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Keywords: carbon tax, carbon cap-and-trade, sustainable production–inventory model

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Article

Sustainable Production–Inventory Model in Technical Cooperation on Investment to Reduce Carbon Emissions

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Abstract: Carbon cap-and-trade and carbon offsets are common and important carbon emission reduction policies in many countries. In addition, carbon emissions from business activities can be effectively reduced through specific capital investments in green technologies. Nevertheless, such capital investments are costly and not all enterprises can afford these investments. Therefore, if all members of a supply chain agree to share the investments in the facilities, the supply chain can reduce carbon emissions and generate more profit. Under carbon cap-and-trade and carbon tax policies, this study proposes a production–inventory model in which the buyer and vendor in the integrated supply chain agree to co-invest funds to reduce carbon emissions. We planned to integrate production, delivery, replenishment, and technology to reduce carbon emissions so as to maximize the total profit of the supply chain system. Several examples are simulated and the sensitivity analysis of the main parameters is carried out. The optimal solutions and joint total profit under various carbon emission policies are also compared. The future carbon emission control trend is expected to enable companies to share risks by co-investing and developing sustainable supply chains.

Keywords: sustainable production–inventory model; carbon cap-and-trade; carbon tax

1. Introduction

Since the industrial revolution, the manufacturing industry has flourished in many countries, resulting in factories producing large amounts of carbon dioxide. In previous eras of economic development, little attention was drawn to gases discharged from manufacturing processes. Carbon emissions from the processes such as production, inventory management, sales, and transportation are the primary sources of greenhouse gasses. Further, factories are one of the main sources of carbon dioxide that causes environmental changes such as ozone layer depletion, the greenhouse effect, and acid rain. This destruction of nature will eventually threaten human health. Therefore, the manufacturing industry must heed the effects of the entire product lifecycle on the environment. Global warming, a toxic environment, and the destruction of the ozone layer are threats to humans and animals. Although carbon emissions have long increased economic growth, authorities should formulate suitable policies to limit its impact on society [1–3]. If we continue to focus only on economic development without considering the ecological impact of manufacturing, these threats will become even more serious.

To reduce gas pollution emissions, developed countries have begun to discuss related threats and establish emission standards. A contract called the "Kyoto Protocol," which regulates the emission of greenhouse gases, was established in 1997 in Kyoto, Japan, and implemented in 2005. In total, 84 countries committed to this protocol. In accordance with these regulations, member governments have actively structured standards to limit greenhouse gas emissions.

These standards include developing new alternative or renewable energy sources (e.g., solar energy), establishing energy saving and carbon reduction regulations (e.g., China's energy conservation law), promoting carbon trading markets (e.g., the European Union's EU Emissions Exchange, the Chicago Climate Exchange, the Tianjin Emissions Exchange in China), and developing carbon taxes (e.g., the UK's climate change tax). In 2015, the United Nations Climate Change Conference (Paris Agreement) was held in Le Bourget, France, which included the 21st Meeting of the Parties to the United Nations Framework Convention on Climate Change and the 11th Meeting of the Parties to the Kyoto Protocol. The expiration of the Kyoto Protocol was extended to 2020, and the agreement of the greenhouse gas reduction standard was approved. The Paris Agreement replaced the Kyoto Protocol after its expiration, becoming a new, legally binding greenhouse gas reduction agreement. Regarding Taiwan, on the first anniversary of the implementation of the Kyoto Protocol, the Environmental Protection Agency of the Executive Yuan actively promoted the Greenhouse Gas Reduction Law and in 2008 adopted the Permanent Energy Policy Framework. The policy is based on the "two highs and two lows" standard, which refers to high efficiency and high value, as well as low emissions and low dependence on energy consumption. This standard aims to improve energy efficiency, reduce carbon emissions, and achieve sustainable energy development. In 2015, the Legislative Yuan enacted the Greenhouse Gas Reduction and Management Law and announced its implementation on 1 July of the same year. The long-term national objective to reduce greenhouse gas emissions is as follows: by 2050, greenhouse gas emissions will be reduced to less than 50% of emissions measured in 2005. Therefore, the regulation of greenhouse gas emissions in Taiwan is the trend of the future, and the control standards established will influence the business strategies of all enterprises in Taiwan. Faced with pressure from governments, customers, and other stakeholders, enterprises must take action to reduce the environmental and social impact of their corporate activities in order to cope with the increasing environmental awareness [4]. However, some researchers have discovered that enterprises can significantly reduce their carbon emissions without drastically increasing costs through operational adjustments, such as adjustments to inventory management [5].

In addition, inventory is an essential component in the progression from production to sales in the manufacturing industry. Inventory is the core component of a company's assets. No matter the size of the enterprise or the type of industry, to meet the needs of the market and customers, inventory is needed. Enterprises usually hold inventory for two reasons: the first reason is to reduce the gap between production and distribution systems, so that the entire manufacturing process and distribution logistics will not affect the interruption of certain operations or activities; and the second reason is to meet the needs of customers, prevent out of stock situations, reduce the risk of increased raw-material prices, and reap the benefits of quantity discounts. Unfortunately, enterprises holding inventory incur certain costs, including various storage management fees, insurance costs, obsolescence elimination, and opportunity costs. Excessive inventory leads to a backlog of funds. Because of the movement of global competition and diverse consumer demands, the lifecycles of products have been shortening. If enterprises hold too much inventory, they risk inventory obsolescence, which can cause large losses to the company. From the perspective of enterprise operations, if profits cannot be acquired from selling inventory, then the cash flow within the enterprise system is reduced, diminishing the short-term liquidity of funds. Therefore, proper inventory management not only reduces unnecessary capital backlog of enterprises but streamlines the utilization of storage space. The establishment of an effective inventory system model can increase the competitiveness of enterprises. Moreover, in highly competitive global markets, companies are unable to independently accomplish all of their goals. Integration of corporate supply chain systems must be achieved to improve the operational efficiency and respond quickly to customer demands; enterprises must also attempt to reduce inventory costs and increase profits to survive and continually develop in this highly competitive environment. Thus, with the formation of the supply chain system, the most crucial matters include effectively integrating supply chain members, developing appropriate production and inventory models, determining optimal production and ordering strategies for partners in the same supply chain, minimizing total inventory-related costs, and maximizing total profit.

Based on these topics of discussion, this study was conducted mainly under the assumption that the supply chain members agreed to invest jointly in technologies to reduce carbon emissions. This study investigated the production–inventory problems of degradable products under policies such as carbon cap-and-trade and carbon taxation. The main novelty of this study is as follows: (1) this paper discussed investment, production, and replenishment decisions, simultaneously; (2) two common carbon emission reduction policies—carbon cap-and-trade and carbon tax—were considered; (3) the concept of a co-investment agreement in carbon emission reduction investment among supply chain members was used in the proposed model; and (4) the concavity of the optimal solution were verified through numerical analysis.

In this paper, we first developed the total profit and carbon emission models for each supply chain member, followed by appropriate integration of systems. Next, we developed optimal solutions for decision making problems of carbon cap-and-trade and carbon tax policies. The primary purpose of this article was to determine the optimal production, transportation, replenishment, and investments in technology for reducing carbon emissions for each supply chain member, thus meeting the objective of simultaneously balancing profit and environmental protection so as to maximize the total profit of the entire supply chain system. Furthermore, we offered a practical example to illustrate the solution method, and the sensitivity analysis of the main parameters was carried out. To develop the sustainable supply chain to meet the inevitable trend of carbon emission control, we expect to provide enterprises with a means of risk sharing through joint investment.

2. Literature Review

Herein, the literature review of this study was mainly divided into two parts: the discussion of relevant manuscripts on carbon emission reduction and the description of the basic concept of a decision support system. The review is as follows.

2.1. Inventory Model for Carbon Emission Limits

The greenhouse effect leads to climate change; therefore, identifying ways to reduce emissions of greenhouse gases (mainly carbon dioxide) is the way forward. International regulations have been established to monitor greenhouse gas emissions; a large proportion of academic studies have also considered the topics of green energy and carbon emission control. Three topics—low energy consumption, low pollution, and low emission—have become worldwide trends that are expected to continue in the future. Through strategic analysis of economic and social development, the concept of a low carbon economy has deeply influenced national political development, foreign trade situations, and employment situations. The amount of research on inventory management problems, such as carbon emission or carbon footprint management, has also increased in recent years. Some articles used the traditional economic order quantity (EOQ) model to develop sustainable batch order models under different carbon emission management policies [6–10]. Zhang and Xu [11] focused on the newsboy model in reference to multi-item products in limited storage spaces. Other relevant studies include studies by [12–14].

In addition, most carbon inventory management models treat carbon emissions as exogenous variables. Nevertheless, carbon emissions can be efficiently reduced by investing in green, ecological design, and green manufacturing concepts in product scheme, manufacturing, inventory, and transportation [15–17]. Reduction in carbon emissions not only mitigates the influence of the greenhouse effect but enables enterprises to reduce other expenses. Currently, there is far too little investment in carbon reduction technologies; not every company can necessarily afford to reduce carbon emissions. Optimal savings and profits can be obtained from a supply chain system in which all members agree to share both the investments in relevant facilities and the benefits of improved carbon emission reduction.

2.2. Supply Chain Production–Inventory Model

In this highly competitive era, the number of competitors in various industries has continued to increase. The integrated management of supply chain systems has significantly assisted enterprises and has thus received a great deal of attention from many companies. The supply chain production–inventory model was first proposed by Goyal [18] and was based on EOQ. Banerjee [19] subsequently established a joint economic lot size model, assuming that the manufacturer produces the quantity ordered by the retailer and delivers the whole batch to the retailer after the completion of all products.

Under the assumption that the manufacturer produces the quantity ordered by the retailer at once, Goyal [20] proposed a method of shipping in batches that steadily increases in size. Ha and Kim [21] believed that goods should be shipped during production to meet the spirit of the just-in-time (JIT) system. Therefore, Hill [22] promoted the model established by Goyal [20], and considered that goods should be shipped during production. Later, Hill [23] further proposed a more general ordering and production strategy model under the assumption that the manufacturer produces the quantity ordered by the retailer all at once, adopting different delivery strategies (the previous number of shipments n increases at a fixed rate; shipments after the nth shipment remain the same quantity as the nth shipment). Goyal and Nebebe [24] proposed a relatively simple production and delivery strategy: the quantity produced by the manufacturer still equals the quantity ordered by the retailer and is delivered in batches, but a smaller quantity is shipped on the first shipment; the sizes of the following shipments are fixed at multiples of the first shipment. Giri and Roy [25] assumed that each manufacturer's shipment volume was several times larger than the previous shipment and, as such, added a volume discount to establish an integrated production–inventory model. Next, the profit function of both partners has been unequivocally proven to be a concave function of investment preservation cost, and the inventory model extends to a more generalized demand function. In addition, Cárdenas-Barrón and Sana [26] developed a production–inventory model for a two-echelon supply chain when demand was dependent on a sales team's initiatives. For other related articles on supply chain integration of production–inventory models, see [27–37] and the references mentioned therein.

3. Symbols and Assumptions

In this article, we considered the same symbols and assumptions that were used in Lu et al. [38]. The relative definitions and assumptions were used as follows.

Symbols:

P	It denotes the vendor's production rate
D	It denotes the buyer's demand rate
A	It denotes the buyer's ordering cost per replenishment cycle
$\hat{A}$	It denotes the amount of fixed carbon emissions per order for the buyer
S	It denotes the vendor's setup cost per production cycle
$\hat{S}$	It denotes the amount of fixed carbon emissions per setup for the vendor
c	It denotes the vendor's product cost per unit
$\hat{c}$	It denotes the amount of associated carbon emissions per unit produced for the vendor
v	It denotes the vendor's supply price per unit
$\hat{v}$	It denotes the amount of associated carbon emissions per unit purchased for the buyer
p	It denotes the buyer's selling price per unit

θ	It denotes the deteriorating rate of the item
h_b	It denotes the buyer's holding cost per unit of time
$\hat{h}_b$	It denotes the amount of carbon emissions per unit of inventory held by the buyer over a period of time
h_v	It denotes the vendor's holding cost per unit of time
$\hat{h}_v$	It denotes the amount of carbon emissions per unit of time for the vendor
C_T	It denotes the buyer's fixed shipping cost per shipment
$\hat{C}_T$	It denotes the amount of fixed carbon emissions per shipment for the buyer
C_t	It denotes the buyer's variable shipping cost per unit
$\hat{C}_t$	It denotes the amount of associated carbon emissions per unit shipped for the buyer
C	It denotes the tax rate per unit of carbon emission
ϖ_b	It denotes the amount of carbon emissions of the buyer per unit of time
ϖ_v	It denotes the amount of carbon emissions of vendor per unit of time
ξ	It denotes the technology investment for reducing carbon emissions, a decision variable
$m(\xi)$	It denotes the proportion of reduced carbon emissions, as a function of ξ
Q	It denotes the buyer's order quantity, a decision variable
T_p	It denotes the length of the first production and shipping quantity from vendor to buyer
T_b	It denotes the length of the buyer's replenishment cycle, a decision variable
T_v	It denotes the length of the vendor's production cycle, a decision variable
T_s	It denotes the length of the vendor's period of production, a decision variable
n	It denotes the number of shipments from vendor to buyer, a decision variable
q	It denotes the shipped quantity from vendor to buyer on each occasion, $q = Q/n$
*	It denotes the optimal value.

Assumptions:

(1) The proposed production–inventory system is developed to consider a single vendor buyer commodity.
(2) Production rate of the vendor is finite and greater than the demand rate. It implies $P > D$.
(3) The buyer orders a large quantity of Q units of commodity and requests the vendor to divide n consignments and deliver q units of goods in each shipment. All shipping costs are borne by the buyer.
(4) The carbon emissions produced by the buyer in the operational activities such as ordering, holding inventory, transportation, and procurement; the carbon emissions produced by the vendor in the operational activities such as purchasing materials, setup, production, and holding of inventory.
(5) Carbon emissions can be reduced through technological investments, and the reduced carbon emission rate is $m(\xi)$ $(0 < m(\xi) < 1)$, where $m(\xi)$ is an increasing function of the investment in carbon emission technology ξ.
(6) The investment in technology to reduce carbon emissions and the resulting benefits are shared between the vendor and buyer. In other words, the proportion of capital investment in carbon emission reduction technologies by the buyer and vendor α and $1-\alpha$, respectively, in which $0 \le \alpha \le 1$.
(7) Neither the vendor nor the buyer will allow any shortage [38,39].

4. Model Formulation

In this article, a comprehensive inventory model for co-investing in carbon emission technologies under carbon cap-and-trade and carbon tax policies is established. Throughout the production cycle, the buyer orders Q units and then asks the vendor to divide them into n batches for shipment. The quantity to be delivered each lot is $q = Q/n$ units. According to the JIT system, the vendor starts shipping during the production period and delivers to the buyer when the production quantity first reaches q units. Then, the vendor ships q units at the expected time, T_b. As the rate of production exceeds the rate of demand, the vendor will stop production and continue to ship normally until the total quantity of delivery reaches $I_{\max}$. The inventory system for the vendor and buyer throughout the production cycle are shown in Figure 1. According to the abovementioned symbols and assumptions, we established the buyer and vendor' total profits and carbon emissions per unit time as follows.

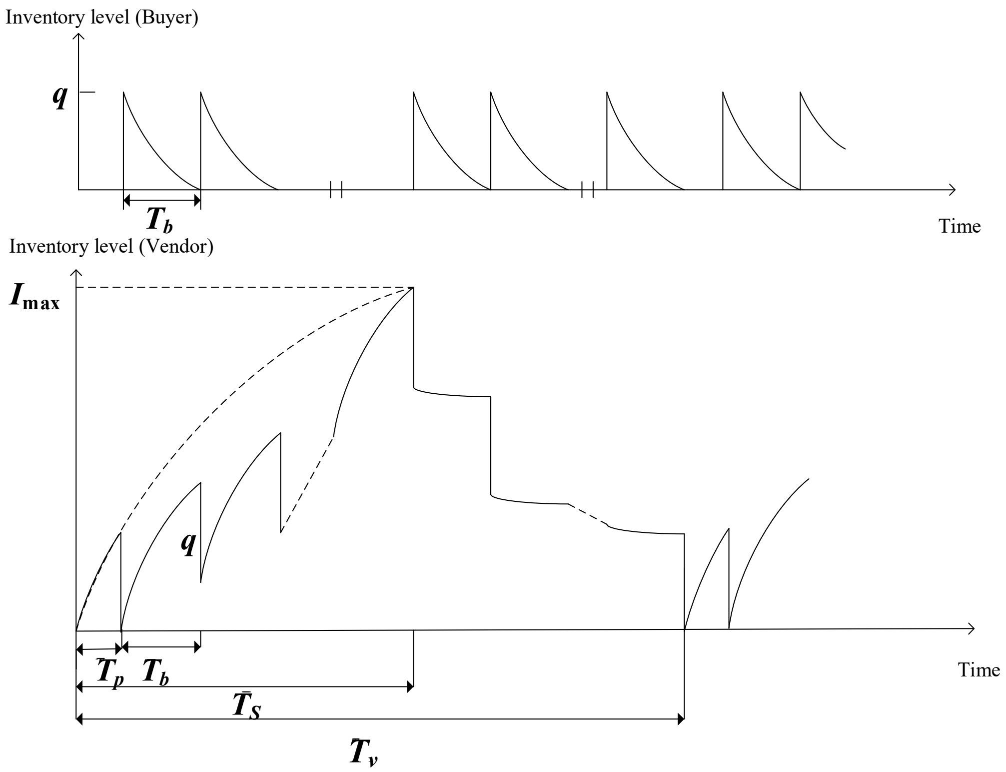

Figure 1. Vendor and buyer's inventory systems throughout the production cycle.

4.1. Buyer's Total Profit Per Unit of Time and the Carbon Emissions Produced

The buyer's inventory level at time t during the replenishment cycle changes with market demand and deterioration of the item (Figure 1), as is presented by following differential equation:

$$\frac{dI(t)}{dt} + \theta I(t) = -D,\ 0 \le t \le T_b. \tag{1}$$

From the differential Equation (1) and boundary condition $I(T_b) = 0$, it can be found that the inventory level of the buyer is

$$I(t) = \frac{D}{\theta}\left[e^{\theta(T_b - t)} - 1\right],\ \ 0 \le t \le T_b. \tag{2}$$

Besides, the order quantity of the buyer, $q = I(0)$, can be calculated from (2) as follows

$$T_b = \frac{1}{\theta}\ln\left[\frac{\theta\, q + D}{D}\right]. \tag{3}$$

The total profit per replenishment cycle for the buyer contains sale revenue, ordering, shipping, purchase, carrying costs, and investments in carbon emission technologies. The calculations of these components are described below:

(a) The buyer's sale revenue per replenishment cycle is

$$pDT_b = \frac{pD}{\theta}\ln\left[\frac{\theta q + D}{D}\right].$$

(b) The buyer's ordering cost per replenishment cycle is A.
(c) The buyer's purchase cost per replenishment cycle is vq.
(d) The buyer's transportation cost per replenishment cycle, including fixed and variable costs, is given by $C_T + C_t q$.
(e) The buyer's holding cost per replenishment cycle is

$$h_b \int_0^{T_b} I(t)dt = h_b \left\{ \frac{q}{\theta} - \frac{D}{\theta^2} \ln\left[\frac{\theta\, q + D}{D} \right] \right\}. \tag{4}$$

(f) Since the investment is jointly undertaken by the buyer and vendor, the fraction of the buyer's investment is α $(0 \leq \alpha < 1)$, so the buyer's investment in the carbon emission reduction technologies per replenishment cycle is $\alpha\xi$.

In summary, the total profit per unit of time for the buyer, $TP_b(q\ ,\ \xi)$, calculated as below:

$$TP_b(q\ ,\ \xi) = (p - \frac{h_b}{\theta})D - \frac{1}{\theta \ln\left[\frac{\theta\, q+D}{D}\right]}[A + C_T + (C_t + v + \frac{h_b}{\theta})q + \alpha\xi]. \tag{5}$$

Subsequently, the carbon emissions from each replenishment cycle of the buyer are related to the ordering, shipping, purchase, and carrying costs, which can be reduced by investing in carbon emission technologies (please refer to Lu et al. [38]). The proportion of reduced carbon emissions is $m(\xi)$; thus, the carbon emissions per replenishment cycle for the buyer, $E_b(q\ ,\ \xi)$, calculated as below:

$$E_b(q\ ,\ \xi) = \frac{[1 - m(\xi)]}{\theta} \left\{ \hat{h}_b D + \frac{1}{\ln\left[\frac{\theta\, q+D}{D}\right]} [\hat{A} + \hat{C}_T + (\hat{C}_t + \hat{v} + \frac{\hat{h}_b}{\theta})q] \right\}. \tag{6}$$

4.2. Vendor's Total Profit Per Unit of Time and the Carbon Emissions Produced

The vendor's inventory level at time t during the time interval $[0, T_s]$ changes with product and deterioration of the item (Figure 2), as presented by following differential equation:

$$\frac{dI_p(t)}{dt} + \theta I_p(t) = P\ ,\ 0 \leq t \leq T_s; \tag{7}$$

with boundary condition $I_p(0) = 0$, the solution of (7) is

$$I_p(t) = \frac{P}{\theta}(1 - e^{-\theta t})\ ,\ 0 \leq t \leq T_s. \tag{8}$$

From Figure 2, $I_p(T_{p)} = q$, which means that

$$T_p = \frac{1}{\theta} \ln\left[\frac{P}{P - \theta q} \right]. \tag{9}$$

In addition, the inventory level of the vendor during the time interval $[T_s, T_v]$ decreases due to deterioration of items, as presented by following differential equation:

$$\frac{dI_d(t)}{dt} + \theta I_d(t) = 0\ ,\ T_s \leq t \leq T_v. \tag{10}$$

Similarly, from Equation (10) and the condition $I_d(T_v) = nq$, it can be found that the inventory level of the vendor during the time interval $[T_s, T_v]$ is

$$I_d(t) = nqe^{\theta(T_v - t)},\ T_s \leq t \leq T_v. \tag{11}$$

From (8), (11), and $I_p(T_s) = I_d(T_s)$, it can be found the value of T_s

$$T_s = \frac{1}{\theta}\ln\left[\frac{P + \theta nq e^{\theta T_v}}{P}\right]. \tag{12}$$

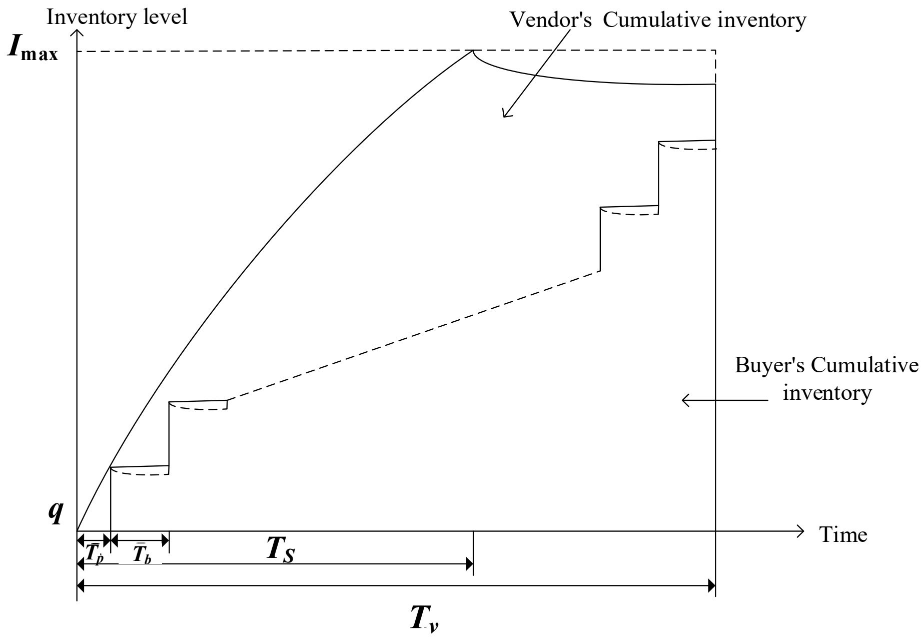

Figure 2. Accumulative inventory of the vendor and buyer.

The total profit per production cycle for the vendor includes sales revenue; costs include setup, production, and holding costs, as well as investment in technology for reducing carbon emissions. These components are calculated as follows:

(a) The vendor's sale revenue per production cycle is $vQ = vnq$.
(b) The vendor's setup cost per production cycle is S.
(c) The vendor's production cost per production cycle is

$$cPT_s = \frac{cP}{\theta}\ln\left[\frac{P + \theta nq e^{\theta T_v}}{P}\right].$$

(d) Holding cost:

It can be found that the total inventory of the vendor for each production cycle is equal to the accumulative inventory of the vendor minus the accumulative inventory of the buyer (Figure 2), which is expressed by $\int_0^{T_s} I_p(t)dt + \int_{T_s}^{T_v} I_d(t)dt - [qT_b(1 + 2 + \cdots + (n-1))]$. Therefore, the vendors' total holding cost per production cycle is

$$\begin{aligned}
&= h_v\left[\int_0^{T_s} \frac{P}{\theta}(1 - e^{-\theta t})dt + \int_{T_s}^{T_v} nq e^{\theta(T_v - t)}dt - \frac{n(n-1)qT_b}{2}\right] \\
&= h_v\left[\int_0^{T_s} \frac{P}{\theta}(1 - e^{-\theta t})dt + \int_{T_s}^{T_v} nq e^{\theta(T_v - t)}dt - \frac{n(n-1)qT_b}{2}\right] \\
&= h_v\left\{\frac{P}{\theta^2}\ln\left[\frac{P + \theta nq e^{\theta T_v}}{P}\right] - \frac{nq}{\theta} - \frac{n(n-1)q}{2\theta}\ln\left[\frac{\theta q + D}{D}\right]\right\}.
\end{aligned}$$

(e) Since the investment is jointly undertaken by the buyer and vendor, the fraction of the vendor's investment is $1-\alpha$ ($0 \leq \alpha < 1$), so the vendor's investment in the carbon emission reduction technologies per production cycle is $(1-\alpha)\xi$.

Accordingly, the vendor's total profit per unit of time, $TP_v(T_v, q, n, \xi)$, is

$$= \frac{1}{T_v}\left\{vnq - S - \frac{cP}{\theta}\ln\left[\frac{P+\theta nqe^{\theta T_v}}{P}\right] - h_v\left\{\frac{P}{\theta^2}\ln\left[\frac{P+\theta nqe^{\theta T_v}}{P}\right] - \frac{nq}{\theta} - \frac{n(n-1)q}{2\theta}\ln\left[\frac{\theta q+D}{D}\right]\right\} - (1-\alpha)\,\xi\right\}. \quad (13)$$

Then, the vendor's carbon emissions per unit time, $E_v(T_v, q, n, \xi)$, is

$$E_v(T_v, q, n, \xi) = \frac{[1-m(\xi)]}{T_v}\left\{\hat{S} + \frac{\hat{c}P}{\theta}\ln\left[\frac{P+\theta nqe^{\theta T_v}}{P}\right] + \hat{h}_v\left\{\frac{P}{\theta^2}\ln\left[\frac{P+\theta nqe^{\theta T_v}}{P}\right] - \frac{nq}{\theta} - \frac{n(n-1)q}{2\,\theta}\ln\left[\frac{\theta\, q+D}{D}\right]\right\}\right\}. \quad (14)$$

When the vendor and buyer decide to share resources for the purpose of undertaking mutually beneficial cooperation, the joint total profit per unit of time, $JTP(T_v, q, n, \xi)$, can be expressed as the sum of total profits per unit of time of the buyer and vendor and is given by $JTP(T_v, q, n, \xi) = TP_b(q, \xi) + TP_v(T_v, q, n, \xi)$.

Because $T_v = T_p + nT_b$, $JTP(T_v, q, n, \xi)$ can be reduced as to $JTP(q, n, \xi)$ from (3) and (9). The objective is to assess the manufacturing, replenishment, and improving the carbon emissions of the vendor and buyer under different carbon cap-and-trade and carbon tax policies after maximizing the total profit of the integrated system. Therefore, the optimization of different carbon emission management policies in this study demonstrated as below.

(1) Carbon cap-and-trade policy

The buyer and vendor are subject to the ϖ_b and ϖ_v caps total carbon emissions under the carbon cap-and-trade policy. If carbon emissions exceed this boundary, products outside the boundary must be purchased at the market price p_c. On the contrary, if emissions do not exceed this boundary, the remainder can be sold at the market price p_c. Assuming carbon emission allowances can be bought and sold in the market, the total profit per unit of time under the carbon cap-and-trade policy, $JTP_{CC}(q,\ n,\ \xi)$, is

$$JTP_{CC}(q,\ n,\ \xi) = JTP(q,\ n,\ \xi) - p_c[E_b(q,\ n,\ \xi) + E_v(q,\ n,\ \xi) - \varpi_b - \varpi_v].$$

The objective of the policy is to determine the optimal order quantity, shipment quantity, and technology investment to reduce carbon emissions under the carbon cap-and-trade policy, so as to maximize the joint profit function $JTP_{CC}(q,\ n,\ \xi)$. Because $\partial JTP_{CC}(q,\ n,\ \xi)/\partial q = 0$ is an integer, we first calculate the values of q and ξ (denoted by $q_{(n)}$ and $\xi_{(n)}$) by solving the equations $\partial JTP_{CC}(q,\ n,\ \xi)/\partial q = 0$ and $\partial JTP_{CC}(q,\ n,\ \xi)/\partial \xi = 0$ for given n. Then, we can use Hessian matrix as follows to check the concavity of the profit function.

$$H = \begin{bmatrix} \frac{\partial^2 JTP(q,\ n,\ \xi)}{\partial q^2} & \frac{\partial^2 JTP(q,\ n,\ \xi)}{\partial q\ \partial \xi} \\ \frac{\partial^2 JTP(q,\ n,\ \xi)}{\partial \xi\ \partial q} & \frac{\partial^2 JTP(q,\ n,\ \xi)}{\partial \xi^2} \end{bmatrix}.$$

For the value of $(q_{(n)}, \xi_{(n)})$, if the first and second determinants of the Hessian matrix (denoted by $|H_1|$ and $|H_2|$) satisfy

$$|H_1| = \left.\frac{\partial^2 JTP(q,\ n,\ \xi)}{\partial q^2}\right|_{(q=q_{(n)}, \xi=\xi_{(n)})} < 0,$$

and

$$|H_2| = \left\{ \frac{\partial^2 JTP(q,\ n,\ \xi)}{\partial q^2} \times \frac{\partial^2 JTP(q,\ n,\ \xi)}{\partial \xi^2} - \left[\frac{\partial^2 JTP(q,\ n,\ \xi)}{\partial q\, \partial \xi} \right]^2 \right\} \Bigg|_{(q=q_{(n)}, \xi=\xi_{(n)})} > 0,$$

then the total profit per unit of time has a maximum value at the point $(q_{(n)},\ \xi_{(n)})$. Due to the difficulty of Hessian matrix, we alternated numerical analysis to verify the concavity. Next, we developed the following algorithm to get the solutions of the buyer and vendor under the carbon cap-and-trade policy.

(2) Carbon tax policy

A carbon tax imposed by external regulators could provide an incentive for companies to take into account environmental costs. A simple tax table is linear and requires enterprises to pay a certain amount of money (in C) per unit of carbon emissions [6]. Thus, the improved model considering the carbon tax policy is $JTP_{CT}(q,\ n,\ \xi)$.

$$JTP_{CT}(q,\ n,\ \xi) = JTP(q,\ n,\ \xi) - C[E_b(q,\ n,\ \xi) + E_v(q,\ n,\ \xi)].$$

The objective of the policy is to determine the optimal order quantity, shipment quantity and technology investment to reduce carbon emissions under the carbon tax regulation, so as to maximize the joint profit function $JTP_{CT}(q,\ n,\ \xi)$.

As in the case of carbon cap-and-trade, identifying the closed forms of q, n, ξ, and assessing the concavity directly is a difficult task. Therefore, we verified the concavity by conducting a numerical analysis and then developed an algorithm to find the buyer and vendor solutions under the carbon tax regulation.

5. Numerical Analysis

To demonstrate the solution procedures and conduct sensitivity analyses of the optimal solutions with respect to major parameters, we used several examples based on Lu et al. [38] and reasonable data.

Example 1. *Let $D = 1000$ units/year, $P = 5000$ units/year, $A = \$200$/order, $\hat{A} = 10$ kg/order, $S = 500$/setup, $\hat{S} = 50$ kg/setup, $c = \$10$/unit, $\hat{c} = 1.5$ kg/unit, $v = \$20$/unit, $\hat{v} = 0.01$ kg/unit, $p = \$50$/unit, $h_b = \$0.5$/unit/year, $\hat{h}_b = 0.01$ kg/unit/year, $h_v = \$0.3$/unit/year, $\hat{h}_v = 0.01$ kg/unit/year, $\theta = 0.1$, $C_T = \$50$/shipment, $\hat{C}_T = 3$ kg/shipment, $C_t = \$3$/unit, $\hat{C}_t = 0.01$ kg/unit, $\omega_b = 5000$ kg/year, $\omega_v = 5000$ kg/year, $p_c = \$0.3$/unit and $\alpha = 0.5$. By using Algorithm 1, the optimal number of shipments and shipping quantity for the vendor under the carbon cap-and-trade policy were $n^* = 1$ and $q^* = 1118.1$ units. The optimal order quantity of the buyer was $Q^* = n^* q^* = 1118.1$ units. In order to check the concavity of the profit function, we calculate the values of the first and second determinants of the Hessian matrix are $|H_1| = -0.0047 < 0$ and $|H_2| = 0.0006 > 0$ which implies the concavity of the profit function is satisfied. The optimal technology investment for reducing carbon emissions was $\xi^* = \$74.0107$ and the optimal joint total profit $JTP_{CC}(q^*, n^*, \xi^*) = \$60{,}130.3$.*

Algorithm 1

Step 1. Set $n = 1$.
Step 2. Identify the values of $q_{(n)}$ and $\xi_{(n)}$ by setting $\partial JTP_{CC}(q,\ n,\ \xi)/\partial q = 0$ and $\partial JTP_{CC}(\ q,\ n,\ \xi)/\partial \xi = 0$.
Step 3. Substitute $q_{(n)}$ and $\xi_{(n)}$ into $JTP_{CC}(q,\ n,\ \xi)$ to obtain $JTP_{CC}(q_{(n)},\ n,\ \xi_{(n)})$.
Step 4. Set $n = n + 1$, and repeat Step 2 to obtain $JTP_{CC}(q_{(n+1)},\ n+1,\ \xi_{(n+1)})$.
Step 5. If $JTP_{CC}(q_{(n+1)},\ n+1,\ \xi_{(n+1)}) < JTP_{CC}(\ q_{(n)},\ n,\ \xi_{(n)})$, then $JTP_{CC}(q^*,\ n^*,\ \xi^*) = JTP_{CC}(q_{(n)},\ n,\ \xi_{(n)})$, and hence $(q^*,\ n^*,\ \xi^*) = (q_{(n)},\ n,\ \xi_{(n)})$ is the optimal solution. Otherwise, return to Step 4.

Example 2. *Data were the same as in Example 1, yet it excluded $C = 0.1$/unit. By using Algorithm 2, the optimal number of shipments and shipping quantity with carbon tax of the vendor were $n^* = 1$ and $q^* = 1086.41$ units. The optimal order quantity of the buyer was $Q^* = n^* q^* = 1086.41$ units. Similarly, to check the concavity of the profit function, we calculated the values of the first and second determinants of* *the Hessian matrix, i.e.,* $|H_1| = -0.0044 < 0$ *and* $|H_2| = 0.0002 > 0$*. This satisfied the concavity of the profit function. The optimal technology investment to reduce carbon emissions was* $\xi^* = \$51.4834$*, and the optimal joint total profit was* $JTP_{CC}(q^*, n^*, \xi^*) = \$60,086.5$*.*

Algorithm 2

Step 1. Set $n = 1$.
Step 2. Identify the values of $q_{(n)}$ and $\xi_{(n)}$ by setting $\partial JTP_{CT}(q, n, \xi)/\partial q = 0$ and $\partial JTP_{CT}(q, n, \xi)/\partial \xi = 0$.
Step 3. Substitute $q_{(n)}$ and $\xi_{(n)}$ into $JTP_{CT}(q, n, \xi)$ to obtain $JTP_{CT}(q_{(n)}, n, \xi_{(n)})$.
Step 4. Set $n = n + 1$ and repeat Step 2 to obtain $JTP_{CT}(q_{(n+1)}, n+1, \xi_{(n+1)})$.
Step 5. If $JTP_{CT}(q_{(n+1)}, n+1, \xi_{(n+1)}) < JTP_{CT}(q_{(n)}, n, \xi_{(n)})$, then $JTP_{CT}(q^*, n^*, \xi^*) = JTP_{CT}(q_{(n)}, n, \xi_{(n)})$, and therefore $(q^*, n^*, \xi^*) = (q_{(n)}, n, \xi_{(n)})$ is the optimal solution. Otherwise, return to Step 4.

Example 3. *We used the same numerical analysis method as Example 1 and analyzed the different investment sharing ratio* $\alpha \in \{0, 0.1, 0.2, 0.3, \ldots, 1\}$ *of the buyer and vendor for the technology investment for carbon emission reduction. Then, optimal values were obtained from the vendor's shipping quantity, carbon emission reduction technology, joint total profit of the buyer and vendor (represented by* $TP_{bCC}{}^*$ *and* $TP_{vCC}{}^*$*, respectively), and joint total profit per unit of time (represented by* $JTP_{CC}{}^*$*). We also assessed the quantity of carbon emission changes by the buyer and vendor. From Table 1, it can be found that large* α *values corresponded to decreased values for each shipment quantity, order quantity, produced carbon emission, and quantity by the buyer. Incremental changes were found in the amount of investment in the carbon emission reduction technologies, total profit of the vendor, joint total profit, and carbon emission produced by the vendor. Notably, the buyer's total profit increased with increases in* α*, which first increased and then decreased.*

Table 1. The optimal solution of various kinds of investment allocation proportion.

α	q^*	Q^*	ξ^*	$TP_{bCC}{}^*$	$TP_{vCC}{}^*$	$JTP_{CC}{}^*$	E_b^*	E_v^*
0	1135.25	1135.25	64.3137	13,855.0	46,156.7	60,011.7	9505.57	5252.24
0.1	1132.09	1132.09	65.8973	13,856.5	46,177.5	60,034.0	9492.27	5244.79
0.2	1128.81	1128.81	67.6252	13,857.8	46,199.2	60,056.9	9478.85	5237.31
0.3	1125.39	1125.39	69.5258	13,858.8	46,221.8	60,080.6	9465.61	5229.82
0.4	1121.83	1121.83	71.6370	13,859.5	46,245.5	60,105.0	9452.26	5222.30
0.5	1118.10	1118.10	74.0107	13,859.8	46,270.4	60,130.3	9438.89	5214.77
0.6	1114.19	1114.19	76.7206	13,859.7	46,296.8	60,156.5	9425.52	5207.22
0.7	1110.05	1110.05	79.8761	13,858.9	46,325.0	60,183.9	9412.16	5199.66
0.8	1105.66	1105.66	83.6507	13,857.3	46,355.3	60,212.6	9398.79	5192.08
0.9	1100.94	1100.94	88.3436	13,854.4	46,388.5	60,242.9	9385.46	5184.49
1	1095.81	1095.81	94.5413	13,849.5	46,425.8	60,275.2	9372.17	5176.90

Example 4. *Data are the same as in Example 1, wherein the sensitivity analysis of optimal solution is carried out, revealing the effect of each parameter on the optimal solution. The numerical analysis results are shown in Table 2.*

Table 2. Sensitivity analysis of individual parameters.

Parameters	Values	q^*	Q^*	ξ^*	$JTP_{CC}{}^*$	E_b^*	E_v^*
D	900	1109.23	1109.23	72.9309	58,727.9	8550.86	5219.48
	950	1113.66	1113.66	73.4802	59,429.0	8995.10	5217.07
	1000	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	1050	1122.56	1122.56	74.5236	60,831.9	9882.26	5212.58
	1100	1127.03	1127.03	75.0199	61,533.7	10325.2	5210.48
P	4500	1070.09	1070.09	74.1805	55,531.7	9454.37	4698.95
	4750	1094.35	1094.35	74.0886	57,829.6	9446.20	4956.93
	5000	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	5250	1141.38	1141.38	73.9454	62,433.5	9432.35	5472.47
	5500	1164.20	1164.20	73.8911	64,739.2	9426.50	5730.04
A	180	1114.64	1114.64	73.9507	60,149.2	9440.33	5215.42
	190	1116.37	1116.37	73.9808	60,139.7	9439.61	5215.1
	200	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	210	1119.83	1119.83	74.0406	60,120.9	9438.19	5214.45
	220	1121.56	1121.56	74.0704	60,111.4	9437.48	5214.13
S	450	1074.01	1074.01	73.2344	60,355.9	9458.45	5223.34
	475	1096.28	1096.28	73.6300	60,241.9	9448.21	5218.93
	500	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	525	1139.52	1139.52	74.3778	60,020.8	9430.41	5210.85
	550	1160.54	1160.54	74.7320	59,913.4	9422.68	5207.15
c	9	1118.10	1118.10	74.0107	65,130.3	9438.89	5214.77
	9.5	1118.10	1118.10	74.0107	62,630.3	9438.89	5214.77
	10	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	10.5	1118.10	1118.10	74.0107	57,630.3	9438.89	5214.77
	11	1118.10	1118.10	74.0107	55,130.3	9438.89	5214.77
h_b	0.4	1128.69	1128.69	74.1929	62,185.5	9434.62	5212.82
	0.45	1123.36	1123.36	74.1014	61,157.8	9436.75	5213.80
	0.5	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	0.55	1112.92	1112.92	73.9209	59,102.9	9441.04	5215.74
	0.6	1107.81	1107.81	73.8320	58,075.6	9443.20	5216.71
h_v	0.2	1129.16	1129.16	74.2010	60,186.7	9434.44	5212.73
	0.25	1123.59	1123.59	74.1054	60,158.4	9436.66	5213.75
	0.3	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	0.35	1112.70	1112.70	73.9171	60,102.3	9441.14	5215.79
	0.4	1107.37	1107.37	73.8244	60,074.4	9443.39	5216.79
C_T	45	1117.24	1117.24	73.9958	60,135.0	9439.25	5214.93
	47.5	1117.67	1117.67	74.0033	60,132.6	9439.07	5214.85
	50	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	52.5	1118.54	1118.54	74.0182	60,127.9	9438.72	5214.69
	55	1118.97	1118.97	74.0257	60,125.6	9438.54	5214.61
C_t	2.7	1121.25	1121.25	74.0650	60,446.8	9437.61	5214.19
	2.85	1119.67	1119.67	74.0378	60,288.5	9438.25	5214.48
	3	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	3.15	1116.54	1116.54	73.9837	59,972.1	9439.54	5215.06
	3.3	1114.99	1114.99	73.9568	59,813.8	9440.18	5215.36
p	45	1118.10	1118.10	74.0107	55,130.3	9438.89	5214.77
	47.5	1118.10	1118.10	74.0107	57,630.3	9438.89	5214.77
	50	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	52.5	1118.10	1118.10	74.0107	62,630.3	9438.89	5214.77
	55	1118.10	1118.10	74.0107	65,130.3	9438.89	5214.77

Table 2. *Cont.*

Parameters	Values	q^*	Q^*	ξ^*	$JTP_{CC}{}^*$	E_b^*	E_v^*
p_c	0.24	1110.01	1110.01	69.4075	60,410.9	9471.28	5232.32
	0.27	1114.15	1114.15	71.8350	60,270.2	9453.38	5222.61
	0.3	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	0.33	1121.91	1121.91	75.9826	59,991.0	9426.90	5208.30
	0.36	1125.58	1125.58	77.7860	59852.1	9416.79	5202.85
$\hat{A}$	8	1110.98	1110.98	73.7117	60,168.6	9314.73	5216.67
	9	1114.55	1114.55	73.8619	60,149.4	9377.00	5215.72
	10	1118.1	1118.1	74.0107	60,130.3	9438.89	5214.77
	11	1121.65	1121.65	74.1584	60,111.2	9500.43	5213.84
	12	1125.18	1125.18	74.3047	60,092.2	9561.61	5212.92
$\hat{S}$	40	1116.33	1116.33	73.9392	60,139.2	9439.86	5185.34
	45	1117.22	1117.22	73.9750	60,134.8	9439.38	5200.07
	50	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	55	1118.99	1118.99	74.0464	60,125.8	9438.41	5229.45
	60	1119.87	1119.87	74.0819	60,121.3	9437.94	5244.11
$\hat{c}$	1.2	1117.39	1117.39	72.5666	60,434.1	9447.74	4206.35
	1.35	1117.75	1117.75	73.3016	60,282.2	9443.16	4710.72
	1.5	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	1.65	1118.45	1118.45	74.6958	59,978.5	9434.91	5718.55
	1.8	1118.77	1118.77	75.3584	59,826.7	9431.18	6222.09
$\hat{h}_b$	0.008	1121.32	1121.32	72.0009	60,561.8	8010.58	5221.08
	0.009	1119.72	1119.72	73.0327	60,346.0	8724.96	5217.75
	0.01	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	0.011	1116.48	1116.48	74.9401	59,914.7	10152.4	5212.10
	0.012	1114.84	1114.84	75.8254	59,699.3	10865.6	5209.68
$\hat{h}_v$	0.008	1118.15	1118.15	74.0105	60,130.5	9438.88	5214.01
	0.009	1118.13	1118.13	74.0106	60,130.4	9438.89	5214.39
	0.01	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	0.011	1118.08	1118.08	74.0109	60,130.2	9438.90	5215.15
	0.012	1118.06	1118.06	74.011	60,130.1	9438.91	5215.53
$\hat{C}_T$	2.4	1115.97	1115.97	73.9216	60,141.8	9401.80	5215.34
	2.7	1117.04	1117.04	73.9662	60,136.0	9420.36	5215.05
	3	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	3.3	1119.17	1119.17	74.0552	60,124.6	9457.39	5214.49
	3.6	1120.23	1120.23	74.0995	60,118.8	9475.86	5214.21
$\hat{C}_t$	0.008	1118.43	1118.43	73.8211	60,173.0	9297.48	5215.34
	0.009	1118.27	1118.27	73.9161	60,151.6	9368.19	5215.05
	0.01	1118.10	1118.10	74.0107	60,130.3	9438.89	5214.77
	0.011	1117.94	1117.94	74.1048	60,108.9	9509.60	5214.50
	0.012	1117.78	1117.78	74.1985	60,087.6	9580.29	5214.22

The following observations were according to the numerical results in Table 2.

(1) When the market demand rate D, the buyer's order cost A, the vendor's setup cost S, and the carbon emissions parameter $\hat{A}$ or $\hat{C}_T$ increases, the shipping quantity, order quantity, investment amount, and the carbon emission produced by the buyer increases, and the joint total profit and the carbon emission produced by the vendor decreases.

(2) When production rate increases, P, the shipping quantity, order quantity, joint total profit, and emissions amount of both the buyer and vendor increases, and the investment amount decreases.

(3) When the vendor's production cost c increases, the joint total profit decreases. However, the shipping quantity, order quantity, investment amount, and emission amount of the buyer and vendor remain unchanged.
(4) When the buyer's carrying cost h_b or variable transportation cost C_t increases, the carbon emission produced by the vendor increases, and the shipping quantity, order quantity, investment amount, joint total profit, and carbon emission produced by the buyer decreases.
(5) With the increase of the vendor's holding costs h_v, the emission produced by the buyer also increases. The buyer's shipping quantity, order quantity, the investment amount, joint total profit, and the emission produced by the vendor are reduced accordingly.
(6) When the fixed shipping cost C_T increases, the shipping quantity, amount of investment, investment amount, and emission amount also increases. Moreover, the order quantity, joint total profit, and emission amount decrease.
(7) As the buyer's selling price per unit of p increases, the joint total profit increases, and the shipping quantity, order quantity, investment amount, and emissions generated by the buyer and vendor remain unchanged.
(8) As the market price p_c increases, the shipping quantity, the order quantity, and the amount of investment increases while the emissions produced by the buyer and vendor, and joint total profit decreases.
(9) When carbon emissions parameters $\hat{S}$ or $\hat{c}$ increase, the shipping quantity, the order quantity, the amount of investment, and the amount of emission produced by the vendor increases while the amount of emission produced by the buyer and joint total profit decreases.
(10) When carbon emissions parameters $\hat{h}_b$, $\hat{c}$, or $\hat{C}_t$ increase, the shipping quantity, the order quantity, the amount of emission produced by the vendor, and joint total profit decreases while the amount of investment and the amount of emission produced by the buyer increases.
(11) When carbon emissions parameter $\hat{h}_v$ increases, the shipping quantity, the order quantity, and joint total profit decrease while the investment amount and emission amount of the buyer and vendor increase.

6. Conclusions

Excessive carbon dioxide emissions contribute to global climate change. As a result, reducing carbon emissions has become a universal goal. Governments and international organizations have adopted different policies (such as carbon cap-and-trade and carbon taxes) to limit carbon emissions. These carbon emission restrictions affect the production, replenishment, and transportation activities of enterprises. The main objective of this article was to identify optimal production, delivery, replenishment, and technology investment strategies to reduce carbon emissions so as to maximize the total profit throughout the supply chain system. The empirical results revealed that when considering a carbon emission policy, whether be it a carbon cap-and-trade policy (Example 1) or a carbon tax (Example 2) policy, the optimal number of shipments for the supply chain system is equal to 1. Moreover, the higher the proportion of the buyer's capital investments in technology for reducing carbon emissions is, the higher the total profit of the integrated supply chain is. Furthermore, although increasing the proportion of sharing may benefit the buyer's total profit in the beginning, when the sharing ratio approaches a certain threshold (as demonstrated in Table 1 when $\alpha = 0.5$), increasing the share proportion reduces the total profit of the retailer. Finally, through a sensitivity analysis, we compiled the effects of various parameter changes on the optimal solution, as displayed in Table 3.

Table 3. Effects of individual parameter changes on the optimal solution.

Parameter \ Optimal Solution	q^*	Q^*	ξ^*	$JTP_{CC}{}^*$	E_b^*	E_v^*
Market Demand Rate	+	+	+	-	+	-
Vendor Product Rate	+	+	-	+	+	+
Buyer Ordering Cost	+	+	+	-	+	-
Vendor Setup Costs	+	+	+	-	+	-
Vendor Production Cost	X	X	X	-	X	X
Buyer Holding Cost	-	-	-	-	-	+
Vendor Holding Cost	-	-	-	-	+	-
Fixed Shipping Cost	+	-	+	-	+	-
Variable Shipping Cost	-	-	-	-	-	+
Selling Price	X	X	X	+	X	X
Market price of the carbon	+	+	+	-	-	-
Carbon emission parameter $\hat{A}$	+	+	+	-	+	-
Carbon emission parameter $\hat{S}$	+	+	+	-	-	+
Carbon emission parameter $\hat{c}$	+	+	+	-	-	+
Carbon emission parameter $\hat{h}_b$	-	-	+	-	+	-
Carbon emission parameter $\hat{h}_v$	-	-	+	-	+	+
Carbon emission parameter $\hat{C}_T$	+	+	+	-	+	-
Carbon emission parameter $\hat{C}_t$	-	-	+	-	+	-

Note: + Indicates a positive effect; - Indicates a negative effect; X Indicates that there is no significant effect.

This paper introduced a practical framework to support academics and practitioners. Academically, the contribution of this study involved the development of a production–inventory model for degraded items under carbon cap-and-trade and carbon tax policies co-invested in technology to reduce carbon emissions. Thus, this study provides a pioneer reference for future studies. From a practical implications point of view, the results of this study firstly indicate that the lot-for-lot delivery method is most beneficial for the sustainability of the supply chain system. Secondly, whether it is investing in technologies to reduce carbon emissions or increasing the proportion of investment sharing, it plays a vital role in the sustainability of the supply chain system. Lastly, our findings can serve as a reference for enterprises regarding the positive effects of risk-sharing through co-investments and its ability to strengthen the supply chain sustainable development.

The following aspects of the article can be extended in future research. First, the article only explores the integrated inventory model with carbon cap-and-trade and carbon tax policies. In the future, emission reduction policies such as carbon quota and carbon offset can be further studied. Moreover, a two-level integrated supply chain model was considered. Nevertheless, in practice, members of a supply chain may not always cooperate with one another, and may even be in competition. Therefore, one future research direction may be to adopt the perspective of competition between supply chain members using game theory to determine a balanced solution for all members of the supply chain. Further, it would be also interesting to consider the issue of distribution of benefits from supply chain integration through supply chain contracts. In addition, the production rate is considered in the proposed model. Sometime, starting at a low production rate and then increasing production capacity after a certain period of time may make holding costs lower [40], i.e., two different rates of production in a production cycle or time-dependent production rate [32] can be considered in future work. Finally, analyses can be conducted on other general scenarios, such as out of stock, volume discount, credit transaction, and changing demand situations.

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References

1. Ioan, B.; Kumaran, R.M.; Larissa, B.; Anca, N.; Lucian, G.; Gheorghe, F.; Horia, T.; Ioan, B.; Rus, M.-I. A Panel Data Analysis on Sustainable Economic Growth in India, Brazil, and Romania. *J. Risk Financ. Manag.* **2020**, *13*, 170. [CrossRef]
2. Bătrâncea, I.; Mozi, R.M.; Lucian, G.; Gheorghe, F.; Horia, T.; Ioan, B.; Rus, M.-I. An Empirical Investigation on Determinants of Sustainable Economic Growth. Lessons from Central and Eastern European Countries. *J. Risk Financ. Manag.* **2020**, *13*, 146. [CrossRef]
3. Larissa, B.; Maran, R.M.; Bătrâncea, I.; Anca, N.; Rus, M.-I.; Horia, T.; Gheorghe, F.; Speranta, M.E.; Dan, M.I. Adjusted Net Savings of CEE and Baltic Nations in the Context of Sustainable Economic Growth: A Panel Data Analysis. *J. Risk Financ. Manag.* **2020**, *13*, 234. [CrossRef]
4. Bouchery, Y.; Ghaffari, A.; Jemai, Z.; Dallery, Y. Including sustainability criteria into inventory models. *Eur. J. Oper. Res.* **2012**, *222*, 229–240. [CrossRef]
5. Benjaafar, S.; Li, Y.; Daskin, M. Carbon footprint and the management of supply chain: Insights from simple models. *IEEE Trans. Autom. Sci. Eng.* **2013**, *10*, 99–116. [CrossRef]
6. Arslan, M.C.; Turkay, M. EOQ revisited with sustainability considerations. *Found. Comput. Decis. Sci.* **2013**, *38*, 223–249. [CrossRef]
7. Chen, X.; Benjaafar, S.; Elomri, A. The carbon–constrained EOQ. *Oper. Res. Lett.* **2013**, *41*, 172–179. [CrossRef]
8. He, P.; Zhang, W.; Xu, X.; Bian, Y. Production lot–sizing and carbon emissions under cap–and–trade and carbon tax regulations carbon tax regulations. *J. Clean. Prod.* **2014**, *103*, 241–248. [CrossRef]
9. Hua, G.; Cheng, T.C.E.; Wang, S. Managing carbon footprints in inventory management. *Int. J. Prod. Econ.* **2011**, *132*, 178–185. [CrossRef]
10. Song, J.; Leng, M. Analysis of the single–period problem under carbon emissions policies. *Int. Ser. Oper. Res. Manag. Sci.* **2012**, *176*, 297–313.
11. Zhang, B.; Xu, L. Multi–item production planning with carbon cap and trade mechanism. *Int. J. Prod. Econ.* **2013**, *144*, 118–127. [CrossRef]
12. Battini, D.; Persona, A.; Sgarbossa, F. A sustainable EOQ model: Theoretical formulation and applications. *Int. J. Prod. Econ.* **2014**, *149*, 145–153. [CrossRef]
13. Bazan, E.; Jaber, M.Y.; Zanono, S. A review of mathematical inventory models for reverse logistics and the future of its modeling: An environmental perspective. *Appl. Math. Model.* **2016**, *40*, 4151–4178. [CrossRef]
14. Tiwari, S.; Daryanto, Y.; Wee, H.M. Sustainable inventory management with deteriorating and imperfect quality items considering carbon emission. *J. Clean. Prod.* **2018**, *192*, 281–292. [CrossRef]
15. Ma, J.; Yin, F.; Liu, Z.; Zhou, X. The eco-design and green manufacturing of a refrigerator. *Procedia Environ. Sci.* **2012**, *16*, 522–529. [CrossRef]
16. Tseng, M.L.; Tan, R.R.; Siriban-Manalang, A.B. Sustainable consumption and production for Asia: Sustainability through green design and practice. *J. Clean. Prod.* **2013**, *40*, 1–5. [CrossRef]
17. Mishra, U.; Wu, J.Z.; Sarkar, B. A sustainable production-inventory model for a controllable carbon emissions rate under shortages. *J. Clean. Prod.* **2020**, *256*, 120268. [CrossRef]
18. Goyal, S.K. An integrated inventory model for a single supplier–single customer problem. *Int. J. Prod. Res.* **1976**, *15*, 107–111. [CrossRef]
19. Banerjee, A. A joint economic–lot–size model for purchaser and vendor. *Decis. Sci.* **1986**, *17*, 292–311. [CrossRef]
20. Goyal, S.K. A one–vendor multi–buyer integrated inventory model: A comment. *Eur. J. Oper. Res.* **1995**, *82*, 209–210. [CrossRef]
21. Ha, D.; Kim, S.L. Implementation of JIT purchasing: An integrated approach. *Prod. Plan. Control* **1997**, *8*, 152–157. [CrossRef]
22. Hill, R.M. The single–vendor single–buyer integrated production–inventory model with a generalized policy. *Eur. J. Oper. Res.* **1997**, *97*, 493–499. [CrossRef]
23. Hill, R.M. The optimal production and shipment policy for the single–vendor single–buyer integrated production–inventory problem. *Int. J. Prod. Res.* **1999**, *37*, 2463–2475. [CrossRef]
24. Goyal, S.K.; Nebebe, F. Determination of economic production–shipment policy for a single–vendor–single–buyer system. *Eur. J. Oper. Res.* **2000**, *121*, 175–178. [CrossRef]
25. Giri, B.C.; Roy, B. A vendor–buyer integrated production–inventory model with quantity discount and unequal sized shipments. *Int. J. Oper. Res.* **2013**, *16*, 1–13. [CrossRef]

26. Cárdenas-Barrón, L.E.; Sana, S.S. A production-inventory model for a two-echelon supply chain when demand is dependent on sales teams' initiatives. *Int. J. Prod. Econ.* **2014**, *155*, 249–258. [CrossRef]
27. Das, B.C.; Das, B.; Mondal, S.K. An integrated inventory model with delay in payment for deteriorating item under Weibull distribution and advertisement cum price–dependent demand. *Int. J. Oper. Res.* **2014**, *20*, 341–368. [CrossRef]
28. Dye, C.Y.; Yang, C.T. Sustainable trade credit and replenishment decisions with credit–linked demand under carbon emission constraints. *Eur. J. Oper. Res.* **2015**, *244*, 187–200. [CrossRef]
29. Ho, C.H.; Ouyang, L.Y.; Su, C.H. Optimal pricing, shipment and payment policy for an integrated supplier–buyer inventory model with two–part trade credit. *Eur. J. Oper. Res.* **2008**, *187*, 496–510. [CrossRef]
30. Kim, S.L.; Ha, D. A JIT lot–splitting model for supply chain management: Enhancing buyer–supplier linkage. *Int. J. Prod. Econ.* **2003**, *86*, 1–10. [CrossRef]
31. Lin, Y.J. An integrated vendor–buyer inventory model with backorder price discount and effective investment to reduce ordering cost. *Comput. Ind. Eng.* **2009**, *56*, 1597–1606. [CrossRef]
32. Sana, S.S. A production–inventory model in an imperfect production process. *Eur. J. Oper. Res.* **2010**, *200*, 451–464. [CrossRef]
33. Wu, K.S.; Ouyang, L.Y. An integrated single–vendor single–buyer inventory system with shortage derived algebraically. *Prod. Plan. Control* **2003**, *14*, 555–561. [CrossRef]
34. Wu, O.Q.; Chen, H. Optimal control and equilibrium behavior of production–inventory systems. *Manag. Sci.* **2010**, *56*, 1362–1379. [CrossRef]
35. Yang, P.C.; Wee, H.M. Economic ordering policy of deteriorated item for vendor and buyer: An integrated approach. *Prod. Plan. Control* **2000**, *11*, 474–480. [CrossRef]
36. Yang, P.C.; Wee, H.M. An integrated multi–lot–size production inventory model for deteriorating item. *Comput. Oper. Res.* **2003**, *30*, 671–682. [CrossRef]
37. Yao, M.J.; Chiou, C.C. On a replenishment coordination model in an integrated supply chain with one vendor and multiple buyers. *Eur. J. Oper. Res.* **2004**, *159*, 406–419. [CrossRef]
38. Lu, C.J.; Yang, C.T.; Yen, H.F. Stackelberg game approach for sustainable production-inventory model with collaborative investment in technology for reducing carbon emissions. *J. Clean. Prod.* **2020**, *270*, 121963. [CrossRef]
39. Shen, Y.J.; Shen, K.F.; Yang, C.T. A production inventory model for deteriorating items with collaborative preservation technology investment under carbon tax. *Sustainability* **2020**, *11*, 5027. [CrossRef]
40. Sivashankari, C.K.; Panayappan, S. Production inventory model for two-level production with deteriorative items and shortages. *Int. J. Adv. Manuf. Technol.* **2015**, *76*, 2003–2014. [CrossRef]