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Review

On State Estimation Modeling of Smart Distribution Networks: A Technical Review

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Abstract: State estimation (SE) is regarded as an essential tool for achieving the secure and efficient operation of distribution networks, and extensive research on SE has been conducted over the past three decades. Nonetheless, the high penetration of distribution generations (DGs) is accompanied by uncertainties and dynamics, and the extensive application of intelligent electronic devices (IEDs) is associated with data processing issues, all of which raise new challenges, and these issues must be taken care of for further development of SE in smart distribution networks. This paper attempts to present a comprehensive literature review of numerous works that address various issues in SE, examining key technical research issues and future perspectives. Hopefully, it will be able to meet the needs for the development of smart distribution networks.

Keywords: state estimation; smart distribution network; distribution generation; uncertainty; smart meter; big data; energy internet

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1. Introduction

Power system SE is defined as "a data processing [system] for transforming system network parameters, redundant real-time measurements (e.g., meter reading and other available information) and pseudo-measurements into an estimate of the state of an electric power system." It has become an essential tool for the operation, control, and management of electric networks worldwide since the initial development of this concept in 1970 [1]. It has strengthened the SCADA systems and eventually led to the development of the EMS [2]. Since its introduction as a way to improve power transmission network reliability and stability by monitoring real-time operation and control, SE has been a hot research topic [3].

Over the past few decades, the discussions and applications of SE at the distribution level have not been of significant interest, mainly because distribution networks have traditionally been designed and operated passively, where power flows are unidirectional and relatively simple to manage [4]. More recently, the reduced cost of instruments required for real-time monitoring, impending deregulation, and a desire to improve power customer services have driven interest and motivation for developing SE at the distribution level. In particular, as noted in related works [5,6], the smart grid's promotion of DERs, such as DGs based on wind and solar generation, PHEVs, and distributed storage units, is transforming passive distribution systems into smart distribution networks with bidirectional power flows (see, Figure 1). The transition calls for advanced monitoring [7,8], control and protection, which must be based on situation awareness of the system conditions [9,10], and a real-time SE technology is becoming increasingly significant in this context [11].

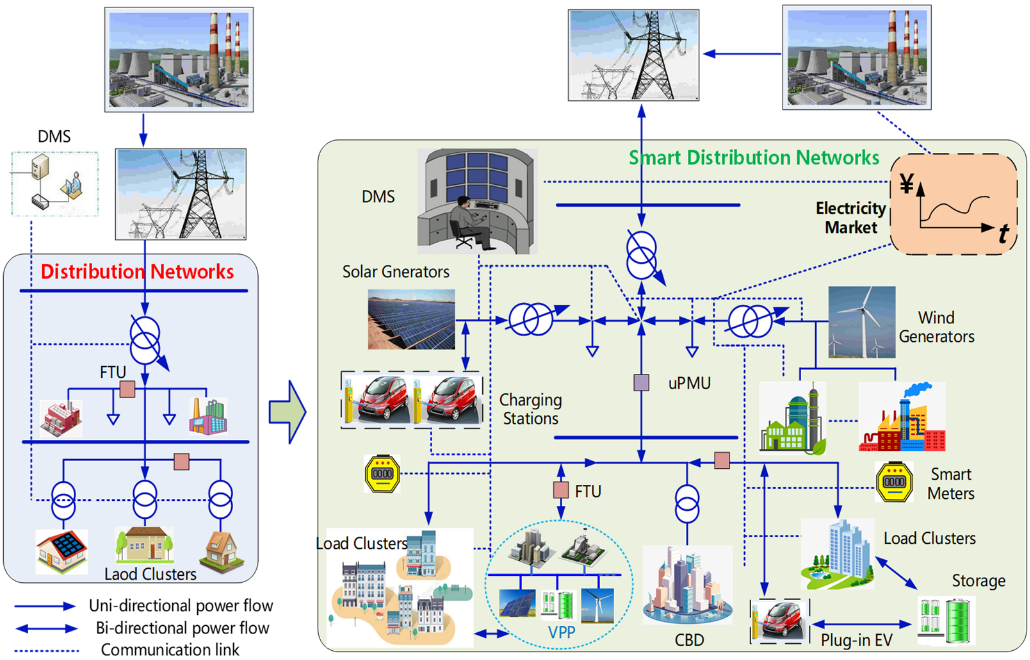

Figure 1. Schematic structures of the conventional and smart distribution networks.

Pioneer works on SE and its related technical issues for distribution networks were conducted in the 1990s [12–14], and multifunctional estimators, adopting branch–current or node–voltage variables in polar or rectangular forms as state variables, were tested [15–17]. Within the background of smart distribution networks, especially given the growing popularity of uncertain DGs, SE processing is also easily affected by the random behavior of electricity customers [18]. Therefore, it is an arduous task to extend traditional SE models to the smart distribution networks due to the following main challenges:

- High network imbalances and high line impedance ratios. The distribution network generally has unbalanced three-phase power lines and loads, and the high penetration of DGs further aggravates the imbalance of system operations. In addition, the line impedance parameter (R/X) ratio is high at the distribution level, resulting in the decoupling relationship between the power and the voltage no longer existing. These factors make simplifying the SE problem to a single-phase model no longer applicable in the distribution network.
- Large-scale networks and intense uncertainty of system parameters. The distribution network has many feeders and dense buses. DGs output and power load demand are uncertain, and the distribution network has many topologies, such as radial and weak-loop. Additionally, the line parameters are susceptible to changes due to the influence of the climate environment. These factors lead to problems such as low calculation efficiency and the low robustness of estimated results in the analysis distribution network SE model.
- Observability problem. Due to the lack of real-time measuring devices, the distribution network is highly unobservable, which requires higher accuracy of the distribution network SE model.

Fortunately, various quality works have been conducted in recent years focusing on improving SE models for the secure and effective operations of smart distribution networks. To the best of the authors' knowledge, there are not many review publications that succeed in combining different strategies while also discussing the suitability of such models and techniques for smart distribution networks. This paper attempts to broadly present various models in SE, review the current status of SE models in smart distribution networks, and highlight some of the most advanced theories currently available for the further development of SE models.

This paper is organized as follows. Section 2 briefly introduces the basic measurement functions of SE. Sections 3 and 4, respectively, survey static SE and dynamic SE models, as well as several advanced methodologies and critical issues regarding the two popular SE models. Section 5 discusses future research directions in this area. Section 6 concludes this paper.

2. Basic Measurement Functions of State Estimation

From the definition of power system SE, "the SE is a procedure for finding feasible state variables $\boldsymbol{x}$ that result in measurement functions $\boldsymbol{h}(\boldsymbol{x})$ which satisfy the measurement constraints", the distribution network SE can estimate system states from limited real-time measurements that may be subject to measurement errors [16]. The measurement system directly affects SE results. Without losing generality, a state vector is identified as $\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^T$, where $x_i = [V_i,\theta_i]$, $i = 1, 2, \cdots, n$, n is the number of buses and V_i, θ_i are, respectively, the voltage magnitude and phase angle at bus i. Two main categories are commonly used for the choice of state variables, namely three-phase bus voltage and three-phase branch current, both of which can be formulated in polar and rectangular coordinates [19]. A measurement vector is identified as $z = [z_1, z_2, \cdots, z_m]^T$, where m is the number of system measurements, including real-time measurements and pseudo-measurements [20,21]. Since there exists an error of measurement, the formulation of system measurements can be expressed as

$$\boldsymbol{z} = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{v} \tag{1}$$

where $\boldsymbol{v} \sim \mathbb{N}(\boldsymbol{0}, \boldsymbol{R})$ is the measurement noise, with a Gaussian distribution of zero mean and covariance matrix $\boldsymbol{R}$. It is defined based on the variances of various measurements, such as $\boldsymbol{R} = \mathrm{diag}(\sigma_1^2, \sigma_2^2, \cdots, \sigma_m^2)$, where σ_τ^2 is the covariance value of the τ^{th} measurement, $\tau = 1, 2, \cdots, m$. $\boldsymbol{h}(\boldsymbol{x})$ is a non-linear function vector relating the system measurements to the state variables, for instance, the three-phase bus power injection measurement equations in the polar coordinates form can be expressed as

$$\begin{cases} P_i^\varphi = U_i^\varphi \left(\sum\limits_{k=1}^{n} \sum\limits_{\gamma=\mathrm{a}}^{\mathrm{c}} U_k^\gamma y_{ik}^{\varphi\gamma} \cos(\theta_i^\varphi - \theta_k^\gamma - \delta_{ik}^{\varphi\gamma}) \right) \\ Q_i^\varphi = U_i^\varphi \left(\sum\limits_{k=1}^{n} \sum\limits_{\gamma=\mathrm{a}}^{\mathrm{c}} U_k^\gamma y_{ik}^{\varphi\gamma} \sin(\theta_i^\varphi - \theta_k^\gamma - \delta_{ik}^{\varphi\gamma}) \right) \end{cases} \tag{2}$$

where P_i^φ and Q_i^φ are the active and reactive power injections at bus i, phase φ, respectively. $U_{i(k)}^{\varphi(\gamma)}$ is the voltage magnitude at bus $i(k)$, phase $\varphi(\gamma)$. $\theta_{i(k)}^{\varphi(\gamma)}$ is the voltage phase angle at bus $i(k)$, phase $\varphi(\gamma)$. $y_{ik}^{\varphi\gamma}$ and $\delta_{ik}^{\varphi\gamma}$ are the magnitude and angle of the $(\varphi\gamma)^{\text{th}}$ element of the $(ik)^{\text{th}}$ admittance sub-matrix, respectively. Similarly, the three-phase current flow measurement equations of the power line in the rectangular coordinates form can be expressed as

$$\begin{cases} I_{ij,\mathrm{re}}^\varphi = \sum\limits_{\gamma=\mathrm{a}}^{\mathrm{c}} \left(G_{ij}^{\varphi\gamma} \left(V_{i,\mathrm{re}}^\gamma - V_{j,\mathrm{re}}^\gamma \right) - B_{ij}^{\varphi\gamma} \left(V_{i,\mathrm{im}}^\gamma - V_{j,\mathrm{im}}^\gamma \right) \right) \\ I_{ij,\mathrm{im}}^\varphi = \sum\limits_{\gamma=\mathrm{a}}^{\mathrm{c}} \left(G_{ij}^{\varphi\gamma} \left(V_{i,\mathrm{im}}^\gamma + V_{j,\mathrm{im}}^\gamma \right) + B_{ij}^{\varphi\gamma} \left(V_{i,\mathrm{re}}^\gamma - V_{j,\mathrm{re}}^\gamma \right) \right) \end{cases} \tag{3}$$

where $I_{ij,\mathrm{re}}^\varphi$ and $I_{ij,\mathrm{im}}^\varphi$ are the real and imaginary parts of the current flow on line ij, phase φ, respectively; $V_{i(j),\mathrm{re}}^\gamma$ and $V_{i(j),\mathrm{im}}^\gamma$ are the real and imaginary parts of voltage at bus $i(j)$, phase γ, respectively; and $G_{ij}^{\varphi\gamma}$ and $B_{ij}^{\varphi\gamma}$ are the conductance and susceptance of the $(\varphi\gamma)^{\text{th}}$ element of the $(ij)^{\text{th}}$ admittance sub-matrix, respectively.

The following sections present the two popular SE models in power distribution networks, namely the static SE model and the dynamic SE model, for the various purposes of the specific measurement function. Several methodologies and critical issues concerning the two SE models are thoroughly examined and discussed.

3. Static State Estimation Models

The static SE focuses on describing the system operating status at the considered instant of time. The following three research aspects, namely, the WLS-based static SE, the LAV-based static SE, and the AI-based static SE, are highly concerned, and scholars worldwide study the corresponding methodologies and issues. It should be mentioned that these static SE models and their related research topics are analyzed as some typical cases, which does not mean that these problems are only involved in the field of static SE. They also exist in the context of research conducted on dynamic SE models.

3.1. Weighted Least Square-Based Static SE

The WLS algorithm is the most widely accepted approach for obtaining static SE in distribution networks, and it aims at minimizing the objective function as

$$\min \boldsymbol{J}(\boldsymbol{x}) = \boldsymbol{v}^T\boldsymbol{v} = [\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})]^T\boldsymbol{W}[\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})] \tag{4}$$

where $\boldsymbol{W} = \boldsymbol{R}^{-1}$ is the system measurement weight matrix, with gradient

$$\frac{\partial \boldsymbol{J}(\boldsymbol{x})}{\partial \boldsymbol{x}} = -2\boldsymbol{H}^T(\boldsymbol{x})\boldsymbol{W}[\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})] = \boldsymbol{0} \tag{5}$$

where $\boldsymbol{H}(\boldsymbol{x}) = \partial\boldsymbol{h}(\boldsymbol{x})/\partial\boldsymbol{x}$ is the non-linear Jacobian matrix obtained by taking partial derivatives of $\boldsymbol{h}(\boldsymbol{x})$ with respect to x. Along with an initial value of x, the step at each iteration ρ becomes

$$\Delta\boldsymbol{x}_\rho = \left[\boldsymbol{G}(\boldsymbol{x}_\rho)\right]^{-1}\boldsymbol{H}^T(\boldsymbol{x}_\rho)\boldsymbol{W}\left[\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}_\rho)\right] \tag{6}$$

$$\boldsymbol{x}_{\rho+1} = \boldsymbol{x}_\rho + \Delta\boldsymbol{x}_\rho, \left|\boldsymbol{x}_{\rho+1} - \boldsymbol{x}_\rho\right| < \boldsymbol{\varepsilon} \tag{7}$$

where $\boldsymbol{G}(\boldsymbol{x}_\rho) = \boldsymbol{H}^T(\boldsymbol{x}_\rho)\boldsymbol{W}\boldsymbol{H}(\boldsymbol{x}_\rho)$ is the so-called gain matrix, and the diagonal elements of this matrix can be used to characterize the accuracy of state estimation.

Due to the high penetration of DERs and their participation in extensive applications of IEDs, various works in recent years have focused on methods to improve the traditional WLS-based static SE models for the secure and effective operations of future distribution networks. The state-of-the-art on research issues and solutions and are presented as follows.

(1) Analysis of the grid connection mechanisms of multiple types of DGs. In order to obtain the full view of distribution networks in terms of sufficient real-time measurements, it is necessary to take the reasonable modeling of unmonitored DGs outputs into consideration and add them to the SE models in the form of novel pseudo-measurements. However, the problem encountered in the smart distribution network SE process is how to reasonably analyze the grid connection mechanisms of multiple types of DGs. Two methods (see Figures 2 and 3) for the connection of different types of DGs to the grid are considered in Reference [22]. Namely, DGs are directly connected to distribution networks and DGs are integrated into distribution networks via a pulse width modulation (PWM) based converter. In Figure 2, $\boldsymbol{P}_0$ and $\boldsymbol{Q}_0$ are the total active and reactive outputs of DGs, respectively. $\boldsymbol{P}_{\text{in}}$ and $\boldsymbol{Q}_{\text{in}}$ are the three-phase total active and reactive powers injected into the AC system, respectively. In Figure 3, $\boldsymbol{P}_0$ and $\boldsymbol{Q}_0$ are the total active and reactive outputs of PWM converters, respectively. $\boldsymbol{M}_0$ represents the modulation coefficient of PWM converters. According to the different DG models and connection methods mentioned above, the operating system states considering DGs are included in state variables as:$\boldsymbol{x}_{AC} = [\boldsymbol{x}, U^{\text{abc}}\delta^{\text{abc}}]$. Let $\boldsymbol{z}_{DG}$ denote the pseudo-measurements setting for DGs, then the augmented measurements vector on the AC side becomes $\boldsymbol{z}_{AC} = [\boldsymbol{z}, \boldsymbol{z}_{DG}]$.

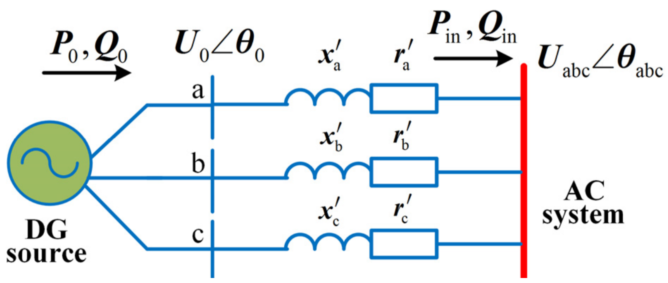

Figure 2. Model of DGs connected to the distribution network directly.

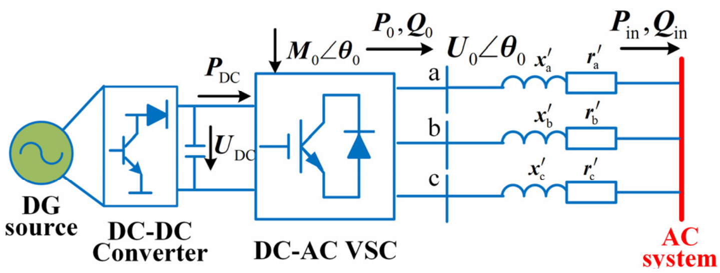

Figure 3. Model of DGs connected to the distribution system via the PWM-based converter.

Further research works on the different types of DGs and methods for their connection to the grid are presented in References [23–25].

(2) Modeling of pseudo-measurements' uncertainty. The measurement errors of pseudo-measurements and real-time measurements are thought to follow the normal distribution in the conventional distribution network SE model. However, the fluctuation of DGs' output no longer obeys the normal distribution, for example, the output of photovoltaics may obey the beta distribution, and the output of wind power may obey the Weibull distribution. In this context, the intermittent connection of DGs' output necessitates the inclusion of more uncertain factors in system pseudo-measurements. The traditional SE model faces significant challenges, and the accuracy of the estimated results may not be sufficient to meet dispatching requirements. This is an urgent problem to be solved. Reasonable mathematical modeling of pseudo-measurements' uncertainty should be considered in the SE model of a smart distribution network. To date, probability-based studies [7,26–30] and fuzzy logic-based studies [31–33] based on a large number of historical datasets have been investigated for the WLS-based SE model in order to eliminate the effect of the stochastic nature of unmonitored DGs' outputs and load power demands on the accuracy of the estimated results. The need for uncertainty models of DG outputs for the WLS-based SE model in distribution networks is stated early in References [7,26]. The statistical profiles of DG and energy prosumers can be different with respect to typical probability density distribution (PDF), and a GMM method is used to represent the mixture of τ Gaussian distribution components (i.e., τ systematic measurements). A simple illustration of the GMM is shown in Figure 4. The PDF $f(y)$ for a random variable modeled by a Gaussian mixture can be expressed as

$$f(y) = \sum_{\tau=1}^{m} \omega_\tau \mathbb{N}\left(y \middle| \mu_\tau, \sigma_\tau^2\right),\ \tau = 1, 2, \cdots, m \tag{8}$$

where ω_τ, μ_τ, and σ_τ^2 are the weight, mean, and variance of the τ^{th} component, respectively. $\mu_{f(y)}$ and $\sigma^2_{f(y)}$ of the Gaussian mixture distribution in Equation (8) are

$$\begin{cases} \mu_{f(y)} = \sum\limits_{\tau=1}^{m} \omega_\tau \mu_\tau \\ \sigma^2_{f(y)} = \sum\limits_{\tau=1}^{m} \omega_\tau \left(\sigma_\tau^2 + (\mu_\tau - \mu_{f(y)})^2\right) \end{cases} \tag{9}$$

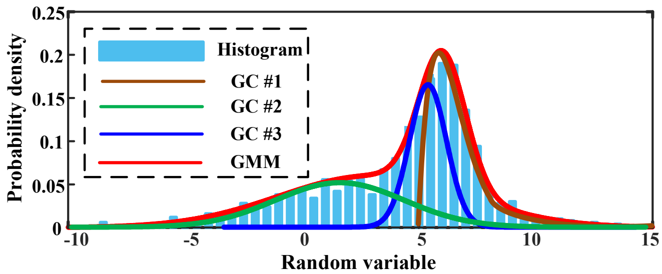

Figure 4. Illustration of Gaussian mixture model (GMM).

A new WLS-based SE model based on Bayes' rule which aimed to perfectly match the uncertainty description of the available input data, is presented in [28]. The method effectively handles the uncertainty of hybrid real-time measurements and takes into account any potential correlations between various DG types and loads. The CMCGD is used in Reference [29] to propose a non-iterative method for SE in distribution networks, and the mean and standard deviation of state variables are obtained in a single step while taking uncertainty, measurement errors, and load correlations into account. A probabilistic approach to observability is proposed in Reference [30], which takes the uncertainty of the WLS-based SE into account and assesses distribution network observability depending on the accuracy of the estimated state variables. A novel SE based on hybridized conventional CSS algorithm with fuzzy logic is proposed in Reference [31] to best handle the problem of high non-linearity and uncertainty in distribution networks. Moreover, an alternative approach to the uncertainty modeling of loads' demand power in the context of WLS-based SE is presented in Reference [33], and these pseudo-measurements are generated from a few real measurements using ANNs in conjunction with typical profiles.

(3) Multi-area WLS-based SE models. The distribution network has the characteristics of a large-scale network and three-phase asymmetry. The grid connection of DGs and the access of controllable loads further increase the dimension of system state variables, which causes the application of traditional centralized SE models to face severe challenges in terms of estimation accuracy and computational efficiency. In order to improve the performance of large-scale smart distribution network SE models, a multi-area computing approach is introduced into the SE models, which can further improve the computational efficiency while ensuring the accuracy of the estimated results. Multi-area schemes are one of the key tools used to improve the efficiency of WLS-based SE models in a large-scale distribution network [6,22,34–39]. A large distribution network can be divided into smaller-sized zones using the overlapping zone approach (OZA), which is proposed in Reference [34]. Each zone executes a local WLS-based SE process on a regular basis, and data exchange and coordination take place as each local SE approaches its near-optimal point. For a distributed real-time measurement system in a multi-area framework, a designed two-step procedure to estimate the status of a large-scale distribution network is presented in Reference [35]. The first step is to divide the network into subareas according to topological constraints and the available measurements. In the second step, data provided by local estimators are further processed in order to refine the knowledge on the operating conditions of the network. Further, a newly designed second step is proposed in Reference [36] in order to further improve the accuracy of the estimated results while reducing the computational burden of the multi-area estimator. To design an interconnected optimal filtering algorithm for a distributed SE algorithm in future power distribution networks, a novel consensus filter-based dynamic SE algorithm with its convergence analysis is proposed in Reference [39], and the optimal local gain is computed after minimizing the mean squared error between the true and estimated states.

(4) Fast-decoupled WLS-based SE models. In addition to adopting a multi-area computing approach, the computational efficiency could also be further improved by reducing the dimensionality of the complex matrix in the WLS-based SE model. A fast decoupled state estimation (FDSE) model for distribution networks is proposed in Reference [40] with

fast convergence and high computational efficiency. The branch ampere measurements are reformulated as active and reactive branch loss measurements directly in the proposed FDSE model. Utilizing the so-called complex *pu* normalization technique and specially chosen system state variables, the performance of the proposed FDSE model can be guaranteed. In Reference [41], a novel formulation of the WLS-based SE model based upon the system voltage drop formula and the system quasi-symmetric matrix is presented, which is suitable for a large-scale distribution network. The system state variables are obtained by using a convenient matrix reduction technique, and the size of the WLS-based SE model is considerably reduced concerning the Jacobian formulation $\boldsymbol{H}$ by considering the exploitation and elimination of interconnecting buses. Meanwhile, a three-phase improved admittance matrix-based (AMB) SE is proposed in Reference [42], and the method features constant coefficient matrices, thus resulting in reduced computational times.

(5) Three-phase linear WLS-based SE models. Moreover, the calculation efficiency could be further improved by linearizing the non-linear measurement function in the WLS-based SE model. An alternative way to improve the computational efficiency of the WLS-based SE model is the so-called linear approach for the non-linear Jacobian matrix $\boldsymbol{H}$ owing to the development of novel measuring devices, for instance, the phasor measurement unit (PMU) and μPMU [43–45]. The estimator follows a complex variable formulation and is intended to incorporate PMU data into SE. A linearized WLS-based SE model for unbalanced distribution networks is first presented in Reference [43]. Afterwards, the performance of different types of linear WLS-based SE models in a large-scale distribution network is assessed in Reference [44]. As a main component of a distribution area monitoring system (DAMS), a novel linear SE is presented in Reference [45], which uses only PMU measurements to calculate complex-valued bus voltages.

(6) Novel representation methods for WLS-based SE gain matrix $\boldsymbol{G}$ and measurement errors. The physical meaning of the gain matrix $\boldsymbol{G}$ in the traditional WLS-based SE model is not clear enough, which makes it difficult to implement the improvement strategy of SE in smart distribution networks. Driven by this motivation, a circuit-based approach that uses an admittance circuit is presented in Reference [46] to represent the effects of measurement accuracies on WLS-based SE errors, and it evaluates the effects of different types of measurements according to the defined indexes of source intensity and relational intensity. Set the measurement Jacobian matrix in Equation (5) as $\boldsymbol{H} = (h_{\tau i})_{m \times n}$, where $h_{\tau i}$ is a relationship between the τ^{th} measurement and the i^{th} state variables, then the $(\tau i)^{\text{th}}$ component of the gain matrix d_{ij} is represented as

$$d_{ij} = \sum_{\tau=1}^{m} \frac{h_{\tau i} h_{\tau j}}{\sigma_\tau^2} \tag{10}$$

As the gain matrix $\boldsymbol{G}$ is also a sparse matrix, it can be described by a reduced and graphical circuit representation. An analytical method for multiple measurement gross error detection, identification, and correction for WLS-based SE in the distribution network is proposed in Reference [47], and extended formulation for Jacobian matrix elements calculation, relating to different load models, is also taken into account.

(7) Novel IEDs-based and informatics-based schemes for WLS-based SE models. Multiple types of IEDs-based and informatics-based schemes have been presented to run the WLS-based SE models in distribution networks [48–55]. The high penetration of DGs may generate unforeseen dynamics which require estimation obtained with the necessary accuracy and updating rate. In this respect, the application of PMU looks promising. Novel WLS-based SE models are proposed in References [48,50], which allow for the synchronized phasor measurements provided by PMU. Meanwhile, an efficient WLS-based SE model to handle the issue of non-synchronized measurements coming from SM is discussed in Reference [51]. Additionally, in order to investigate whether a conflict exists between the SM data access and the functional requirements, the impact of the SM data aggregation restriction on the WLS-based SE performance in terms of the SE accuracies is assessed in

Reference [53]. Simulation results show that the restriction yields an indifferent impact on the estimated voltage magnitude. A straightforward SE based on the CS technique and l_1-norm minimization is proposed in Reference [55] by solving linear equations without any iterative process.

3.2. Least Absolute Value-Based Static SE

When the systematic measurement noise has a Gaussian distribution with zero mean, the solution to WLS-based SE will be an unbiased estimator of the state vector x. However, when gross errors exist, the result can be heavily biased [56]. In order to repress the impact of gross errors on the systematic state, several robust SE models are presented, including the well-known LAV-based SE [57–60] and M-estimators [61], which are expressed in the following Equation (11) and Equation (12), respectively.

$$\begin{cases} \min \sum_{\tau=1}^{m} \|v_\tau\|_1 \\ \text{s.t. } z = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{v} \end{cases} \tag{11}$$

where $\|\cdot\|_1$ is the basic l_1 norm, and the simplex method is commonly used to solve the LAV-based robust SE.

$$\begin{cases} \min \sum_{\tau=1}^{m} \Phi(v_\tau) \\ \text{s.t. } \boldsymbol{z} = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{v} \end{cases} \tag{12}$$

where $\Phi(v_\tau)$ is a chosen function of the measurement residual v_τ. Newton's method is commonly used to solve the M-estimator [62].

More recently, in order to prevent the effect of measurement errors, the different types and locations of measurements, as well as temporary failure of the meter communication system, several improved approaches for LAV-based SE models haven been introduced. A framework of a robust SE model is proposed in Reference [63] at the medium distribution level. A new machine learning algorithm is also developed that is able to provide reliable pseudo-measurements for SE models instead of conventional forecasts. Subsequently, a robust SE model for power distribution networks is proposed in Reference [64], and the uncertainty modeling of pseudo-measurements is described using PDF approaches. Additionally, network parameters are considered within a confidence interval. Recently, a novel robust SE model is proposed to estimate the voltage at specific low-voltage buses using NNs trained on buses' pseudo-measurements from the substation level only [65]. Numerical simulation results all show that the proposed model is not sensitive to the level of PV generation.

3.3. Artificial Intelligence-Based Static SE

The use of AI algorithms to calculate static state variables in distribution networks has been rarely presented in References [66–70]; for instance, an HPSO-based SE model is proposed in Reference [67], and the SE model can estimate the load and DG output values at each bus by minimizing the difference between the measured and the calculated voltages and currents. In order to improve pseudo-measurements within the prior hypothesis and compensate for limited measuring instruments at the distribution level, an NN-based SE model is proposed in Reference [69] so as to obtain better results from a static state estimator.

Future distribution networks will have the issue of rapid voltage fluctuation due to the DGs' output power being unpredictable. The OLTC in the primary substation is an important subject of voltage control actions. It is important to include the tap position as an estimated variable in the static SE models in order to perform network control more effectively [71,72]. A full three-phase static SE model combining discrete and continuous state variables (e.g., voltage magnitudes and phase angles, as well as tap positions) is proposed in Reference [71]. An HPSO method is applied in order to obtain the solution of an optimization problem. The contribution lies in handling the complexity of the unbalanced network and correctly computing the discrete values of the tap as additional system-estimated variables. However, the HPSO method always takes an unacceptably longer solution time for a reasonably sized distribution network model. In later research [72], the OO method is replaced with the HPSO method for static SE models including discrete and continuous-state variables. An ANN-based static SE model is presented in Reference [73], which can estimate voltage amplitude and phase angle with high accuracy and take different switch states as input. The author's contributions include proposing a solution to generate useful training data by the scene generator and the super parameters of NN architecture. The authors in Reference [74] use historical data to train an NN to "learn initialization in Gauss-Newton" of the static SE model. The simulation results show that a reasonable design of the NN training cost function could improve the accuracy of the estimated results. After that, a new learning NN with consideration of the network structure is proposed in Reference [75], and it could reduce the number of coefficients needed by the device for system measurements of network state. This can prevent the excessive planning stage and reduce the complexity. In Reference [76], the authors develop a method for a harmonic SE model of a distribution network based on Bayesian learning framework, which utilizes SM and µPMU measurement data. It includes procedures such as analysis of power flow, demand forecasting using recurrent NN, and sparse Bayesian learning for SE calculations.

4. Dynamic State Estimation Models

DySE, also treated as FASE, is a recursive estimation methodology based on several measurement snapshots in a time sequence [77]. The EKF algorithm, which evolved from the basic KF, is often used to determine the probable consecutive state of distribution networks [77–80]. The EKF defines the relationship between the states and the measurements and the state transition function via the following non-linear functions, $\boldsymbol{h}(\boldsymbol{x})$ and $\boldsymbol{f}(\boldsymbol{x})$, respectively.

$$\boldsymbol{z}_t = \boldsymbol{h}(\boldsymbol{x}_t) + \boldsymbol{v}_t, \boldsymbol{x}_t = \boldsymbol{f}(\boldsymbol{u}_t, \boldsymbol{x}_{t-1}) + \boldsymbol{\vartheta}_t \tag{13}$$

where $\boldsymbol{u}_t$ is a set of control variables of the system at time step t, and $\boldsymbol{\vartheta}_t \sim \mathbb{N}(\boldsymbol{0}, \boldsymbol{B})$ is the uncertainty introduced by the transition. The aim of EKF is to approximate the exact belief $bel(\boldsymbol{x}_t)$ via a Gaussian distribution described by mean $\boldsymbol{\mu}_t$ and covariance σ_t carried out in the following two stages.

- Prediction

$$\overline{\boldsymbol{x}}_t = \overline{\boldsymbol{x}}_{t-1}, \overline{\sigma}_t = \overline{\sigma}_{t-1} + \boldsymbol{B} \tag{14}$$

- Estimation

$$\boldsymbol{K}_t = \overline{\sigma}_t \boldsymbol{H}^T(\overline{\boldsymbol{x}}_t) \left[\boldsymbol{H}(\overline{\boldsymbol{x}}_t)\overline{\sigma}_t \boldsymbol{H}^T(\overline{\boldsymbol{x}}_t) + \boldsymbol{R}\right]^{-1} \tag{15}$$

$$\boldsymbol{x}_t = \overline{\boldsymbol{x}}_t + \boldsymbol{K}_t[\boldsymbol{z}_t - \boldsymbol{h}(\overline{\boldsymbol{x}}_t)],\ \sigma_t = [I - \boldsymbol{K}_t \boldsymbol{H}(\overline{\boldsymbol{x}}_t)]\overline{\sigma}_t \tag{16}$$

where $\boldsymbol{K}_t$ is the so-called Kalman gain matrix. The EKF can be utilized instead of the WLS method if there is a priori knowledge about the system state, and this foreknowledge is incorporated by the covariance matrix $\boldsymbol{B}$.

For future power distribution networks, the increasing development of multiple dynamic, active components has posed new challenges to the system's reliability and stability, and more enhanced dynamic and real-time monitoring infrastructures are required. Driven by this motivation, the following two aspects are highly studied, and scholars worldwide study the corresponding issues and methodologies. Similarly, these related researches are analyzed as some typical cases for the analysis of dynamic SE models.

Although EKF could solve several nonlinear DySE issues, the accuracy would decrease with the increase of the system nonlinearity. Therefore, the UKF is proposed to improve the estimated accuracy on highly nonlinear power systems [81], where the system nonlinearity is seized by the unscented transformation (UT) approach. UT needs to randomly generate a set of sigma points distributed near the current estimation point symmetrically, and each sigma point has its own weights on the mean matrix and the covariance matrix. Assuming that a total of $2n$+1 sigma points are treated as $\boldsymbol{X}$, it can be obtained from the columns of the $\sqrt{\eta \mathbf{P}}$ as

$$\begin{cases} \mathbf{X}_0 = \mathbf{m} \\ \mathbf{X}_i = \mathbf{m} + \sqrt{\eta \mathbf{P}},\ i = 1, 2, \cdots, n \\ \mathbf{X}_i = \mathbf{m} - \sqrt{\eta \mathbf{P}},\ i = n+1, n+2, \cdots, 2n \end{cases} \tag{17}$$

with weights

$$\begin{cases} W_0^{\mathrm{m}} = \frac{\lambda}{\mathrm{n}+\lambda} \\ W_0^{c} = \frac{\lambda}{\mathrm{n}+\lambda} + (1 - \alpha^2 + \beta) \\ W_i^{\mathrm{c}} = W_i^{\mathrm{m}} = \frac{1}{2(n+\lambda)} \qquad i = 1, 2, \cdot \cdot \cdot, 2n \end{cases} \tag{18}$$

where $\boldsymbol{P}$ is a semidefinite matrix; $\boldsymbol{W}^{\mathrm{m}}$ and $\boldsymbol{W}^{\mathrm{c}}$ are weights matrix of the mean and the covariance, respectively; λ is a scaling; α and β are parameters and are constants.

A SR-UKF method is improved from the basic UKF and has better numerical stability; it generates sigma points by directly estimating the square root of the covariance matrix, thus avoiding the semi-definite constraint of the covariance matrix [82].

4.1. Novel Dynamic State Estimation Models under the Influence of Communication Network

Although several dynamic SE models based on KF and EKF methodologies have been proposed at the distribution level, most of these prior efforts do not consider a particular fact, namely that SE is dependent on real-time or quasi real-time measurements, which are typically transmitted through a communication network [83–87]. This issue is addressed in Reference [83] by establishing a specific communication platform based on the information-centric networking concept. The proposed C-DAX enables flexible and scalable (re)configuration of PMU data communication for the seamlessness and full observability of network conditions in dynamic scenarios. Subsequently, a novel dynamic SE model is proposed in Reference [83]. The SE model is localized within an automation platform which performs real-time numerical system stability, computational efficiency, and bad data processing. An EKF-based dynamic SE model for distribution networks is presented in Reference [85], considering time-varying and stochastic properties of a selected system model. Conditions of the communication network are quantified for the stability of the error covariance matrix and the original Kalman filter equations. Subsequently, the feasibility of implementing dynamic SE for future distribution networks in FPGAs by presenting a SDKF-based operational prototype is presented in Reference [87], and the suitability of the SDKF for an FPGA implementation by means of a computational complexity analysis is highlighted.

4.2. Dynamic State Estimation Models for Both Radial and Weakly Meshed Power Distribution Networks

The topological configuration of distribution networks may also transition from a mainly radial to a more weakly meshed topology in order to minimize the power loss and improve the voltage profile, etc. However, most of the existing works on dynamic SE models only apply to radial networks and do not consider the application of these models to a weakly meshed network [88–91]. Under this background, the development of a DQSE based on hybrid measurement data (e.g., SM data as well as traditional SCADA data) in distribution networks is proposed in Reference [88]. In particular, the proposed DQSE implementation can work for both radial or weakly meshed distribution networks, as distribution networks may be reconfigured following the clearing of a fault. Furthermore, a novel dynamic SE model for meshes practically available in distribution networks is presented in Reference [89], which is accessed by embedding the power flow equations of the buses which are part of the loops. A real-time monitoring tool implemented on a Brazilian distribution utility is presented in Reference [91]. The implemented tool comprises two steps to provide load values for service restoration software in multiple situations and different topological networks.

5. Future Directions

In the sections above, the state-of-the-art for popular SE models in power distribution networks has been examined. The transition to more intelligent and sustainable distribution networks yields more complex and dynamic results, so more efficient monitoring situations and control systems for distribution networks would be required. To date, multiple SE models have been increasingly complicated, not only as a result of special network characteristics and low redundancy of real-time measuring instruments but also due to the geographical extension of system networks, the wide application of IEDs, and the high number of integrations of DGs. Hence, challenges remain and novel monitoring solutions are required. In what follows, various new research areas for SE models in future distribution networks are discussed, and potential solutions are shared.

5.1. Distributed Interval-Based State Estimation Models in Smart Distribution Networks

To date, solutions have been explored for WLS-based SE models in balanced distribution networks considering uncertainty, and these solutions are typically dependent on pre-defined PDF parameters or fuzzy information on uncertainty modeling for non-monitored DGs and loads. However, it may be difficult for DSO to obtain detailed and precise data in most practical situations [92]. Additionally, these approaches could provide probabilistic solutions for system reliability [93].

In many circumstances, although the exact values or comprehensive PDF information on measurements are unknown due to noise, the lower and upper bounds of these measurements can be specified [94,95]. On the other hand, the increasing penetration of DGs and new types of loads will further extend the dimensions of distribution networks, affecting the computational burden of SE models. Considering the above limitations, the interval arithmetic (IA) approach will be referred to as an effective alternative to resolve uncertain descriptions, and the distributed IA-based SE models for smart distribution networks considering measurement uncertainty will be worth investigating in addition to the hypothetical framework (see Figure 5).

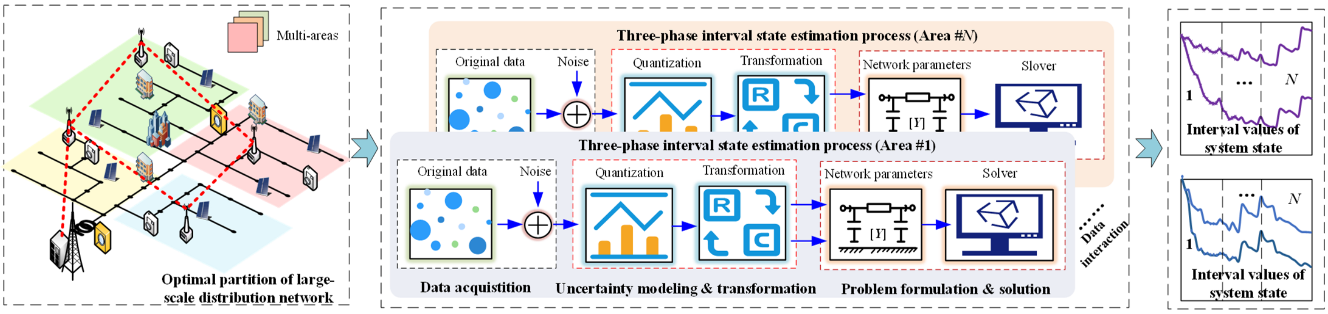

Figure 5. Framework of the distributed IA-based SE approaches in smart distribution systems.

As shown in Figure 5, in order to meet the demand for improving the calculation efficiency of the SE model for large-scale distribution networks, it is necessary to first divide the distribution network into several sub-areas. The process of achieving the optimal partitioning of a large-scale distribution network refers to the existing partitioning method of the transmission network by defining the equivalent electrical distance and abstracting load buses, power lines, DGs, etc. On this basis, the edge division and community discovery algorithm are used to define the value of extended modularity. The number of sub-areas is clarified in order to realize the decoupling of large-scale distribution networks into several small areas. The data acquisition process mainly includes pseudo-measurement data in the sub-area, the FTU, the μPMU, and other real-time measurement data, thus forming a multi-source measurement dataset and taking it as the input of ISE. The uncertainty modeling and transformation process is aimed at the sub-area, mainly including the interval modeling of measurement data in different sub-areas and the transformation of multi-source measurement data. At this point, it is of great significance to note that if the these multi-source measurement data are used for SE process without integration, issues such as data incompatibility will occur, and even the accuracy of integration SE will be reduced. The problems of and solutions to the interval state estimation (ISE) model are based on the interval modeling and analysis of multi-source measurement data in different sub-areas combined with the line impedance parameters expressed in the form of interval numbers, all to establish multi-type real-time measurement data simultaneously. A mathematical model for local ISE of the distribution network is established, where the pseudo-measurement data such as PV output and load are taken into account. In order to improve the solution efficiency of the overall SE model and reduce the conservatism of the interval results, each sub-area is used as the basic analysis unit to form the distribution network ISE subtask, and the improved interval optimization method is used to solve the proposed ISE model. Completing the effective interaction of interval information between adjacent sub-areas, and the global interval state information of the distribution network under the interference of multiple uncertain factors could be outputted.

For smart distribution networks, the representation of measurement uncertainty as confidence intervals can offer advantages over the previous approaches with probabilistic and fuzzy noises. Obtaining the interval bounds of state variables under uncertain environments could provide system operators the confidence that the real status of the smart distribution network is not exceeding the interval, which is useful, as it provides great insights for operation.

5.2. Data-Driven State Estimation Models in Distribution Networks

The construction of distribution networks encourages a growing number of intelligent sensors to be installed [96], which can cause DSO to perform high-density monitoring and control on the user side. On the other hand, an unprecedentedly large amount of data is contained. It will be a challenge to use these multi-source data to effectively and accurately achieve full network observation. Advanced big data analytic tools and approaches (e.g., cloud computing platforms [97–99] and multi-dimensional data fusion techniques [100–102], etc.) need to be developed for in-depth data mining of the massive amount of collected data, including the unified capture, filtering, and integration of structured data (e.g., SCADA data, μPMU data, network parameters, and topological data) and non-structured data (e.g., power user behavior analysis information, environmental data, wind speed, and solar radiation value). For the identification and processing of multi-source bad data, the effective information needed for SE could be extracted as much as possible so as to further enhance situational awareness. Data-driven SE models in future distribution networks are roughly divided into the following parts:

- Filtering and preconditioning the original data uploaded by multiple types of sensors by means of data clustering and classification, generating the initial feature matrices (e.g., admittance matrix $\boldsymbol{Y}$) required by the SE program according to the network parameters.
- Merging feature matrices with single characteristics at different time instants into ones with multiple characteristics employing data fusion methods, which is the bottleneck for the application of the current big data concept, can be specifically applied to practical power systems' engineering.
- Processing high-dimensional and spatiotemporal feature matrices of SE on low-dimensional platforms via analytical schemes of multiple time scales. With this, the detection of multi-source bad data and the real-time monitoring of system operating status could be achieved smoothly.

5.3. Global State Estimation Models Integrating Transmission and Distribution Networks

The high penetration of DGs makes the smart distribution network more flexible and changeable, and the connection between the transmission network and the distribution network is increasingly close. In order to realize the accurate estimation of the development trend of the operation state, it is urgent to improve the state awareness of the distribution network under the background of transmission and distribution coordination and lay a technical foundation for the situation security assessment and preventive control under various uncertain scenarios. In the EMS, transmission network SE models are often treated as the backbone of advanced power applications, such as bad data identification, contingency analysis, and reactive power optimization analysis. While in the DMS, SE can provide state variables data for voltage regulation control, and active load sharing, etc. It is significant to notice that different voltage levels often belong to different grid operators, and there is the question of how to break this barrier from the technical and algorithm perspectives. Until recently, SE in T&D networks has been separately analyzed, and the boundary between T&D networks is often ignored. In fact, T&D networks are connected by a large number of electrical apparatus and interact with each other, and the evolutions in the smart grid operation and control sector will require a closer interaction between EMS and DMS. In this respect, attention from both academia and industry towards a state estimator that includes MV distribution feeders and high voltage transmission networks for estimating the consistent global state of the whole power system is urgently needed [103]. A potential feasibility framework is shown in Figure 6. Before this, an appropriate modeling solution and a communication channel considering the boundary conditions for integrated T&D networks should be developed.

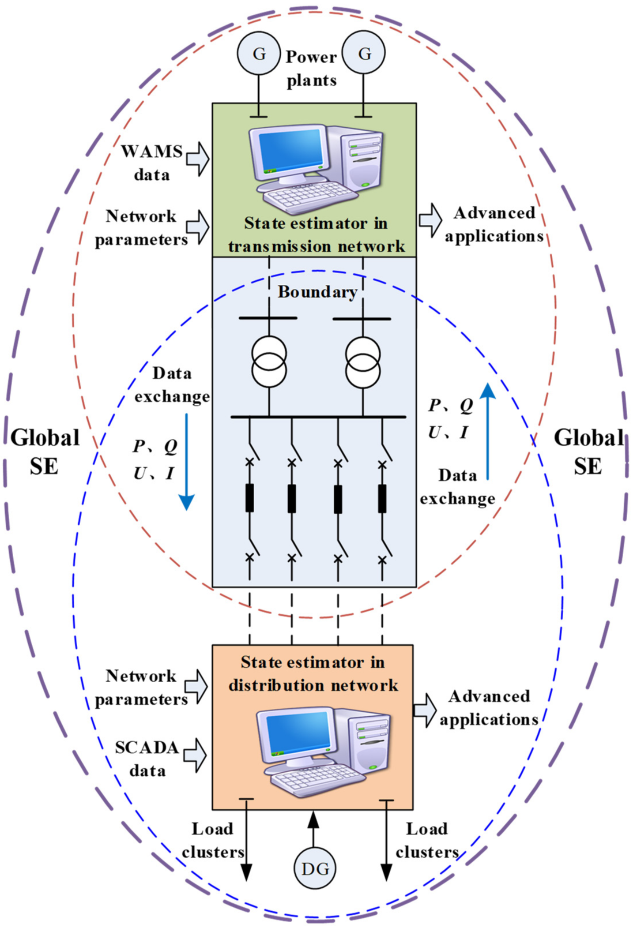

Figure 6. The framework of global SE for integrated T&D systems.

Due to the physical features of T&D networks, a mixed three-sequence/three-phase modeling approach for integrated T&D networks' SE and dynamic simulation is worth developing. In the meantime, the fusion methods of measurement data collected by PMU and remote terminal units (RTU), etc., at the transmission level, and μPMU, FTU, and smart meters, etc., at the distribution level also need to be considered. Speeding up the related research of global SE models integrating T&D networks helps to acquire more accurate and real system state information and provides better service for other advanced applications in smart grids.

5.4. False Data Injection Attacks on Urban Cyber–Physical Energy System

Conventionally, energy systems such as electricity, heat, and gas, have been operated and optimized independently. However, the concepts of IoT and EI make the interaction between urban (distribution) electricity, heat, and gas networks increasingly common [104,105]. In this context, hackers can take loopholes in the integrated energy systems' bad data detection and identification program, maliciously tamper with the SE results, and seriously endanger safe and reliable system operation. Therefore, it is important to analyze the data loopholes and formulate FDIAs when building a secure and smart energy system.

In general, the methods of FDIAs can be classified into two categories: manipulating data collection and disrupting data communication. After that, based on the defense capability of the existing grid defense measurement, the defensive measures against FDIAs are divided into three categories, which are the detection process, the identification process, and the containment process. Due to the challenges of high network imbalance, high line impedance ratio, and the great difference between measuring data at the distribution level, especially for the urban area, determining how to resist the influence of FDIAs on the SE process and ensure the reliability of the estimated results while taking into account the accuracy of SE for different types of energy networks is one of the issues that needs to be considered in the field of SE models in future.

6. Conclusions

In smart distribution networks, power flows are more complex and bi-directional due to the high penetration of DGs and the extended applications of IED, which requires extending the monitoring and control circle. The SE technique will play an essential role in the DMS of smart distribution networks. This paper has presented a survey of the fundamentals, models, and state-of-the-art of SE in distribution networks. Furthermore, it has outlined a few brief possible future research directions, including distributed interval arithmetic-based SE models in distribution networks, data-driven SE models in distribution networks, global SE models coordinated with transmission networks, and SE models for cyber-physical energy systems. SE, as one of the most fundamental technologies, can track the state of smart distribution networks and provide reliable data for advanced applications such as optimal scheduling, network reconfiguration, and the fault recovery, which is a necessary precondition for secure and reliable system operation. New technological challenges and issues in the 21st century require new SE models and application breakthroughs. This is indeed an exciting research field to get into.

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Abbreviations

SE	State Estimation	DERs	Distributed Energy Resources
DGs	Distribution Generations	PHEVs	Plug-in Hybrid Electric Vehicles
IEDs	Intelligent Electronic Devices	AMI	Advanced Metering Infrastructure
SCADA	Supervisory Control and Data Acquisition	FTU	Feeder Terminal Units
EMS	Energy Management System	μPMU	Micro-Phasor Measurement Units
DSO	Distribution System Operators	VM	Voltage Magnitude
WLS	Weighted Least Square	LAV	Least Absolute Value
AI	Artificial Intelligence	SG	Synchronous Generator
IG	Induction Generator	GMM	Gaussian Mixture Model
PDF	Probability Density Function	CMCGD	Conditional Multivariate Complex Gaussian Distribution
CSS	Charged System Search	OZA	Overlapping Zone Approach
FDSE	Fast Decoupled State Estimation	AMB	Admittance Matrix-based
DAMS	Distribution Area Monitoring System	CS	Compressive Sensing
NNs	Neural Networks	HPSO	Hybrid particle swarm optimization
OLTC	On-Load Tap Changer	OO	Ordinal Optimization
DySE	Dynamic State Estimation	FASE	Forecasting-Aided SE
EKF	Extended Kalman Filter	KF	Kalman Filter
FPGAs	Field-Programmable Gate Arrays	SDKF	Sequential Discrete Kalman Filter
DQSE	Distributed Quasi-Dynamic SE	IA	Interval Arithmetic
DMS	Distribution Management Systems	T&D	Transmission and Distribution
IoT	Internet of Things	EI	Energy Internet
FDIAs	False Data Injection Attacks	UKF	Unscented KF
UT	Unscented Transformation	SR-UKF	Square-Root UKF

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