


Article

Mechanism of Casing String Curvature Due to Displacement of Surface Strata

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Abstract: One of the main problems of well operation is the risk of uncontrolled leakage of hydrocarbons into the environment. This problem is especially relevant for the long-term operation of wells. The idea for this study was inspired by a real industrial problem that the authors of the article were involved in solving. At several operating gas wells, an abnormal slope of the production tree occurred, which raised the question of the safety of their further operation. An analysis of known studies and current regulatory documents did not allow us to assess the safety of using a gas well based on the measured kinematic parameters of production tree deviations. A mathematical model for the deformation of a package of casing strings when the surface layer of the rock is displaced is developed in the article. A boundary value problem is formulated for differential equations of pipe bending on an elastic foundation. Based on the results of solving this problem, an unambiguous relationship was established between the maximum bending stress in the surface pipe and the angle of inclination of the production tree. The quantitative characteristics of the connection depend on the geometric and mechanical properties of the pipes and on the thickness and mechanical parameters of the formations. It was established that the existing inclination of the production tree can be achieved due to the beginning of the plastic bending of the surface pipe under the slickenside, which does not exclude the exhaustion of the safety margin of the surface pipe and indicates the possible operation of the casing string in a pre-emergency state. In general, the obtained results develop analytical approaches to assessing the behavior of underground structures of a production well in unstable soil bodies.

Keywords: conductor casing; moving surface strata; curvature; stresses calculation



Citation: Dutkiewicz, M.; Shatskyi, I.; Martsynkiv, O.; Kuzmenko, E. Mechanism of Casing String Curvature Due to Displacement of Surface Strata. *Energies* **2022**, *15*, 5031. <https://doi.org/10.3390/en15145031>

Academic Editor: Dameng Liu

Received: 4 June 2022

Accepted: 6 July 2022

Published: 10 July 2022

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1. Introduction

In the mining industry, significant attention is paid to safety and production efficiency. In particular, this is reflected in the annual growth of efforts and costs aimed at ensuring the integrity of oil and gas wells [1–3]. The life cycle of a well includes several stages, such as design, construction, operation and abandonment. One of the main problems at the stage of well operation is the risk of uncontrolled leakage of hydrocarbons into the environment. This problem is especially relevant for the long-term operation of wells. The human factor has been identified as the most common cause of catastrophic accidents in the oil and gas industry [4]. However, there are threats that are relatively independent of human impact, for example, the destruction of casing strings due to landslides [5,6] or the collapse of deep wells built in unstable formations such as mudstones, bischofites, shales and layers of plastic mineral salt [7–11].

The idea of the presented study was the result of the emergence of a real industrial problem, in the solution of which the authors of the article were directly involved. Several active gas wells experienced an abnormal inclination of the production tree while the wells continued to produce. Because of this, the issue of the safety of further operation of the mentioned wells became extremely acute. An analysis of known studies and current regulatory documents did not allow us to assess the safety of using a gas well according to the parameters of the production tree deviations measured on the daylight surface. It turned out that there are no methods for such an assessment. Our study aims to fill the identified theoretical gap.

The problem of casing string failure due to collapse or shear exists in many oil and gas fields in a number of countries and can lead to emergencies, premature shut-in of wells and significant economic damage [12]. Ensuring the reliability and durability of casing strings is closely related to their strength calculations. Across the range of issues for assessing the strength of casing strings, the most difficult are the issues of calculating collapse resistance to external pressure. Research on this problem is carried out simultaneously in two main areas: theoretical and experimental studies to determine the critical pressure of casing collapse as well as determination of the design external pressure for casing strings [13,14]. It should be recognized that the problem of determining the design external pressure for casing strings has not found a final solution to date. Difficulties in solving this problem lie in the fact that at present there are almost no means for measuring the actual pressures in the casing string at different stages of well construction and operation. Therefore, in the methods of calculating casing strings, when determining the design external pressure, there are always a number of assumptions, and design schemes do not always correspond to the real object of study [15].

In [16], based on experimental and numerical results, expressions were obtained for calculating the shear strength of the casing pipe at various irregularity coefficients. The authors of [17] obtained regularities of the effect of axial length on the casing string failure. Probabilistic estimates of the strength of casing strings are studied in the work [18]. Finite element modeling of various scenarios of static and dynamic loads of casing strings was carried out in [19,20]. A comparative analysis of widely used methods for predicting the destruction of casing pipes under combined loads is presented in [21].

One of the key factors positively influencing the life of a well is the centering of casing strings, which ensures the quality of annulus cementing [22–25]. The greatest success is achieved through the use of centralizers with non-linear rigidity characteristics, as described in [26–28]. Other problems of centering of long objects are discussed in [29,30].

To assess the stress–strain state and limit equilibrium in zones of displacement of the surface strata and in areas of unstable or damaged foundation, it is appropriate to use the methods developed for analyzing the integrity of underground pipelines as extended rod-shell systems [31–35]. In particular, for pipelines built in mountainous areas, such calculations of strength parameters can be carried out according to the kinematic characteristics of the movements of the damaged foundation [36,37]. On the basis of simple schemes for the interaction of rods with the surrounding rock, many other studies involving innovative methods of well construction can be carried out [38–42]. Models of rod [43–45] and plate [46–48] structures resting on elastic or inelastic Winkler foundations are effective for such engineering simulations. The results of limit state calculations based on 1D models can be refined, taking into account the stress concentration in a pipe thread [49–53] and through the application of fracture mechanics methods to closable crack-like defects in bent and compressed cylindrical shells [54–59]. In this case, the model of contact of crack edges along a line has proven itself well [60–63].

Therefore, the initiation of this study is the inclination of the production tree at the production well Bytkiv-40, located in the Carpathians on the territory of the Bytkiv gas condensate field (Figure 1). The directly measured deviation of the upper part of the surface string from the vertical reaches 6° and is associated with shear processes. Within the area where the well is located, a planned-altitude geodetic survey, ground-based geophysical

surveys using electrometric methods [64], as well as a detailed complex interpretation of well logging data for the upper part of the section [65] were performed. The geophysical prerequisites for landslide hazard include anomalies of geophysical parameters, which, by their physical content, are unconditionally associated with the activation of landslides, in particular, zones of low electrical resistance, lithologically consistent with clays, which are potential slickensides with a depth of up to 7 m.



Figure 1. Production tree incline.

The interpretation of the described problem does not fit into the typical algorithms that are offered to the manufacturers by the current directive documents. Bibliosemantic analysis showed that today there is no method for analytical evaluation of the safe use of underground structures of a production well by the parameters of production tree inclines measured on the daylight surface.

The aim of this work is to explain the possible mechanism of casing string curvature in the surface displacement zone and to develop a model for estimating the maximum stresses from the kinematic parameters of the production tree inclinations.

2. Materials and Methods

2.1. Output Data and Problem Statement

We study the formulated problem of estimating the stress state of casing pipes according to ground measurements of the incline of the production tree on the basis of a well-tested model of the interaction of rods with the base through the Winkler interlayer [66,67].

Direct measuring (Figure 2) set the angle of inclination of the surface casing to the vertical:

$$\theta_0 = \arcsin \frac{113}{1180} \approx 0.096 \approx 0.1 \text{ rad} \approx 5.5^\circ$$

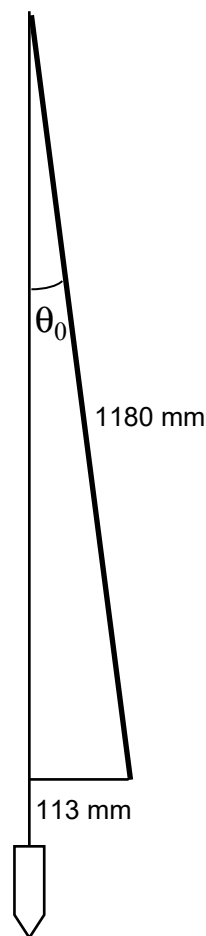


Figure 2. Angle θ_0 measurement scheme.

According to geophysical studies, the probable displacement of the upper rock layer occurred above the slickenside at depth $b = 7$ m.

Let us consider the behavior of a casing string in surface rock layers. We direct the axis Oz with the beginning on the daylight surface along the axis of the column deep into the Earth (Figure 3).

A cemented surface casing with a conductor casing is modeled by a piecewise homogeneous rod with tubular cross sections, which in the shear section ($0 \leq z \leq b$) bends by the forces of plastic resistance of the rock, and below the slickenside ($z > b$) interacts with bedrock according to Winkler's law with an elastic bed coefficient k . The reinforcing effect of the cement stone on the mechanical behavior of the casing in the cemented areas is not taken into account. At small angles of production tree inclination, the interaction of the conductor with the internal uncemented technical, production and flow strings from above through the wellhead equipment flanges and through contact at depth is neglected. Such assumptions significantly simplify the solution of the problem, transferring it to the category of linear ones.

Let us investigate the distribution of forces, moments, angle of rotation and transverse movement of the column and estimate the magnitude of the maximum stresses.

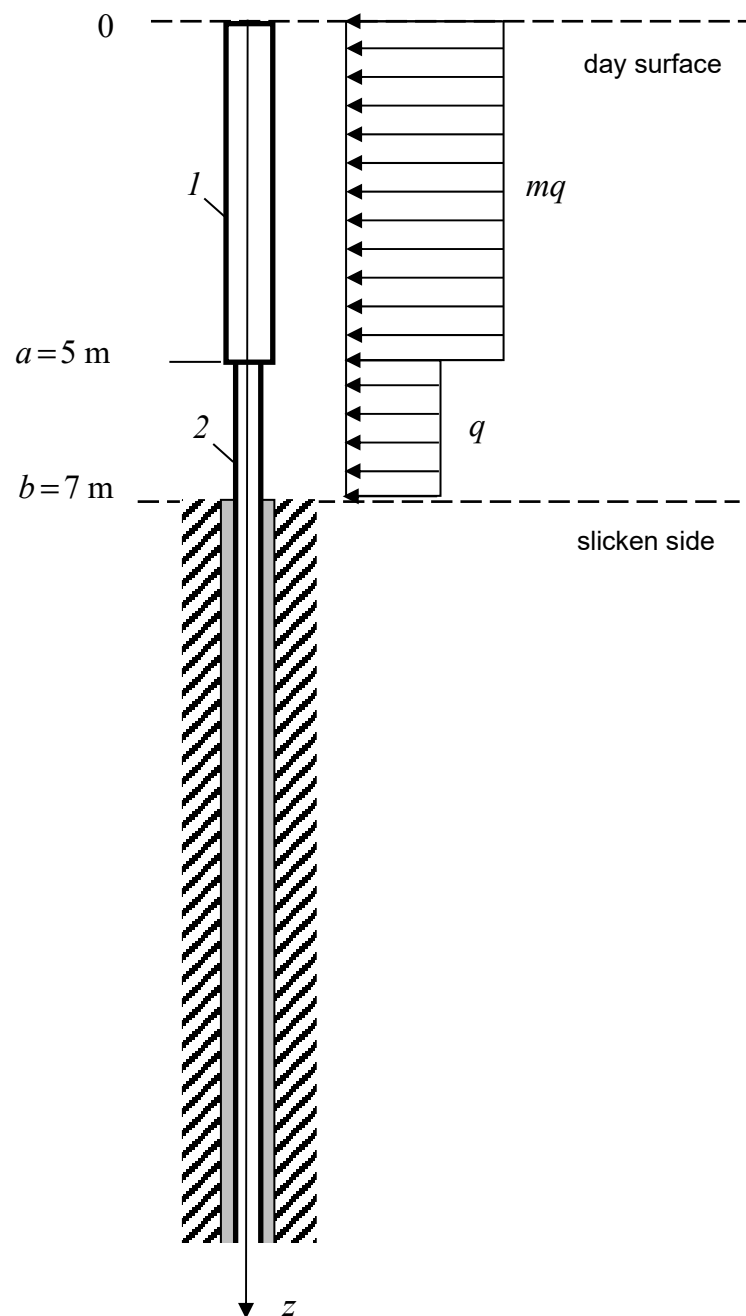


Figure 3. Calculation scheme of the problem: 1—surface casing, 2—conductor casing.

2.2. Boundary Value Problem of String Bend

The boundary value problem for the system of differential equations of string bending has the following form:

(a) equation in the region:

$$EJy'''' = mq, \quad z \in (0, a); \quad (1)$$

$$EJ_2y'''' = q, \quad z \in (a, b); \quad (2)$$

$$EJ_2y'''' + ky = 0, \quad z \in (b, \infty); \quad (3)$$

(b) boundary and matching conditions:

$$M(0) = 0, \quad Q(0) = 0; \quad (4)$$

$$y(a-0) = y(a+0), \quad y'(a-0) = y'(a+0),$$

$$M(a-0) = M(a+0), \quad Q(a-0) = Q(a+0); \quad (5)$$

$$y(b-0) = y(b+0), \quad y'(b-0) = y'(b+0),$$

$$M(b-0) = M(b+0), \quad Q(b-0) = Q(b+0); \quad (6)$$

$$M(\infty) = 0, \quad Q(\infty) = 0. \quad (7)$$

where

y is a horizontal displacement of the string axis;

$q = \sigma_Y D_2$, σ_Y is a plastic resistance of the rock;

$m = D_1/D_2$, D_1 , D_2 are outer diameters of the conductor and the surface casing;

a is a conductor length;

b is a depth of the slickenside;

$EJ_1 \approx \frac{\pi}{16} D_1^3 t_1$, $EJ_2 \approx \frac{\pi}{16} D_2^3 t_2$ are the bending rigidity of the pipes of the conductor and surface casing, respectively;

t_1 and t_2 are their wall thickness;

E is Young's modulus of pipe material;

$EJ = EJ_1 + EJ_2 = EJ_2(1+n)$ is a total rigidity of the conductor with a coaxial surface pipe ($n = \frac{EJ_1}{EJ_2} \approx \left(\frac{D_1}{D_2}\right)^3 \frac{t_1}{t_2}$, if the pipes are connected perfectly, $n = 0$, if the pipes slide mutually, and only the windage of the conductor is taken into account);

M is a bending moment;

Q is a transverse force;

k is a bedrock coefficient.

An analytical study of the boundary value problem (Equations (1)–(7)) was performed using the method of initial parameters [68], which turned out to be effective in simulating casing rod centralizers [69].

3. Results and Analysis

Using the technique of initial parameters [60], solutions of Equations (1)–(3) satisfying conditions (4), (5), and (7) were constructed in the following form:

when $z \in [0, b]$

$$Q(z) = -q(mz - (m-1)(z-a)H(z-a)),$$

$$M(z) = -q\left(m\frac{z^2}{2!} - (m-1)\frac{(z-a)^2}{2!}H(z-a)\right),$$

$$y'(z) = \theta_0 - \frac{q}{EJ_2}\left(\frac{m}{1+n}\frac{z^3}{3!} + \frac{mn}{1+n}\frac{z^3 - a^3}{3!}H(z-a) - (m-1)\frac{(z-a)^3}{3!}H(z-a)\right),$$

$$y(z) = -\Delta + \theta_0 z - \frac{q}{EJ_2}\left(\frac{m}{1+n}\frac{z^4}{4!} + \frac{mn}{1+n}\left(\frac{z^4 - a^4}{4!} - a^3(z-a)\right)H(z-a) - (m-1)\frac{(z-a)^3}{3!}H(z-a)\right); \quad (8)$$

when $z \in [b, \infty)$

$$Q(z) = \frac{2}{\gamma} q b e^{-\frac{z-b}{\gamma}} \left((A+B) \cos \frac{z-b}{\gamma} + (B-A) \sin \frac{z-b}{\gamma} \right),$$

$$M(z) = 2q b^2 e^{-\frac{z-b}{\gamma}} \left(-B \cos \frac{z-b}{\gamma} + A \sin \frac{z-b}{\gamma} \right),$$

$$y'(z) = \frac{q b^2 \gamma}{EJ_2} e^{-\frac{z-b}{\gamma}} \left((B-A) \cos \frac{z-b}{\gamma} - (A+B) \sin \frac{z-b}{\gamma} \right),$$

$$y(z) = \frac{q b^2 \gamma^2}{EJ_2} e^{-\frac{z-b}{\gamma}} \left(A \cos \frac{z-b}{\gamma} + B \sin \frac{z-b}{\gamma} \right). \quad (9)$$

where

$\gamma = \left(\frac{4EJ_2}{kD_2}\right)^{1/4}$ is a characteristic length parameter that decreases with an increase in the degree of sticking of the string in the bedrock;

k is a coefficient of subgrade reaction (Pa/m);

θ_0, Δ, A, B are arbitrary constants;

$H(z)$ is the Heaviside function.

Satisfying the matching conditions of (6), we obtain

$$A = -\frac{1}{4}\left(m\left(1 + 2\frac{\gamma}{b}\right) - (m-1)(1-\alpha)\left(1 - \alpha + 2\frac{\gamma}{b}\right)\right),$$

$$B = \frac{1}{4}\left(m - (m-1)(1-\alpha)^2\right),$$

$$\theta_0 = \frac{qb^3}{3!EJ_2}\left(3!\frac{\gamma}{b}(B-A) + \frac{m}{1+n} + \frac{mn}{1+n}(1-\alpha^3) - (m-1)(1-\alpha)^3\right),$$

$$\frac{\Delta}{b} = \theta_0 - \frac{qb^3}{4!EJ_2}\left(4!\frac{\gamma^2}{b^2}A + \frac{m}{1+n} + \frac{mn}{1+n}(1-\alpha^4 - 4!\alpha^3(1-\alpha)) - (m-1)(1-\alpha)^4\right), \quad (10)$$

and $\alpha = a/b$.

Thus, Equations (8)–(10) represents solution of the problem i at a given load q .

Let us investigate the function of the bending moment to the extremum. From the condition $Q = 0$ we find a stationary point $z_* = b + \gamma \arctg \frac{A+B}{A-B}$.

Therefore,

$$\left|M\right|_{\max} = \left|M\right|(z_*) = qb^2\sqrt{2(A^2 + B^2)}e^{-\arctg \frac{A+B}{A-B}}.$$

This moment corresponds to the maximum stress in the surface pipe $\sigma_{\max} = \frac{|M|_{\max}D_2}{2J_2}$ or

$$\sigma_{\max} = \frac{qb^2D_2}{2J_2}\sqrt{2(A^2 + B^2)}e^{-\arctg \frac{A+B}{A-B}}. \quad (11)$$

Excluding from relations of Equations (10) and (11) the value qb^2 , we obtain the relationship between the highest stress in the surface string and the angle of inclination of the production tree θ_0 , which is the only parameter available for direct measurement:

$$\sigma_{\max} = \frac{3}{2}E\theta_0\frac{D_2}{b}F,$$

$$F = \frac{2\sqrt{2(A^2 + B^2)}e^{-\arctg \frac{A+B}{A-B}}}{3!\frac{\gamma}{b}(B-A) + \frac{m}{1+n} + \frac{mn}{1+n}(1-\alpha^3) - (m-1)(1-\alpha)^3}.$$

In a similar way, a relationship was established between the horizontal displacement of the top of the surface pipe and its angle of rotation:

$$\Delta = b\theta_0G,$$

$$G = 1 - \frac{1}{4}\frac{4!\frac{\gamma^2}{b^2}A + \frac{m}{1+n} + \frac{mn}{1+n}(1-\alpha^4 - 4!\alpha^3(1-\alpha)) - (m-1)(1-\alpha)^4}{3!\frac{\gamma}{b}(B-A) + \frac{m}{1+n} + \frac{mn}{1+n}(1-\alpha^3) - (m-1)(1-\alpha)^3}.$$

Numerical studies of the solution were performed for specific parameters of the casing pipes of the Bytkiv-40 well: $E \approx 2 \cdot 10^{11}$ Pa, $D_1 = 529$ mm, $D_2 = 325$ mm, $t_1 \approx t_2 \approx 10$ mm, $a = 5$ m, $b = 7$ m. Then $n \approx 1.628$, $m \approx 4.312$, $\alpha = 5/7$.

According to the results of Equations (8)–(10), the distribution of the transverse force, bending moment, angle of rotation and lateral displacement were studied (Figure 4). The graphical results of the force and deformation analysis of the interaction of underground structures of a borehole with a soil body are presented in a dimensionless form, since in this case one solved dimensionless option corresponds to a whole group of possible dimensional

problems. Therefore, in Figure 4, going from left to right, we have: the results of the force analysis—the distribution of the shear force (Figure 4a) and the distribution of the bending moment (Figure 4b) along the length of the underground well structure; the results of the deformation analysis—the distribution of the angles of rotation of the cross sections of the system (Figure 4c) and the distribution of transverse displacements (Figure 4d). The transverse force is equal to zero on the daylight surface, as a whole it changes nonmonotonically, changes symbol once and has a kink in the area of the slickenside. The bending moment behaves nonmonotonically and has an extremum outside the slickenside. The rotation angle graph has linear and non-linear sections, which are associated with the design feature of the underground part of the well. The angles of rotation over the entire interval of the study retain a constant symbol, which determines the interval of monotonous change in the graph of transverse displacements of the underground structure of the well. The maximum values of the angle of rotation and transverse displacements are reached on the daylight surface.

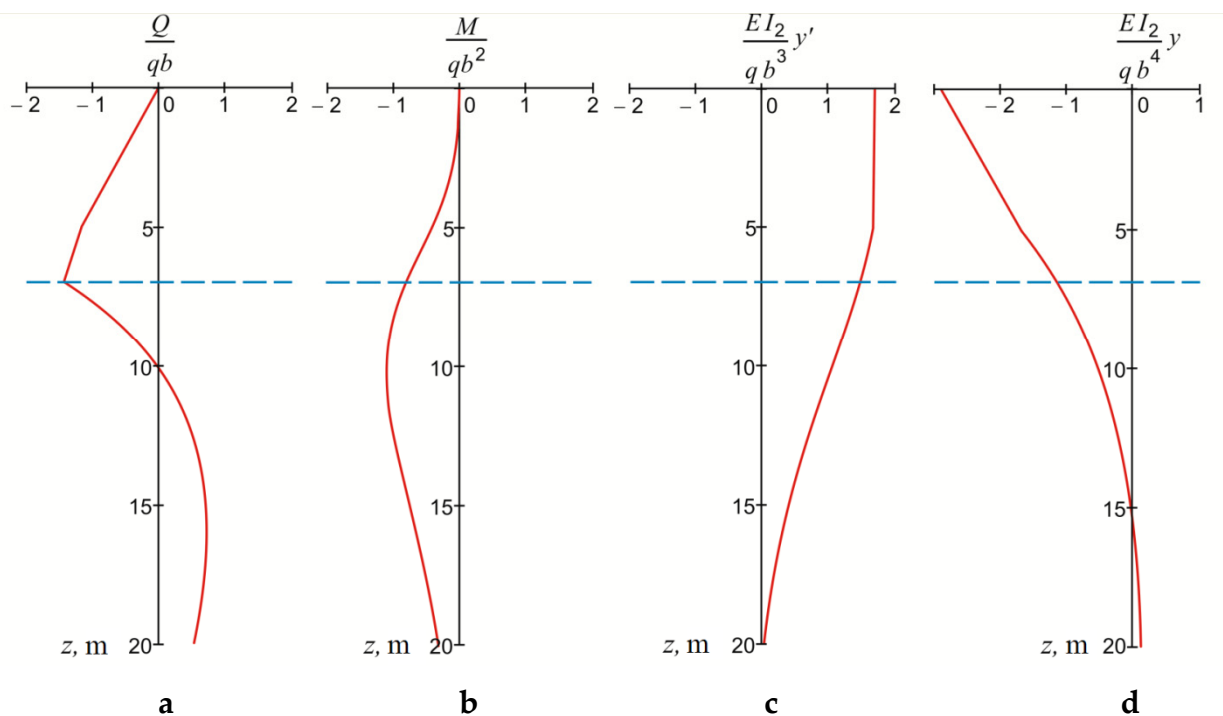


Figure 4. Diagrams of transverse force, bending moment, angle of rotation and lateral displacement at $\gamma/b = 1$; $z = b = 7$ m —slickenside. (a) the distribution of the shear force; (b) the distribution of the bending moment; (c) the distribution of the angles of rotation of the cross sections of the system; (d) the distribution of transverse displacements.

At different values of the coefficient of subgrade reaction of the layered bedrock, corresponding to the values $\gamma/b = 2$; 1; 0.5, we found the value of the maximum stress in the surface pipe and the lateral displacement of the surface pipe at the wellhead, achieved at a given angle of inclination: $\theta_0 \approx 0.1$ rad $\approx 5.5^\circ$ (Table 1).

Table 1. The results of the calculation of mechanical parameters.

Coefficient of Subgrade Reaction k , Pa/m	$\frac{\gamma}{b}$	Maximum Stresses σ_{\max} , MPa	Angle of Inclination θ_0 , rad	Lateral Displacement on the Surface Δ , m
$4.3 \cdot 10^3$	2	150	0.1	1.82
$6.9 \cdot 10^4$	1	293	0.1	1.20
$1.1 \cdot 10^6$	0.5	540	0.1	0.95

It is difficult to establish the value of the coefficient of subgrade reaction of the layered bedrock with a cement layer of long-term operation. Therefore, in the calculations, the value k varied over a wide range. A change of value k by a factor of 16 corresponds to a change of value γ by a factor of two.

As can be seen from Table 1, increasing the degree of sticking of casing pipes in the bedrock at a given θ_0 leads to a significant increase in maximum stresses in the wall of the surface pipe. For real $\theta_0 \approx 0.1$ rad, $\Delta \approx 1$ m and $k \approx 10^5$ – 10^6 Pa/m stresses within the string may reach up to $\sigma_{\max} \approx 350$ – 500 MPa. At the same time, according to [18], the yield strength for casing pipes of strength group D is $\sigma_Y = 373$ – 552 MPa.

Thus, production tree inclination $\theta_0 \approx 0.1$ rad can be achieved due to the beginning of the plastic bending of the surface pipe under the slickenside, which does not exclude the exhaustion of the margin of safety of the surface pipe and indicates the possible operation of the casing in the limit state, and this is, in turn, a sign of a pre-emergency situation.

4. Conclusions

In order to explain the mechanism of casing string curvature as a result of the displacement of the surface strata and to assess the safety of pipes in unstable ground, a calculation scheme for determining the stress in the casing pipe based on the observed deflection of the ground part of the well structure is proposed for the first time.

A mathematical model was developed for the deformation of a package of casing strings when the upper rock layer is displaced. Based on the results of solving the boundary value problem for the differential equation of bending, a relationship was established between the maximum bending stress in the surface pipe and the slope angle of the production tree. The quantitative characteristics of this connection depend on the geometric and mechanical properties of the pipes and the thickness and mechanical parameters of the formations. It was established that the existing inclination of the production tree can be achieved due to the beginning of the plastic bending of the surface pipe under the slickenside, determined at a depth of 7 m, which does not exclude the exhaustion of the margin of safety of the surface pipe and indicates the possible operation of the casing string in the limit state, and this is, in turn, a sign of pre-emergency situations.

Passive waiting can lead to further plastic bending of the surface casing and the destruction of the integrity of the strings. Therefore, it is recommended to continue instrumental observations of the inclination of the string, perform another survey near the well in order to determine the halo of gas contamination, and carry out engineering work to reduce the slope load on the well. In the future, when designing a well, it will be necessary to provide for the study of the upper part of the section in order to prevent the exploitation of the territory in complicated geological, hydrogeological and geomorphological conditions.

Future studies of the described problem should be developed, taking into account the inelastic properties of the unstable foundation.

Author Contributions: Conceptualization, M.D. and I.S.; methodology, I.S. and M.D.; software, O.M.; validation, M.D., O.M., E.K.; formal analysis, M.D., O.M.; investigation, I.S., E.K., M.D.; resources, E.K., M.D.; writing—original draft preparation, O.M., I.S.; writing—review and editing, M.D., I.S.; visualization, O.M.; supervision, M.D.; project administration, M.D., E.K.; funding acquisition, M.D., I.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

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