional reinforcements, since they guarantee full accommodation of EV), and as the flexibility grows the optimal decision becomes D_4 (mix of conventional and smart investments), and for values of flexibility greater than or equal to 60%, the optimal decision is D_3 (only smart charging), which indicates the potential of smart charging to completely displace conventional investments for high values of flexibility.

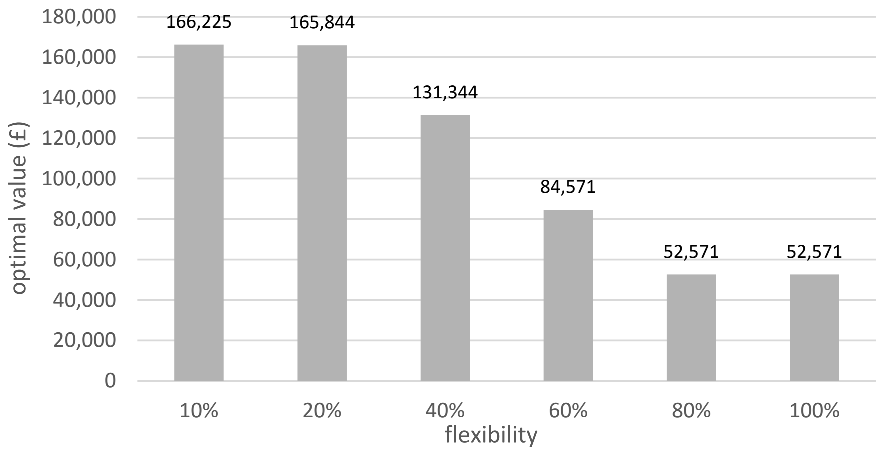

Figure 9. Optimal Value of the Objective Function as a function of the Smart Charging flexibility, using BIF.

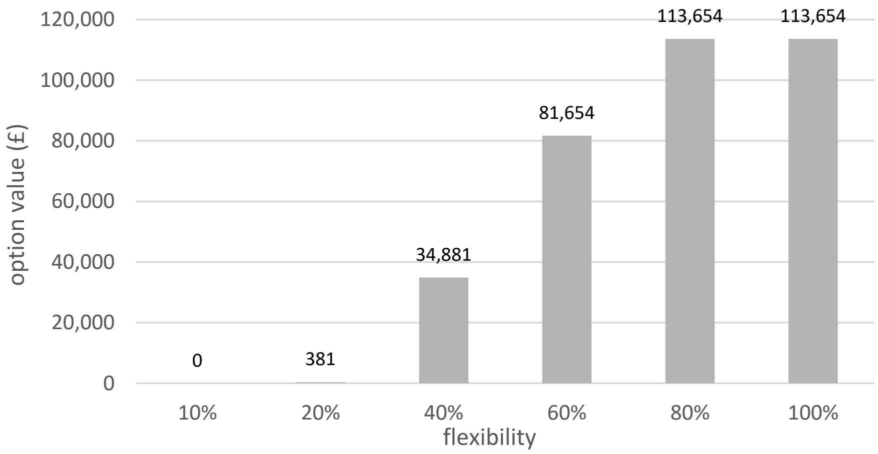

Figure 10. Option Value of Smart Charging of EV as a function of the Smart Charging flexibility, using BIF.

5.2. Sensitivity Analysis on Social Cost

In this section, the cost of not being able to accommodate the full capacity of connected EV (i.e., social cost), takes on a series of values between 1000 £/MWh and 1 £m/MWh as depicted in the first column of Table 4 below. For each case, the optimal value of the objective function is found (second column), the optimal investment decision (column 3) and the option value of Smart Charging (column 4). Figures 11 and 12 are the equivalent illustrations for this table. Note that all other input parameters remain at their initial values.

Table 4. Sensitivity analysis on social cost, using BIF.

Social Cost (£/MWh)	Optimal Value (£) of Objective Function (i.e., min E_j)	Optimal Decision	Option Value of Smart Charging (£)
1000	5600	2	0
10,000	56,000	2	0
25,000	100,571	3	65,654
50,000	131,344	4	34,881
100,000	166,225	1	0
500,000	166,225	1	0
1,000,000	166,225	1	0

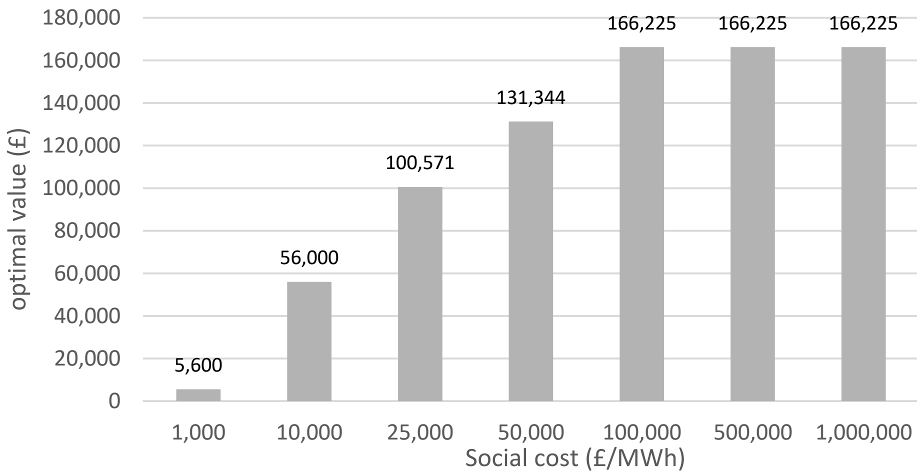

Figure 11. Optimal Value of the Objective Function as a function of social cost, using BIF.

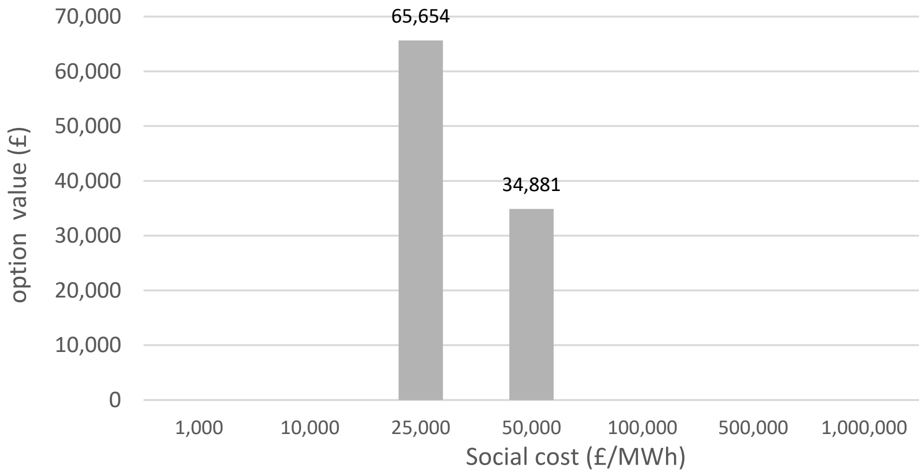

Figure 12. Option Value of Smart Charging as a function of social cost, using BIF.

5.3. Sensitivity Analysis on Scenario Probabilities

In this section, the scenario probabilities take on a series of values as depicted in the first column of Table 5 below. For each case, the optimal value of the objective function is found (4th column), the optimal investment decision (column 5) and the option value of Smart Charging (column 6). Figures 13 and 14 are the equivalent illustrations for this table. Note that all other input parameters remain at their initial values.

Table 5. Sensitivity analysis on the scenario probabilities, using BIF.

p_1	p_2	p_3	Optimal Value of Objective Function i.e., E_j	Optimal Decision	Option Value of Smart Charging (£)
0.40	0.35	0.25	131,344	4	34,880
0.10	0.10	0.80	37,714	3	128,510
0.25	0.25	0.50	94,285	3	71,939
0.40	0.40	0.20	132,201	4	34,023
0.10	0.80	0.10	69,714	3	96,510
0.25	0.50	0.25	105,714	3	60,510
0.40	0.20	0.40	128,772	4	37,452
0.80	0.10	0.10	166,225	1	0
0.50	0.25	0.25	142,058	4	24,166
0.20	0.40	0.40	84,571	3	81,653

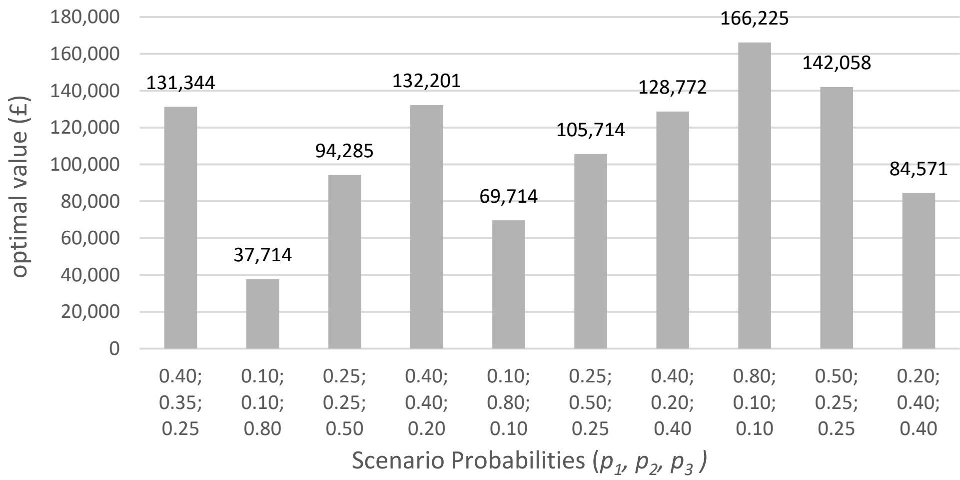

Figure 13. Optimal Value of the Objective Function as a function of scenario probabilities, using BIF.

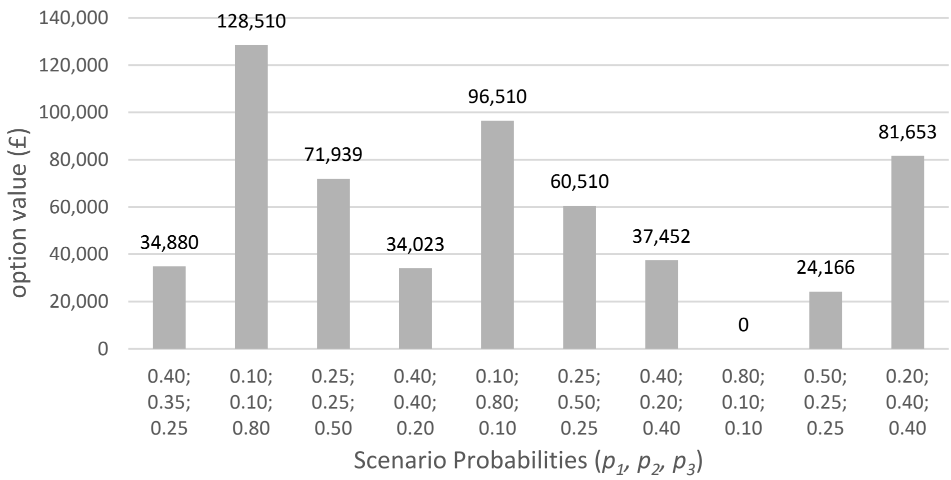

Figure 14. Option Value of Smart Charging of EV as a function of scenario probabilities, using BIF.

We can observe that when the probability of scenario 1 occurring is 80% then decision D_1 is optimal and, therefore, the option value of smart charging is zero; this is because conventional reinforcement is the only technology that guarantees output that has zero social cost under this scenario of high load growth. On the other hand, when the probability of S_1 is less than or equal to 25%, the optimal decision is D_3 because this decision involves some social cost if S_1 occurs, which has relatively small likelihood. When the probability of scenario S_1 takes a value between 40% and 50% then the optimal decision becomes equal to D_4, which involves smaller social cost than D_3 (which is selected for smaller value of probability of S_1) since conventional reinforcement allows for zero social cost.

6. Comparison of the Backwards Induction Framework (BIF) with the Stochastic Optimization Framework (SOF)

6.1. Basic Case Study

The SOF has been established in literature for application to advanced models for power system planning under uncertainty. As outlined in Section 2, this method explores all possible combinations for investment and system operation to find the optimal solution that yields the minimum expected total system costs. Authors in [2] present an example of an advanced SOF for the network expansion problem with demand side response investments and scenario tree representation of uncertainty. Furthermore, [33] proposes the modelling of EV smart charging for the integration in SOF for network expansion planning. In the current work, a modified version of the model in [2] and the smart charging modelling in [33] have been used to validate the optimality of the solution generated from the proposed BIF approach; this validation is confirmed by comparing the solutions of the two planning methods.

The case study presented in Section 4 translates to a two-stage scenario tree, where the first stage corresponds to the present decision point, and the second stage is the uncertainty realization point. All case study assumptions have been kept the same. For the base case study, the results obtained using SOF are identical to the results obtained using the BIF, i.e., identical optimal decisions, option values, and optimal values for the objective function. In the next subsection, other case studies are presented showing differences in the two frameworks.

6.2. Sensitivity Analysis

The same sensitivity analyses presented in Section 5 are performed using the SOF, allowing the comparison of the two frameworks. Specifically, the figures below present the optimal system cost (optimal value) under SOF and BIF for different levels of flexibility (Figure 15), social cost (Figure 16), and scenario probabilities (Figure 17).

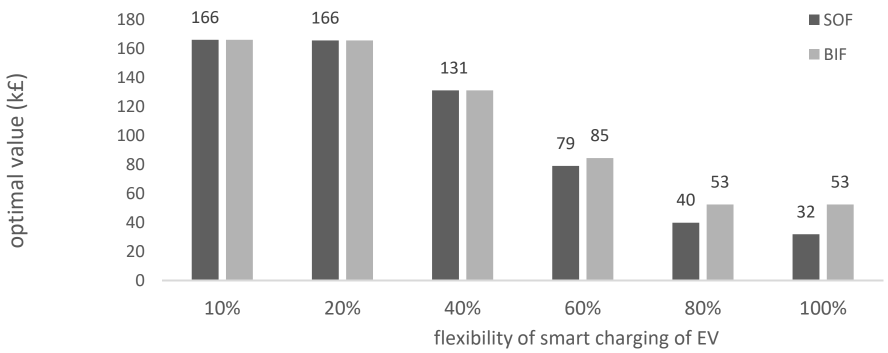

Figure 15. Optimal total expected system cost as a function of smart charging flexibility, under SOF and BIF.

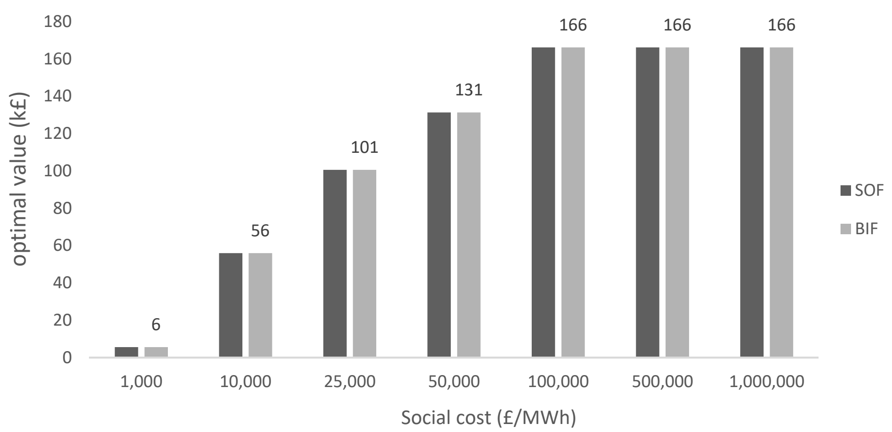

Figure 16. Optimal total expected system cost as a function of social cost, under SOF and BIF.

In almost all cases the optimal solutions obtained using SOF and BIF are identical. Specifically, in Figures 16 and 17 all optimal costs are the same, while in Figure 15 there are some cases where the SOF yields a better solution (smaller cost) as can be observed by the height of the bars. Since the BIF is not based on mathematical optimization as opposed to SOF, the optimal expected system cost obtained using the latter can never be worse (i.e., greater) than that obtained using BIF; it can either be equal or better (i.e., smaller) as is the case in this sensitivity analysis.

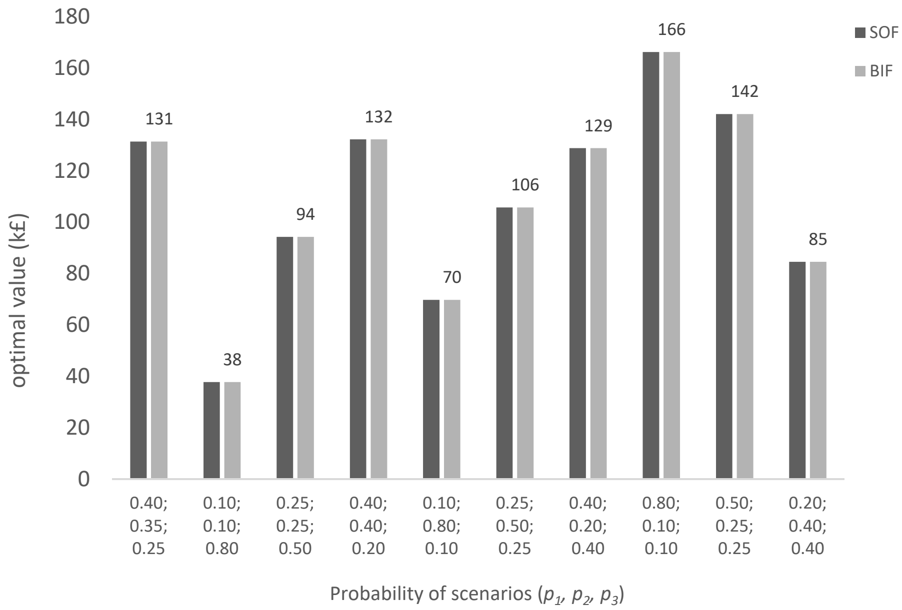

Figure 17. Optimal total expected system cost as a function of scenario probabilities, under SOF and BIF.

Figure 18 shows that as the investment cost of Smart Charging of EV increases, which means that the technology becomes more expensive to invest in, its Option Value drops; this applies for both values of flexibility (40% and 80%). Expectedly, a higher level of flexibility lead to higher Option Value as it leads to a greater amount of EV peak demand that can be reduced, thereby leading to reduced investment in expensive conventional reinforcement. As a result, with the increase in the flexibility level, a higher value for the investment cost of Smart Charging is needed, for its Option Value to become zero.

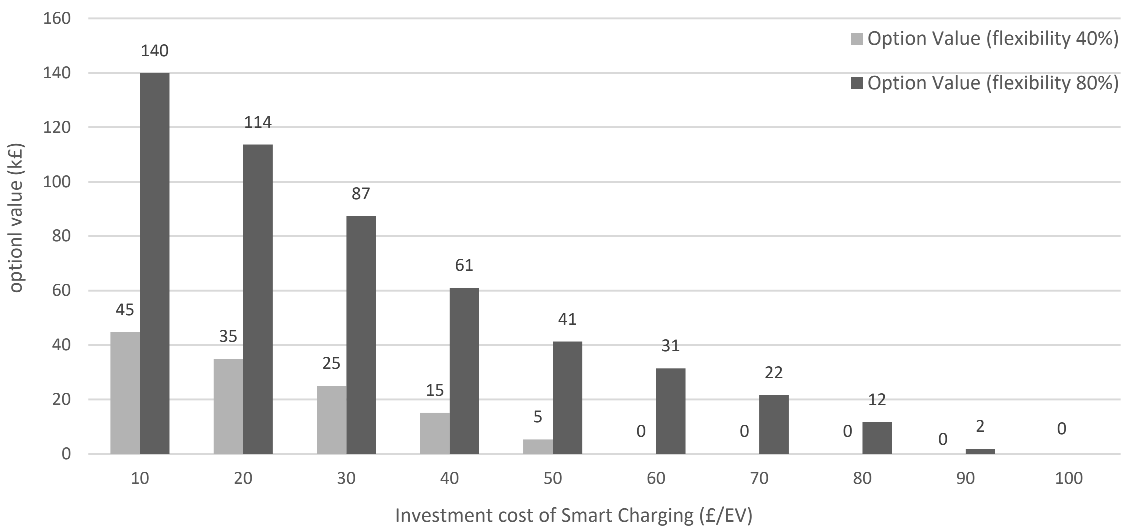

Figure 18. Option Value of investing in Smart Charging of EV (vertical axis) as a function of its investment cost (horizontal axis), for different values of Smart Charging flexibility, conducted via BIF.

The comparison of the proposed BIF with the SOF demonstrates that, in almost all cases, both approaches lead to the same optimal solution. Therefore, the proposed BIF is validated using the well-established SOF.

The main drawback of SOF is its complexity, since it arrives at the optimal solution using algorithms that are based on mathematical optimization theory. Whereas, the BIF is a heuristic approach, akin to natural decision-making, and which has the benefit of transparency and simplicity, which may be particularly attractive characteristics of a planning methodology.

In addition, as opposed to BIF, the SOF conducts an exhaustive search of possible feasible solutions in order to identify the optimal solution which involves evaluating each and every investment combination, i.e., every combination of types of technologies, locations of deployment, timing of connections, and magnitudes of investment. As a result, the SOF tends to require significant computational power, resulting in large solution times, depending on the dimensions of the problem.

7. Conclusions and Future Work

This paper presents the Backwards Induction Framework (BIF), which is used for investment decision-making under uncertainty. To the best of the authors' knowledge, this is the first application of this framework in the context of power systems, in the literature. This framework employs the backwards induction technique to identify the optimal investment solution, which is the one that corresponds to the minimum expected system cost. The framework quantifies the Option Value of a smart technology, and specifically of investing in Smart Charging of EV. Unlike regular stochastic optimization modeling approaches, the BIF does not conduct an exhaustive search i.e., it does not examine the entire set of possible combinations of investment decisions, with respect to timing, magnitude, and location, but rather it examines specific investment strategies decided a priori. In this context, the BIF can be used to provide a first insight into the investment requirements of an electricity network, under uncertainty. In addition, it can provide such insight much faster than with complex stochastic optimization models that may take even days or weeks to yield the optimal solution.

Furthermore, a case study is presented where the BIF is applied to electricity distribution network planning under uncertain future EV penetration. The analysis shows that the Smart Charging technology has significant Option Value due to the fact that it possesses flexibility to deal with uncertainty and the optimal decision is to invest in a mix of smart and conventional technologies. Sensitivity analyses are conducted on this solution as well. First, these studies indicate that the greater the flexibility of Smart Charging, the greater its Option Value, and vice versa. Moreover, as the social cost of not accommodating EV capacity increases, conventional reinforcement becomes the optimal investment, given that additional network capacity can guarantee full accommodation of EV demand under all scenarios as opposed to Smart Charging of EV that may not achieve full accommodation of power flows under high load-growth scenarios. Moreover, the probability of scenario 1 affects the optimal solution; when it is high, then decision D_1 is optimal and, therefore, the option value of smart charging is zero. On the other hand, when the probability of S_1 is relatively small, the optimal decision is D_3 with some social cost appearing in the solution. The Backwards Induction Framework has also been compared with the Stochastic Optimization Framework for the case study presented in this paper, yielding optimal solutions that are for the most part identical to each other. Since the BIF is not based on mathematical optimization as opposed to SOF, the optimal expected system cost obtained using the latter is never worse (i.e., greater) than that obtained using BIF.

These findings can have significant policy implications. Specifically, current network planning standards do not yet provide an explicit formal framework for the calculation of the Option Value of investing in smart technologies. This may pose an obstacle in the establishment of a level-playing field, where all candidate investment technologies can be compared based on the entire value that they bring to the system operation and investment, including their Option Value, which is the value of the flexibility that they possess to deal with uncertainty. Therefore, an update of the planning standards is necessary by formally including methodologies that can assist in this direction. However, the complexity that is inherent in the proposed stochastic optimization frameworks used for the calculation of the Option Value may become a barrier to their implementation, because the higher the modeling complexity, the lower the model transparency is. In this context, the BIF has the benefit of transparency given its simplicity, while at the same time it can provide a first insight at the optimal investment strategies.

Future work includes the development of multi-stage Backwards Induction frameworks that can span across many years into the future, with the horizon discretized in more than two stages. In addition, it is of interest to the authors to include a greater variety of novel Smart Technology options, such as Soft Open Points [34–36] and Dynamic Line Rating systems [37] as well as Vehicle-to-Grid and Vehicle-to-Building technologies [33,38]. The authors are also interested in comparing the BIF with the Least Worst Regret framework and analyzing the factors that drive the differences in the solutions produced with these two frameworks.

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