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Numerical Calculation Scheme of Neutronics-Thermal-Mechanical Coupling in Solid State Reactor Core Based on Galerkin Finite Element Method

Baoxin Yuan *, Jie Zheng, Jian Wang, Herong Zeng, Wankui Yang, Huan Huang and Songbao Zhang

China Academy of Engineering Physics, Institute of Nuclear Physics and Chemistry, Mianyang 621900, China

* Correspondence: bxyuan@caep.cn

Abstract: It is of practical significance to study the multi-physical processes of solid state nuclear systems for device design, safety analysis, and operation guidance. This system generally includes three multi-physical processes: neutronics, heat transfer, and thermoelasticity. In order to analyze the multi-physical field behavior of solid state nuclear system, it is necessary to analyze the laws of neutron flux, temperature, stress, and other physical fields in the system. Aiming at this scientific goal, this paper has carried out three aspects of work: (1) Based on Galerkin's finite element theory, the governing equations of neutronics, heat transfer, and thermoelasticity have been established; (2) a neutronics-thermal-mechanical multi-physical finite element analysis code was developed and verified based on benchmark examples and third-party software for multi-physical processes; (3) for a solid state nuclear system with a typical heat pipe cooled reactor configuration, based on the analysis code developed in this work, the neutronics-thermal-mechanical coupling analysis was carried out, and the physical field laws such as neutron flux, temperature, stress, etc., of the device under the steady-state operating conditions were obtained; and (4) finally, the calculation results are discussed and analyzed, and the focus and direction of the next work are clarified.

Keywords: finite element method; multiple physical; neutronics; heat transfer; thermoelasticity



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1. Introduction

It is of practical significance to study the multi-physical processes of solid state nuclear systems for device design, safety analysis, and operation guidance. The multi-physical processes include the transients of neutronics, heat transfer, and thermoelasticity. A fission reaction is the heat source in the system, and the heat source will affect the temperature field, which will lead to stress and strain, and the strain will in turn affect the neutron field. In order to analyze the multi-physical field behavior of a solid state nuclear system, it is necessary to analyze the laws of physical fields such as neutron flux, temperature, and stress in the system.

Fiorina, C et al. [1] reported the work of a time-dependent neutron thermomechanical coupling solver and the simulation calculation of prompt critical pulse in Godiva device on ICONE2014. The report pointed out that PSI developed a brand-new multi-physics computing platform for reactor analysis based on OpenFOAM, which used a finite volume grid that was aimed at the tight coupling calculation among neutronics, thermodynamics, and fluid dynamics. In the first stage, a steady 3D discrete ordinate/thermal solver was developed. In the second stage, the solver was upgraded to a time-dependent version. During the testing process, firstly, the step reactivity of an infinite plate critical configuration with highly enriched uranium was simulated, and then the ultra-prompt critical power pulse of the Godiva device was simulated. The power equivalent of the device in a super-prompt critical transient state was 10 GW, and the temperature rise that was caused by the transient state led to the expansion of the sphere, which finally led to the decrease of reactivity. The whole transient process was in the millisecond order. The calculated

results of the neutronics-thermal-mechanical coupling solver were in good agreement with the experimental results. Ma, Y et al. [2–6] designed the neutron-heat-force coupling scheme of a heat pipe cooled reactor based on RMC and ANSYS. RMC was a neutron solver based on the Monte Carlo method that was developed by Tsinghua University. Using this solver, the neutron calculation of a heat pipe cooled reactor was carried out, and the heat source distribution in the core was obtained. The RMC calculation results were transmitted to ANSYS through APDL language. After that, the thermodynamic calculation was completed in the thermodynamic module of ANSYS, which integrated the MC code and FEM code to form a set of neutron-thermal-force coupling analysis schemes for a solid nuclear system. Guo, Y et al. [7–9] conducted a lot of research on the technical system of the heat pipe cooled reactor. They proposed a transient multi-physical analysis model of the heat pipe cooled reactor based on OpenFOAM, conducted a numerical analysis of the startup process of the high-temperature heat pipe reactor, and also studied the heat pipe itself. Li, J et al. [10] proposed a design scheme of a multi-purpose heat pipe cooled microreactor with a concentration rate of 19.75%, which can be operated for more than 5 years without refueling at a power of 2 MW. Liu, L et al. [11] proposed a heat pipe cooled nuclear reactor (HPR) which has great potential in underwater application. Aiming at the conceptual design scheme of the nuclear silent reactor, the thermal analysis and mechanical analysis were carried out on the three-dimensional solid model by using the finite element analysis software FLUENT and MECHANICAL. Alawneh, L.M et al. [12] proposed the conceptual design of a heat pipe cooled yttrium hydride microreactor. The neutronics calculation was carried out by using Monte Carlo code MCNP6.2 and Serpent code, and the thermal hydraulic analysis and calculation were carried out by using STAR CCM+ multi-physical computational fluid dynamics (CFD) software. The analysis showed that the reactor can operate safely for more than 11 years and produce 3 MW thermal power. Li, S.N et al. [13] proposed a medium temperature heat pipe cooled reactor (MHR) with extremely high inherent safety. Mercury heat pipes are used to cool solid reactors using uranium zirconium hydride fuel rods, which results in limited core temperature and low requirements for high temperature materials. Krecicki, M et al. [14] aimed at the neutron-thermal coupling problem of nuclear thermopropulsion reactor and developed a neutron-thermal hydraulics-thermoelasticity calculation scheme for the whole core coupling of a low enriched uranium nuclear thermopropulsion reactor. This work showed in detail how the thermoelasticity module was added to the existing NTP thermal analysis code to solve the thermoelasticity equation. Based on the Basilisk multi-physics framework, the whole reactor coupling calculation was realized, and the thermodynamic feedback was given quantitatively to reduce the fuel temperature and the pressure drop in the flow channel. This work provided a multi-physics solution for the future nuclear propulsion system.

In view of the fact that an increasing number of commercial CAE analysis software can be used to analyze heat transfer and thermoelastic problems, there is no essential difficulty in designing numerical schemes for neutron-thermal-mechanical coupling problems based on finite difference, finite volume, and meshless methods, but in order to facilitate the connection and mutual verification between neutronics solver and commercial CAE software. This work carries out the numerical scheme of neutron-thermal-mechanical coupling based on the Galerkin finite element method. The Galerkin finite element method is one of many finite element methods and its main idea is to seek the solution of a mathematical physics problem through the weighted residual method; interested readers can consult the relevant textbooks of computational mathematics. In addition, there are excellent cases for reference in neutronics calculation based on the finite element method (Kong, B et al. [15,16], Hosseini, S.A et al. [17–19], Zhang, C et al. [20]). On the basis of the existing framework of FEMN code (Yuan, B et al. [21]), the work mainly includes the following contents:

1. Galerkin finite element models of neutronics, thermal, and thermoelasticity;
2. Test and verification of neutronics model;
3. Test and verification of thermal model;
4. Test and verification of thermoelasticity model;

5. A coupling calculation example of 2D heat pipe cooled reactor analysis;
6. Conclusion and prospect.

2. Basic Theory

2.1. Equations of Neutronics

Using the improved quasi-static calculation method, the point reactor equation can be obtained as follows:

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \bar{\beta}(t)}{\Lambda(t)} n(t) + \sum_{l=1}^N \lambda_l \bar{C}_l(t) \quad (1)$$

$$\frac{d\bar{C}_l(t)}{dt} = \frac{\bar{\beta}_l(t)}{\Lambda(t)} n(t) - \lambda_l \bar{C}_l(t) \quad (2)$$

where $n(t)$ is the amplitude function, $\rho(t)$ is reactivity, $\bar{\beta}(t)$ is the effective delayed neutron fraction, $\Lambda(t)$ is the neutron generation time, and λ_l is the decay constant of the delayed neutron precursor of group l , and $\bar{C}_l(t)$ is the concentration of the delayed neutron precursor of group l .

Wherein the coefficient of the point reactor dynamics equation is:

$$\rho(t) = \frac{1}{FT(t)} \sum_{g=1}^G \iint_{\Omega} \varphi_g^* \left[D_g \nabla^2 \Psi_g - \sum_g^R \Psi_g + \sum_{g'=1}^{g-1} \sum_{g'-g} \Psi_{g'} + \chi_g F(\vec{r}, t) \right] dx dy \quad (3)$$

$$F(\vec{r}, t) = \sum_{g'=1}^G \left(\nu \sum_f \right)_{g'} \Psi_{g'} \quad (4)$$

$$FT(t) = \sum_{g=1}^G \iint_{\Omega} \varphi_g^* \chi_g F(\vec{r}, t) dx dy \quad (5)$$

$$\bar{\beta}_l(t) = \frac{1}{FT(t)} \sum_{g=1}^G \iint_{\Omega} \varphi_g^* \beta_l \chi_g F(\vec{r}, t) dx dy \quad (6)$$

$$\bar{\beta}(t) = \sum_{l=1}^N \bar{\beta}_l(t) \quad (7)$$

$$\Lambda(t) = \frac{1}{FT(t)} \sum_{g=1}^G \frac{1}{v_g} \iint_{\Omega} \varphi_g^* \Psi_g dx dy = \frac{1}{FT(t)} \quad (8)$$

$$\sum_{g=1}^G \frac{1}{v_g} \iint_{\Omega} \varphi_g^* \Psi_g dx dy = 1 \quad (9)$$

$$\bar{C}_l(t) = \frac{1}{\Lambda(t) \cdot FT(t)} \sum_{g=1}^G \chi_g \iint_{\Omega} \varphi_g^* C_l dx dy = \sum_{g=1}^G \chi_g \iint_{\Omega} \varphi_g^* C_l dx dy \quad (10)$$

where Ω is the calculation domain, φ_g^* is the adjoint flux of group g , D_g is the diffusion coefficient of group g , Ψ_g is the shape function of group g , $\sum_g^R$ is the total cross section of group g , $\sum_{g'-g}$ is the intergroup scattering cross section of group g' to g , χ_g is the fission share of group g , and $\left(\nu \sum_f \right)_{g'}$ is the fission cross section of group g' .

The point reactor equation is solved in small steps and the transient flux equation is solved in large steps:

$$\begin{aligned} & \iint_{\Omega} -v D_g \left(t_{j+1}^B \right) \nabla^2 \Psi_g \left(t_{j+1}^B \right) dx dy + \iint_{\Omega} v \left[\sum_g^R \left(t_{j+1}^B \right) + \frac{1}{v_g} \left(\frac{1}{\Delta t_j^B} + \frac{1}{n} \frac{\partial n}{\partial t} \left(t_{j+1}^B \right) \right) \right] \Psi_g \left(t_{j+1}^B \right) dx dy \\ &= \iint_{\Omega} v \sum_{g'=1}^{g-1} \sum_{g' \rightarrow g} \left(t_{j+1}^B \right) \Psi_{g'} \left(t_{j+1}^B \right) dx dy + \iint_{\Omega} v (1 - \beta) \chi_g \sum_{g'=1}^G \left(\nu \sum_f \right)_{g'} \left(t_{j+1}^B \right) \Psi_{g'} \left(t_{j+1}^B \right) dx dy \\ &+ \iint_{\Omega} v \frac{\chi_g}{n \left(t_{j+1}^B \right)} \sum_{l=1}^N \lambda_l C_l \left(t_{j+1}^B \right) dx dy + \iint_{\Omega} v \frac{1}{v_g} \frac{\Psi_g \left(t_j^B \right)}{\Delta t_j^B} dx dy \end{aligned} \quad (11)$$

where v represents the arbitrary shape function that is introduced by the Galerkin finite element method, v_g is the neutron velocity of group g , and N represents the total number of groups of delayed neutron precursor.

2.2. Equations of Heat Transfer

Heat transfer dynamics:

$$k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + q_V - \rho c \frac{\partial T}{\partial t} = 0 \quad (12)$$

where k was the thermal conductivity of the material, q_V was the heat source, and ρc is the basic thermophysical parameter.

After derivation, the final finite element equation in Galerkin form is:

$$\frac{\partial J^D}{\partial T_l} = \iint_{\Omega} k \left(\frac{\partial W_l}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial W_l}{\partial y} \frac{\partial T}{\partial y} \right) dx dy - \oint_{\Gamma} k W_l \frac{\partial T}{\partial n} ds - \iint_{\Omega} W_l q_V dx dy + \iint_{\Omega} W_l \rho c \frac{\partial T}{\partial t} dx dy \quad (13)$$

where W_l represents the arbitrary shape function that is introduced by the Galerkin finite element method.

The transient temperature field was solved by the backward difference scheme:

$$\left\{ \frac{\partial T}{\partial t} \right\}_t = \frac{1}{\Delta t} (\{T\}_t - \{T\}_{t-\Delta t}) + O(\Delta t) \quad (14)$$

The discrete form of the finite element equation at time t :

$$[K] \{T\}_t + [N] \left\{ \frac{\partial T}{\partial t} \right\}_t = \{P\}_t \quad (15)$$

The format for easier solution could be written as:

$$[K] \{T\}_t + \frac{[N]}{\Delta t} \{T\}_t - \{P\}_t = \frac{[N]}{\Delta t} \{T\}_{t-\Delta t} \quad (16)$$

$$\left([K] + \frac{[N]}{\Delta t} \right) \{T\}_t = \{P\}_t + \frac{[N]}{\Delta t} \{T\}_{t-\Delta t} \quad (17)$$

where $\{T\}$ is the temperature matrix, $[K]$ is the thermal conductivity coefficient matrix, $[N]$ is the time term coefficient matrix, $\{P\}$ is the heat source coefficient matrix, O is the error matrix, and Δt is the time step.

2.3. Equations of Thermoelasticity

The basic elasticity equation of plane thermal stress problem:

$$\begin{cases} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + X = \rho \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + Y = \rho \frac{\partial^2 v}{\partial t^2} \end{cases} \quad (18)$$

where σ_x and σ_y are normal stresses, τ_{xy} and τ_{yx} are shear stresses, X and Y are volume forces, and u and v are deformations. Considering the influence of the temperature field on the strain after heating, the first law of thermodynamics can be expressed as follows:

$$\frac{\partial T}{\partial t} = a \nabla^2 T - \frac{\beta T_0}{\rho c} \frac{\partial e}{\partial t} \quad (19)$$

where T_0 is the initial temperature of the object, $e = \varepsilon_x + \varepsilon_y$ is the total strain, $\beta = \frac{aE}{1-2\mu}$ is a physical parameter composed of linear expansion coefficient a , and the tension-compression elastic modulus E and Poisson coefficient μ .

In general, the term $\frac{\beta T_0}{\rho c} \frac{\partial e}{\partial t}$ can be omitted, and this value is meaningful only in the case of high-speed thermal shock. Therefore, for low speed dynamic thermal problems, the heat conduction equation of elastomers can be reduced to:

$$\frac{\partial T}{\partial t} = a \nabla^2 T \quad (20)$$

This equation was the transient heat transfer equation that was described in Section 2.2, which did not include displacement and could be solved separately. For the basic elasticity equation of the plane thermal stress problem, the acceleration term was again ignored, and the quasi-static problems were as follows:

$$\begin{cases} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + X = 0 \\ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + Y = 0 \end{cases} \quad (21)$$

The Galerkin finite element equation of thermoelasticity of the external element written as only including node displacement:

$$\begin{cases} \frac{\partial I^D}{\partial u_1} = \iint_{\Omega} \left\{ \frac{E}{1-\mu^2} \left[\left(\frac{\partial u}{\partial x} + \mu \frac{\partial v}{\partial y} \right) - (1+\mu) \alpha \Delta T \right] \frac{\partial W_1}{\partial x} + \frac{E}{2(1+\mu)} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \frac{\partial W_1}{\partial y} - X W_1 \right\} dx dy - \oint_{\Gamma} Q_x W_1 ds \\ \frac{\partial I^D}{\partial v_1} = \iint_{\Omega} \left\{ \frac{E}{1-\mu^2} \left[\left(\frac{\partial v}{\partial y} + \mu \frac{\partial u}{\partial x} \right) - (1+\mu) \alpha \Delta T \right] \frac{\partial W_1}{\partial y} + \frac{E}{2(1+\mu)} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \frac{\partial W_1}{\partial x} - Y W_1 \right\} dx dy - \oint_{\Gamma} Q_y W_1 ds \end{cases} \quad (22)$$

3. Neutronics-Thermal-Mechanical Calculation Verification

Based on the mathematical model in Section 2, the corresponding modules of neutronics, heat transfer, and thermoelasticity are developed in FEMN. Together with the FEMN code framework, the above modules form a multi-physical coupling numerical calculation scheme for heat pipe cooled reactors. The specific block diagram is shown in Figure 1. From the block diagram, it can be seen that the extended FEMN mainly includes three calculation modules: neutronics, heat transfer, and thermoelasticity. The calculation accuracy and error band of these three modules will directly determine the computing power of the extended FEMN and affect the accuracy of the two-dimensional calculation of the solid core of the heat pipe cooled reactor. Therefore, this section will gradually use benchmark examples, commercial software, and analytical solutions to calculate and verify the three modules.

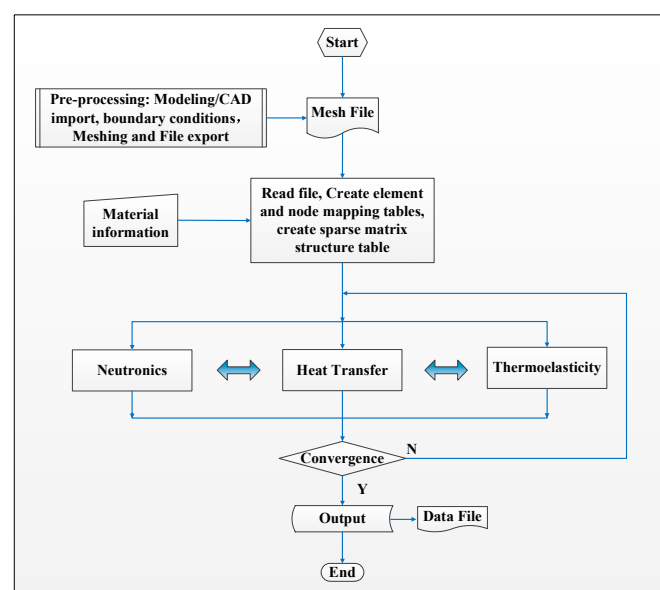


Figure 1. Numerical calculation scheme of neutronics-thermal-mechanical coupling.

3.1. Neutronics Calculation Verification

(1) Steady-state problem of neutronics

The IAEA 2D PWR benchmark example is selected for the steady-state calculation test of the neutronics module in FEMN. The geometric layout and group constants of this benchmark can be referred to in [22]. The grid division is shown in Figure 2. The calculation results of k_{eff} calculated by neutronics module of FEMN code are shown in Table 1, and the calculation results of power distribution are shown in Figure 3.

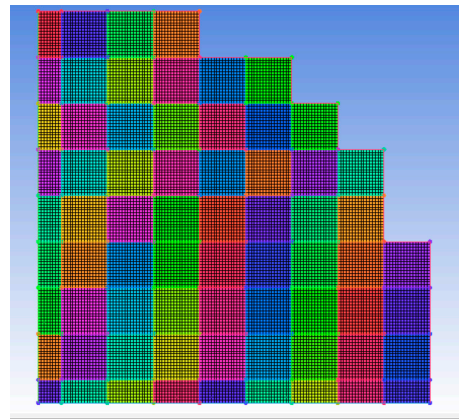


Figure 2. The mesh of benchmark IAEA 2D PWR.

Table 1. The k_{eff} calculation result of IAEA-2D PWR.

Item				Reference Value/Calculated Value			
k_{eff}				1.02901/1.02955			
0.7456	1.3097	1.4537	1.2107	0.6100	0.9351	0.9343	0.7549
0.7451	1.3074	1.4517	1.2091	0.6102	0.9349	0.9351	0.7567
−0.07%	−0.18%	−0.13%	−0.13%	0.03%	−0.03%	0.08%	0.24%
1.3097	1.4351	1.4799	1.3149	1.0697	1.0361	0.9504	0.7358
1.3074	1.4329	1.4779	1.3134	1.0688	1.0361	0.9512	0.7376
−0.18%	−0.15%	−0.13%	−0.11%	−0.09%	0.00%	0.08%	0.24%
1.4537	1.4799	1.4694	1.3451	1.1792	1.0705	0.9752	0.6921
1.4517	1.4779	1.4677	1.3438	1.1787	1.0707	0.9761	0.6946
−0.13%	−0.13%	−0.12%	−0.09%	−0.04%	0.02%	0.09%	0.37%
1.2107	1.3149	1.3451	1.1929	0.9670	0.9064	0.8461	
1.2091	1.3134	1.3438	1.1920	0.9666	0.9069	0.8482	
−0.13%	−0.11%	−0.09%	−0.08%	−0.04%	0.05%	0.25%	
0.6100	1.0697	1.1792	0.9670	0.4706	0.6856	0.5972	
0.6102	1.0688	1.1787	0.9666	0.4712	0.6861	0.5993	
0.03%	−0.09%	−0.04%	−0.04%	0.12%	0.07%	0.36%	
0.9351	1.0361	1.0705	0.9064	0.6856	0.5849		
0.9349	1.0361	1.0707	0.9069	0.6861	0.5870		
−0.03%	0.00%	0.02%	0.05%	0.07%	0.36%		
0.9343	0.9504	0.9752	0.8461	0.5972			
0.9351	0.9512	0.9761	0.8482	0.5993			
0.08%	0.08%	0.09%	0.25%	0.36%			
0.7549	0.7358	0.6921	−R				
0.7567	0.7376	0.6946	−C				
0.24%	0.24%	0.37%	−Error%				

Figure 3. Calculation result of power distribution of IAEA-2D PWR benchmark example.

It can be seen from the calculation results in Table 1 that the calculated value of k_{eff} is very close to the reference value, with an error of only 44 pcm. According to the calculation results of power distribution in Figure 3, the calculation error of the neutron flux is controlled within 0.5%.

(2) Transient heat conduction neutronics

The benchmark TWIGL was a two-dimensional and two-group transient problem based on diffusion theory, which was used to simulate an ignition breeder reactor with a side length of 165 cm and no reflector. The geometry, cross-section parameters, and delayed neutron parameters of this benchmark example can be referred to in [23]. It contained two independent transient problems, one of which was linear perturbation problem and the other was step perturbation problem. For the linear perturbation problem, the linear perturbation begins at zero time, and the Σ_{a2} in region 1 is linearly reduced by 2.333% in the time from $t = 0.0$ to $t = 0.2$ s to introduce a perturbation. For the step perturbation problem, the absorption cross section of the second group of region 1 is reduced by 0.0035 at zero time to introduce a step change.

The benchmark is meshed by commercial CAD pre-processing software ICEM-CFD, which is shown in Figure 4. The neutron-thermal-thermoelastic coupling scheme involves three calculation steps that are described in Section 2.1 to Section 2.3, all of which use the same set of grids. A neutronics calculation of this benchmark was carried out by using the multi-physical analysis code that was developed in this work, with a fixed step size of 0.0001 s. For linear perturbation and step perturbation, the variation characteristics of relative power calculated with time are shown in Figure 5. It could be seen that the calculation results were in good agreement with those that were given by public codes such as TDTORT, CONQUEST, DOT4-T, etc. given in the reference.

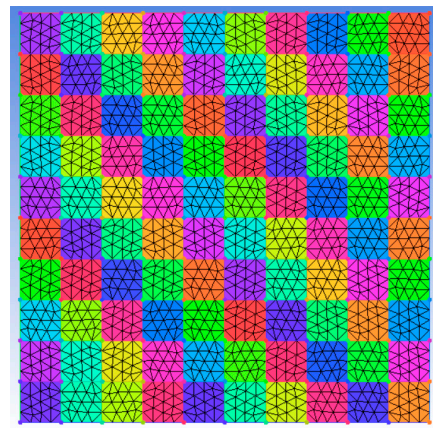


Figure 4. The mesh of benchmark TWIGL.

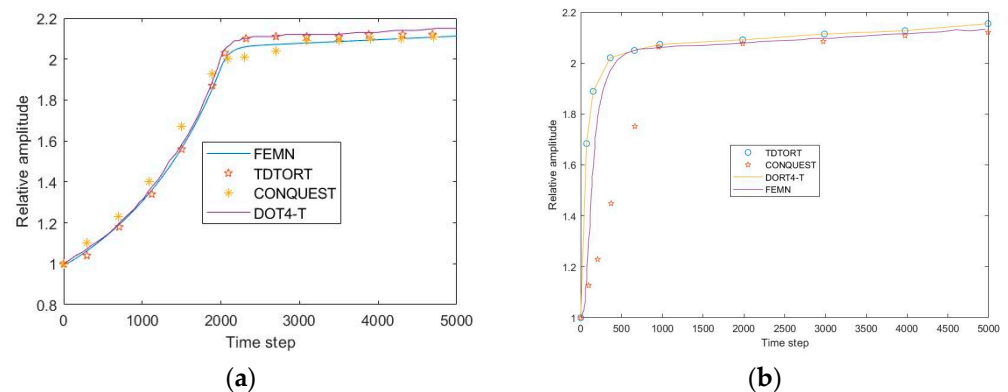


Figure 5. Neutronics calculation results of benchmark TWIGL ((a) linear perturbation and (b) step perturbation).

3.2. Heat Transfer Calculation Verification

(1) Steady-state problem of uniform internal heat source

As shown in Figure 6, in the rectangular region ($0 < x < a, 0 < y < b$), there is a heat source with the intensity $g(x, y) = g_0$:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{g_0}{k} = 0 \quad (0 < x < a, 0 < y < b) \quad (23)$$

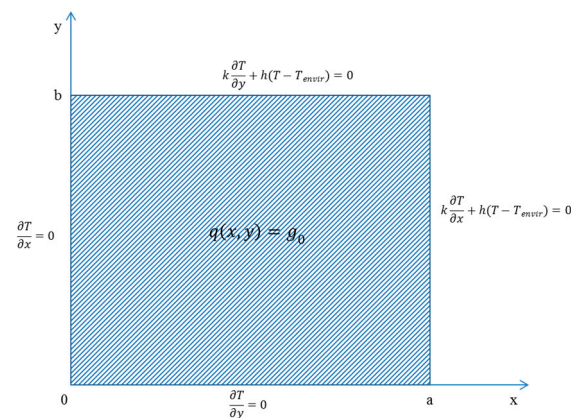


Figure 6. Heat conduction problem with uniform internal heat source.

The boundary conditions:

$$\begin{cases} \frac{\partial T}{\partial x} = 0, & x = 0 \\ \frac{\partial T}{\partial y} = 0, & y = 0 \end{cases} \quad (24)$$

$$\begin{cases} k \frac{\partial T}{\partial x} + h(T - T_{envir}) = 0, & x = a \\ k \frac{\partial T}{\partial y} + h(T - T_{envir}) = 0, & y = b \end{cases} \quad (25)$$

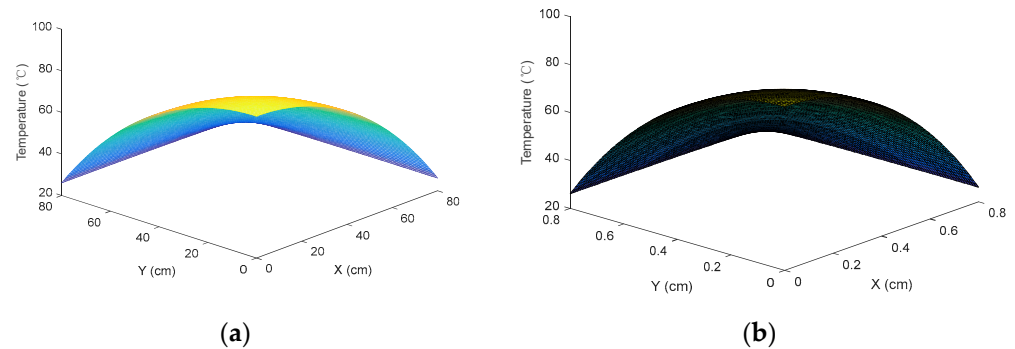
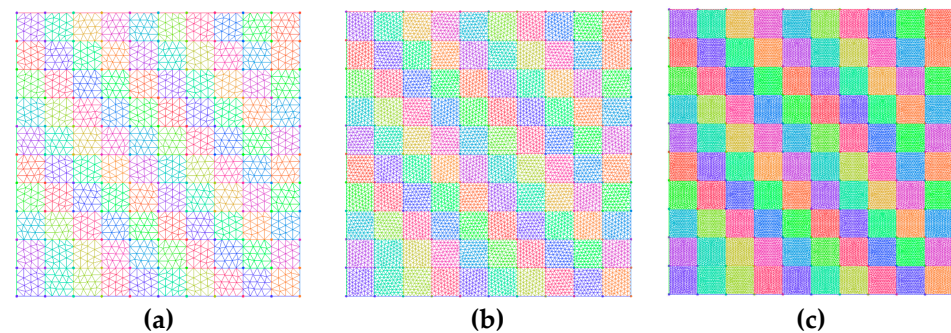
After derivation, the analytical solution of this kind of problem can be obtained as follows:

$$T = \sum_{m=1}^{\infty} \frac{-\frac{g_0 h^2}{k^3 \beta_m^4} \cos(\beta_m a)}{N(\beta_m) [\beta_m \sinh(\beta_m b) + H \cosh(\beta_m b)]} \cos(\beta_m x) \cosh(\beta_m y) - \frac{g_0 x^2}{2k} + \frac{g_0 a^2}{2k} + \frac{g_0 a}{Hk} + T_{envir} \quad (26)$$

For the steady-state problem with a uniform internal heat source, the input parameters are shown in Table 2, the steady-state temperature field that was calculated by the steady-state thermal module of FEMN code is shown in Figure 7a, and the steady-state temperature field that was obtained by analytical solution according to formula (26) is shown in Figure 7b. From the qualitative comparison of Figure 7a,b, the calculated results of FEMN tend to be consistent with the analytical solutions for the steady-state heat conduction of uniformly distributed internal heat sources. For further quantitative comparison, as shown in Figure 8a–c, the grid number was increased from 3200 to 56600, and the maximum and minimum temperature of the steady-state temperature field are shown in Table 3. From the results, FEMN can achieve a certain calculation accuracy when the grid number is 3200, and the error between the calculated value of FEMN and the analytical solution is less than 0.5%.

Table 2. Input parameters for steady-state problem of uniform internal heat source.

Parameter		Value
1	a	0.8 m
2	b	0.8 m
3	k	60.5 W/(m·°C)
4	h	10,000 W/(m ² ·°C)
5	g ₀	0.02 W/cm ²

**Figure 7.** Heat transfer calculation results for steady-state problem of uniform internal heat source ((a) FEMN solution and (b) analytical solution).**Figure 8.** Grid refinement for the steady-state problem of uniform internal heat source ((a) mesh number (3200), (b) mesh number (13,800), and (c) mesh number (56,600)).**Table 3.** Comparison of the upper and lower limits of the temperature field.

Computing Method		Maximum Temperature/°C	Minimum Temperature/°C
1	FEMN/3200 elements	88.2596	25.1031
2	FEMN/12800 elements	88.2840	25.0714
3	FEMN/56600 elements	88.2905	25.0317
4	Analytic solution	88.2872	25.0521

(2) Steady-state problem of non-uniform internal heat source

As it is difficult to solve the steady-state heat conduction problem with a non-uniform internal heat source under the second kind of boundary conditions, the grid heat source is calculated by the neutronics module of FEMN, which is provided to the steady-state thermal module of FEMN and the workbench module of ANSYS, respectively. Finally, the steady-state temperature field that was calculated by the steady-state thermal module of FEMN and ANSYS Workbench combined with APDL parametric programming technology is shown in Figure 9a,b.

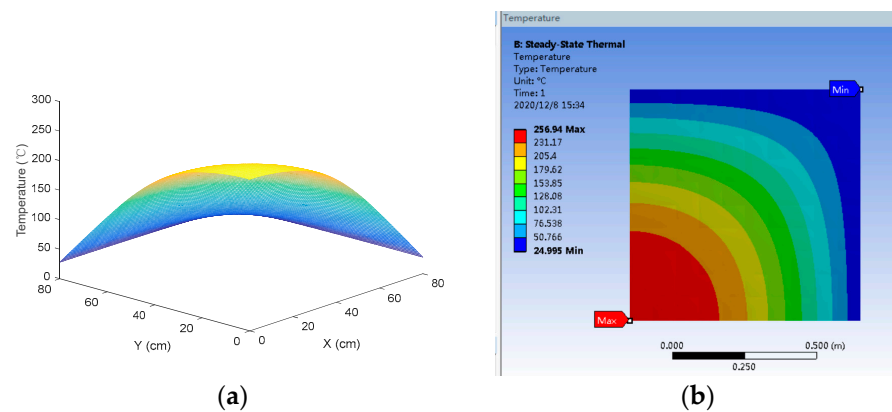


Figure 9. Heat transfer calculation results for the steady-state problem of non-uniform internal heat source ((a) FEMN solution, and (b) analytical solution).

From the qualitative comparison of Figure 9a,b for the steady-state heat conduction problem of non-uniform internal heat source, the calculation results of FEMN steady-state heat module and ANSYS workbench tend to be consistent. For further quantitative comparison, the maximum and minimum temperature of the steady-state temperature field are shown in Table 4. From the results, the calculation ability of the steady-state thermal module of FEMN is further verified, and the error between the calculated value of FEMN and ANSYS workbench is less than 0.05%.

Table 4. Comparison of the upper and lower limits of temperature field.

Computing Method		Maximum Temperature/°C	Minimum Temperature /°C
1	FEMN/3200 elements	256.940	24.990
2	ANSYS Workbench	256.940	24.995

(3) Transient heat conduction problem

The geometry of the transient heat conduction problem is shown in Figure 6, and the input parameters are shown in Table 5. The transient heat conduction is calculated by the FEMN transient thermal module and ANSYS workbench, respectively, and the final transient temperature field is shown in Figure 10a,b.

Table 5. Input parameters for the transient heat conduction problem.

Parameter		Value
1	a	0.8 m
2	b	0.8 m
3	k	$3 \cdot \text{W}/(\text{m} \cdot ^\circ\text{C})$
4	h	$10,000 \cdot \text{W}/(\text{m}^2 \cdot ^\circ\text{C})$
5	ρ	$10,900 \text{ (kg}/(\text{m}^3)\text{)}$
6	C	$328 \text{ (J}/(\text{kg} \cdot ^\circ\text{C})\text{)}$
7	g_0	$2.0 \text{ W}/\text{cm}^2$

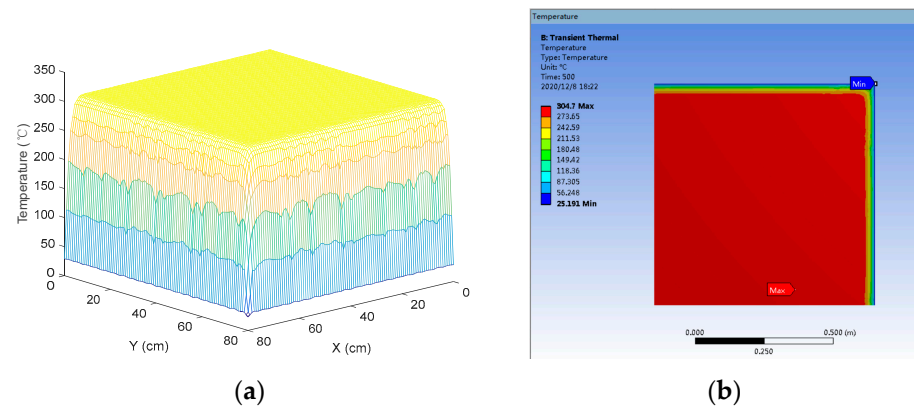


Figure 10. Heat transfer calculation results for the transient heat conduction problem ((a) FEMN solution and (b) analytical solution).

From the qualitative comparison of Figure 10a,b, the calculation results of the FEMN transient thermal module and ANSYS workbench tend to be consistent for transient heat conduction problems. For further quantitative comparison, the maximum and minimum temperature of the transient temperature field is shown in Table 6. From the results, the transient heat conduction calculation ability of the FEMN is reliable, and the error between the calculated value of FEMN and ANSYS workbench is less than 3%.

Table 6. Comparison of the upper and lower limits of the temperature field.

	Computing Method	Maximum Temperature/°C	Minimum Temperature /°C
1	FEMN/3200 elements	305.05	25.820
2	ANSYS Workbench	304.70	25.191

3.3. Thermoelasticity Calculation Verification

The two-dimensional thermoelastic problem is calculated and confirmed, the geometry of the problem is shown in Figure 6, and the input parameters are shown in Table 7. The thermoelastic calculation is carried out by the FEMN mechanics module and ANSYS workbench, respectively, and the node displacement that was obtained by the FEMN mechanics module and ANSYS workbench is shown in Figures 11 and 12.

Table 7. Input parameters for thermoelasticity problem.

	Parameter	Value
1	a	0.8 m
2	b	0.8 m
3	E	2.0×10^{11} Pa
4	μ	0.3
5	a	1.2×10^{-5}
6	Temperature of model	300 °C

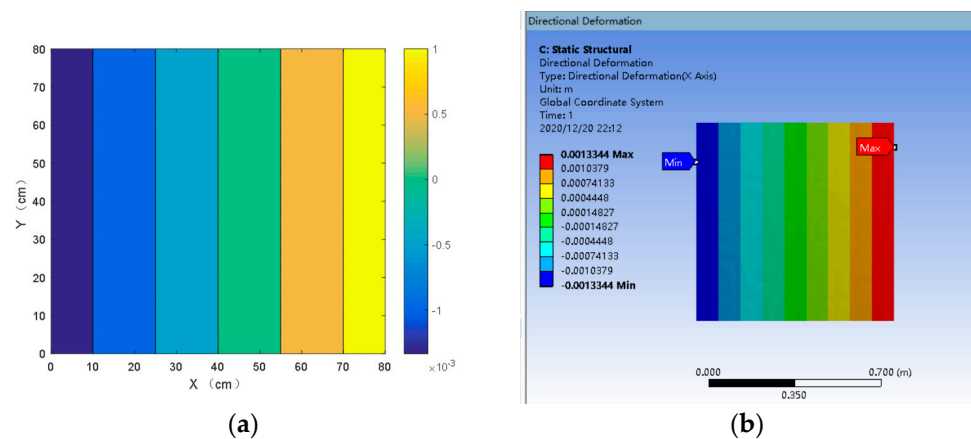


Figure 11. Thermoelasticity calculation results of x direction displacement ((a) FEMN solution and (b) analytical solution).

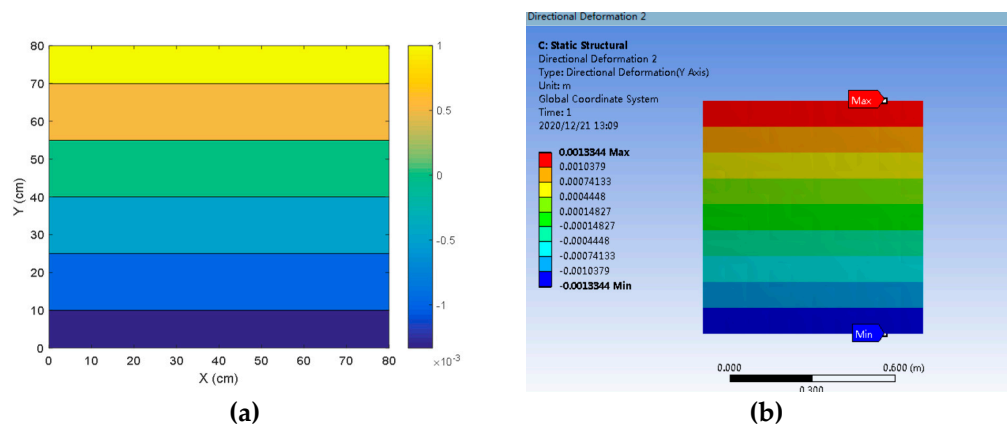


Figure 12. Thermoelasticity calculation results of y direction displacement ((a) FEMN solution and (b) analytical solution).

From the qualitative comparison of Figures 11 and 12, the calculation results of the mechanics module of FEMN and ANSYS workbench tend to be consistent for thermoelastic problems. For further quantitative comparison, the maximum positive and negative displacements of the nodes are shown in Table 8. From the results, the calculation ability of the mechanics module of the FEMN is credible, and the error between the calculated value of FEMN and ANSYS workbench is less than 0.05%.

Table 8. Comparison of maximum positive and negative displacements of the nodes.

Computing Method		Maximum Positive Displacement/m		Maximum Negative Displacement/m	
		X Direction	Y Direction	X Direction	Y Direction
1	FEMN/3200 elements	0.0013348	0.0013350	−0.0013342	−0.0013340
2	ANSYS Workbench	0.0013344	0.0013344	−0.0013344	−0.0013344

4. Coupling Calculation Example

Figure 13 shows a schematic diagram of a typical heat pipe cooled reactor for nuclear thermal propulsion, in which the assembly is designed in a hexagonal shape with a cylindrical heat pipe arranged in the center of the assembly. The seven layers of the core from the center to the outside are composed of fuel assemblies of different abundances, and the outermost layer is composed of reflector assemblies. The specific technical parameters of the core are shown in Table 9.

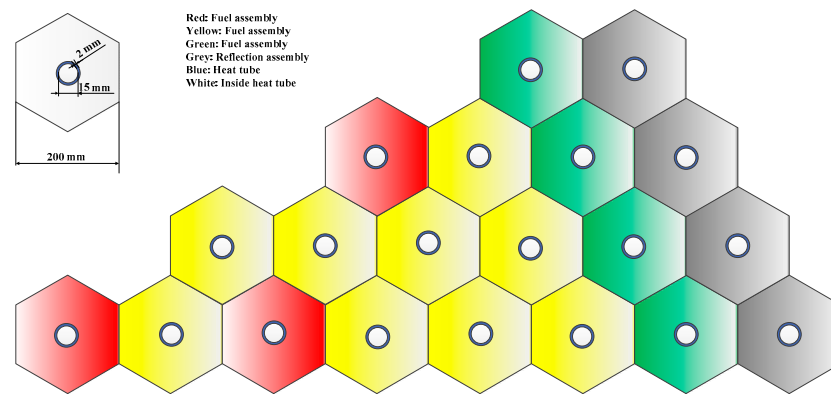


Figure 13. Heat pipe cooled reactor for nuclear thermal propulsion.

Table 9. Parameters of the heat pipe cooled reactor for nuclear thermal propulsion.

	Parameter	Value
1	Distance between fuel assemblies	200 mm
2	Inner diameter of heat pipe	30 mm
3	Wall thickness of heat pipe	2 mm
4	Fuel type	UN fuel
5	Heat pipe material	INCONEL 617
6	Number of Type I fuel assemblies	13
7	Number of Type II fuel assemblies	78
8	Number of Type III fuel assemblies	36
9	Number of reflection assemblies	42

The heat pipe cooled reactor for nuclear thermal propulsion works at steady-state rated power, and the transient problem of neutronics-thermal-thermoelasticity coupling is reduced to a static problem. After the solid core reaches the critical point, a stable distribution of neutron field is formed, which interacts with the fission nucleus to form a stable distribution of nuclear heat. The nuclear heat constitutes the source term of steady-state heat conduction and forms a stable temperature field in the core structure material. Finally, the structural material forms a stable displacement field and stress field under the action of temperature field.

Therefore, the partial derivative term of time of the transient neutronics model described in Section 2.1 can be set to 0, and the dynamics problem degenerates into a steady-state neutronics problem. According to the neutron diffusion model, the steady-state neutronics problem can be described as:

$$-\nabla \cdot D_g \nabla \phi_g + \Sigma_g^R \phi_g = \sum_{g'=1}^G \Sigma_{g' \rightarrow g} \phi_{g'} + \frac{\chi_g}{k_{eff}} \sum_{g'=1}^G \left(\nu \Sigma_f \right)_{g'} \phi_{g'} \quad (27)$$

where ϕ_g is the neutron flux of group g , D_g is the diffusion coefficient of group g , Σ_g^R is the total cross-section of group g , $\Sigma_{g' \rightarrow g}$ is the intergroup scattering cross-section of group g' to g , χ_g is the fission share of group g , $\left(\nu \Sigma_f \right)_{g'}$ is the fission cross-section of group g' , and k_{eff} is the effective multiplication coefficient.

According to the classical Galerkin finite element method, the mathematical model of the above steady-state problem can be obtained:

$$\iint_{\Omega} -v \nabla \cdot D_g \nabla \phi_g dS + \iint_{\Omega} v \Sigma_g^R \phi_g dS = \iint_{\Omega} v \sum_{g'=1}^G \Sigma_{g' \rightarrow g} \phi_{g'} dS + \iint_{\Omega} v \frac{\chi_g}{k_{eff}} \sum_{g'=1}^G \left(\nu \Sigma_f \right)_{g'} \phi_{g'} dS \quad (28)$$

where Ω is the calculation domain and v is the arbitrary shape function that is introduced by the Galerkin finite element method.

According to the core geometry given in Figure 13 and the material information given in Table 9, the group constant of the heat pipe cooling reactor is obtained by using the neutron cross-section production code and substituted into the finite element mathematical model of the steady-state problem, and the flux distribution shown in Figure 14 can be obtained by FEMN.

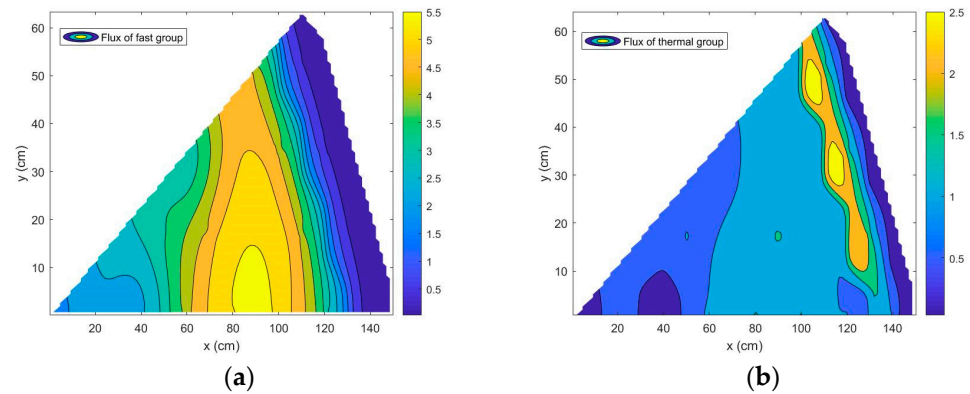


Figure 14. Relative flux distribution of 1/12 Core ((a) fast group and (b) thermal group).

It can be seen from Figure 14a,b that although the group constants of the area where the heat pipe is located are quite different from those of other types of assemblies, because the diffusion model is used and the average free path of neutrons of this reactor type is relatively long, the flux distribution of both fast group and thermal group is relatively flat, and the heat pipe layout will not lead to flux distortion.

In the same way, for the steady state problem, the partial derivative term to time contained in the temperature field equation that is described in Section 2.2 is set to 0:

$$\iint_D k \left(\frac{\partial W_l}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial W_l}{\partial y} \frac{\partial T}{\partial y} \right) dx dy - \oint_{\Gamma} k W_l \frac{\partial T}{\partial n} ds = \iint_D W_l q_v dx dy \quad (29)$$

The whole device operates steadily under the rated nuclear thermal power of 20 KW, and the grid heat generation rate that is calculated by the neutronics calculation module of the FEMN is transferred to the heat transfer calculation module. The module performs the steady-state temperature field calculation as described in the formula (24), and finally obtains the steady-state temperature under the working condition as shown in Figure 15.

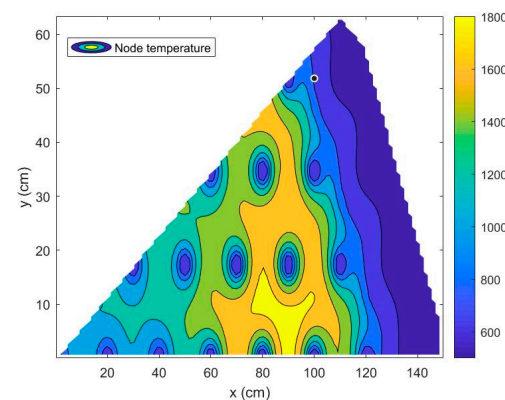


Figure 15. Temperature distribution of 1/12 core.

It can be seen from Figure 15 that due to the limitation of the working mechanism of the heat pipe, the wall temperature of the heat pipe is stable at about 750 °C, so the distribution of the temperature field is obviously different from the relatively smooth neutron field that is shown in Figure 14, and there is an obvious depression at the location

of the heat pipe. It is this depression, however, that enables this reactor to stably transfer heat and supply it to the propulsion system.

In the same way, for steady-state problems, the time partial derivative term contained in the thermoelastic equation that is described in Section 2.3 is set to 0:

$$\begin{cases} \iint_D \left\{ \frac{E}{1-\mu^2} \left[\left(\frac{\partial u}{\partial x} + \mu \frac{\partial v}{\partial y} \right) - (1+\mu)\alpha\Delta T \right] \frac{\partial W_1}{\partial x} + \frac{E}{2(1+\mu)} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \frac{\partial W_1}{\partial y} - XW_1 \right\} dx dy - \oint_{\Gamma} Q_x W_1 ds = 0 \\ \iint_D \left\{ \frac{E}{1-\mu^2} \left[\left(\frac{\partial v}{\partial y} + \mu \frac{\partial u}{\partial x} \right) - (1+\mu)\alpha\Delta T \right] \frac{\partial W_1}{\partial y} + \frac{E}{2(1+\mu)} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \frac{\partial W_1}{\partial x} - YW_1 \right\} dx dy - \oint_{\Gamma} Q_y W_1 ds = 0 \end{cases} \quad (30)$$

The whole device operates stably under the rated nuclear thermal power of 20 KW, and the grid temperature that is calculated by the heat transfer calculation module of FEMN is transferred to the thermoelastic calculation module. The module performs the thermoelastic calculation according to Formula (26), and finally obtains the node displacement and stress under this working condition as shown in Figures 16 and 17.

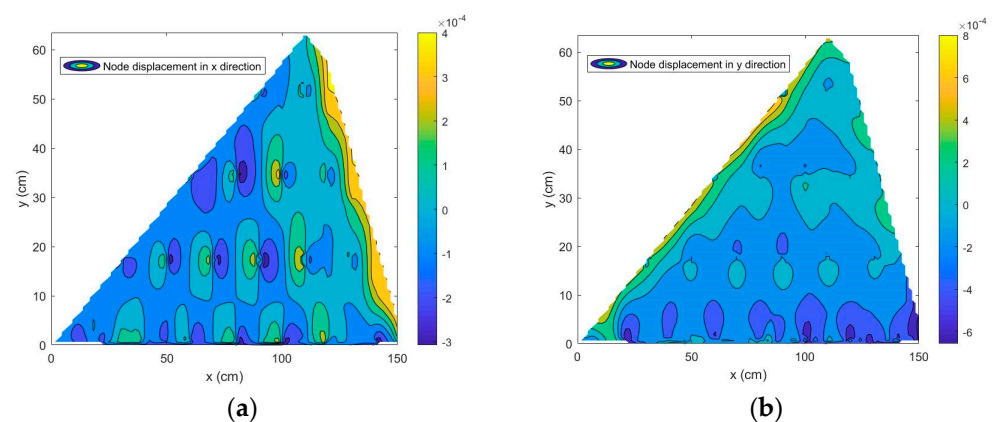


Figure 16. Node displacement of 1/12 core ((a) x direction and (b) y direction).

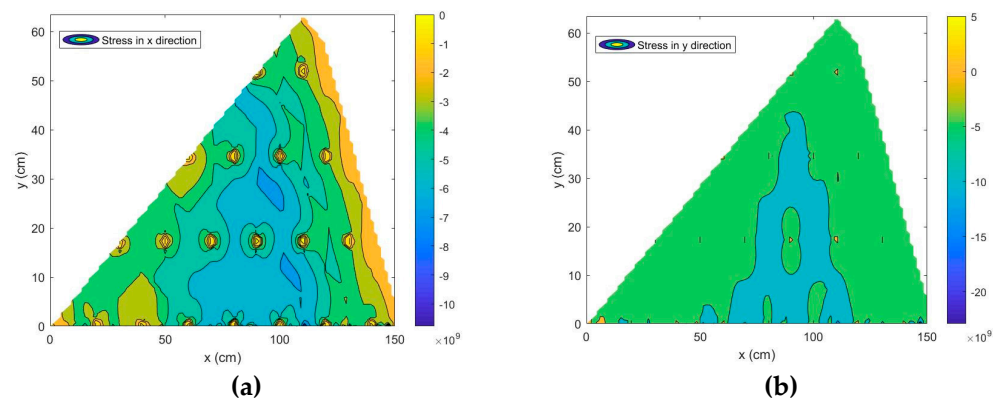


Figure 17. Stress distribution of 1/12 core ((a) x direction and (b) y direction).

It can be seen from Figure 13 that the distribution of node displacement and stress in the core is obviously different from the relatively smooth neutron field that is shown in Figure 11, but there are some similarities with the temperature field that is shown in Figure 12, which can be explained that the outer boundary of this reactor type is sliding and relatively free, so the node displacement and stress are mainly dominated by the temperature field.

5. Conclusions and Prospect

Aiming at the multi-physical process analysis of solid-state nuclear system, a coupled numerical calculation scheme of neutronics-thermal-thermoelasticity of solid-state core based on the Galerkin finite element method is proposed. Through the above research, derivation, and calculation, the following conclusions are drawn:

- (1) The coupling analysis of neutronics, heat transfer, and thermoelasticity of heat pipe cooled reactor based on the Galerkin finite element method is a feasible technical route, based on which the variation rules of physical fields such as neutron flux, temperature, displacement, and stress during stable operation of the heat pipe cooled reactor can be obtained.
- (2) The neutronics, heat transfer, and thermoelasticity modules of the FEMN coupling code were verified by benchmark examples or commercial programs. It can be seen that the error between the predicted values and reference values of key parameters such as flux, system eigenvalue, temperature, and displacement were controlled within 3%. This calculation accuracy is acceptable in the engineering design or conceptual design of the heat pipe cooling reactor type.
- (3) Heat pipe cooled reactors have received great attention in special propulsion application scenarios because of their simplified system, advanced performance, and new design features. In recent years, the industry has put forward a variety of heat pipe cooled reactor design schemes, but there is still a lot of room for improvement in the computing power and confidence of the FEMN. Therefore, for the high performance and high confidence performance analysis of a heat pipe cooled solid core, the comprehensive research and capability improvement of technical units such as three-dimensional calculation, transport calculation, heat pipe start-up, operation, and failure analysis of FEMN will be carried out in the future to strengthen its analysis capability.

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