

Article

Axisymmetric Flow and Heat Transfer in $\text{TiO}_2/\text{H}_2\text{O}$ Nanofluid over a Porous Stretching-Sheet with Slip Boundary Conditions via a Reliable Computational Strategy

Tahir Naseem ¹, Azeem Shahzad ², Muhammad Sohail ^{3,*} and Sameh Askar ⁴¹ Research Institute for Natural Sciences, Hanyang University, Seoul 04763, Republic of Korea² Department of Basic Sciences, University of Engineering and Technology, Taxila 47050, Pakistan³ Department of Applied Mathematics and Statistics, Institute of Space Technology, P.O. Box 2750, Islamabad 44000, Pakistan⁴ Department of Statistics and Operations Research, College of Science, King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia

* Correspondence: muhammad_sohail111@yahoo.com

Abstract: In this investigation, the motion of $\text{TiO}_2/\text{H}_2\text{O}$ nano-structures towards heated and porous sheets by considering the MHD effect and partial slip at the boundary is inspected. The non-linear PDEs that correspond to the basic conservation laws are converted into ODEs with the help of suitable similarity transformation. Furthermore, the shooting method is used to solve these transformed ODEs and boundary conditions. The impact of thermophoresis properties has been shown graphically and the effect of these properties on the skin friction coefficient (C_f) and Nusselt number (Nu) are given in table form. The comparison between the present exploration and published work is carried out and validation among results is prepared. The enhancement in thermophysical parameters showed contrary results to the velocity profile of the $\text{TiO}_2/\text{H}_2\text{O}$ nanofluid as compared with temperature profile. Moreover, it is observed that the higher estimation in the velocity slip parameter retards the flow and an enhancement in volume fraction increases the fluid's temperature. Furthermore, it has been discovered that the geometry of nanoparticles has a major impact on the flow behaviour. The temperature distribution diminishes when the shape of the nanoparticles changes from platelet to spherical.

Keywords: nanofluid; porous medium; heat transfer; stretching sheet; shooting method; partial slip



Citation: Naseem, T.; Shahzad, A.; Sohail, M.; Askar, S. Axisymmetric Flow and Heat Transfer in $\text{TiO}_2/\text{H}_2\text{O}$ Nanofluid over a Porous Stretching-Sheet with Slip Boundary Conditions via a Reliable Computational Strategy. *Energies* **2023**, *16*, 681. <https://doi.org/10.3390/en16020681>

Academic Editor: Abderrahmane Baïri

Received: 19 October 2021

Revised: 16 November 2021

Accepted: 23 November 2021

Published: 6 January 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

It is a general fact that the solid has a greater thermal conductivity as compared to standard fluids just as $\text{C}_2\text{H}_6\text{O}_2$, water, and mineral oil which causes the low heat transfer properties. Therefore, a contemporary way to improve the heat transportation in fluids is to produce a heat transfer medium by mixing the two substances. The easiest way is to suspend solid materials of micro-sized particles whose thermal conductivity of metal in the conventional fluids and new fluids are known as nanofluids. Naturally, the metals (Cu, Al), oxides (TiO_2 , Al_2O_3 , CuO, SiO_2), carbides (SiC), nitrides (SiN, AlN), or nanometals (graphite, carbon nanotubes) are the nanoparticles used in nanofluids. Maxwell [1] tried to calculate the solid-liquid mixture's thermal conductivity. He did not consider the impact of size and shape factor of the nanoparticles in the analysis for the proposed correlation. Maxwell's work was extended by Hamilton et al. [2] by introducing the new concept "Sphericity" in Maxwell's proposed equations. However, this idea didn't guarantee the clogging, sedimentation, and stability of the suspension. Choi et al. [3] introduced the idea of effective heat transfer in fluids during the study of mixing nanoparticles into traditional fluids in new coolants and cooling technologies. They opened the new era for the researcher in the field of nanofluids. Various disciplines and industries used nanofluids for their products such as in drug delivery [4,5], the automotive industry [6–9], heating and cooling

processes [10–13], nuclear reactors [14], heat exchangers [15,16], oscillating heat pipes, micro-channel heat sinks, and heat transport of nanofluids in porous mediums [17–20], and the references mentioned therein.

Magnetic hydrodynamic heat and mass transmission of nanofluids has caught the interest of scientists in the last year. Sohail et al. [21] used the Cattaneo–Christov theory for heat and mass transfer to investigate the Sutterby nanofluid over a stretching cylinder. They discovered that raising the magnetic parameter improves the heat and concentration fields, whereas the reverse is true for the velocity field. Furthermore, as both solutal and thermal parameters increase, the thermal and concentration fields decrease. Naz et al. [22] explored the behavior of Carreau nanofluid past a flat cylinder in the occupancy of gyrotactic microorganisms and in the magnetic field. Behavior of physical thermophoresis parameters on velocity, thermal, mass transportation, impact of motile organism's density number, and fluxes are shown graphically. It is analyzed that bio convection parameter and curvature enhances the mass transportation rate of microorganisms. Khan et al. [23] discussed entropy generation and heat absorption/generation of Sisko-nanomaterial over stretchable surface with MHD stagnation point flow. Different thermophoresis parameters and their effects on velocity, temperature, and concentration profiles are depicted graphically. They observed that the velocity profile increases monotonically against the velocity ratio parameter. Temperature rises as thermophoretic variables rise, while decay is measured as the Prandtl number rises. Concentration reduces with the increase for heterogeneous cases while the reverse behavior is noted for homogeneous cases. By using 2D MHD flow of Hybrid nanoparticles in the presence of forms of nanoparticles of micropolar dusty fluid, Ghadikolaei et al. [24] discovered that when the value of shape factors increases, the local Nusselt number in prescribed heat flux increases. Nadeem et al. [25] investigated MHD Williamson nanofluid flow over a hot surface and discovered that Williamson fluid had a lower thermal conductivity than MHD Williamson nanofluid.

Khanafer et al. [26] discussed heat transportation performance of nanofluids by presenting the Nusselt number and heat transfer correlation of volume fraction for different Grashoff numbers. Oztop et al. [27] considered partially heated rectangular enclosures filled with nanofluids. They studied natural convection heat transportation with Cu nanoparticles and obtained the highest values of heat transfer. Hamad et al. [28] investigated fluid transportation across a porous membrane as well as heat transportation of an incompressible nanofluid over a semi-finite vertical stretching surface with a magnetic effect. They discussed that as the magnetic parameter increases, the boundary layer thickness also increases while the reverse behavior is observed for the momentum boundary layer thickness. Heris et al. [29] studied the convective heat transfer of oxide nanofluid and observed that there are different parameters to increase the heat transportation, as well as enhance the disorderly variation of nanoparticles, thermal conductivity, and fluctuations. Shah et al. [30] considered a thin film flow of nanofluid past a horizontal stretching disk by considering the effect of thermal radiation and magnetic properties. They discovered that as the magnetic parameters are increased, the thickness of the thin film nanofluid decreases.

The phenomenon of velocity slip has been investigated by taking into account a number of factors. The interface of slip velocity between the fluid and solid boundary appears as a result of nanoparticles, enticing researchers to explore the slip condition. Wang [31], Andersson [32], Ariel et al. [33], Ariel [34], Yang [35], and Abbas et al. [36] discussed problems related to flow considering slip boundary conditions. Mukhopadhyay [37] analyzed the fluid flow past a porous nonlinearly stretching surface with slip boundary condition and observed that with the enhancement of the slip parameter, the velocity profile decreases. Noghrehabadi et al. [38] studied partial slip at the boundary of nanofluid flow over a stretching sheet. They found that for increasing the value of the slip parameter, the Nusselt and Sherwood numbers decrease. Raisi et al. [39] numerically studied slip and non-slip conditions on nanofluids and found that the slip velocity coefficient increases only for greater Reynolds number which affects the rate of heat transfer. Yap et al. [40] analyzed the Williamson nanofluid flow with thermal and velocity slip conditions by taking the

Buongiorno model and carried out the study of heat and mass transportation analysis. They showed that with the increase of thermal and velocity slip parameters, the thickness of boundary layer decreases.

The heat and mass transportation of MHD nanofluids with radiation was explored by Prasad et al. [41], who observed that the diffusion thermo/radiation absorption parameter increases velocity, temperature, and skin friction for Cu nanoparticles. This is because of Copper's higher conductivity compared to TiO_2 . Zangoee et al. [42] used AGM to study the MHD nanofluid ($\text{TiO}_2 - \text{GO}$) between two radiative stretchable rotating disks and showed that the temperature increased with the concentration decreasing function of the Reynolds number. Islam et al. [43] studied nanofluids both theoretically and experimentally. They used water-ethylene glycol based TiO_2 nanofluids as coolants in PEM fuel cells and observed that with the increase of temperature and concentration of nanoparticles, the increase in thermal and electrical conductivity of 50/50 water-ethylene glycol based TiO_2 nanofluids were almost linear. Bobbo et al. [44], considered water based SWCNH TiO_2 nanoparticales and analyzed the viscosity experimentally as well as theoretically. They concluded that both have Newtonian behavior. Fedele et al. [45] investigated the viscosity and thermal conductivity of water-based nanofluids including TiO_2 nanoparticles and discovered that the $\text{TiO}_2/\text{H}_2\text{O}$ nanofluid's thermal characteristics made it appropriate for heat transfer applications. Fakour et al. [46] studied the heat transportation past an unsteady stretching elastic sheet by taking nanofluid thin film flow and demonstrated that the $\text{H}_2\text{O} - \text{Al}$ nanofluid gave better heat transfer as compared to other types of nanofluids.

Azeem et al. [47] explored unsteady, incompressible axisymmetric flow and heat transfer in a liquid film across a radially extending sheet using the magnetic effect. They came to the conclusion that raising the value of magnetic and unsteadiness parameters causes a decrease in the thickness of the film boundary layer. The velocity and thermal conductivity of a fluid are both increased when unsteadiness parameters are increased. They also noticed a decrease in temperature profile when the Prandtl and Eckert numbers increased. Jawad et al. [48] considered hybrid nanofluid ($\text{Al}_2\text{O}_3 - \text{Cu}/\text{H}_2\text{O}$) with a magnetic field by using Tiwari and Das model in stagnation region. They analyzed that escalating the values of volume fraction enhances the flow profile across the stream and cross-stream direction. They showed that increasing the value the of nanoparticle's shape and thermal radiation enhances the temperature profile. Sobia et al. [49] discussed the heat transportation of unsteady thin film flow of nanofluid ($\text{SiO}_2/\text{H}_2\text{O}$) passed over stretching surface with convective condition at boundary. They came to the conclusion that platelet-shaped SiO_2 nanoparticles improve the flow and heat transfer rates. They also discovered that increasing the volume fraction and Biot number raises the temperature profile, but the slip parameter has the opposite effect. Sikdar et al. [50] completed an experimental study of TiO_2 Nanofluid's electrical conductivity and found that enhancement in electrical conductivity increases the temperature and concentration profile.

For more insight, there are many researchers who investigated the concept of heat transfer and fluid flow of nanofluid by using a single phase and two-phase model by taking different conditions and nanoparticles [51–56]. Sohail et al. [57] performed significant achievement in the field of thermal energy by using tri-hybrid nanoparticles over a stretching surface via a finite element method. Nazir et al. [58] discussed the thermal aspects' enhancement into Hall and ion slip currents in Carreau Yasuda fluid over a porous cone using the finite element method. Sohail et al. [59] used variable properties in viscoelastic liquid in concentration and thermal energy effects.

To the author's knowledge, no investigation has been described in literature investigating the analyzed axisymmetric flow with heat transportation of $\text{TiO}_2/\text{H}_2\text{O}$ nanofluid passed a radially stretching porous surface with partial slip boundary condition also taking consideration of MHD effect and viscous dissipation. This work is organized as follows: the literature review is given in segment 1, the formulation of governing equations of the problem is presented in segment 2, the adopted method is listed and applied in seg-

ment 3, segment 4 mentions the physical interpretation, and a summary of the performed exploration is listed in segment 5.

2. Mathematical Description of the Problem

Assume the unsteady axisymmetric 2D motion of nanofluid on a radially stretched porous surface under role of magnetic field. The fluid flow begins when the sheet is radially stretched. U and T reveal wall velocity and temperature. Schematic presentation of nanofluid flow over a porous stretching sheet and the coordinate system is depicted in following Figure 1.

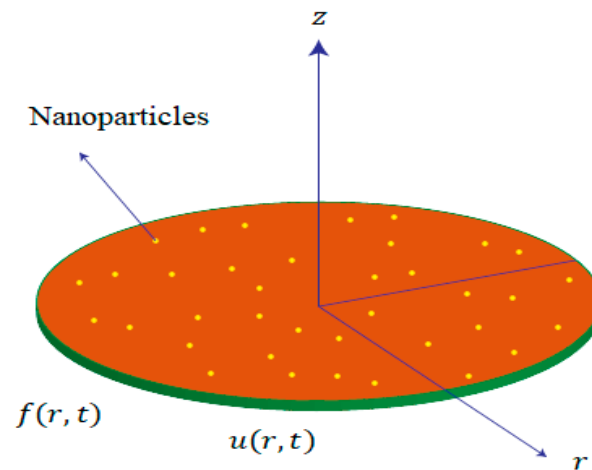


Figure 1. Geometry of desired model.

Base fluid water (H_2O), titanium dioxide (TiO_2) was considered as nanoparticles. Table 1 illustrates the thermal properties [47]. Moreover, uniform structures of nanoparticles are addressed. Furthermore, the phase of nanofluid and nanoparticles are considered in thermal equilibrium with stretching velocity $U = br / (1 - \alpha t)$, where b and α are dimensional constants. $U(r, t)$, reflects the fixed elastic sheet at origin when radially stretched with force in positive r —direction and effective stretching rate $b / (1 - \alpha t)$ increase with time as $0 \leq \alpha \leq 1$. Temperature profile on the porous radially stretching sheet is formulated as

$$T = T_0 - T_r \left[br^2 / 2\nu_f \right] (1 - \alpha t)^{-\frac{3}{2}}, \quad (1)$$

where T_0 , T_r , and ν_f are the surface temperature, the constant reference temperature, such as $0 \leq T_r \leq T_0$, and kinematic viscosity of the water (H_2O), respectively. The variable magnetic field is assumed to be

$$B(r) = \frac{B_0}{(1 - \alpha r)^{1/2}}, \quad (2)$$

which is applied z —direction.

Table 1. Properties (Thermo-physical) regarding H_2O and TiO_2 .

Nanoparticle/Based Fluid	$\rho(\text{kg/m}^3)$	$C_p(\text{J/kgK})$	$K(\text{W/mK})$	$\sigma(\Omega\text{m})^{-1}$
TiO_2	4250	686.2	8.9538	0.125
H_2O	997.1	4179	0.613	5.5

The mathematical form of the physical model mentioned above is given as [47]

$$\frac{\partial}{\partial r}(u) + \frac{1}{r}(u) + \frac{\partial}{\partial z}(w) = 0, \quad (3)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = \nu_{nf} \frac{\partial^2 u}{\partial z^2} - \frac{\sigma_{nf}}{\rho_{nf}} (B(r))^2 u, \quad (4)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \alpha_{nf} \frac{\partial^2 T}{\partial z^2} + \frac{\mu_{nf}}{\rho_{nf} C_p} \left(\frac{\partial u}{\partial z} \right)^2, \quad (5)$$

where u and w are velocity components. The physical properties characterizing the H_2O and TiO_2 , namely, density (ρ_{nf}), thermal diffusivity (α_{nf}), viscosity (ν_{nf}), electrical conductivity (σ_{nf}) and (volume fraction) ϕ . For thermal conductivity (K_{nf}) and dynamic viscosity (μ_{nf}), Hamilton and Crosser's models are used which are [2,60]

$$\alpha_{nf} = \frac{K_{nf}}{(\rho C_p)_{nf}}, \quad \rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s, \quad \sigma_{nf} = (1 - \phi)\sigma_f + \phi\sigma_s, \quad (6)$$

$$\mu_{nf} = \mu_f \left(1 + A^1 \phi + A^2 \phi^2 \right), \quad \frac{K_{nf}}{K_f} = \frac{K_s + (nm - 1)K_f + (nm - 1)(K_s - K_f)\phi}{K_s + (nm - 1)K_f - (K_s - K_f)\phi}. \quad (7)$$

The shape factor $nm = \frac{3}{\phi}$; ϕ is the ratio of surface areas of sphere to particle. Table 2 presents the shape factor and viscosity coefficients corresponding to various shapes of nanoparticles which is utilized in this research.

Table 2. Shapes factor and viscosity co-efficient corresponding to five distinct nanoparticles.

Nanoparticle Shapes	Blade	Brick	Cylinder	Platelets	Sphere
A_1	14.6	1.9	13.5	37.1	2.5
A_2	123.3	471.4	904.4	612.6	6.5
Shape factor(nm)	8.26	3.72	4.82	5.72	3.0

The corresponding boundary conditions of the differential equations for the given system are

$$u = k \left(\frac{\partial u}{\partial z} \right) + U, \quad 0 = v, \quad T = T_s, \quad \text{at } z = 0, \quad u \rightarrow 0, \quad T \rightarrow T_\infty, \quad \text{as } z \rightarrow \infty. \quad (8)$$

The following similarity transformations are introduced for simplification of problem.

$$\psi = -r^2 U Re^{-\frac{1}{2}} f(\eta), \quad \theta(\eta) = \frac{(T - T_s)}{T_r \left[\frac{br^2}{2\nu_f} \right] (1 - \alpha t)^{-\frac{3}{2}}}, \quad \eta = \frac{z}{r} Re^{\frac{1}{2}}, \quad Re = \frac{rU}{\nu_f}. \quad (9)$$

A (stream function) ψ defined by

$$u = -\frac{1}{r} \frac{\partial \psi}{\partial z} = U f'(\eta), \quad w = \frac{1}{r} \frac{\partial \psi}{\partial r} = -2U f(\eta) Re^{-\frac{1}{2}}. \quad (10)$$

Using similarity variables (9), Equations (4) and (5) take the form

$$\varepsilon_1 f''' - M \varepsilon_2 f' - S \left[f' + \frac{\eta}{2} f'' \right] - f'^2 + 2f f'' = 0, \quad (11)$$

$$\frac{\varepsilon_2}{Pr} \theta'' + Ec \varepsilon_1 (f'')^2 - \frac{S}{2} [3\theta + \eta \theta'] - 2f'\theta + 2f\theta' = 0, \quad (12)$$

subject to transform boundary conditions

$$f(0) = 0, f'(0) = 1 + kf''(0), \theta(0) = 1, f'(\infty) = 0, \theta(\infty) = 0. \quad (13)$$

The thermo-physical parameters Pr , S , Ec , and M appear in Equations (11) and (12) are mathematically defined as

$$Pr = \frac{\nu}{\alpha_{nf}}, S = \frac{a}{b}, Ec = \frac{U^2}{Cp(T_s - T)}, \text{ and } M = \frac{\sigma_f B_0^2}{\rho_{nf} b}. \quad (14)$$

Finally, $\varepsilon_1, \varepsilon_2$, and ε_3 are constants which are defined by

$$\varepsilon_1 = \frac{(1 + A^1 \varphi + A^2 \varphi^2)}{(1 - \varphi + \varphi \frac{\rho_s}{\rho_f})}, \varepsilon_2 = \frac{\frac{K_{nf}}{K}}{1 - \varphi + \varphi \frac{(\rho Cp)_s}{(\rho Cp)_f}}, \varepsilon_3 = \frac{1 - \varphi + \varphi \frac{\sigma_s}{\sigma_f}}{1 - \varphi + \varphi \frac{\rho_s}{\rho_f}}. \quad (15)$$

The desired temperature gradient and drag force coefficient are

$$\begin{cases} C_f = \frac{2\mu_{nf}(\frac{\partial u}{\partial z})_{z=0}}{\rho_f U^2} = Re^{-\frac{1}{2}} (1 + A^1 \varphi + A^2 \varphi^2) f''(0), \\ Nu = \frac{-x K_{nf}(\frac{\partial T}{\partial z})_{z=0}}{T_s - T_0} = Re^{\frac{1}{2}} \frac{K_{nf}}{K} \theta'(0). \end{cases} \quad (16)$$

3. Numerical Scheme for Solution

Several numerical [27,39,52,55,58,59] and analytical schemes [4,5,21,22,28,31,57] are available to handle the nonlinear complex problems arising in mathematical physics. The exact solution is not possible for every problem due to highly nonlinearity and as well as due to the involvement of complex geometry and mixed boundary conditions. The solution of a nonlinear ODE system (11–12) subject to boundary conditions (13) is calculated using numerical technique known as BVP4C. In this case, the third- and second-order nonlinear ODEs are given by Equations (11) and (12), respectively, and have been reduced to first-order differential equations as follow and the better understanding of the used scheme is explained through Figure 2.

$$f = y_1, f' = y_2, f'' = y_3, f''' = \frac{1}{\varepsilon_1} [M \varepsilon_3 y_2 + S \{y_2 + \frac{\eta}{2} y_3\} + y_2^2 - 2y_1 y_2], \quad (17)$$

$$\theta = y_4, \theta' = y_5, \theta'' = \frac{Pr}{\varepsilon_2} \left[\frac{S}{2} \{3y_4 + \eta \varepsilon_0\} + 2y_2 y_4 - 2y_1 y_5 - Ec \varepsilon_1 y_3^2 \right], \quad (18)$$

corresponding boundary condition becomes

$$y_1(0) = 0, y_2(0) = 1 + k y_3(0), y_4(0) = 1, y_2(\infty) = 0, y_5(\infty) = 0. \quad (19)$$

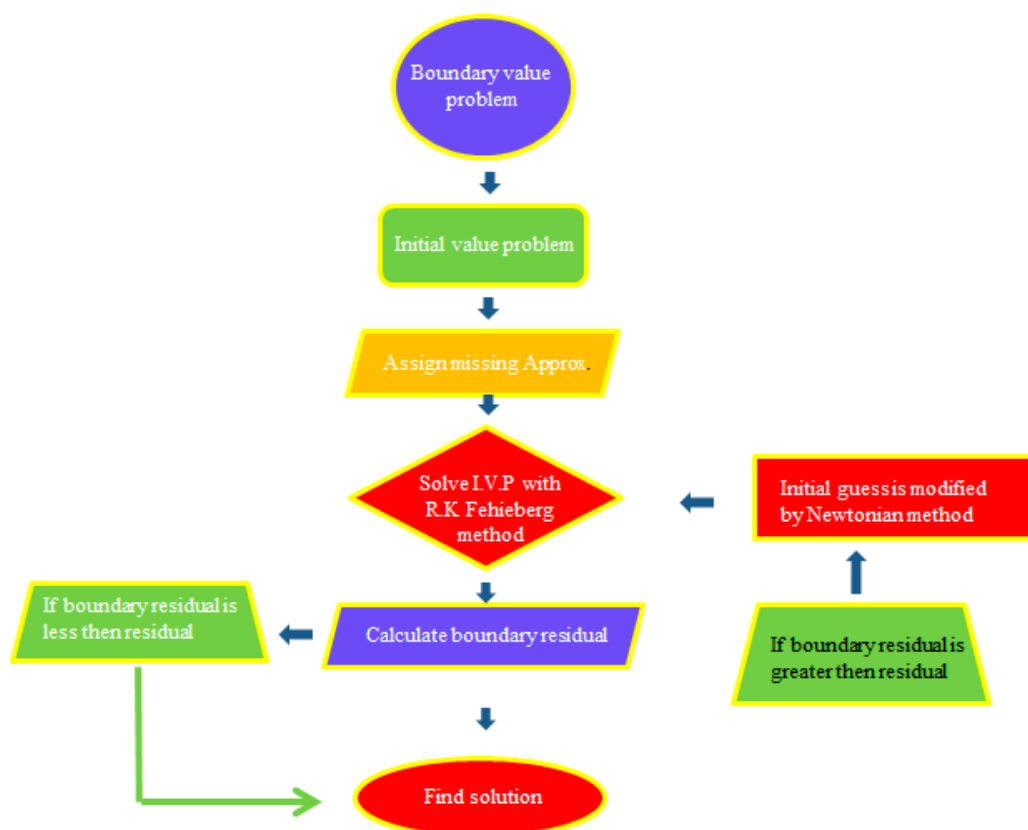


Figure 2. Flow scheme of numerical approach.

The iterative procedure will end with the appropriate level of precision.

4. Results and Discussion

This includes the simulation results of titanium dioxide-water nanofluid unsteady flow and heat transfer over a radially stretching surface (Figure 1). We set some fundamental parameters for this analysis, changed one of them and then composed a response. The results so obtained are compared for skin friction ($-f''(0)$) and local Nusselt number ($-\theta'(0)$), respectively, with different values of unsteadiness parameter (S), magnetic parameter (M), slip parameter (k), nanoparticles volume fraction (φ), Prandtl number (Pr) and Eckert number (Ec) by taking nanoparticle shapes like brick, cylinder, blade, sphere and platelets are depicted in Tables 3 and 4.

The effect of S , k and φ on the velocity field is investigated here, taking into account the various shapes of titanium dioxide nanoparticles in base fluid water. S has an effect on the velocity profile, as shown in Figure 3a–e. The velocity of nanofluid in region (boundary layer) declines using higher values of S . Thickness related to momentum is said to decrease the function against the values of S . Therefore, fluid is known as thick when S is increased. It is evident that the impact of S on the velocity profile is more effective by using platelet-shaped TiO_2 for the nanofluid as compared to the other shapes of the nanoparticles. It is found that the maximum velocity of TiO_2/H_2O nanofluid is attained with platelet-shaped particles than that of cylinder, blade, bricks, and sphere shaped particles. It is also seen that the minimum velocity of TiO_2/H_2O nanofluid is captured with sphere shaped nanoparticles than that of the other shapes of nanoparticles. Figure 4a–e represent the influence of velocity slip parameter (k). It is evident from the figures that with the increase of k , the horizontal velocity decreases. When platelets shape nanoparticles are employed, the velocity is determined to be the highest when compared to other shapes of nanoparticles used. Figure 5a–e shows the effect of volume fraction of the nanofluid on the velocity

profile. It is analyzed that the enhancement in the velocity field occurs due to increase in nanoparticles volume fraction.

Table 3. The computed results for local skin friction.

<i>S</i>	<i>M</i>	<i>k</i>	φ	<i>Pr</i>	<i>Ec</i>	$(-f''(0)Present)$				
0.2	1.0	0.5	0.02	7.56	1.0	Blade	Brick	Cylinder	Platelets	Sphere
0.0	—	—	—	—	—	0.7155365	0.7347624	0.6740443	0.6335091	0.7680837
0.2	—	—	—	—	—	0.7350176	0.7545665	0.6928182	0.6515231	0.7884168
0.4	—	—	—	—	—	0.7535168	0.7733531	0.7106818	0.6686982	0.8076700
—	—	0.0	—	—	—	1.281081	1.339648	1.1615725	0.9274630	1.4460915
—	—	0.5	—	—	—	0.7350176	0.7545665	0.6928966	0.5906568	0.7884168
—	—	1.0	—	—	—	0.5254160	0.5357161	0.5027803	0.4414587	0.5532537
—	—	—	0.00	—	—	0.4317496	0.4317460	0.4317460	0.4317460	0.4317460
—	—	—	0.02	—	—	0.4113289	0.4177996	0.3968889	0.3821424	0.4287374
—	—	—	0.4	—	—	0.3912077	0.3891333	0.3520232	0.3406492	0.4255740
—	—	—	—	4.0	—	0.7350176	0.7545665	0.6928966	0.6516882	0.7884168
—	—	—	—	6.0	—	0.7350176	0.7545665	0.6928966	0.6516882	0.7884168
—	—	—	—	8.0	—	0.7350176	0.7545665	0.6928966	0.6516882	0.7884168
—	—	—	—	—	0.0	0.7350176	0.7545665	0.6928966	0.6516882	0.7884168
—	—	—	—	—	0.5	0.7350176	0.7545665	0.6928966	0.6516882	0.7884168
—	—	—	—	—	1.0	0.7350176	0.7545665	0.6928966	0.6516882	0.7884168

Table 4. The computed results for local Nusselt number.

<i>S</i>	<i>M</i>	<i>k</i>	φ	<i>Pr</i>	<i>Ec</i>	$(-\theta'(0)Present)$				
0.2	1.0	0.5	0.02	7.56	1.0	Blade	Brick	Cylinder	Platelets	Sphere
0.0	—	—	—	—	—	2.0944532	2.1456869	2.0959472	2.0502516	2.1668261
0.2	—	—	—	—	—	2.3907456	2.4500127	2.3922862	2.3423895	2.4767264
0.4	—	—	—	—	—	2.6594658	2.7258783	2.6612633	2.6074317	2.7571685
—	—	0.0	—	—	—	1.6787138	1.729614	1.6293238	2.0918449	1.7842033
—	—	0.5	—	—	—	2.3907456	2.4500127	2.391635	2.6116387	2.4767264
—	—	1.0	—	—	—	2.4113003	2.4565463	2.4523227	2.6497208	2.454599
—	—	—	0.00	—	—	2.4072763	2.4073228	2.4073228	2.4073237	2.4073254
—	—	—	0.02	—	—	2.3428233	2.3792423	2.4041469	2.4122462	2.3632343
—	—	—	0.4	—	—	2.2790864	2.3740099	2.3774746	2.3587298	2.319692
—	—	—	—	4.0	—	1.7602862	1.8001545	1.7720755	1.744038	1.8113522
—	—	—	—	6.0	—	2.1417315	2.1932347	2.1470323	2.1053742	2.2136552
—	—	—	—	8.0	—	2.4555829	2.5168879	2.4553144	2.4022715	2.5452625
—	—	—	—	—	0.0	3.2971464	3.3367739	3.4221096	3.4752914	3.2918987
—	—	—	—	—	0.5	2.843946	2.8933933	2.9068724	2.9081209	2.8843125
—	—	—	—	—	1.0	2.3907456	2.4500127	2.391635	2.3409504	2.4767264

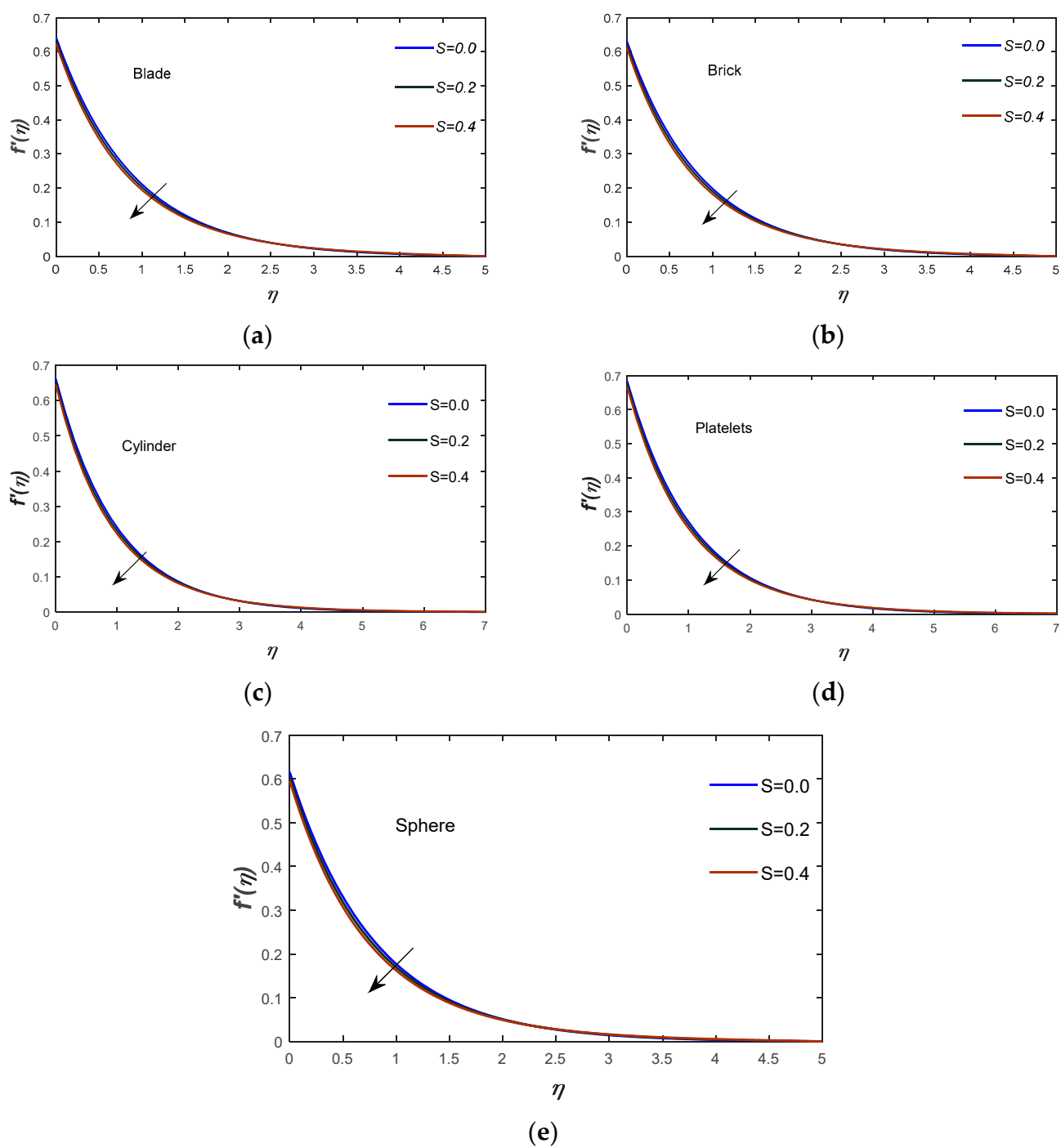
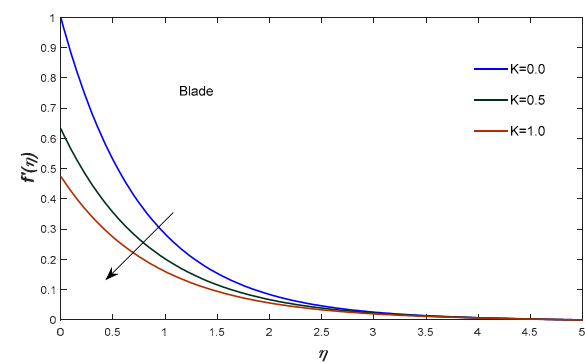
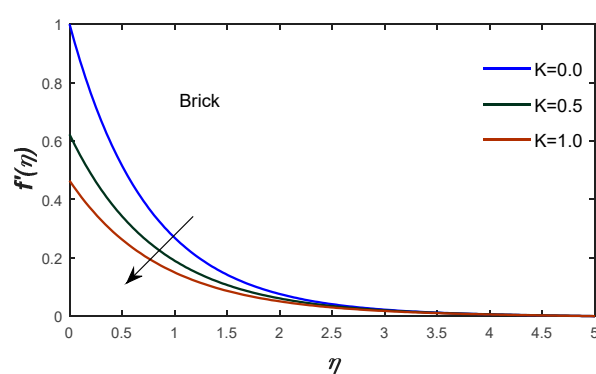


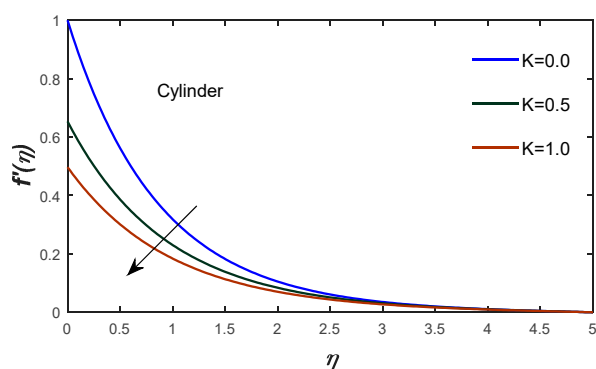
Figure 3. (a–e). Impact of S on $f'(\eta)$ by using various shapes factors of nanoparticles for fixed $M = 1.0$, $Pr = 6.0$, $k = 0.5$, $Ec = 1.0$, $\varphi = 0.02$.



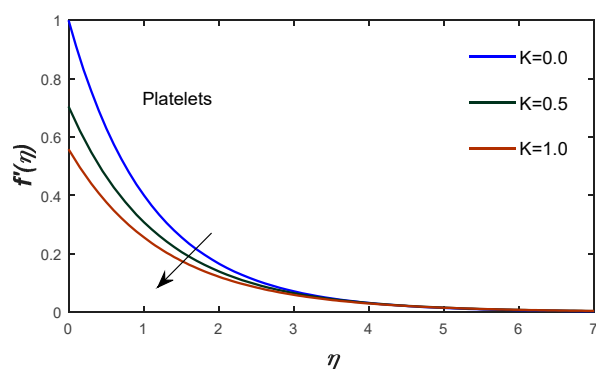
(a)



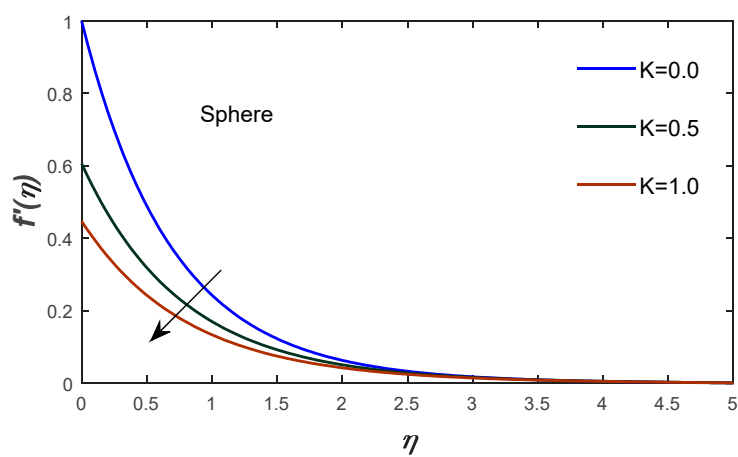
(b)



(c)



(d)



(e)

Figure 4. (a–e). Impact of on $f'(\eta)$ by using various shapes factors of nanoparticles for fixed $M = 1.0, Pr = 6.0, S = 0.5, Ec = 1.0, \varphi = 0.02$.

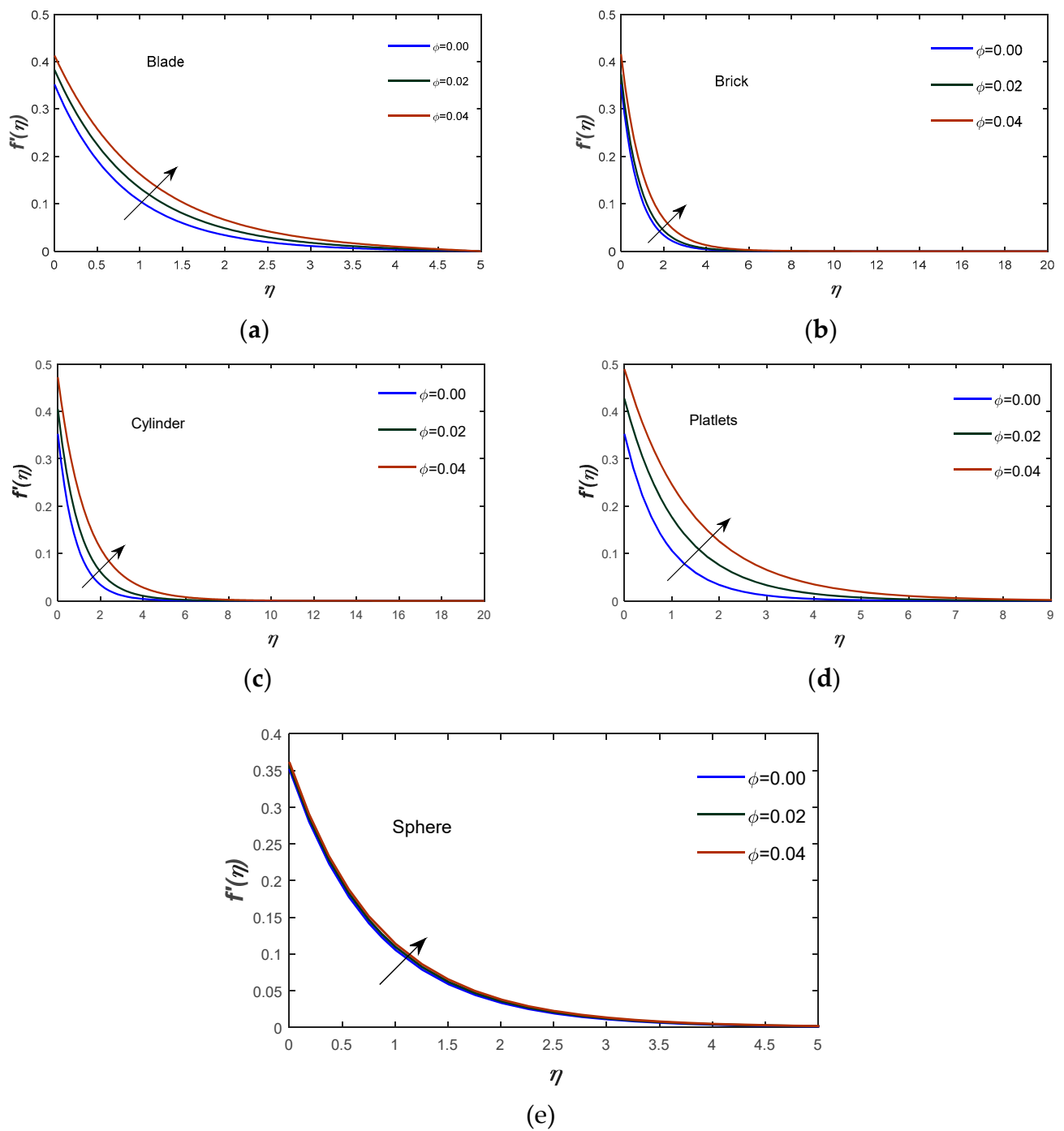


Figure 5. (a–e). Impact of ϕ on $f'(\eta)$ by using various shapes factors of nanoparticles for fixed $M = 1.0$, $Pr = 6.0$, $k = 0.5$, $Ec = 1.0$, $S = 0.5$.

The impact of S , k , ϕ , Pr , and Ec on temperature field is predicted here by taking the different shapes of nanoparticles of titanium dioxide with base fluid water. Figure 6a–e indicate that S had an impact on temperature profile. With increasing values of the steadiness parameter S , the nanofluid temperature decreases in the boundary layer region. Figure 7a–e demonstrate the effects of velocity slip parameter (k) on the temperature; the graphical results showed that with the increase of k , the temperature decreases. It is estimated that heat energy becomes reduce against distribution in k . This reducing impact into heat energy is due to role porosity of surface. Numbers of pores is known as reducing function into heat energy. Layers associated within heat energy are declined versus k . Moreover, the impact of volume fraction of nanoparticles of different shapes on the temperature field $\theta(\eta)$ is predicted, in Figure 8a–e for $\text{TiO}_2/\text{H}_2\text{O}$ nanofluid. It has been discovered that using nano-sized particles of various shapes in water increases thermal conductivity, resulting in an increase in the heat transfer rate. Figure 9a–e demonstrated that as the Prandtl number (Pr) varied, the thermal diffusion rate slowed, resulting in a lower temperature profile at all times. Thermal layers at boundary have decreasing function against Prandtl number. This decreasing function among heat energy and Prandtl number is developed due to using concept of Prandtl number. According to concept of Prandtl number, it is ratio among momentum and thermal layers at boundary of surface. Therefore, an increment in Prandtl number creates reduction in thermal energy. Momentum layers are based on distribution of Prandtl number. Thus, momentum layers at boundary are known as decreasing function versus an impact of Prandtl number. Physically, the fluids with larger values of Prandtl number have small thermal conductivity and high viscosity, due to which fluid become thick, and causes the decrease in the velocity field. It is concluded that with the increase of Eckert number causes the increases in temperature field as can be seen in Figure 10a–e. Figure 11a indicates that the Titanium dioxide/water nanofluid attains its maximum velocity when we used the platelets shape nanoparticles of Titanium dioxide than cylindrical than blade than brick shapes nanoparticles and then minimum in the case of sphere shape nanoparticle. Figure 11b depicts that the Titanium dioxide/water nanofluid attains its maximum temperature when we used the platelet-shaped nanoparticles of Titanium dioxide rather than cylindrical, blade or brick-shaped nanoparticles, and then the minimum in sphere shaped particles. It is noticed that the maximum production in heat energy occurs because of the maximum amount of viscous dissipation. Physically, viscous dissipation is defined as the rate of work done in view of movement of nanoparticles. Higher viscous dissipation brings an enhancement in the rate of work done of nanoparticles. It means that large viscous dissipation creates more heat energy in the particles. Moreover, a directly proportional relation is found between the Eckert number and thermal energy. Therefore, temperature profile increases in line with higher values of Eckert number. Table 5 present the comparative study. From the analysis, it is obvious that the obtained results are in good settlement with the findings mentioned in studies reported by [47].

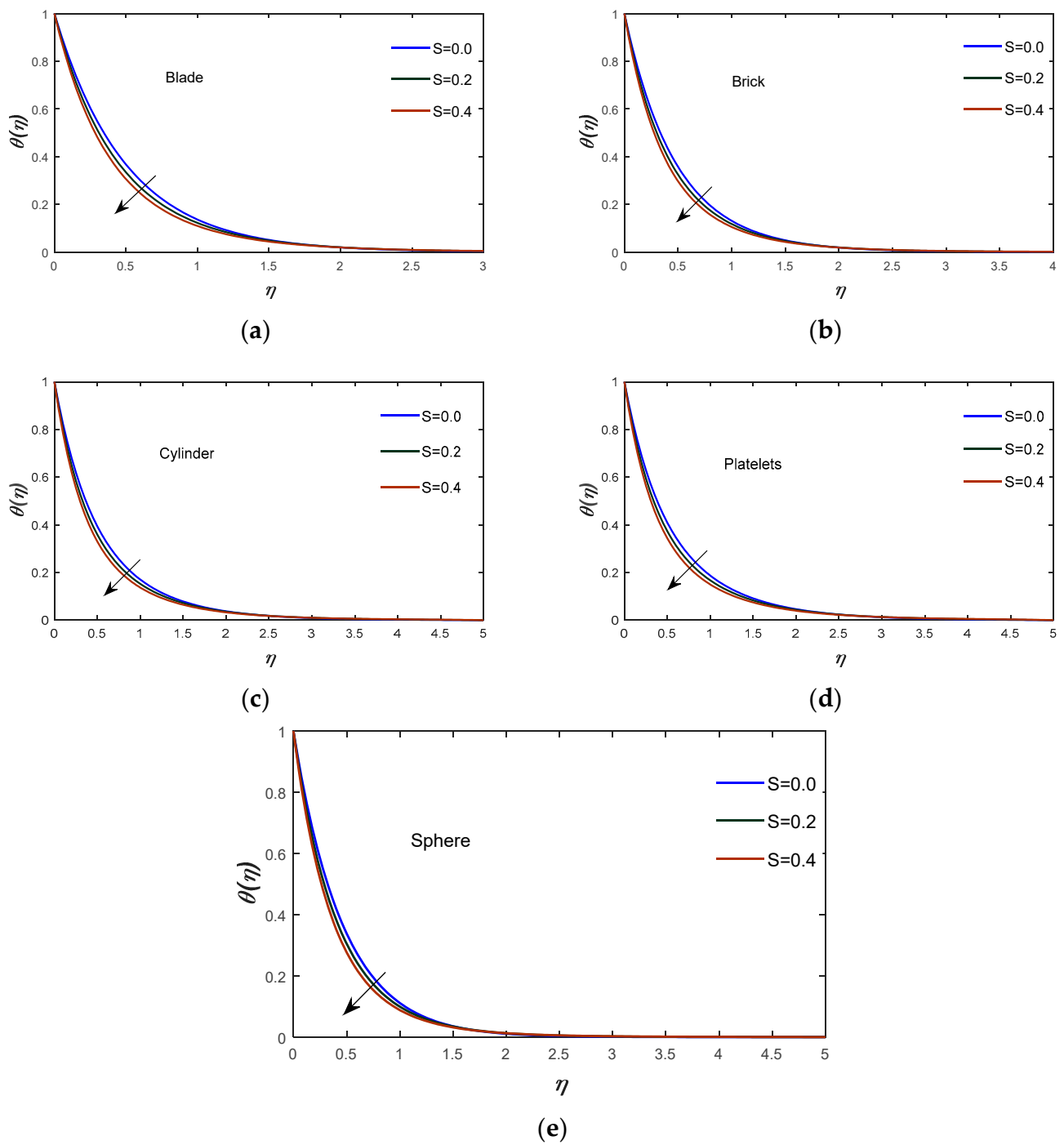


Figure 6. (a–e). Impact of S on $\theta(\eta)$ by using various shapes factors of nanoparticles for fixed $M = 1.0, Pr = 6.0, k = 0.5, Ec = 1.0, \varphi = 0.02$.

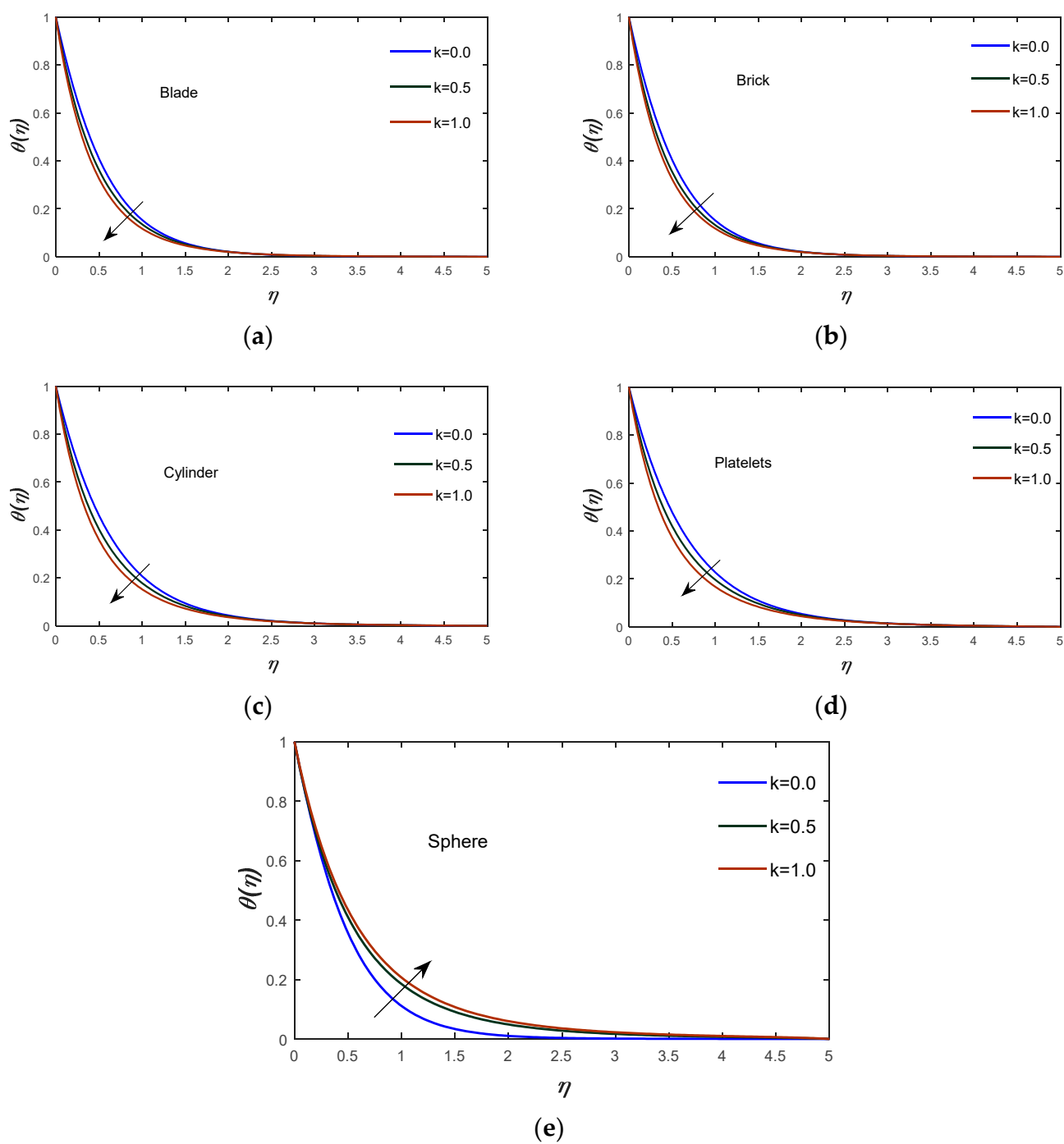
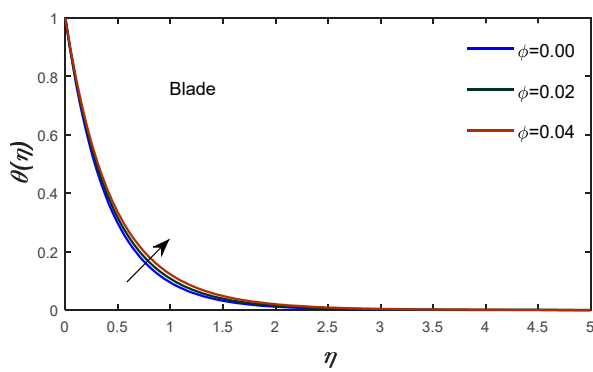
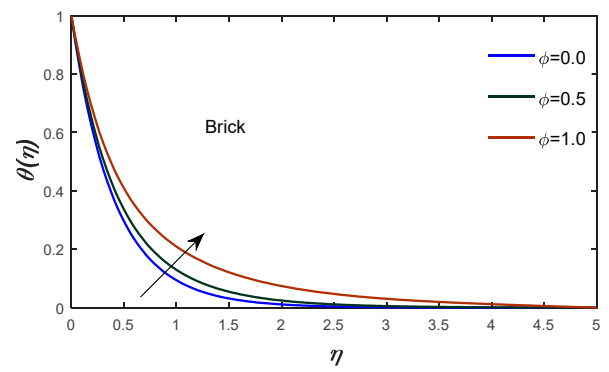


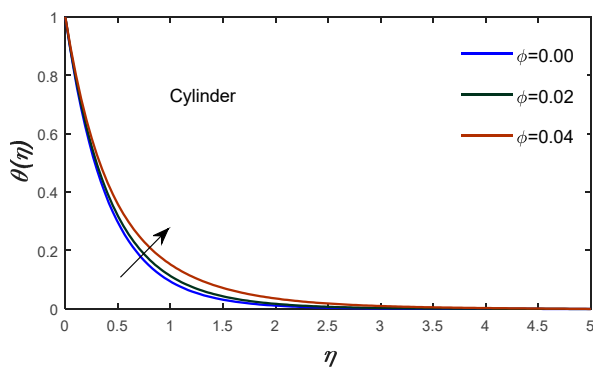
Figure 7. (a–e). Impact of K on $\theta(\eta)$ by using various shapes factors of nanoparticles for fixed $M = 1.0$, $Pr = 6.0$, $S = 0.5$, $Ec = 1.0$, $\varphi = 0.02$.



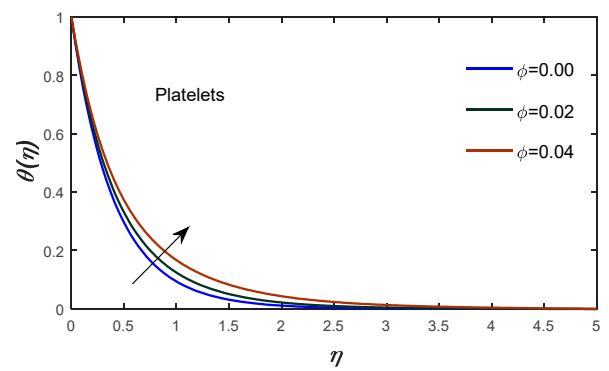
(a)



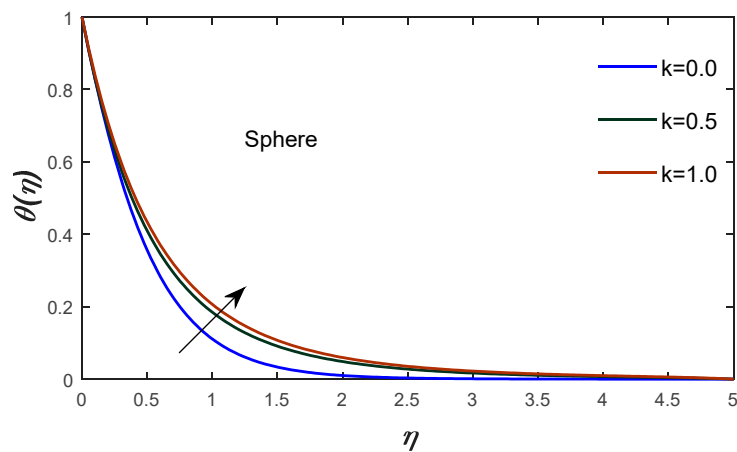
(b)



(c)

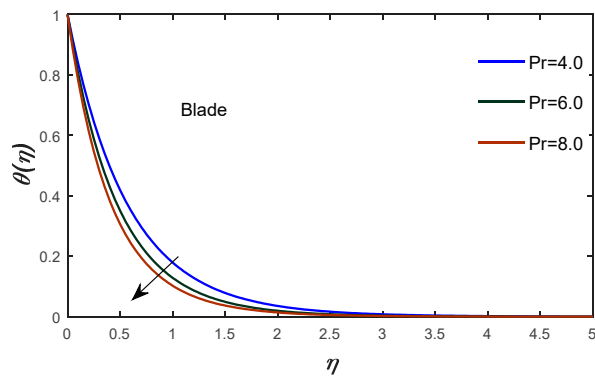


(d)

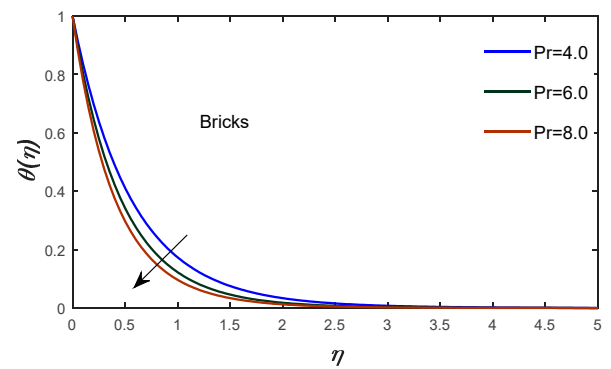


(e)

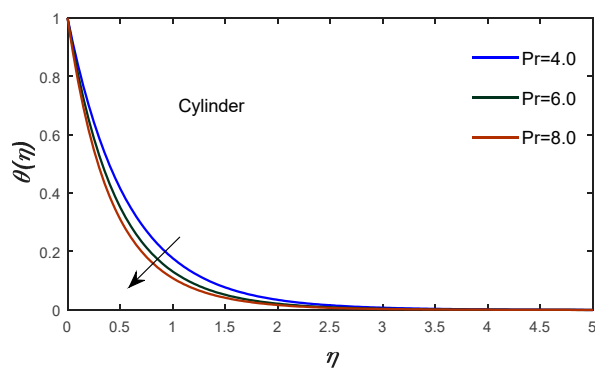
Figure 8. (a–e). Impact of K on $\theta(\eta)$ by using various shapes factors of nanoparticles $M = 1.0$, $Pr = 6.0$, $S = 0.5$, $Ec = 1.0$, $k = 0.5$.



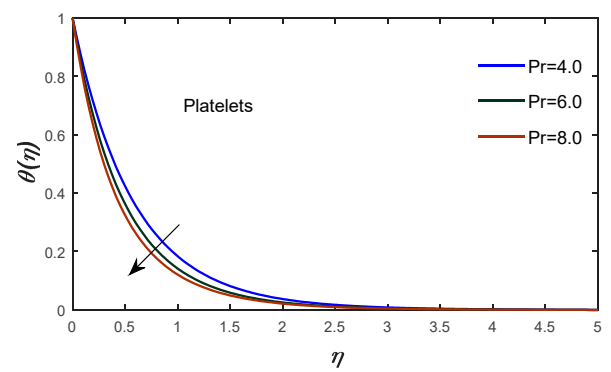
(a)



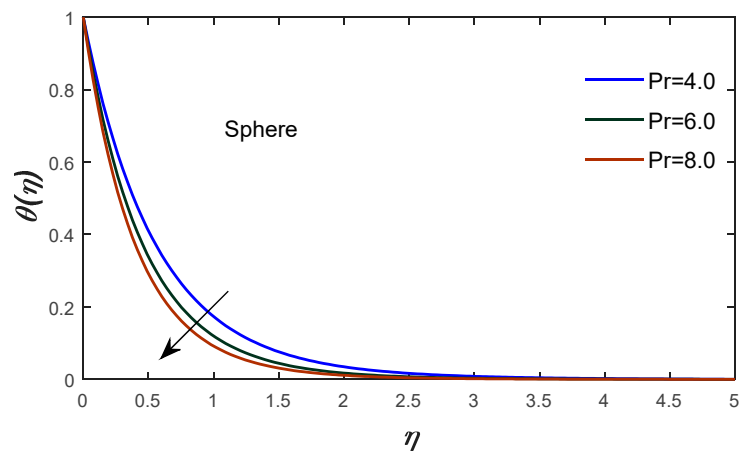
(b)



(c)



(d)



(e)

Figure 9. (a–e). Impact of Pr on $\theta(\eta)$ by using various shapes factors of nanoparticles for fixed $M = 1.0$, $k = 0.5$, $S = 0.5$, $Ec = 1.0$, $\varphi = 0.02$.

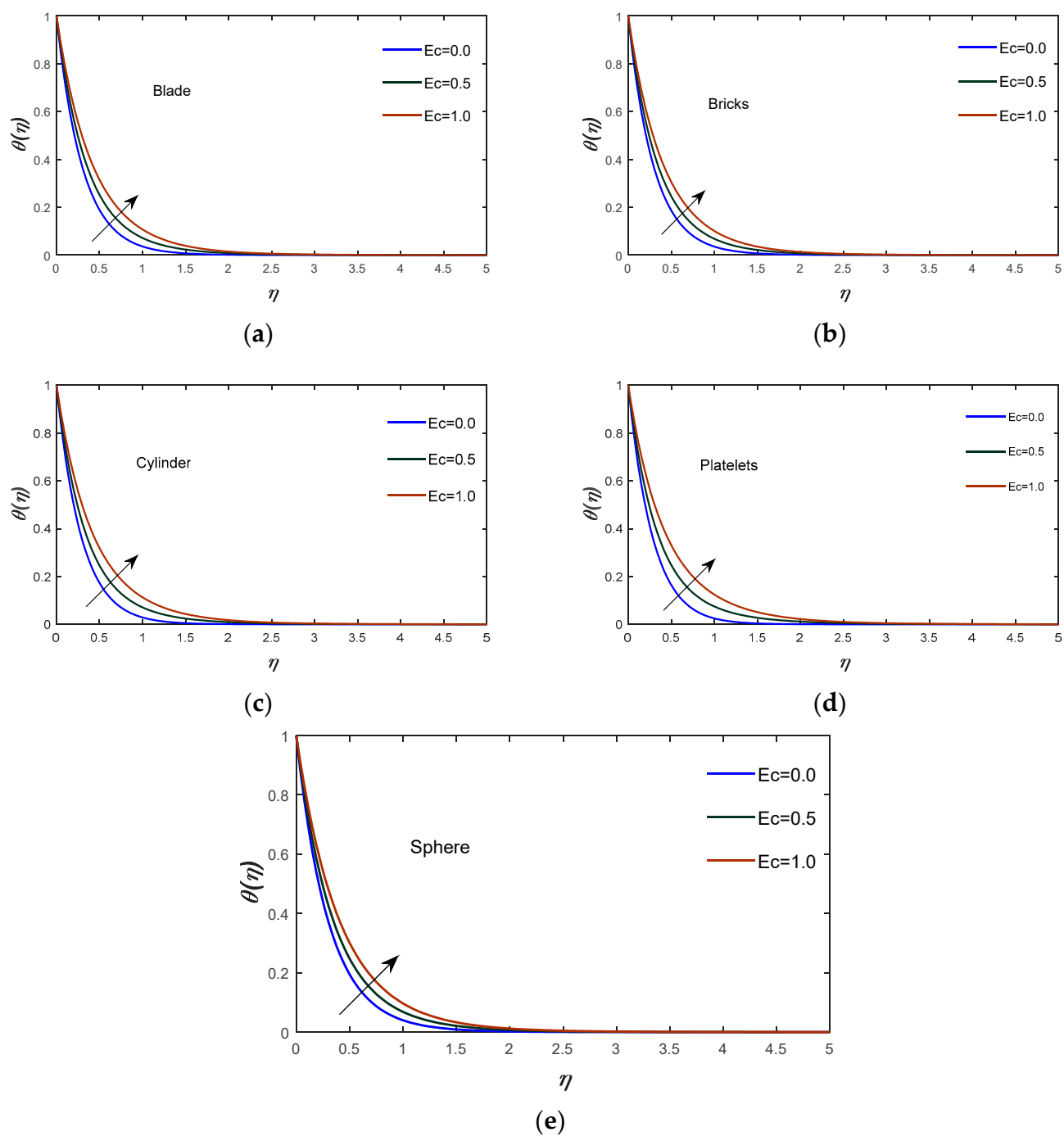


Figure 10. (a–e). Impact of Pr on $\theta(\eta)$ by using various shapes factors of nanoparticles for fixed $M = 1.0$, $Pr = 6.0$, $S = 0.5$, $k = 0.5$, $\varphi = 0.02$.

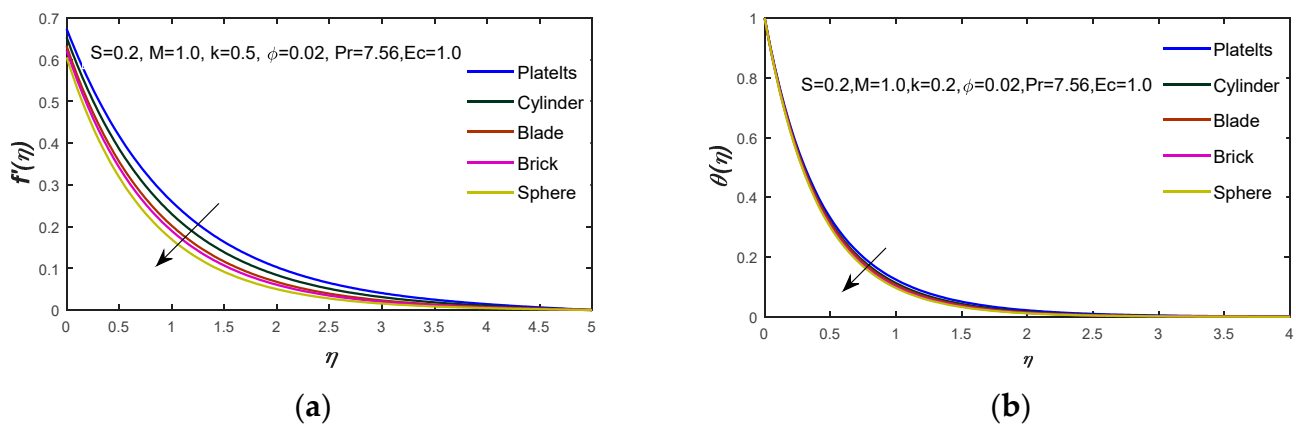


Figure 11. (a,b). Impact of various shapes factors of nanoparticles on $f'(\eta)$ and $\theta(\eta)$.

Table 5. Impacts of pertinent parameters on Skin fraction coefficient.

M	S	Pr	Ec	$-f''(0)$	$-f''(0)$
				Reference [47]	Present
0	1.0	0.7	0.2	1.2784767	1.1353469
1	-	-	-	1.6237276	1.5146317
2	-	-	-	1.9067150	1.8155125
1	0.8	0.7	0.2	1.674472	1.6107074
-	1.0	-	-	1.6650959	1.6571311
-	1.2	-	-	1.6186823	1.7025145

5. Key Findings of Performed

Axisymmetric viscous flow in 2D of unsteady and incompressible TiO_2/H_2O nanofluid over a porous radially stretching sheet with velocity slip conditions at a boundary is analyzed here in this article. From the contemporary investigations, the main interpretations are

- Dimensionless velocity $f'(\eta)$ shows a decreasing nature for the unsteadiness parameter (S) and velocity slip parameter (k), while, the reverse behavior for volume fraction (ϕ);
- Augmenting values of velocity slip and unsteadiness parameters retards the velocity field;
- Higher estimation of the volume fraction boosts the fluid's temperature;
- For a rise in the volume fraction parameter (ϕ) causes the enhancement in temperature profile on the other hand the reverse behavior is observed in case of increasing the unsteadiness parameter (S) and velocity slip parameter (k);
- Higher values of the Prandtl number (Pr) results to decrease the temperature field and connected layer thickness;
- The increasing effect of the Eckert number (Ec) on the temperature field is observed;
- It is concluded from the obtained results that the impact of the Eckert number and Prandtl number is opposite in nature;
- On the platelet and sphere-shaped nanoparticles, the velocity profile of TiO_2 nanofluid reaches its maximum and minimum, respectively, while thermal conductivity follows a similar pattern.
- Maximum surface force is developed against higher values of the magnetic number, while the minimum production of surface force is generated with respect to higher values of the unsteadiness parameter;

- A significant performance of nanoparticles is observed for enhancement of thermal conductivity and production of heat energy.

Author Contributions: Conceptualization, M.S. and T.N.; methodology, T.N.; software, A.S.; validation, S.A., A.S. and T.N.; formal analysis, M.S.; investigation, S.A.; resources, T.N.; data curation, A.S.; writing—original draft preparation, T.N. and M.S.; writing—review and editing, T.N.; S.A. and A.S.; visualization, A.S.; supervision, M.S.; project administration, M.S.; funding acquisition, S.A. All authors have read and agreed to the published version of the manuscript.

Funding: Research Supporting Project number (RSP-2021/167), King Saud University, Riyadh, Saudi Arabia.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The datasets generated/produced during and/or analyzed during the current study/research are available from the corresponding author on reasonable request.

Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Nomenclature

S	Unsteadiness parameter
k	Slip parameter
ϕ	Solid volume fraction of nanofluid
M	Magnetic parameter
Pr	Prandtl number
Ec	Eckert number
η	Similarity Variable
Re	Reynolds number
ρ_f	Stream function
u, w	Velocity components
ρ_f	Density of base fluid
μ_f	Dynamic viscosity
σ_f	Electrical conductivity
$Re^{-\frac{1}{2}}C_f$	Local skin friction coefficient
k_f	Thermal conductivity
C_f	Skin friction coefficient
ρ_s	Density of nanoparticles
σ_s	Nanoparticle's electrical conductivity
μ_s	Nanoparticle's dynamic viscosity
k_s	Nanoparticle's thermal conductivity
$(\rho C_p)_s$	Nanoparticle's heat capacitance
μ_{nf}	Nano fluid's Viscosity
ρ_{nf}	Nano fluid's Density
k_{nf}	Nano fluid's Thermal conductivity
$(\rho C_p)_{nf}$	Nano fluid's heat capacitance
$\epsilon_1, \epsilon_2, \epsilon_3$	Constants
$Re^{-\frac{1}{2}}Nu$	Local Nusselt number

References

- Maxwell, J.C. *A Treatise on Electricity and Magnetism*; Clarendon Press: Oxford, UK, 1873; Volume 1.
- Hamilton, R.L.; Crosser, O.K. Thermal conductivity of heterogeneous two-component systems. *Ind. Eng. Chem. Fundam.* **1962**, *1*, 187–191. [[CrossRef](#)]
- Choi, S.U.; Eastman, J.A. *Enhancing Thermal Conductivity of Fluids with Nanoparticles* (No. ANL/MSD/CP-84938; CONF-951135-29); Argonne National Lab.: Lemont, IL, USA, 1995.
- Ghasemi, S.E.; Vatani, M.; Hatami, M.; Ganji, D.D. Analytical and numerical investigation of nanoparticle effect on peristaltic fluid flow in drug delivery systems. *J. Mol. Liq.* **2016**, *215*, 88–97. [[CrossRef](#)]
- Abbas, M.A.; Bai, Y.Q.; Rashidi, M.M.; Bhatti, M.M. Application of drug delivery in magnetohydrodynamics peristaltic blood flow of nanofluid in a non-uniform channel. *J. Mech. Med. Biol.* **2016**, *16*, 1650052. [[CrossRef](#)]
- Leong, K.Y.; Saidur, R.; Kazi, S.N.; Mamun, A.H. Performance investigation of an automotive car radiator operated with nanofluid-based coolants (nanofluid as a coolant in a radiator). *Appl. Therm. Eng.* **2010**, *30*, 2685–2692. [[CrossRef](#)]
- Kakavand, H.; Molana, M. A numerical study of heat transfer characteristics of a car radiator involved nanofluids. *Heat Transf.—Asian Res.* **2018**, *47*, 88–102. [[CrossRef](#)]
- Molana, M. On the nanofluids application in the automotive radiator to reach the enhanced thermal performance: A review. *Am. J. Heat Mass Transf.* **2017**, *4*, 168–187. [[CrossRef](#)]
- Kumar, V.; Pandya, N.; Pandya, B.; Joshi, A. Synthesis of metal-based nanofluids and their thermo-hydraulic performance in compact heat exchanger with multi-louvered fins working under laminar conditions. *J. Therm. Anal. Calorim.* **2019**, *135*, 2221–2235. [[CrossRef](#)]
- Nithyadevi, N.; Gayathri, P.; Chamkha, A. Three dimensional MHD stagnation point flow of Al-Cu alloy suspended water based nanofluid with second order slip and convective heating. *Int. J. Numer. Methods Heat Fluid Flow* **2017**, *27*, 2879–2901. [[CrossRef](#)]
- Molana, M. A comprehensive review on the nanofluids application in the tubular heat exchangers. *Am. J. Heat Mass Transf.* **2016**, *3*, 352–381. [[CrossRef](#)]
- Alsabery, A.I.; Chamkha, A.J.; Saleh, H.; Hashim, I.; Chanane, B. Effects of finite wall thickness and sinusoidal heating on convection in nanofluid-saturated local thermal non-equilibrium porous cavity. *Phys. A Stat. Mech. Its Appl.* **2017**, *470*, 20–38. [[CrossRef](#)]
- Izadi, S.; Armaghani, T.; Ghasemiasl, R.; Chamkha, A.J.; Molana, M. A comprehensive review on mixed convection of nanofluids in various shapes of enclosures. *Powder Technol.* **2019**, *343*, 880–907. [[CrossRef](#)]
- Buongiorno, J.; Hu, L.W.; Kim, S.J.; Hannink, R.; Truong, B.; Forrest, E. Nanofluids for enhanced economics and safety of nuclear reactors: An evaluation of the potential features, issues, and research gaps. *Nucl. Technol.* **2008**, *162*, 80–91. [[CrossRef](#)]
- Li, Z.; Sheikholeslami, M.; Jafaryar, M.; Shafee, A.; Chamkha, A.J. Investigation of nanofluid entropy generation in a heat exchanger with helical twisted tapes. *J. Mol. Liq.* **2018**, *266*, 797–805. [[CrossRef](#)]
- Sakhri, N.; Menni, Y.; Chamkha, A.J.; Lorenzini, E.; Kaid, N.; Ameer, H.; Bensafi, M. Study of heat and mass transfer through an earth to air heat exchanger equipped with fan in South West of Algeria. *Int. J. Heat Technol.* **2019**, *37*, 689–695. [[CrossRef](#)]
- Chamkha, A.J. Unsteady laminar hydromagnetic flow and heat transfer in porous channels with temperature-dependent properties. *Int. J. Numer. Methods Heat Fluid Flow* **2001**, *11*, 430–448. [[CrossRef](#)]
- Chamkha, A.J. Flow of two-immiscible fluids in porous and nonporous channels. *J. Fluids Eng.* **2000**, *122*, 117–124. [[CrossRef](#)]
- Chamkha, A.J. Non-Darcy fully developed mixed convection in a porous medium channel with heat generation/absorption and hydromagnetic effects. *Numer. Heat Transf. Part A Appl.* **1997**, *32*, 653–675. [[CrossRef](#)]
- Umavathi, J.C.; Chamkha, A.J.; Mateen, A.; Al-Mudhaf, A. Unsteady oscillatory flow and heat transfer in a horizontal composite porous medium channel. *Nonlinear Anal. Model. Control* **2009**, *14*, 397–415. [[CrossRef](#)]
- Sohail, M.; Naz, R. Modified heat and mass transmission models in the magnetohydrodynamic flow of Sutterby nanofluid in stretching cylinder. *Phys. A Stat. Mech. Its Appl.* **2020**, *5549*, 124088. [[CrossRef](#)]
- Naz, R.; Tariq, S.; Sohail, M.; Shah, Z. Investigation of entropy generation in stratified MHD Carreau nanofluid with gyrotactic microorganisms under Von Neumann similarity transformations. *Eur. Phys. J. Plus* **2020**, *135*, 178. [[CrossRef](#)]
- Khan, M.I.; Hayat, T.; Alsaedi, A.; Qayyum, S.; Tamoore, M. Entropy optimization and quartic autocatalysis in MHD chemically reactive stagnation point flow of Sisko nanomaterial. *Int. J. Heat Mass Transf.* **2018**, *127*, 829–837. [[CrossRef](#)]
- Ghadikolaie, S.S.; Hosseinzadeh, K.; Hatami, M.; Ganji, D.D. MHD boundary layer analysis for micropolar dusty fluid containing Hybrid nanoparticles (Cu-Al₂O₃) over a porous medium. *J. Mol. Liq.* **2018**, *268*, 813–823. [[CrossRef](#)]
- Nadeem, S.; Hussain, S.T. Analysis of MHD Williamson Nano Fluid Flow over a Heated Surface. *J. Appl. Fluid Mech.* **2016**, *9*, 729–739. [[CrossRef](#)]
- Khanafer, K.; Vafai, K.; Lightstone, M. Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. *Int. J. Heat Mass Transf.* **2003**, *46*, 3639–3653. [[CrossRef](#)]
- Oztop, H.F.; Abu-Nada, E. Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. *Int. J. Heat Fluid Flow* **2008**, *29*, 1326–1336. [[CrossRef](#)]
- Hamad, M.A.A. Analytical solution of natural convection flow of a nanofluid over a linearly stretching sheet in the presence of magnetic field. *Int. Commun. Heat Mass Transf.* **2011**, *38*, 487–492. [[CrossRef](#)]
- Heris, S.Z.; Etemad, S.G.; Esfahany, M.N. Experimental investigation of oxide nanofluids laminar flow convective heat transfer. *Int. Commun. Heat Mass Transf.* **2006**, *33*, 529–535. [[CrossRef](#)]

30. Shah, Z.; Dawar, A.; Kumam, P.; Khan, W.; Islam, S. Impact of nonlinear thermal radiation on MHD nanofluid thin film flow over a horizontally rotating disk. *Appl. Sci.* **2019**, *9*, 1533. [\[CrossRef\]](#)
31. Wang, C.Y. Flow due to a stretching boundary with partial slip—an exact solution of the Navier–Stokes equations. *Chem. Eng. Sci.* **2002**, *57*, 3745–3747. [\[CrossRef\]](#)
32. Andersson, H.I. Slip flow past a stretching surface. *Acta Mech.* **2002**, *158*, 121–125. [\[CrossRef\]](#)
33. Ariel, P.D.; Hayat, T.; Asghar, S. The flow of an elastico-viscous fluid past a stretching sheet with partial slip. *Acta Mech.* **2006**, *187*, 29–35. [\[CrossRef\]](#)
34. Ariel, P.D. Two dimensional stagnation point flow of an elastico-viscous fluid with partial slip. *ZAMM-J. Appl. Math. Mech./Z. Angew. Math. Mech. Appl. Math. Mech.* **2008**, *88*, 320–324. [\[CrossRef\]](#)
35. Yang, F. Slip boundary condition for viscous flow over solid surfaces. *Chem. Eng. Commun.* **2009**, *197*, 544–550. [\[CrossRef\]](#)
36. Abbas, Z.; Wang, Y.; Hayat, T.; Oberlack, M. Slip effects and heat transfer analysis in a viscous fluid over an oscillatory stretching surface. *Int. J. Numer. Methods Fluids* **2009**, *59*, 443–458. [\[CrossRef\]](#)
37. Mukhopadhyay, S. Analysis of boundary layer flow over a porous nonlinearly stretching sheet with partial slip at the boundary. *Alex. Eng. J.* **2013**, *52*, 563–569. [\[CrossRef\]](#)
38. Noghrehabadi, A.; Pourrajab, R.; Ghalambaz, M. Effect of partial slip boundary condition on the flow and heat transfer of nanofluids past stretching sheet prescribed constant wall temperature. *Int. J. Therm. Sci.* **2012**, *54*, 253–261. [\[CrossRef\]](#)
39. Raisi, A.; Ghasemi, B.; Aminossadati, S.M. A numerical study on the forced convection of laminar nanofluid in a microchannel with both slip and no-slip conditions. *Numer. Heat Transf. Part A Appl.* **2011**, *59*, 114–129. [\[CrossRef\]](#)
40. Kho, Y.B.; Hussanan, A.; Mohamed, M.K.A.; Salleh, M.Z. Heat and mass transfer analysis on flow of Williamson nanofluid with thermal and velocity slips: Buongiorno model. *Propuls. Power Res.* **2019**, *8*, 243–252. [\[CrossRef\]](#)
41. Prasad, P.D.; Kumar, R.K.; Varma, S.V.K. Heat and mass transfer analysis for the MHD flow of nanofluid with radiation absorption. *Ain Shams Eng. J.* **2018**, *9*, 801–813. [\[CrossRef\]](#)
42. Zangoee, M.R.; Hosseinzadeh, K.; Ganji, D.D. Hydrothermal analysis of MHD nanofluid (TiO₂-GO) flow between two radiative stretchable rotating disks using AGM. *Case Stud. Therm. Eng.* **2019**, *14*, 100460. [\[CrossRef\]](#)
43. Islam, M.R.; Shabani, B.; Rosengarten, G. Electrical and thermal conductivities of 50/50 water-ethylene glycol based TiO₂ nanofluids to be used as coolants in PEM fuel cells. *Energy Procedia* **2017**, *110*, 101–108. [\[CrossRef\]](#)
44. Bobbo, S.; Fedele, L.; Benetti, A.; Colla, L.; Fabrizio, M.; Pagura, C.; Barison, S. Viscosity of water based SWCNH and TiO₂ nanofluids. *Exp. Therm. Fluid Sci.* **2012**, *36*, 65–71. [\[CrossRef\]](#)
45. Fedele, L.; Colla, L.; Bobbo, S. Viscosity and thermal conductivity measurements of water-based nanofluids containing titanium oxide nanoparticles. *Int. J. Refrig.* **2012**, *35*, 1359–1366. [\[CrossRef\]](#)
46. Fakour, M.; Rahbari, A.; Khodabandeh, E.; Ganji, D.D. Nanofluid thin film flow and heat transfer over an unsteady stretching elastic sheet by LSM. *J. Mech. Sci. Technol.* **2018**, *32*, 177–183. [\[CrossRef\]](#)
47. Shahzad, A.; Gulistan, U.; Ali, R.; Iqbal, A.; Benim, A.C.; Kamran, M.; Khan, S.U.D.; Khan, S.U.D.; Farooq, A. Numerical Study of Axisymmetric Flow and Heat Transfer in a Liquid Film over an Unsteady Radially Stretching Surface. *Math. Probl. Eng.* **2020**, *2020*, 6737243. [\[CrossRef\]](#)
48. Ahmed, J.; Shahzad, A.; Farooq, A.; Kamran, M.; Khan, S.U.D.; Khan, S.U.D. Radiative heat transfer in Homann stagnation-point flow of hybrid nanofluid. *Appl. Nanosci.* **2020**, *10*, 5305–5314. [\[CrossRef\]](#)
49. Bibi, S.; Elahi, Z.; Shahzad, A. Impacts of different shapes of nanoparticles on SiO₂ nanofluid flow and heat transfer in a liquid film over a stretching sheet. *Phys. Scr.* **2020**, *95*, 115217. [\[CrossRef\]](#)
50. Sikdar, S.; Basu, S.; Ganguly, S. Investigation of electrical conductivity of titanium dioxide nanofluids. *Int. J. Nanopart.* **2011**, *4*, 336–349. [\[CrossRef\]](#)
51. Pushpa, B.V.; Sankar, M.; Mebarek-Oudina, F. Buoyant Convective Flow and Heat Dissipation of Cu–H₂O Nanoliquids in an Annulus through a Thin Baffle. *J. Nanofluids* **2021**, *10*, 292–304. [\[CrossRef\]](#)
52. Marzougui, S.; Mebarek-Oudina, F.; Magherbi, M.; Mchirgui, A. Entropy generation and heat transport of Cu–water nanoliquid in porous lid-driven cavity through magnetic field. *Int. J. Numer. Methods Heat Fluid Flow* **2021**, ahead-of-print. [\[CrossRef\]](#)
53. Shafiq, A.; Mebarek-Oudina, F.; Sindhu, T.N.; Abidi, A. A study of dual stratification on stagnation point Walters’ B nanofluid flow via radiative Riga plate: A statistical approach. *Eur. Phys. J. Plus* **2021**, *136*, 407. [\[CrossRef\]](#)
54. Swain, K.; Mebarek-Oudina, F.; Abo-Dahab, S.M. Influence of MWCNT/Fe₃O₄ hybrid nanoparticles on an exponentially porous shrinking sheet with chemical reaction and slip boundary conditions. *J. Therm. Anal. Calorim.* **2021**, 1–10. [\[CrossRef\]](#)
55. Mebarek-Oudina, F.; Fares, R.; Aissa, A.; Lewis, R.W.; Abu-Hamdeh, N.H. Entropy and convection effect on magnetized hybrid nano-liquid flow inside a trapezoidal cavity with zigzagged wall. *Int. Commun. Heat Mass Transf.* **2021**, *125*, 105279. [\[CrossRef\]](#)
56. Mebarek-Oudina, F.; Reddy, N.K.; Sankar, M. Heat source location effects on buoyant convection of nanofluids in an annulus. In *Advances in Fluid Dynamics*; Springer: Singapore, 2021; pp. 923–937.
57. Sohail, M.; Nazir, U.; Bazighifan, O.; El-Nabulsi, R.A.; Selim, M.M.; Alrabaiah, H.; Thounthong, P. Significant Involvement of Double Diffusion Theories on Viscoelastic Fluid Comprising Variable Thermophysical Properties. *Micromachines* **2021**, *12*, 951. [\[CrossRef\]](#) [\[PubMed\]](#)
58. Nazir, U.; Sohail, M.; Hafeez, M.B.; Krawczuk, M. Significant Production of Thermal Energy in Partially Ionized Hyperbolic Tangent Material Based on Ternary Hybrid Nanomaterials. *Energies* **2021**, *14*, 6911. [\[CrossRef\]](#)

59. Nazir, U.; Sohail, M.; Selim, M.M.; Alrabaiah, H.; Kumam, P. Finite element simulations of hybrid nano-Carreau Yasuda fluid with hall and ion slip forces over rotating heated porous cone. *Sci. Rep.* **2021**, *11*, 19604. [[CrossRef](#)] [[PubMed](#)]
60. Vanaki, S.M.; Mohammed, H.A.; Abdollahi, A.; Wahid, M.A. Effect of nanoparticle shapes on the heat transfer enhancement in a wavy channel with different phase shifts. *J. Mol. Liq.* **2014**, *196*, 32–42. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.