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Article

Study on Nonlinear Stochastic Process of Deck Slamming on Floating Offshore Platform

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Abstract: In this paper, a semi-analytic method is introduced to predict the deck-slamming probability and corresponding loads. This method is based on a nonlinear statistical approach that takes into account the linear and second-order components of the relative wave elevation up to the second order. The linear and second-order wave elevation is assumed to be a two-term Volterra series. The joint probability density function of the relative wave elevation and velocity are formulated using the Hermite-moment method, and the probability distributions of the wave crest and relative wave velocity are calculated. These probability distributions are verified using the data sampled from the linear and second-order relative wave elevation. Based on this formulation, the probabilities of deck slamming and slamming-induced loads are estimated. This method is applied to a tension leg platform (TLP) model, and the effects of the second-order component of the relative wave elevation on the deck slamming are investigated.

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1. Introduction

As recent offshore platforms are required to be operated in harsher environments, deck slamming is of great interest for structural design. Deck slamming, in which free surface hits the bottom of a deck, can cause large impulsive loads on an offshore structure. Therefore, it is important to consider this phenomenon in the design stage of such structures. Many classification societies have provided rules and guidance regarding the deck clearance of offshore structures [1–3]. They state that the considerations of the probability of deck slamming occurrence and wave impact pressure are necessary to determine the deck clearance.

The interpretation of the deck slamming phenomenon requires an understanding of the relative wave elevation, which takes into account the absolute wave elevation and the platform motion. However, it is well known that the second-order components can take up more than 30% of the total elevation under certain extreme sea states [4], and the second-order components of the platform motion should also be included to represent the platform characteristics [5]. Therefore, consideration of the second-order components is essential for an accurate interpretation of the relative wave elevation.

For this reason, it is difficult to predict the relative wave elevation based on conventional linear theory. Thus, the deck slamming phenomenon is typically analyzed using the model test and computational fluid dynamics (CFD) simulations, which can express nonlinearity. However, because these methods are time-consuming and expensive, some studies have been conducted on efficiently estimating the deck slamming phenomenon using a statistical method. This analysis can be used to quickly predict the deck slamming occurrence, and the number of selections for the model test and CFD simulation can there-

fore be reduced. In addition, there is an advantage of an easy application during the early design stage.

First, since the 1940s, studies on statistical analysis methods for nonlinear problems have been conducted. Kac and Siegert [6] and Bedrosian and Rice [7] investigated nonlinear signals with second-order components in the communication field. In the above studies, they first developed a probability distribution by using an eigenvalue analysis for the signals represented as a two-term Volterra series. Later, Neal [8] brought this method to the ocean engineering field. Neal [8] expressed the eigenvalue problem with the frequency-domain transfer functions and wave spectrum and studied the probability distribution of the second-order hydrodynamic force. His approach was a pioneering work in ocean engineering, particularly introducing an eigenvalue approach for nonlinear statistical signals which Gaussian probability is not valid any more. However, his approach is limited to narrow-banded signals. Furthermore, Marthinsen and Winterstein [9] applied the Hermite-moment method using statistical moments calculated from the eigenvalue problem and analyzed the second-order load and response of the tension leg platform (TLP).

Studies on predicting a deck slamming using the statistical method began in the 1960s. Ochi [10] and Ochi and Motter [11] studied the prediction of slamming characteristics for ships based on linear theory. In the above studies, the slamming characteristics are estimated by conducting a statistical modeling of the relative wave elevation and velocity using frequency-domain transfer functions at the analysis point. Using the joint probability distribution, they calculated the probability of a slamming occurrence and the expected impact pressure. However, because this approach is based on the linear wave theory, it has a limitation in that this statistical method does not properly consider the nonlinearity of the relative wave elevation. Studies considering nonlinearity have since been conducted to predict the relative wave elevation. Manuel et al. [12] and Sweetman and Winterstein [13] applied the statistical analysis technique of nonlinear problems to the deck slamming phenomenon and analyzed the relative wave elevation. They assumed the relative wave elevation as a two-term Volterra series and conducted the statistical modeling of the relative wave elevation using an eigenvalue analysis and the Hermite-moment method. Finally, they presented the probability distribution of the peak values of air gap from this model. However, there is a limitation in that studies on the impact pressure have not been carried out.

This paper summarizes the stochastic analysis of Nam [14], introduced in the author's degree dissertation, extending the works of Nam and Kim [5]. This paper introduces an improved nonlinear stochastic approach which complements the two approaches of [12] and [13] by suggesting a statistical relationship between relative wave elevation and velocity. In order to predict the occurrence of deck slamming, not only the relative wave elevation but also the relative velocity are needed. In the case of [10] and [11], the study was limited to Gaussian process, and only wave elevation was considered in the case of [12] and [13]. This study extends their studies to non-Gaussian distribution of the relative wave velocity, which is essential to define the occurrence of deck slamming and predict the instantaneous slamming pressure.

In this study, a nonlinear stochastic approach is used to efficiently predict the probability of deck slamming occurrence and slamming pressure. The proposed method is based on the assumption of potential flow. Under this assumption, the relative wave elevation can be represented as a two-term Volterra series, and the statistical moments are calculated from the eigenvalue analysis. By applying the Hermite-moment method to the relative wave elevation, a statistical modeling of the relative wave elevation and velocity is conducted. From these results, the joint probability density function of the relative wave elevation and velocity is derived, and the probability distributions are calculated. Finally, using the probability distributions of the relative wave elevation and velocity, the probability of deck slamming occurrence and slamming pressure are estimated. This method is applied to the TLP model and is verified using the data sampled from the time series of a nonlinear wave elevation up to the second order. Furthermore, based on the computational results, the

effect of the second-order components of the relative wave elevation on the deck slamming is investigated.

2. Mathematical Background

2.1. Mathematical Modeling

Prior to defining the relative wave elevation, the motion of the offshore structure is defined in a coordinate system (O - xyz) which has origin at the center of the structure and mean water level, as shown in Figure 1.

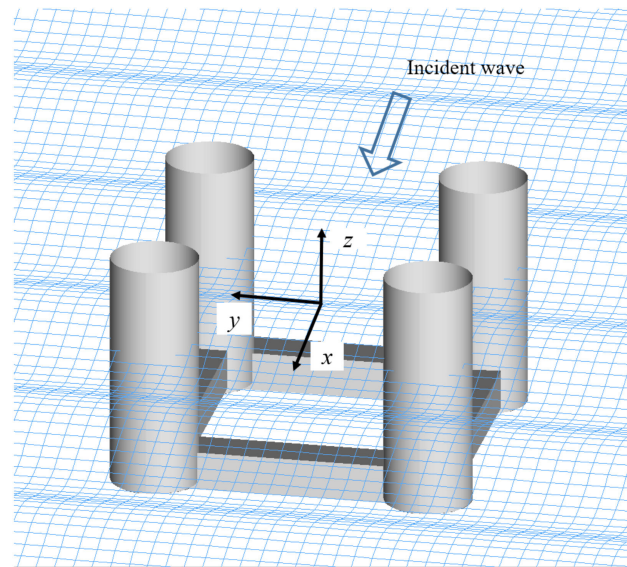


Figure 1. Coordinate system for offshore structure.

In this study, the relative wave elevation is defined as the wave elevation viewed from the offshore structure, taking into account the platform motion and absolute wave elevation. If we assume that the platform motion is small, then the relative wave elevation at the analysis point (x, y) can be calculated using Equation (1).

$$\eta_R(x, y, t) \equiv \eta(x, y, t) - \zeta_3 - y\zeta_4 + x\zeta_5 \quad (1)$$

In the above equation, η_R is the relative wave elevation, η is the absolute wave elevation, and ζ_i is the i th mode component of the platform motion, i.e., $i = 1, 2, 3$ means translational motion and $i = 4, 5, 6$ means rotational motion. In this case, ζ_3 , ζ_4 , and ζ_5 are heave, roll, and pitch motion of the platform. By defining the relative waves as above, the occurrence of the deck slamming phenomenon is interpreted when the relative waves have values above the deck clearance.

Next, mathematical modeling of the nonlinear relative wave elevation with the second-order term is conducted for a statistical analysis. As the basic assumption for the application of the statistical method used in this study, the nonlinear relative wave elevation can be expressed as a two-term Volterra series. Under the assumption of a potential flow, the nonlinear relative wave elevation up to the second order can be represented as a two-term Volterra series using the frequency-domain transfer functions.

$$\eta_R(t) = \text{Re} \left[\sum_{j=1}^{\infty} A_j H_R^{(1)}(\omega_j) e^{i\omega_j t} \right] + \text{Re} \left[\sum_{j=1}^{\infty} \sum_{k=1}^{\infty} A_j A_k H_R^{(2)}(\omega_j, \omega_k) e^{i(\omega_j + \omega_k)t} \right] + \text{Re} \left[\sum_{j=1}^{\infty} \sum_{k=1}^{\infty} A_j A_k^* H_R^{(2)}(\omega_j, -\omega_k) e^{i(\omega_j - \omega_k)t} \right] \quad (2)$$

In Equation (2), ω_j and A_j indicate the discretized frequency component and complex amplitude of ω_j , respectively, and $H_R^{(i)}$ is the i th order frequency-domain transfer functions of the relative wave elevation. The first term of Equation (2) is the linear component, and

the second and third terms are the sum and difference-frequency components. In addition, $H_R^{(i)}$ can be calculated from Equation (1) as follows:

$$H_R^{(i)} = H_W^{(i)} - H_{M,3}^{(i)} - yH_{M,4}^{(i)} + xH_{M,5}^{(i)} \quad (3)$$

In the above equation, $H_W^{(i)}$ and $H_{M,k}^{(i)}$ are the i th order frequency-domain transfer functions of the absolute wave elevation and the k th mode platform motion. These frequency-domain transfer functions are computed from commercial software that can conduct a frequency-domain analysis of a second-order potential flow.

It should be mentioned that the application of the two-term Volterra series, i.e., up to the second order, is limited to simulate the fully nonlinear waves and it cannot simulate very harsh wave conditions such as wave breaking. However, this theory can cover a significant part of nonlinearity of wave run-up [4].

2.2. Statistical Modeling

To statistically analyze η_R expressed as a two-term Volterra series, the eigenvalue analysis method is used. An eigenvalue analysis is a typical method for calculating the statistical characteristics of nonlinear properties. By solving the eigenvalue problem for a two-term Volterra series, the characteristic function and moment-generating function of nonlinear properties, which can be used for a statistical analysis, can be obtained. Neal [8] defined the eigenvalue problem with the frequency-domain transfer function as follows:

$$\int_{-\infty}^{\infty} K(\omega_1, \omega_2) \psi_j(\omega_2) d\omega_2 = \lambda_j \psi_j(\omega_1) \quad (4)$$

$$K(\omega_1, \omega_2) = \sqrt{S(\omega_1)S(\omega_2)} H_R^{(2)}(\omega_1, -\omega_2) \quad (5)$$

where $K(\omega_1, \omega_2)$ is the Hermitian kernel, which is defined by the energy spectrum of the input wave $S(\omega)$ and transfer functions. From the above analysis, η_R can be rewritten using the standard Gaussian process, $u_j(t)$:

$$\eta_R(t) = \sum_{j=1}^{2n} c_j u_j(t) + \sum_{j=1}^{2n} \lambda_j u_j^2(t) \quad (6)$$

$$c_j = \left| \int_{-\infty}^{\infty} H_R^{(1)}(\omega) \sqrt{S(\omega)} \psi_j^*(\omega) d\omega \right| \quad (7)$$

In the above equations, n is the number of discretized frequencies, and c_j and eigenvalue λ_j are calculated through the eigenvalue problem. We can then formulate the moment-generating function using the statistical characteristics of $u_j(t)$ [15].

$$M(\theta) = \frac{1}{\prod_{j=1}^{2n} (1 - 2\lambda_j \theta)^{1/2}} e^{\sum_{j=1}^{2n} \frac{c_j^2 \theta^2}{2(1-2\lambda_j \theta)}} \quad (8)$$

Moreover, the statistical moments of $\eta_R(t)$ are obtained as follows [9]:

$$m_{\eta_R} = \sum_{j=1}^{2n} \lambda_j, \quad \sigma_{\eta_R}^2 = \sum_{j=1}^{2n} (c_j^2 + 2\lambda_j^2), \quad \alpha_3 = \sum_{j=1}^{2n} \left(\frac{6c_j^2 \lambda_j + 8\lambda_j^3}{\sigma_{\eta_R}^3} \right), \quad \alpha_4 = 3 + \sum_{j=1}^{2n} \frac{48(c_j^2 \lambda_j^2 + \lambda_j^4)}{\sigma_{\eta_R}^4} \quad (9)$$

These four statistical moments are used to conduct the statistical modeling of $\eta_R(t)$. In this study, the Hermite-moment model is used, which can transform the non-Gaussian process into a standard normal process $u(t)$ by using Hermite-polynomial expansion [16].

Through the Hermite-moment method, the statistical analysis for nonlinear variables can easily be performed. When applying this method to $\eta_R(t)$, the result is as follows:

$$\eta_R = g(u) = m_{\eta_R} + \kappa\sigma_{\eta_R} \left[u + \sum_{n=3}^4 \hat{h}_n He_{n-1}(u) \right] \quad (10)$$

In the above equation, $He_{n-1}(u)$ is the n th-order Hermite-polynomial. The coefficient κ is a scale factor, and \hat{h}_3 and \hat{h}_4 are the shape factors of a non-Gaussian distribution. The coefficients κ , \hat{h}_3 , and \hat{h}_4 are computed by matching the statistical moments on each side of Equation (10) up to the fourth order through the following [17]:

$$\begin{aligned} \kappa^2 \left(1 + 2\hat{h}_3^2 + 6\hat{h}_4^2 \right) &= 1 \\ \kappa^3 \left(8\hat{h}_3^3 + 108\hat{h}_3\hat{h}_4^2 + 36\hat{h}_3\hat{h}_4 + 6\hat{h}_3 \right) &= \alpha_3 \\ \kappa^4 \left(\begin{aligned} &60\hat{h}_3^4 + 3348\hat{h}_4^4 + 2232\hat{h}_3^2\hat{h}_4^2 \\ &+ 60\hat{h}_3^2 + 252\hat{h}_4^2 + 1296\hat{h}_4^3 + 576\hat{h}_3^2\hat{h}_4 + 24\hat{h}_4 + 3 \end{aligned} \right) &= \alpha_4 \end{aligned} \quad (11)$$

Equation (11) is the coupled nonlinear equation; thus, the coefficients are calculated numerically. The details can be found in Yang et al. [18].

The probability of deck slamming occurrence can be calculated from the statistical model of η_R . However, the estimation of deck slamming pressure requires an analysis of the relative wave velocity. Therefore, statistical modeling of a relative wave velocity is conducted by differentiating the statistical model of $\eta_R(t)$.

$$\dot{\eta}_R = \frac{dg(u)}{dt} = \kappa\sigma_{\eta_R} \left[1 + \sum_{n=3}^4 (n-1)\hat{h}_n He_{n-2}(u) \right] \dot{u} \quad (12)$$

In Equation (12), $\dot{\eta}_R$ is the relative wave velocity, and \dot{u} is the time differential of u . Here, it is well known that \dot{u} , the time differential of the standard normal variable, also follows the normal process and is statistically independent with u [19]. Equation (12) also shows that u and \dot{u} should be considered simultaneously when statistically analyzing $\dot{\eta}_R$. Thus, $\dot{\eta}_R$ is formulated by using the joint probability density function of u and \dot{u} , expressed as Equation (13).

$$P_n(u, \dot{u}) = \frac{1}{2\pi\sigma_{\dot{u}}} \exp \left[-\frac{1}{2} \left(u^2 + \frac{\dot{u}^2}{\sigma_{\dot{u}}^2} \right) \right] \quad (13)$$

In Equation (13), $\sigma_{\dot{u}}$ indicates the standard deviation of \dot{u} , and can be calculated from the second spectral moment of u as Equation (14). The spectral moment u can be obtained from Equations (15) and (16).

$$\sigma_{\dot{u}}^2 = \int_0^\infty S_{\dot{u}}(\omega) d\omega = \int_0^\infty \omega^2 S_u(\omega) d\omega = m_{u,2} \quad (14)$$

$$S_{\eta_R}(\omega) = \left| H_R^{(1)}(\omega) \right|^2 S(\omega) + 8 \int_0^\infty \left| H_R^{(2)}(\omega - \mu, \mu) \right|^2 S(|\omega - \mu|) S(|\mu|) d\mu \quad (15)$$

$$S_{\eta_R}(\omega) = \kappa\sigma_{\eta_R} \left[S_u(\omega) + \sum_{n=3}^4 (n-1)! \hat{h}_n^2 [S_u(\omega)]_{n-1} \right] \quad (16)$$

In Equations (15) and (16), $S_{\eta_R}(\omega)$ and $S_u(\omega)$ are the spectral densities of η_R and u , respectively, and $[S_u(\omega)]_n$ is the n -fold convolution. $S_{\eta_R}(\omega)$ is calculated using the linear and second-order frequency-domain transfer function as Equation (15), and $S_u(\omega)$ is computed from the relationship between the spectral density of η_R and u as shown in Equation (16) [20]. From these equations, the spectral moment u can be calculated using

Equation (17), and the details can be found in Lim and Kim [21]. Thus, we can obtain the value of $\sigma_{\dot{u}}$ from Equation (17).

$$\begin{aligned} m_{\eta_R,1} &= (\kappa\sigma_{\eta_R})^2 \left[(1 + 4\hat{h}_3^2 + 18\hat{h}_4^2) m_{u,1} \right] \\ m_{\eta_R,2} &= (\kappa\sigma_{\eta_R})^2 \left[(1 + 4\hat{h}_3^2 + 18\hat{h}_4^2) m_{u,2} + (4\hat{h}_3^2 + 36\hat{h}_4^2) m_{u,1}^2 \right] \end{aligned} \quad (17)$$

Finally, the joint probability density function of η_R and $\dot{\eta}_R$ can be formulated as follows by transformation from the joint probability density function of u and \dot{u} .

$$P(\eta_R, \dot{\eta}_R) = P_n(u, \dot{u}) \left| \frac{\partial(u, \dot{u})}{\partial(\eta_R, \dot{\eta}_R)} \right| = \left[\frac{1}{g'(g^{-1}(\eta_R))} \right]^2 P_n \left(g^{-1}(\eta_R), \frac{1}{g'(g^{-1}(\eta_R))} \dot{\eta}_R \right) \quad (18)$$

In the above equation, $\left| \frac{\partial(u, \dot{u})}{\partial(\eta_R, \dot{\eta}_R)} \right|$ indicates a Jacobian determinant, and $g^{-1}(\eta_R)$ is an inverse function of $g(u)$. The value of $g^{-1}(\eta_R)$ can also be formulated numerically from Equation (10), the details of which can be found in Yang et al. [18].

2.3. Prediction of Deck Slamming Occurrence

The deck slamming phenomenon occurs when η_R has a value over the deck clearance. Therefore, the probability of a deck slamming occurrence is calculated from the fact that the crest of η_R will be higher than the deck height. The wave crest defined by Forristall [22] is the maximum value of η_R when zero up-crossing; thus, the probability distribution of the crest of η_R can be calculated as the ratio of the average up-crossing rate to zero up-crossing rate [20].

$$\Pr(\text{wave crest} > z) = \frac{f_z^+}{f_0^+} = \exp \left\{ -\frac{1}{2} [g^{-1}(z)]^2 \right\} \quad (19)$$

$$f_z^+ = \int_0^\infty f_{z, \dot{\eta}_R}^+ d\dot{\eta}_R = \int_0^\infty P(z, \dot{\eta}_R) \dot{\eta}_R d\dot{\eta}_R \quad \dot{\eta}_R > 0 \quad (20)$$

In the above equation, f_z^+ is the rate when η_R passes the value of height z , and $f_{z, \dot{\eta}_R}^+$ is the rate when η_R passes the value of height z with velocity $\dot{\eta}_R$. To calculate the probability of a deck slamming occurrence, assuming that the offshore structure is operated with the deck clearance a_0 and duration T_R , a deck slamming occurring at least once can be interpreted as a complementary event of all wave crests of less than the deck height. Therefore, the probability of a deck slamming occurrence is calculated as follows:

$$\Pr[\max(\eta_R(t) | 0 \leq t \leq T_R) > a_0] = 1 - [\Pr(\text{wave crest} < a_0)]^{N_c} = 1 - \left[1 - \exp \left\{ -\frac{1}{2} [g^{-1}(a_0)]^2 \right\} \right]^{N_c} \quad (21)$$

$$N_c = \frac{T_R}{T_z} \quad (22)$$

where N_c is the number of wave crests, and T_z is the zero up-crossing period.

Next, the deck slamming pressure is calculated from the relation between the impact pressure and relative wave velocity. First, the probability distribution of upward passing $\dot{\eta}_R$ when η_R has the value of the deck clearance, that is, when a deck slamming occurs, can be obtained as follows:

$$P(\dot{\eta}_R | \eta_R = a_0) = \frac{f_{a_0, \dot{\eta}_R}^+}{f_{a_0}^+} = \frac{P(a_0, \dot{\eta}_R) \dot{\eta}_R}{\int_0^\infty P(a_0, \dot{\eta}_R) \dot{\eta}_R d\dot{\eta}_R} = \frac{1}{\sigma_{\dot{u}}^2 [g'(g^{-1}(a_0))]^2} \dot{\eta}_R \exp \left[-\frac{1}{2} \left(\frac{\dot{\eta}_R}{\sigma_{\dot{u}} [g'(g^{-1}(a_0))]} \right)^2 \right] \quad \dot{\eta}_R > 0 \quad (23)$$

The deck slamming pressure can be estimated using the time variation of momentum proposed by Kaplan [23], or the empirical model of Cuomo [24]; however, the equation from Det Norske Veritas (DNV) [25] is used in this study because of the limitation on the

analysis of the deck slamming area and wetting time. DNV proposed a simple method to estimate the impact pressure as follows:

$$P_s = \frac{1}{2} \rho C_V [\dot{\eta}_R | \eta_R = a_0]^2 \quad (24)$$

In the above equation, P_s is the slamming pressure, and C_V is the slamming coefficient. C_V is dependent on the local geometry information at the contact point of wave and deck such as local shape and surface slope/angle. DNV proposed using 0.5 for C_V in the head sea and 10.0 for oblique waves at 45° , and the present study also adopts these values because the aim of this study is not to accurately predict the pressure but to propose a simple statistical method for prediction. From Equations (23) and (24), the probability distribution of the slamming pressure is calculated.

$$P(P_s) = \frac{1}{\rho C_V \sigma_u^2 [g'(g^{-1}(a_0))]^2} \exp \left[-\frac{1}{\rho C_V \sigma_u^2 [g'(g^{-1}(a_0))]^2} P_s \right] \quad (25)$$

To provide the reference impact pressure for the determination of the deck clearance, an analysis taking into account both the probability of the deck slamming occurrence and slamming pressure is necessary. Therefore, the maximum average impact pressure is presented in this study. During time T_R , the number of deck slamming occurrences N can be calculated from Equations (21) and (22).

$$N = \Pr(\text{wave crest} > a_0) \times N_c = \exp \left\{ -\frac{1}{2} [g^{-1}(a_0)]^2 \right\} \frac{T_R}{T_z} \quad (26)$$

From Equations (25) and (26), the mean value of the impact pressure corresponding to the upper probability of one over N times can be calculated. In this study, this value is defined as a term of the maximum average impact pressure, and implies maximum impact pressure the offshore platform may experience during time T_R .

$$P_s^{1/N} = \rho C_V \sigma_u^2 [g'(g^{-1}(a_0))]^2 \ln N \quad (27)$$

$$\overline{P_s^{1/N}} = P_s^{1/N} + \rho C_V \sigma_u^2 [g'(g^{-1}(a_0))]^2 \quad (28)$$

The overall procedure for this analysis can be summarized as follows:

Using the input data of offshore structure, compute the linear and quadratic transfer function for motion responses and relative wave elevation at the specified points;

- Using the input data of wave spectrum, generate the Hermit Kernel;
- Solve the eigenvalue problem and obtain the eigenvalues;
- Calculate the probability density function using characteristic function and statistical moments;
- Generating mapping function for Hermit-moment method;
- Predict the probability of deck slamming occurrence.

It must be mentioned that this analysis aims at the prediction of deck slamming occurrence, not the analysis of whole slamming occurrence. That is, the fluid-structure interaction during deck slamming is beyond of this study.

3. Validation of Proposed Method

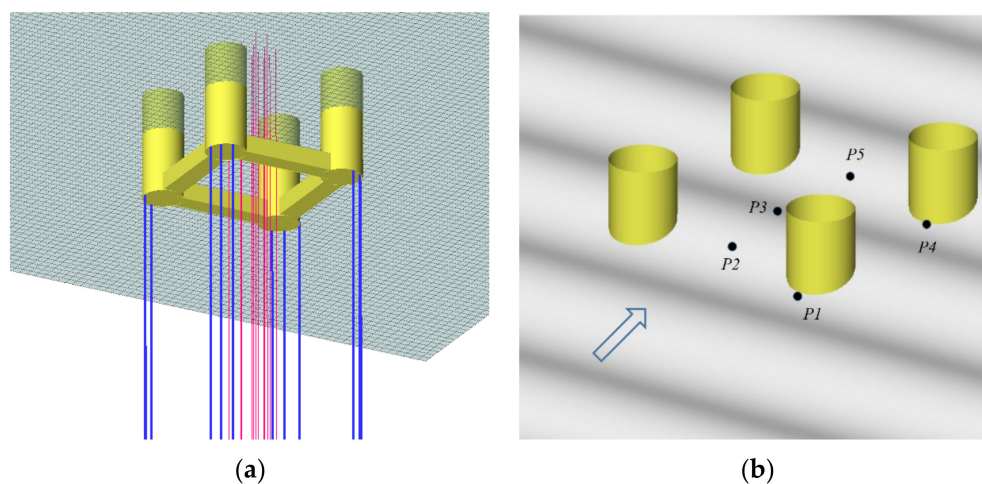
3.1. Computational Model

In this study, the proposed stochastic approach is applied to the TLP model to verify the method and investigate the effect of a nonlinear relative wave on the deck slamming occurrence. The TLP model consists of four cylindrical columns and rectangular pontoons and the major dimensions of the TLP is summarized in Table 1 [14,26].

Table 1. Principal dimensions of tension leg platform (TLP) model.

Column diameter	19.52	m
Pontoon length	41.48	m
Draft	31.42	m
Deck clearance	30.00	m
Displacement	35290	ton
Total length of tendons	1800	m
Pre-tension	10960	kN

Figure 2a shows the geometry of the TLP model, and Figure 2b shows the analysis points where the statistical method used are applied. P1 and P4 indicate the front of the fore and rear columns, and P2 and P5 are the middle of the fore and rear pontoons, respectively. In addition, P3 is the center of the deck. In this study, the application of the TLP focuses on a specific random wave condition as an example case. To this end, the JONSWAP spectrum with a significant wave height of 13 m and a peak period of 14 s is considered with a coefficient γ of 3. Nam et al. [5] carried out different sea state for the same model, and it was found that the condition showed the highest probability of deck slamming among the Northern North Sea. In this study, this wave condition is chosen for application example. It is assumed that the wave heading is 0° , as shown in Figure 2b. Table 2 shows the locations of P1~P5.

**Figure 2.** Geometry of computational model. (a) is TLP model, and (b) is five analysis locations around the platform.**Table 2.** Locations of the observation points.

Observation Point	(x, y, z) (m)
P1	(40.27, 30.50, 0.00)
P2	(30.50, 0.00, 0.00)
P3	(0.00, 0.00, 0.00)
P4	(−20.74, 30.50, 0.00)
P5	(−30.50, 0.00, 0.00)

The frequency-domain transfer functions at the analysis points are computed using the commercial software program WADAM developed by DNV-GL. As well known, WADAM is one of the most reliable frequency-domain solvers up to the second-order problem. This software adopts the potential-based approach which applies the wave Green function on the wetted body surface. In the case of the second-order solver, the distribution of the Green function on free surface near the body is needed. From these analyses, the frequency-domain transfer functions of platform and absolute wave elevation can be calculated. The

detailed theory for solving the second-order radiation/diffraction analysis can be found in a study by Lee [27].

Figure 3 shows the computed linear frequency-domain solutions of motion responses and the relative wave elevation η_R at the observed points. The motion responses in Figure 3a show the typical pattern of TLP under a strong underwater pretension on the body. Consequently, as indicated in Figure 3b, the value of the linear transfer function around the low frequency converges to 1.0 at all points. This indicates that the vertical motion in the area of long wavelength is small, and thus the relative wave elevation is similar to the absolute wave elevation.

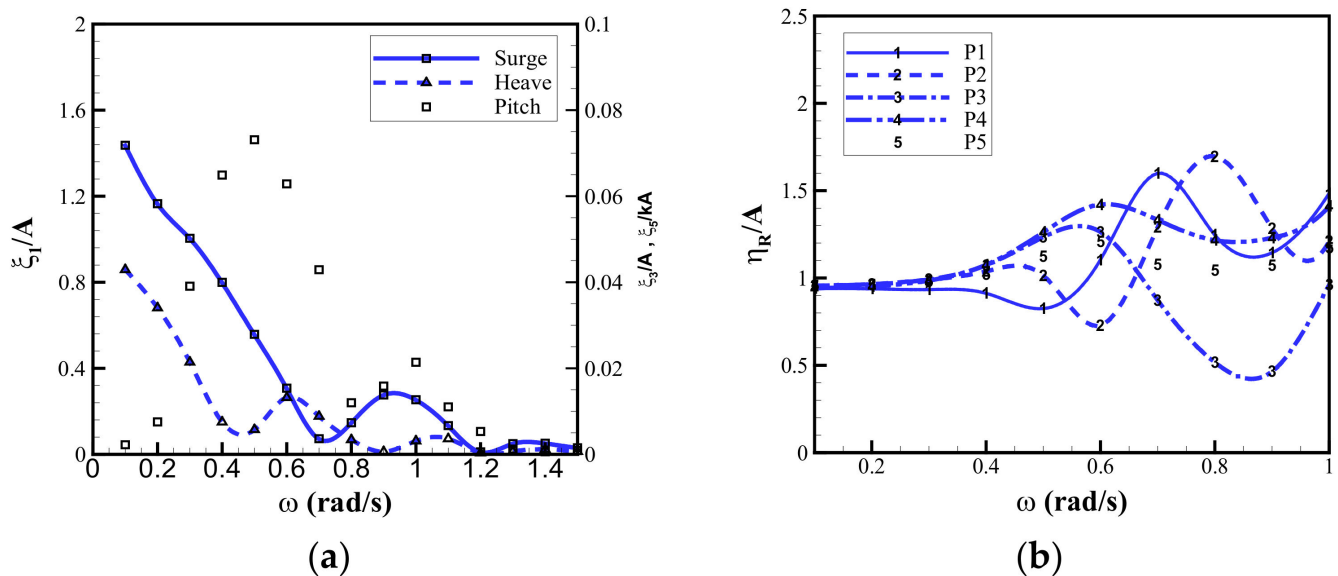


Figure 3. Magnitude of linear RAO (response amplitude operator) of motion and relative wave elevation at five locations. (a) is platform motion, and (b) is relative wave elevation.

It should be noted that the responses of the motion and of relative elevation are dependent on the wave spectrum. That is, the frequency range within which the wave energy concentrates will play a critical role in the actual responses of the motion and wave elevation. For example, if the energy of the ocean wave is confined to the range of 0.3–0.6 rad/s, the linear component of η_R can be predicted to be larger at P4 than at the other points. It is believed that there is an effect of standing wave between two columns, which results in larger wave run-up than other locations. This is a very interesting results which are related to so-called the trapped mode of offshore columns, and it can be investigated more thoroughly as another research theme.

Figure 4 shows the quadratic transfer functions (QTFs) of the difference and sum-frequency components of the relative wave elevation. The computational results in Figure 4 are corresponding to the analysis on P1–P5. Comparing the frequency-domain transfer functions of P1–P5, it can be seen that the sum-frequency components fluctuate more clearly than the difference-frequency QTFs. This is mostly due to more local behavior of the absolute wave elevation around the body in the case of the sum-frequency solution. Figure 5 shows the Hermitian kernels at the locations P1, P3, and P5, respectively.

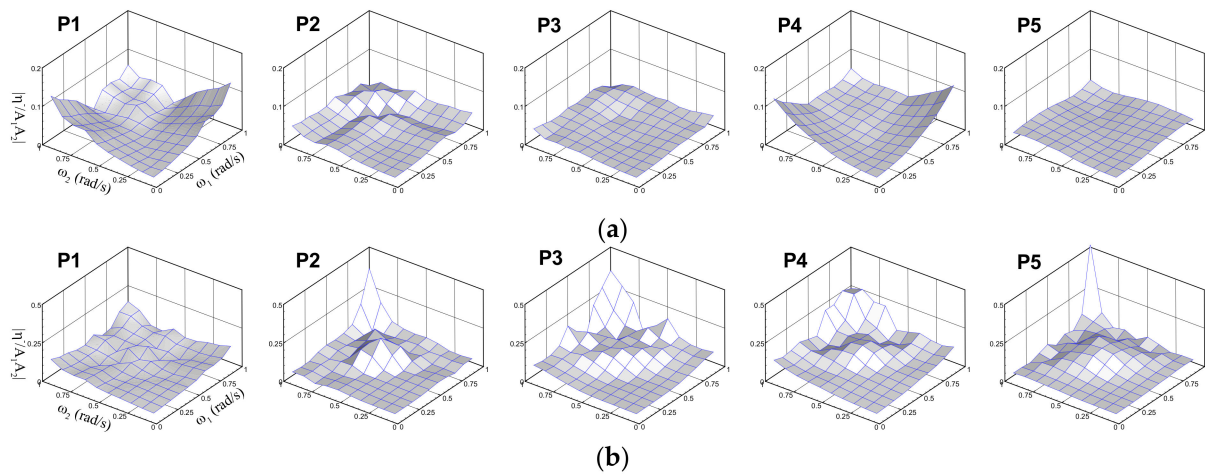


Figure 4. Quadratic transfer functions of relative wave elevation at five points. (a) is difference-frequency component, and (b) is sum-frequency component.

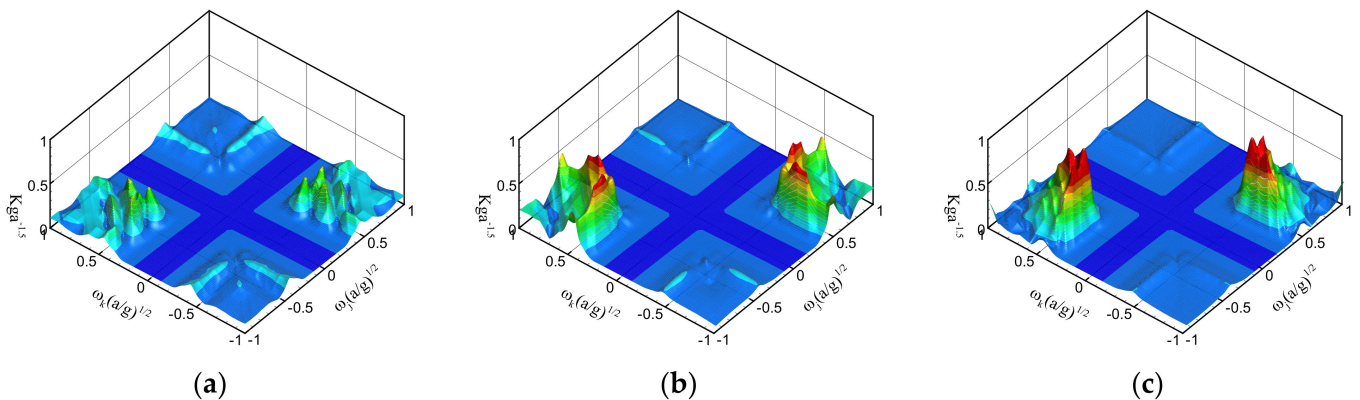


Figure 5. Hermitian kernel. (a–c) are the calculated Hermitian kernel on P1, P3, and P5.

3.2. Verification of Present Method

Validation of the proposed statistical model is conducted by comparing the derived probability distribution and data sampled from a time series of η_R . Because the presented method is based on the joint probability density function, the distribution in two-dimension should be compared; however, the conditional probability distributions are validated for easier comparison. The probability of deck slamming occurrence is related to η_R ; thus, it is verified by comparing the wave crest distribution of η_R . Moreover, the deck slamming pressure is related to $\dot{\eta}_R$, and thus it is verified by comparing the conditional probability distribution of $\dot{\eta}_R$ and the upward passing velocity distribution.

First, as mentioned earlier, the validation of η_R is conducted by comparing the derived wave crest distribution with the sampled data. The sampled data are extracted from the 9-h time series of η_R , expressed as Equation (2), and these data are compared with the crest distribution of η_R calculated from Equation (19). This analysis was conducted at the analysis point P4, where the nonlinear component is expected to be larger.

Figure 6 shows examples of time signals of the linear, second-order difference-frequency and sum-frequency components, as well as the total elevation of vertical motion and the relative wave elevation at P4. All these signals are from the time conversion of frequency-domain solutions obtained by using the WADAM program.

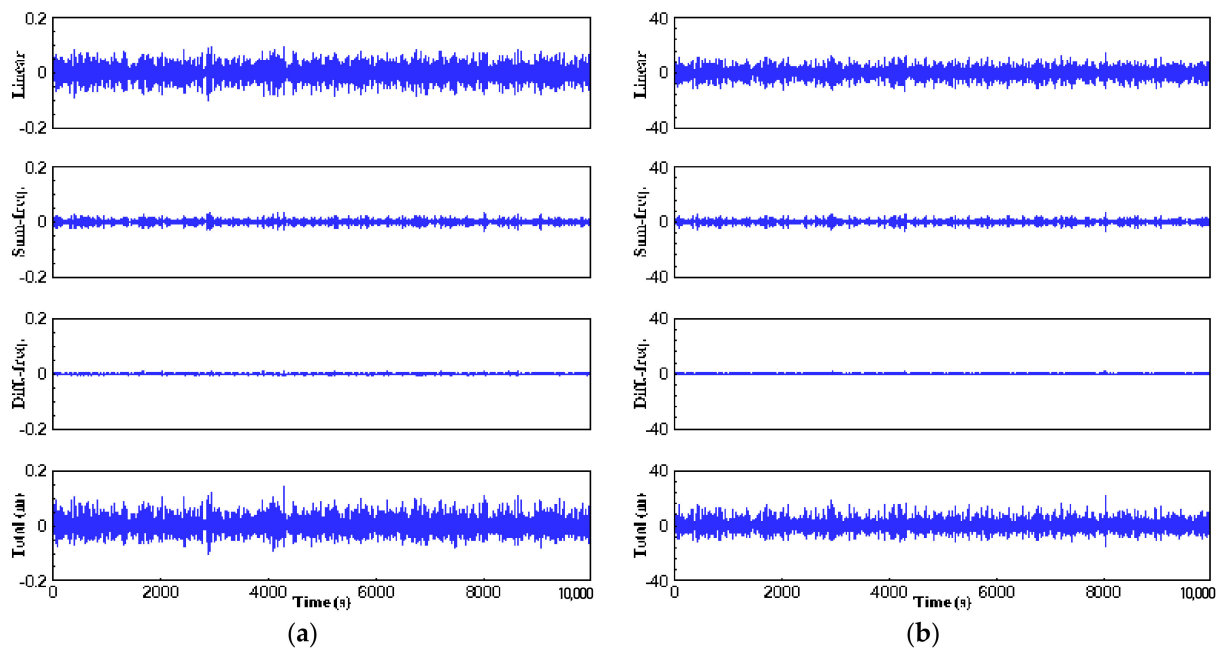


Figure 6. Components of platform motion and relative wave elevation signals at P4. (a) is vertical platform motion signals, and (b) is relative wave elevation signals.

It is clear that the sum-frequency component is the major contributor to the second-order effect. Figure 7 shows the exceedance probability distribution of crest of η_R at P4. Because η_R contains nonlinear components up to the second order, it has non-normal properties. Therefore, the Rayleigh distribution, which represents the peak distribution of linear signals, is insufficient for a nonlinear wave elevation. As Figure 7 indicates, the present nonlinear scheme achieves a much better correspondence with the sampled peak distribution from the actual time signal. For a quantitative comparison, the statistical moments of the sampled data and the derived probability distribution of η_R are summarized in Table 3. It is confirmed that the statistical results of the present analytical method show a very reasonable agreement with the results obtained from the sampled data.

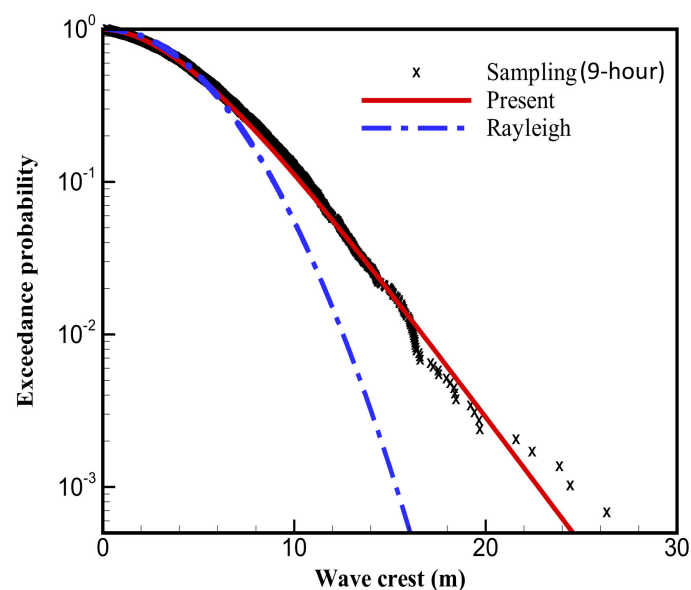


Figure 7. Exceedance probability distribution of crest of η_R : P4 location, sampling results are from 9-h signal.

Table 3. Statistical moments of probability distribution of $\dot{\eta}_R$: P4 location, sampling results are from 9-h signal.

Parameter	Sampled from Signal (S)	Present Analytic Method (A)	Ratio (A/S)
Mean	0.19	0.20	1.05
Standard deviation	4.05	4.08	1.01
Skewness	0.48	0.52	1.08
Kurtosis	3.47	3.96	1.14

Figure 8 shows the probability distribution of $\dot{\eta}_R$ at P4, based on a total of 1000 h of time series data of η_R . In this case, the relative wave velocities are for the cases when η_R is more than 5 and 10 m, considering up to the second-order component. Figure 9 also shows the probability distribution of $\dot{\eta}_R$ at P4. These cases are conditional probabilities for the upward passing velocity value of η_R at 5 and 10 m. That is, the values of $\dot{\eta}_R$ are considered when η_R is over the specified value, which aims to consider the condition of a slamming occurrence, where η_R is higher than the freeboard of the deck. Values of 5 and 10 m were chosen in this study for validation purposes. As Figures 8 and 9 show, it is obvious that the probability model from the present method is not perfect but expresses well the conditional probability of $\dot{\eta}_R$. It is clear that the present statistical method predicts higher upward passing values. This difference may be mostly due to the limit of the present analytic method, particularly in mapping the non-Gaussian process to the standard normal process $u(t)$ by using a Hermite-polynomial expansion. The slight differences in the mean and skewness in Figure 8 cause the difference in the up-crossing statistics in Figure 9. However, it must be noted that the present method is purely analytic, and no full simulation or time-signal generation is needed. Therefore, this present method can be accepted as an extremely good approximation without a heavy simulation.

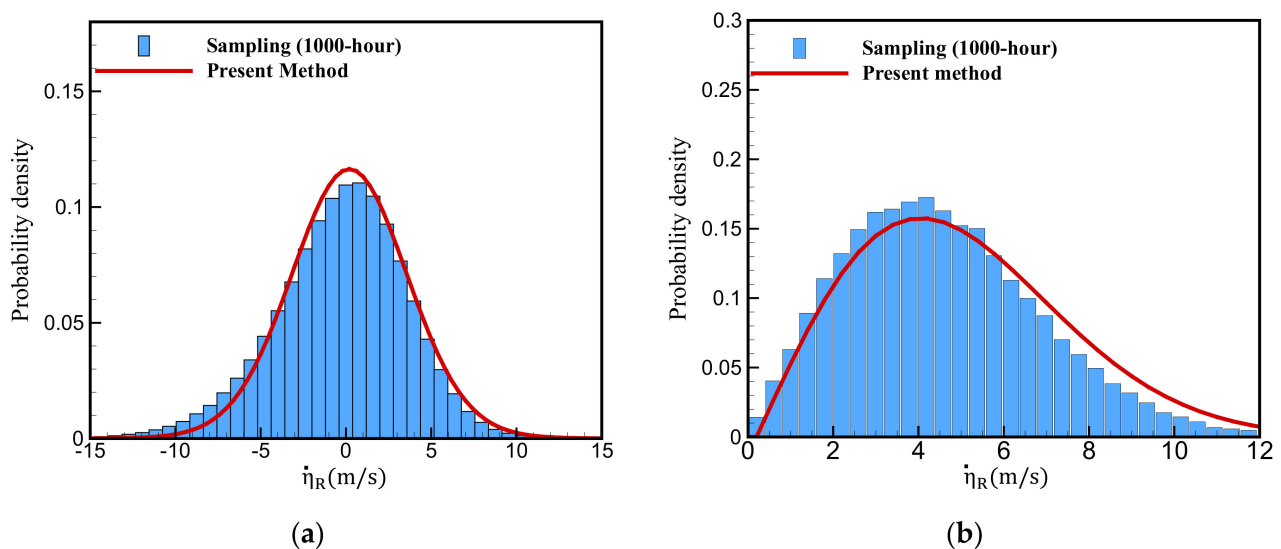


Figure 8. Conditional probability distribution of $\dot{\eta}_R$: P4. (a) is the case when η_R is more than 5 m, and (b) is the case when η_R is more than 10 m.

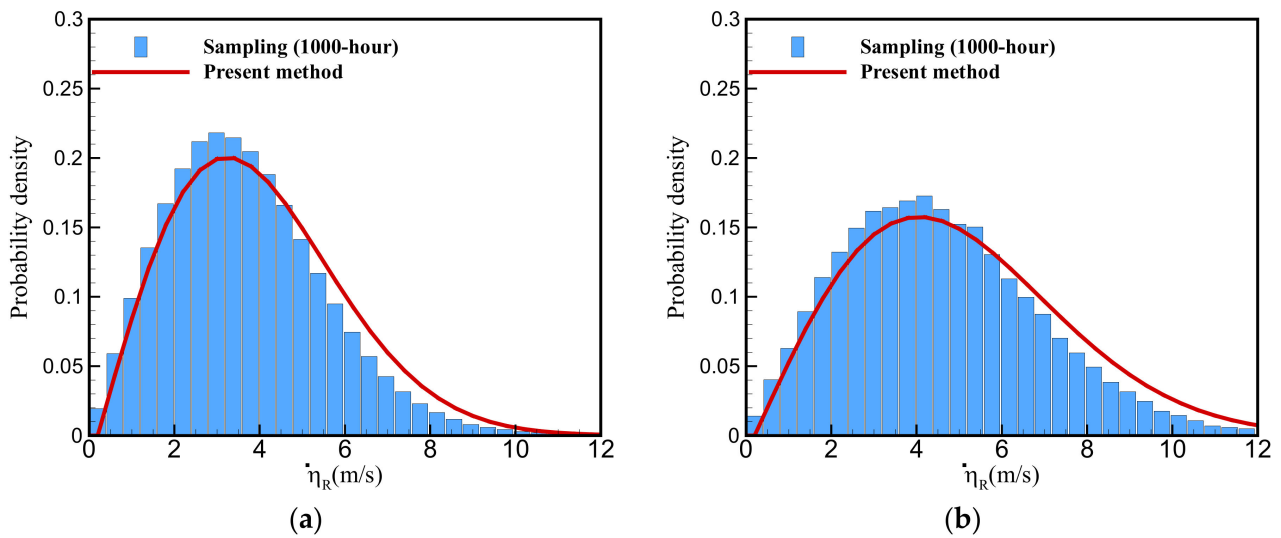


Figure 9. Conditional probability distribution of $\dot{\eta}_R$: P4. (a) is the case when η_R at 5 m, and (b) is the case when η_R at 10 m.

In the case of linear theory, the relative wave elevation and velocity are statistically independent. However, in the case of nonlinear problems, such an assumption is invalid. Figure 10 shows the cumulative probability distribution of $\dot{\eta}_R$ for η_R at P4 with and without second-order components. In the case of the linear component of η_R , it can be seen that the distribution of $\dot{\eta}_R$ is almost the same regardless of the condition of η_R . This result corresponds with the linear theory, in which the wave elevation and wave velocity are statistically independent. The methodology of Ochi and Motter [11] is based on this theory, and in this case, the slamming impact pressure distribution is also the same regardless of η_R . However, considering up to the second-order component of η_R , the distribution of $\dot{\eta}_R$ is changed for each condition of η_R . This means that if nonlinear components are included, η_R and $\dot{\eta}_R$ are not statistically independent, and the second-order components of η_R can affect the probability distribution of the deck slamming pressure.

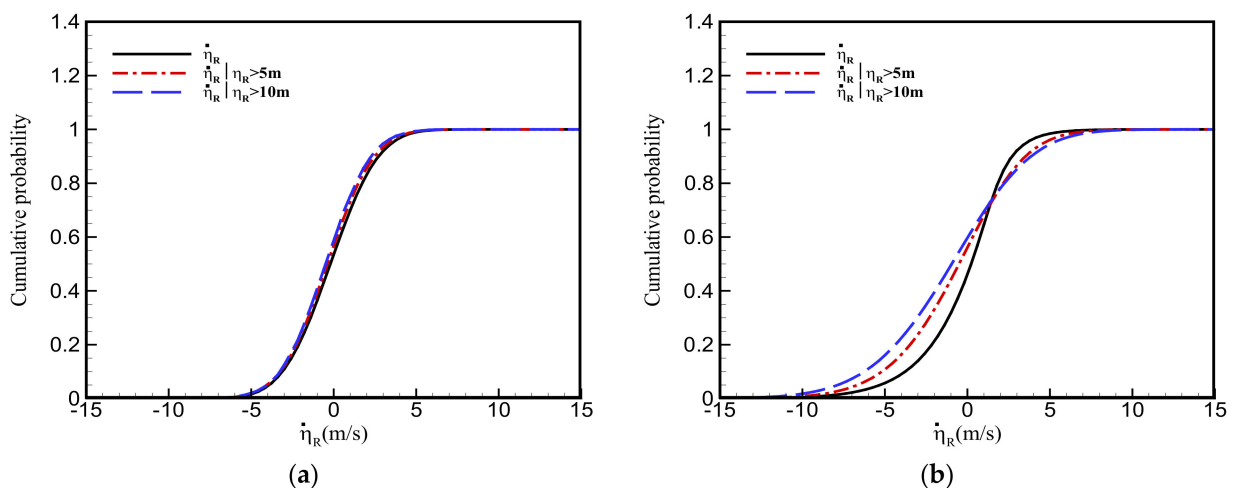


Figure 10. Cumulative probability distribution of $\dot{\eta}_R$ for η_R depending on the presence of 2nd-order components: P4. (a) is the case only considering the linear component, and (b) is the case considering the linear and 2nd-order component.

3.3. Prediction of Deck Slamming Occurrence

To observe the occurrence of deck slamming, the exceedance probability of the relative wave elevation is predicted using the verified statistical method for the 3-h duration of the sea states, T_R . As analyzed by Nam et al. [5], the wave elevations become largest at P4

and the smallest at P1. Figure 11 shows the exceedance probability distribution of deck slamming occurrence according to the deck clearance calculated from Equation (21). In this study, a specific height is not given as a freeboard height, and the exceedance probability of η_R for different deck clearances is observed and can thus be helpful for examining the design of the deck height. Figure 11 compares the probability of deck clearance when only linear theory is applied, and all linear and second-order components are considered. Linear theory clearly provides an extreme under-prediction, and thus the second-order component is important.

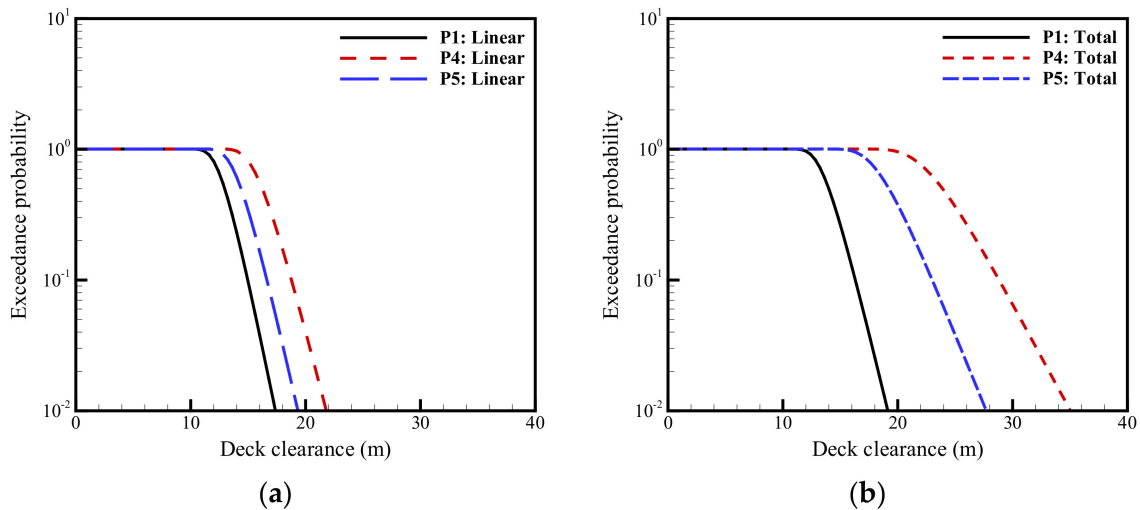


Figure 11. Exceedance probability distribution of deck slamming occurrence in 3-h according to deck clearance. (a) is the case only considering the linear component, and (b) is the case considering the linear and 2nd-order component.

Figure 12 shows an example of the more detailed components at P4. It is well known that the sum-frequency component plays an important role in the wave run-up around a body, e.g., [4]. This trend is clearly shown in Figure 12. The sum-frequency component shares a major contribution with a linear wave run-up. A comparison of the exceedance probability for different deck clearances at each point is shown in Figure 13. This is the same result as in Case 2 of Nam et al. [5]. As mentioned, P4 was shown to have the highest possibility of deck slamming, and P1 has the least. As Nam et al. [5] showed, this tendency is the same for other sea states.

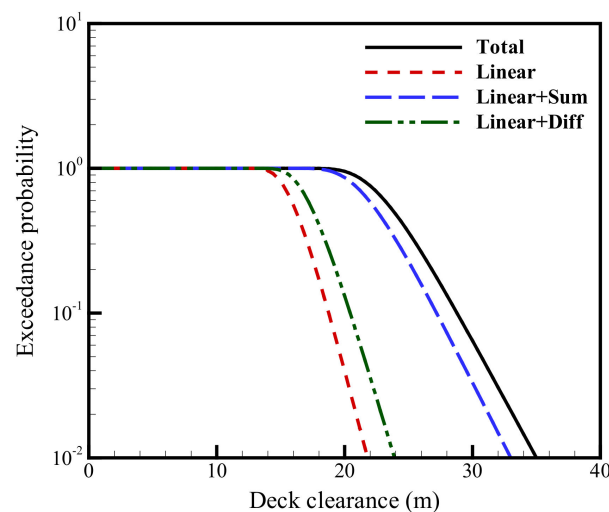


Figure 12. The detailed component of exceedance probability distribution: P4.

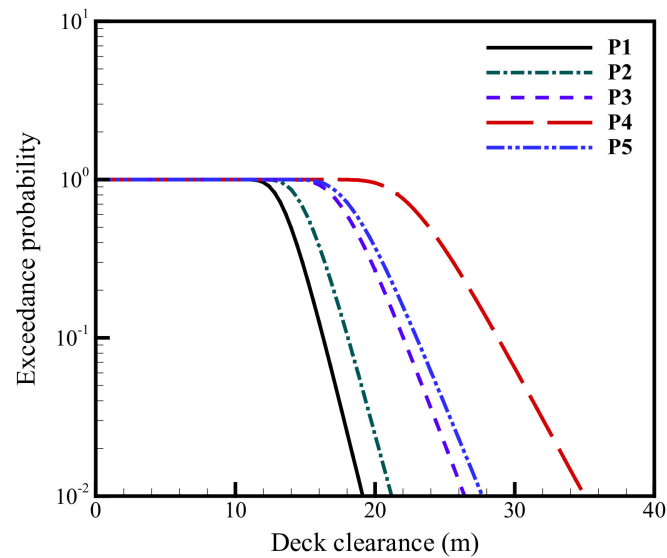


Figure 13. Comparison of exceedance probability at five locations.

The slamming pressure can be predicted using Equation (28). Figure 14 shows the predicted maximum slamming pressure over a 3-h duration at P4, where the largest slamming pressure is highly probable. It should be mentioned that the largest wave elevation does not guarantee the largest impact pressure. The slamming pressure is dependent on the magnitude of the relative velocity at the slamming occurrence. For the same reason, the slamming pressure is different for different deck clearances. Figure 14 shows the range of deck clearance and corresponding pressure at which deck slamming may occur more than once within a 3-h time period. In particular, this figure compares the slamming pressure with and without the second-order components. The difference is extremely clear, and thus the linear solution is insufficient. The same observation can be found in Figure 15, which shows the slamming pressure for different exceedance probabilities. In this case, a 15-m deck height is assumed. The difference with and without the second-order contribution is significant, as indicated in Figure 14.

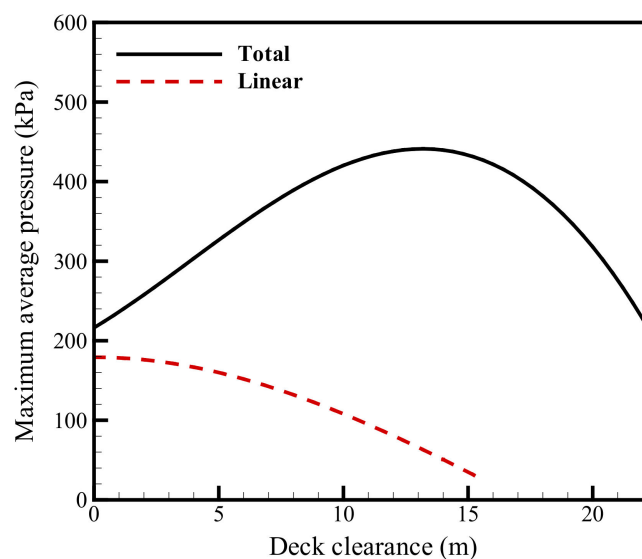


Figure 14. Comparison of maximum slamming pressure between linear and total solutions: P4, for different deck clearance.

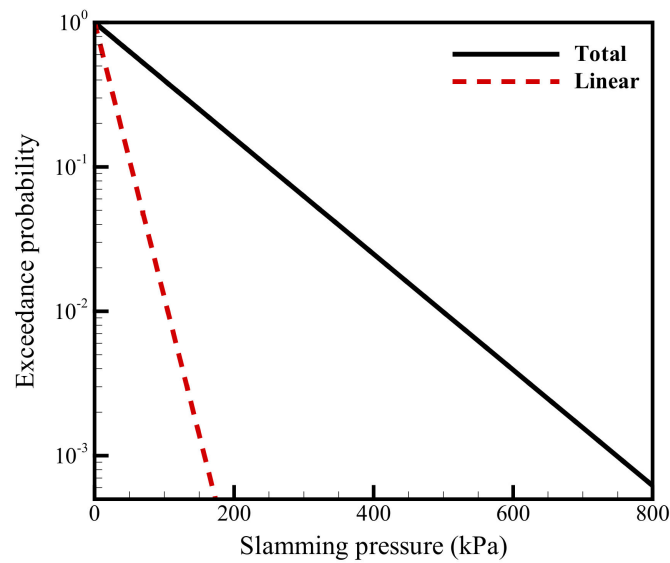


Figure 15. Comparison of slamming pressure between linear and total solutions: P4, 15 m deck clearance.

From Figure 14, it can be seen that the slamming pressure does not always increase as the deck clearance increases. There is a peak in the slamming pressure. In general, a larger wave elevation has a higher potential for a larger velocity; therefore, a larger slamming pressure can be assumed for a larger deck clearance. However, this is the case when the probability of a slamming occurrence is the same. Under actual conditions, a larger deck clearance provides less possibility of slamming. That is, the value of N in Equation (26) decreases. An easier interpretation is possible using Figure 16, which shows the pressure at deck slamming occurrence. Here, the pressure is defined as the value averaged from one over N time occurrence of deck slamming. In these figures, A, B, C, and D represent deck clearances of 5, 10, 15, and 20 m, respectively. The pressure at D, that is, a 20-m deck clearance, is less than that of B and C. Looking at the right-side result in Figure 16, it can be seen that the exceedance probability value of D is much larger than that of B and C, meaning that the probability of one over N is larger as slamming occurrence number N is lower. Thus, the maximum average pressure does not show a monotonic increase or decrease as the deck clearance increases.

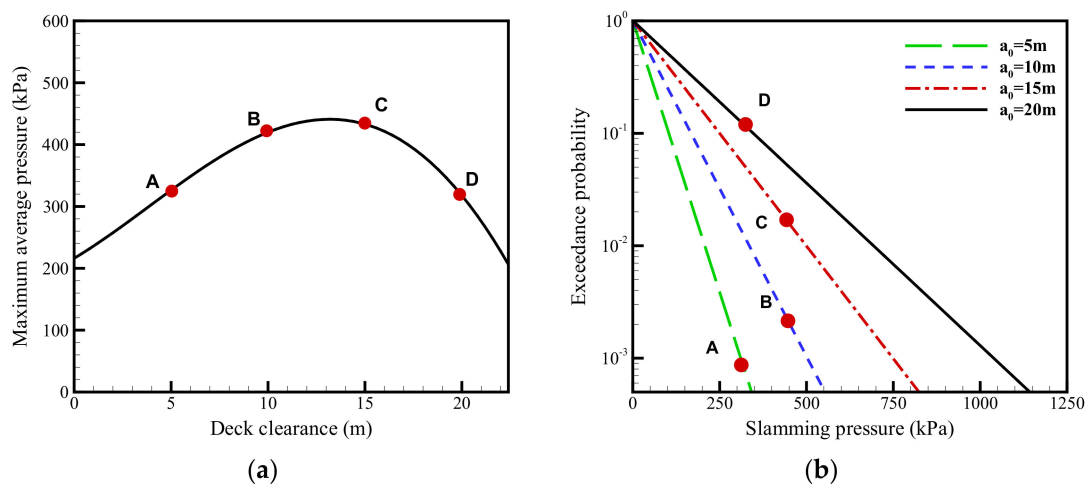


Figure 16. Maximum average pressure and slamming pressure distribution for different deck heights: P4. (a) is the slamming pressure and (b) is the corresponding probability.

4. Conclusions

In this study, a nonlinear statistical method is proposed to predict the deck slamming phenomenon, considering the second-order term of the relative wave elevation. To verify the proposed method, the probability distribution of the relative wave elevation and the velocity are compared with the sampled data extracted from the two-term Volterra time series. The proposed stochastic approach is applied to the TLP model, and the deck slamming occurrence is predicted for a total of five analysis points. Furthermore, the effect of second-order components on the probability of deck slamming occurrence and slamming pressure are investigated. From this study, the following conclusions were drawn:

- Owing to the second-order effect, the probability distribution of the relative wave elevation shows the non-normality of the statistical properties. The proposed method represents such characteristics well. The same result is obtained from the crest distribution.
- It can be confirmed that the relative wave velocity is not statistically independent of the relative wave elevation when considering up to the second-order component. Thus, the probability distribution of the relative wave velocity changes according to the conditions of the relative wave elevation, and the proposed method also represents this dependency well. However, because of the limitation of statistical modeling of the relative wave velocity, there was some difference in estimating the probability distribution of upward passing relative wave velocity.
- From the predicted probability of deck slamming occurrence, it can be seen that a larger probability is shown in the rear part of the offshore platform rather than at the fore part. The largest probability of deck slamming was observed in the front of the rear column. This result is due to an increase in the second-order components of the relative wave elevation at the rear part of the offshore structure.
- It can be seen that the second-order component, particularly the sum-frequency component, plays a critical role in the slamming pressure. For the predicted maximum average pressure during the duration of the sea states, the impact pressure increases as the deck clearance rises to a certain value, and decreases thereafter. The increase and increase of the slamming pressure are dependent on not only the magnitude of the relative velocity but also the probability of occurrence.

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References

1. American Bureau of Shipping. *Air Gap and Wave Impact Analysis for Semi-Submersibles*; American Bureau of Shipping: Houston, TX, USA, 2018.
2. Lloyd's Register. *Rules and Regulations for Classification of Offshore Units*; Lloyd's Register: London, UK, 2018.
3. Det Norske Veritas (DNV-GL). *Structural Design of Offshore Units—WSD Method*; DNV-GL: Oslo, Norway, 2017.
4. Kriebel, D.L. Nonlinear wave interaction with a vertical circular cylinder. Part II: Wave Run-up. *J. Ocean Engin.* **1992**, *19*, 75–99. [[CrossRef](#)]
5. Nam, H.; Lim, D.H.; Kim, Y. Stochastic approach on the prediction of deck slamming occurrence for column-stabilizing platforms. In Proceedings of the Offshore Technology Conference Asia 2018, Kuala Lumpur, Malaysia, 20–23 March 2018.

6. Kac, M.; Siegert, A.J.F. An explicit representation of a stationary Gaussian process. *Ann. Math. Statist.* **1947**, *18*, 438–442. [[CrossRef](#)]
7. Bedrosian, E.; Rice, S.O. The output properties of volterra systems (nonlinearsystems with memory) driven by harmonic and gaussian inputs. *Proc. IEEE* **1971**, *59*, 1688–1707. [[CrossRef](#)]
8. Neal, E. Second-order hydrodynamic forces due to stochastic excitation. In Proceedings of the 10th Symposium on Naval Hydrodynamics, Cambridge, MA, USA, 24–28 June 1974.
9. Marthinsen, T.; Winterstein, S.R. *Second-Order Load and Response Statistics for Tension-Leg Platforms*; RMS-9, Rel. Marine Struct. Prog.; Stanford University: Stanford, CA, USA, 1992.
10. Ochi, M.K. Prediction of occurrence and severity of ship slamming at sea. In Proceedings of the 5th Symposium on Naval Hydrodynamics, Bergen, Norway, 10–12 September 1964.
11. Ochi, M.K.; Motter, L.E. Prediction of Slamming Characteristics and Hull Responses for Ship Design. In Proceedings of the Annual Meeting of SNAME, New York, NY, USA, 15–17 November 1973.
12. Manuel, L.; Sweetman, B.; Winterstein, S.R. Analytical predictions of the air gap response of floating structures. *J. Offshore Mech. Arct. Eng.* **2001**, *123*, 112–117. [[CrossRef](#)]
13. Sweetman, B.; Winterstein, S.R. Non-Gaussian air gap response models for floating structures. *J. Eng. Mech.* **2003**, *129*, 302–309. [[CrossRef](#)]
14. Nam, H. Stochastic Approach on the Prediction of Deck Slamming Occurrence and Impact Pressure for Offshore Structures. Master’s Thesis, Seoul National University, Seoul, Korea, February 2019.
15. Naess, A. The statistical distribution of second-order slowly-varying forces and motions. *Appl. Ocean Res.* **1986**, *8*, 110–118. [[CrossRef](#)]
16. Winterstein, S.R.; Ude, T.C.; Kleiven, G. Springing and slow-drift responses: Predicted extremes and fatigue vs. simulation. In Proceedings of the 7th International Conference on the Behaviour of Offshore Structures, Cambridge, MA, USA, 12–15 July 1994.
17. Gurley, K.R.; Tognarelli, M.A.; Kareem, A. Analysis and simulation tools for wind engineering. *Probabilistic Eng. Mech.* **1997**, *12*, 9–31. [[CrossRef](#)]
18. Yang, L.; Gurley, K.R.; Prevatt, D.O. Probabilistic modeling of wind pressure on low-rise buildings. *J. Wind Eng. Ind. Aerodyn.* **2013**, *114*, 18–26. [[CrossRef](#)]
19. Kim, C.H. *Nonlinear Waves and Offshore Structures*, 1st ed.; World Scientific: Singapore, 2008; p. 23.
20. Winterstein, S.R. Nonlinear vibration models for extremes and fatigue. *J. Eng. Mech.* **1988**, *114*, 1772–1790. [[CrossRef](#)]
21. Lim, D.H.; Kim, Y. A comparative study of probabilistic models for second-order hydrodynamic responses of offshore platforms. In Proceedings of the 32nd International Workshop on Water Waves and Floating Bodies, Dalian, China, 23–36 April 2017.
22. Forristall, G.Z. Wave crest distributions: Observations and second-order theory. *J. Phys. Oceanogr.* **2000**, *30*, 1931–1943. [[CrossRef](#)]
23. Kaplan, P.; Murray, J.J.; Yu, W.C. Theoretical Analysis of Wave Impact Forces on Platform Deck Structures. In Proceedings of the 14th International Conference on Offshore Mechanics & Arctic Engineering, Copenhagen, Denmark, 18–22 June 1995.
24. Cuomo, G.; Tirindelli, M.; Allsop, N.W.H. Wave-in-deck loads on exposed jetties. *Coast. Eng.* **2007**, *54*, 657–679. [[CrossRef](#)]
25. Det Norske Veritas (DNV). *Recommended Practice—Environmental Conditions and Environmental Loads*; DNV-GL: Oslo, Norway, 2010.
26. Lim, D.H. Study on Statistical Analysis of Nonlinear Hydrodynamic Loads and Responses for the Design of Floating Offshore Structures 2019. Ph.D. Thesis, Seoul National University, Seoul, Korea, February 2019.
27. Lee, C.H. *WAMIT Theory Manual*; Massachusetts Institute of Technology: Cambridge, MA, USA, 1995.