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Keywords: combined heat and power; improved artificial bee colony algorithm; non-convex optimization; valve point effects

1. Introduction

1.1. Motivation and Problem Statement

Heat is considered to be a byproduct of power generation in conventional power generation systems and when it is not fully used that results in lower efficiency. Co-generation systems or combined heat and power (CHP) generation systems use the heat from a power plant and send it around to interested consumers. Thus, co-generation plants can produce both heat and electricity with better energy efficiency and fuel usage [1]. In recent years, CHP systems have attracted more attention due to their higher efficiency (up to 85%), network loss reduction, and rapid return of investment [2,3] compared to conventional systems. The complexity of the economic dispatch problem will be increased by including the CHP systems. Hence, it is necessary to propose appropriate solution procedure to obtain optimal schedules for both heat and power.

1.2. Review of Related Works

Several methods were presented in the literature to solve the CHP economic dispatch (CHPED) problem, which can fall into two main categories: classical optimization methods and heuristic search methods. In the first category, classical Lagrangian-based approaches are used to solve the CHPED problem. In the approximated models, the CHPED can be formulated as a linear programming problem [4]; however, in the case of realistic models the problem is a non-convex and nonlinear model due to the objective function and feasible operation region of the CHP units [5]. Therefore, in the most of cases the CHPED is modeled as a non-convex problem. In these cases, the classical Lagrangian-based approaches may not be able to find the global optimal solution and hence the obtained heat-power schedules may be sub-optimal. Also, the need for gradients and in some cases Hessian matrix of the problem constraints increases the computational burden of these techniques. Some examples are dual partial-separable programming method [6], quadratic program method [6], Lagrange relaxation technique [7], and Lagrangian relaxation with surrogate sub-gradient multiplier updating technique [8]. A more recent approach is semi-definite programming method which was proposed in [9]. In the case of convex problem, this approach gives the optimal solution, and semi-definite programming relaxation of non-convex problem provides a strong calculable bound to the optimal value. A numerical procedure, which uses a direct analytical approach to solve the CHPED problem, was proposed in [10]. Another technique with branch and bound algorithm was recorded in [11] which uses the generalized reduced gradient technique. Most of these numerical methods have approximately the same solutions, but the respective computational loads and CPU times are different.

Most of the recently proposed approaches to solve the CHPED problem are heuristic search methods (second category). In [12] the genetic algorithm was applied to solve the CHPED problem. Genetic algorithm has been used in [13] for daily operation scheduling of CHP units. A solution using a selective particle swarm optimization approach was presented in [14]. In [15] an approach based on time varying acceleration coefficients particle swarm optimization (TVAC-PSO) has been proposed to deal with CHPED problem. Stochastic particle swarm optimization algorithm that takes into account random variations in power and heat demands was used in [16]. Specific evolutionary approaches used to solve the CHPED include harmony search algorithms [3,17], improved ant colony search algorithm [18], enhanced firefly algorithm [19], direct search method [20], artificial immune system [21], bee colony optimization [22], differential evolution [23], an augmented Lagrange combined with Hopfield neural network [24]. Also, [25] optimizes heat and power from CHP units and expected wind power, by stochastic particle swarm optimization approach. These heuristic search methods have the ability of well handling non-convex problems; however, due to the fact that they are population-based and they have stochastic nature, their convergence to the optimal solution is not ensured, and they may be trapped in a local optimal or even a non-optimal solution. The comprehensive review of the application of different stochastic search algorithms for solution of CHPED is provided in [26].

As was previously mentioned, the implemented classical and mathematical-based optimization methods are not efficient for solving nonlinear and non-convex optimization problems. On the other hand, the meta-heuristic algorithms can find better results in comparison with classical optimization methods in non-convex optimization problems. By investigating the literature in CHPED problem solution, it can be observed that different heuristic algorithms yield different solutions. A better solution for CHPED problem has a great economic saving in system operation cost. Hence, it is required to improve the capabilities of heuristic algorithms such that more optimal solutions (i.e., solutions with lower costs) attained for non-convex CHPED problems. It is worth mentioning that some exact gradient-based mathematical programming algorithms, such as [27–31], have not implemented for CHPED in the literature. Therefore, it is not possible to judge their performance in comparison with the meta-heuristic optimization algorithms, and it can be considered in future works.

1.3. Contributions

Many literature works are listed in the previous section that concentrated on the solution of the CHPED problem. Given that the CHPED problem is a non-convex and nonlinear problem of optimization, there is no mathematical or metaheuristic algorithm that can guarantee the optimal global solution to these problems. Because of high economic saving potential of better algorithms, this paper focuses on solution methodology of CHPED problem. In this study, a related problem in the literature [12,15,20–24] is used to compare the results obtained with the methods previously applied to CHPED problem. In this paper a method based on improved artificial bee colony (IABC) algorithm is proposed to solve the CHPED problem. ABC is a heuristic optimization technique, which is based on the intelligent search behavior of honey bee swarm. It provides a population-based search procedure in which individuals (which called foods positions) are modified by the artificial bees, which their aim is to discover the places of food sources with the highest nectar amount [32]. The main contributions of this work can be summarized as follows:

1. Proposing an improved version of artificial bee colony algorithm for dealing with non-convex optimization problems.
2. Studying the effectiveness and performance of the proposed algorithm using normal and large-scale test systems and benchmark functions.
3. Implementation of the proposed algorithm on CHPED problem with different sizes and characteristics.
4. Compared with available methods in the literature, achieving feasible and better results for large-scale CHPED test systems.

1.4. Paper Organization

The rest of this paper is organized as follows. Section 2 provides the mathematical formulation of the CHPED problem considering valve-point effects, transmission losses and regional heat dispatch. Section 3 describes the proposed IABC algorithm. Section 5 gives the step by step procedure of the proposed IABC algorithm for solving the CHPED problem. Several case studies are presented in Section 6. Finally, conclusions are given in Section 7.

2. Chp Economic Dispatch Problem Formulation

The considered co-generation system in this study consists of power-only units, heat-only units and CHP units. The objective of the CHPED problem is to minimize total cost of serving the heat and power demands. The total cost can be stated as sum of the costs of generating heat and power as follows [20].

$$\min\ TC = \sum_{e=1}^{N_{po}} C_e(P_e^{po}) + \sum_{c=1}^{N_{chp}} C_c(P_c^{chp}, H_c^{chp}) + \sum_{h=1}^{N_{ho}} C_h(H_h^{ho}) \quad \$/h \tag{1}$$

The cost functions of the different units can be expressed using the following quadratic functions.

$$C_e(P_e^{po}) = a_e^{po}(P_e^{po})^2 + b_e^{po}P_e^{po} + c_e^{po} \quad \$/h \tag{2}$$

$$C_c(P_c^{chp}, H_c^{chp}) = a_c^{chp}(P_c^{chp})^2 + b_c^{chp}P_c^{chp} + c_c^{chp} + d_c^{chp}(H_c^{chp})^2 + e_c^{chp}H_c^{chp} + f_c^{chp}H_c^{chp}P_c^{chp} \ \$/h \tag{3}$$

$$C_h(H_h^{ho}) = a_h^{ho}(H_h^{ho})^2 + bho_h H_h^{ho} + c_h \quad \$/h \tag{4}$$

The quadratic function approximation (2) is widely used in the literature for modelling the cost function of power-only units [17]. Usually, an absolute sinusoidal term is added to the quadratic cost function for modeling the valve-point effects [33] as follows.

$$C_e(P_e^{po}) = a_e^{po}(P_e^{po})^2 + b_e^{po}P_e^{po} + c_e^{po} + |d_e^{po}\sin(f_e^{po}(P_e^{po,min} - P_e^{po}))| \tag{5}$$

Therefore, the CHPED problem becomes non-convex. The objective function (1), should be minimized subject to the following technical constraints [19]:

$$\sum_{e=1}^{N_{po}} P_e^{po} + \sum_{c=1}^{N_{chp}} P_c^{chp} = P^D + P^L \tag{6}$$

$$P^L = \sum_{e=1}^{N_{po}} \sum_{m=1}^{N_{po}} P_e^{po} B_{em} P_m^{po} + \sum_{e=1}^{N_{po}} \sum_{c=1}^{N_{chp}} P_e^{po} B_{ec} P_c^{chp} + \sum_{c=1}^{N_{chp}} \sum_{n=1}^{N_{chp}} P_c^{chp} B_{cn} P_n^{chp} \tag{7}$$

$$\sum_{c=1}^{N_{chp}} H_c^{chp} + \sum_{kh=1}^{N_{ho}} H_h^{ho} = H^D \tag{8}$$

$$P_e^{po,min} \leq P_e^{po} \leq P_e^{po,max} \quad e = 1, \ldots, N_{po} \tag{9}$$

$$H_h^{ho,min} \leq H_h^{ho} \leq H_h^{ho,max} \quad h = 1, \ldots, N_{ho} \tag{10}$$

$$P_c^{chp,min}(H_c^{chp}) \leq P_c^{chp} \leq P_c^{chp,max}(H_c^{chp}) \quad c = 1, \ldots, N_{chp} \tag{11}$$

$$H_c^{chp,min}(P_c^{chp}) \leq H_c^{chp} \leq H_c^{chp,max}(P_c^{chp}) \quad c = 1, \ldots, N_c \tag{12}$$

where (6) models the power production and consumption balance. The power transmission system loss is calculated using $B-$ matrix coefficients using (7). The heat production and demand balance is modeled using (8). The capacity limits of the power-only units and heat-only units are bounded using (9) and (10), respectively. The production limits of heat and power generation of CHP units are modeled using (11) and (12). It is observed from these equations that the upper and lower limits of power generation of CHP units are functions of produced heat (or vice versa). This heat-power dual dependency is presented using feasible operation region (FOR) for a specific CHP. The FOR of CHP units represents either a convex region or non-convex region as described in [34]. In the case of non-convex region, the CHPED problem becomes more complicated, due to non-convexity in both the objective function and the constraints. Typical FORs of CHP units is presented in Figures 1 and 2. As it can be observed, Figure 1 represents a convex region while Figure 2 represents a non-convex region [35].

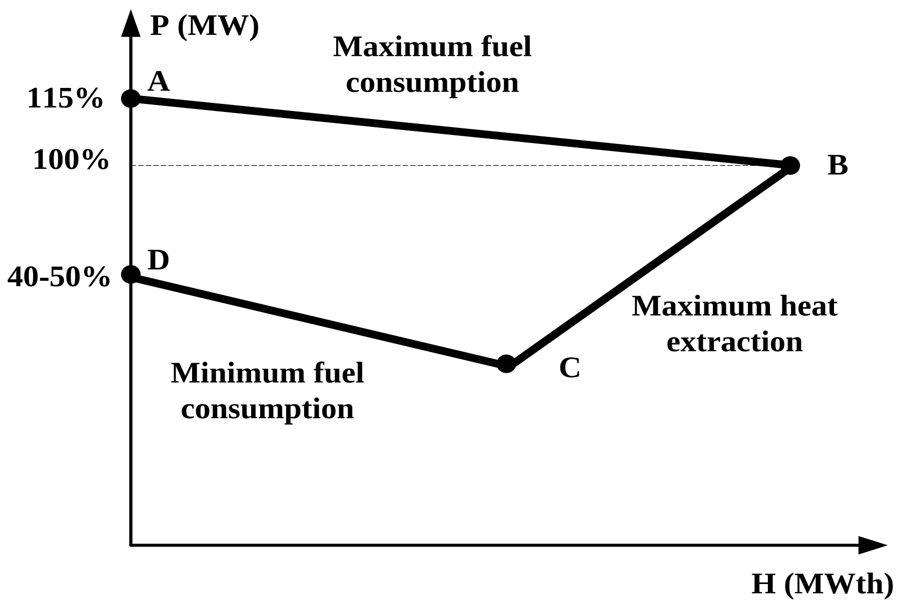

Figure 1. Convex feasible operation region (FOR) of CHP units.

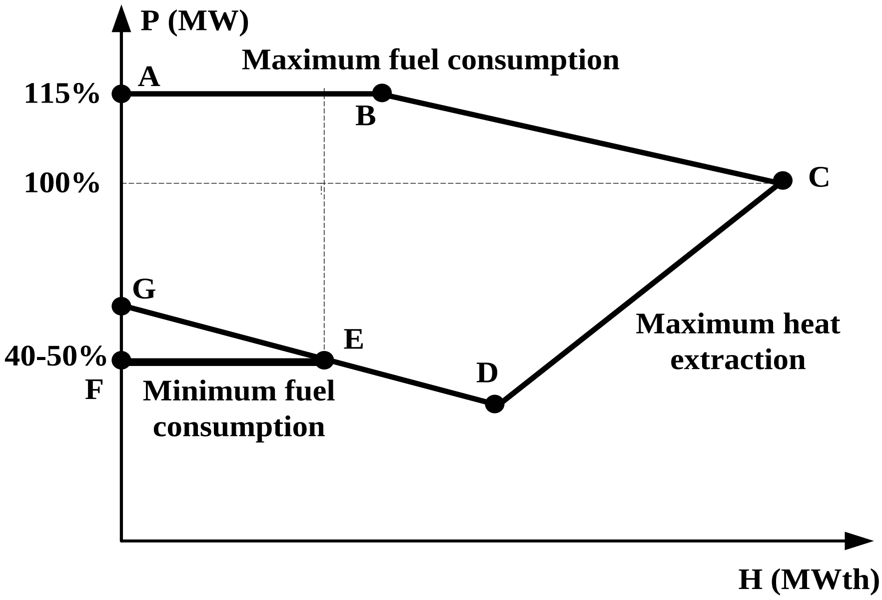

Figure 2. Non-convex feasible operation region (FOR) of CHP units.

3. Improved Artificial Bee Colony Algorithm

3.1. Original Abc Algorithm

This algorithm is based on particle swarm intelligence that is inspired by the behavior of honey bees finding food. Bee colony algorithm first was proposed by Karaboga [32]. In this algorithm there are three categories of bees, i.e., employed, onlooker and scout bees. The population of employed bees and onlooker bees are equal (i.e., half of the colony).

Employed bees are responsible for exploiting the nectar sources and providing the waiting bees (onlooker bees) in the hive with information about the nature of the locations of the food source which they exploit. Onlooker bees wait in the hive and decide to exploit a food source based on knowledge exchanged by the bees they are working. Scouts either search the area randomly to find a new food source according to their internal motivation or based on potential external clues [36]. The process of finding food source in honey bee colony can be divided into three parts [32]:

1. Employed bees discover food sources and determine the quality of nectar and share its location with others bees.
2. Onlooker bees decide based on the quality of the food sources found by employed bees and follow the location of food sources of employed bees.
3. If the food source of an employed bee is abandoned, it becomes scout bee and discover new food source randomly.

In ABC algorithm, each multi-dimensional particle (or food source) is shown as follows [32].

$$X = \left[x_1, \ldots, x_j, \ldots, x_D\right] \tag{13}$$

Thus, the i-th particle is shown as follows.

$$X_i = \left[x_{i,1}, \ldots, x_{i,j}, \ldots, x_{i,D}\right] \tag{14}$$

Here, SN is number of artificial bees and D is the number of optimization variables (or problem dimension), $i \in \{1, \ldots, SN\}$ and $j \in \{1, \ldots, D\}$.

That bee employed is associated with a single place of food source. Therefore, the number of places of food supply is equal to the number of bees employed. In each iteration of ABC algorithm, employed bees discover food sources as follows [32].

$$x_{i,j}^{new} = x_{i,j}^{old} + rand(-1,1) \times (x_{i,j}^{old} - x_{k,j}^{old}) \tag{15}$$

In the above equation k is a random integer that it selected from the set $\{1, ..., SN\}$. After production of new solutions in each iteration, the fitness (nectar) function is calculated from the following expression [32].

$$Fit(X_i) = \begin{cases} \frac{1}{1+f(X_i)} & f(X_i) \geq 0 \\ 1 + abs(f(X_i)) & f(X_i) < 0 \end{cases} \tag{16}$$

where $f(X_i)$ is the objective function value for X_i to be minimized. After calculation of objective function fitness, if $Fit(X_i^{new}) > Fit(X_i^{old})$ then X_i^{old} is replaced with X_i^{new}.

The onlooker bees select the employed bees location based on the fitness value of their corresponding food sources. For this purpose, the possibility of choosing the food source location is calculated as follows.

$$P_i = \frac{Fit(X_i)}{\sum_{k=1}^{SN} Fit(X_k)} \tag{17}$$

As the nectar quantity of food sources (fitness of solutions) increases, so does the number of onlookers visiting them, which facilitates convergence to the optimal solution [36].

For each iteration a random real number is generated for each source within the range [0,1]. If the probability (Pi in (17) associated with this source is greater than this random number, the onlooker bee modifies the location of this source of food by using (15). After the food source is evaluated from (16), if the fitness value is improved, then the onlooker bee replaces the old food source location by the new one, otherwise it keeps the old location.

If after a certain number of iterations, employed bee's food source location does not improved, the food source location is abandoned and this location is replaced with a random new location by the scout bee from:

$$x_{i,j}^{Scout} = x_j^{\min} + rand(0,1) \times (x_j^{\max} - x_j^{\min}) \tag{18}$$

where $x_j^{\max}$ and $x_j^{\min}$ are upper and lower bounds for j-th decision variable x_j, respectively.

3.2. Improved Abc (Iabc) Algorithm

The ABC algorithm has been implemented successfully in various optimization problems such as in hydroelectric generation estimation [37] and parameter estimation of solar cells [38]. However, it still attracts the attention of many researchers to improve its performance. Most of these methods modify Equation (15). For example, Kraboga proposed a new search equation for employed bees as follows [36].

$$x_{i,j}^{new} = \begin{cases} x_{i,j}^{old} + rand(-1,1) \times \left(x_{i,j}^{old} - x_{k,j}^{old}\right), & R_{i,j} \leq MR \\ x_{i,j}^{old}, & Otherwise \end{cases} \tag{19}$$

In the above equation MR is modification rate which is equal to 0.8, $R_{i,j}$ is a uniformly distributed random number in the interval $[0, 1]$.

Also, in [39] Gao and Liu proposed a new search equation as follows.

$$x_{i,j}^{new} = x_j^{best} + rand(-1,1) \times (x_{r_1,j}^{old} - x_{r_2,j}^{old}) \tag{20}$$

where r_1 and r_2 are mutually different random integers selected from the set $\{1, ..., SN\}$. x_j^{best} is the individual x_j corresponds to the particle with the best fitness in the current population.

The results reported using the above modifications indicate that both of the above search rules are very effective approaches in the optimization problems solved by ABC algorithm [39].

In this paper, a hybrid search technique is proposed which combines the above search formulas as follows.

$$x_{i,j}^{new} = \begin{cases} x_j^{best} + rand(-1,1) \times (x_{r_1,j}^{old} - x_{r_2,j}^{old}), & R_{i,j} \leq MR \\ x_{i,j}^{old}, & Otherwise \end{cases} \tag{21}$$

The main distinguishing features of the proposed IABC algorithm are as follows:

1. As it is observed from Equation (21), as the difference between $x_{r_1,j}$ and $x_{r_2,j}$ decreases, the disturbance of position $x_{i,j}$ decreases. Therefore, the length of step is adaptively reduced by approaching to an optimal solution, and hence the algorithm converges to the optimal solution.
2. It is observed from Equation (18) that the algorithm automatically jumps form local optimal or even non-optimal points, since scout bees are generated when no progress made in the search for a specified food source (or solution).
3. Onlooker bees capability included in this algorithm enables comparison of the behavior of all food sources (or solutions) simultaneously. In other words, it is observed from Equation (17) that if a specified solution (or food source) i has a small P_i, then it is a good solution, and hence it is not updated by onlooker bees. Otherwise, it is replaced with new position by onlooker bees.

4. Investigation of Iabc Algorithm on the Benchmark Functions

To investigate the performance of proposed IABC algorithm, two studies are conducted here as follows.

4.1. Study-I: Investigations on Six Benchmark Functions

In this study, six well-known benchmark functions which have different characteristics are examined. These functions which have been employed in [39,40] for evaluation of ABC algorithm and its variants, are given in Table 1.

The proposed IABC algorithm is applied to the above benchmark functions. Similar to the settings of GABC algorithm given in [40], the following settings are considered for the proposed IABC algorithm evaluation: Population size (or SN) is 80, maximum iterations number ($Iter_{max}$) is 5000 and the number of trial runs is 30.

For the above settings, the mean value and standard deviation of the results are presented in Table 2. The obtained results are compared with MABC [39], ABC [40] and GABC [40]. It can be observed from Table 2 that the proposed IABC algorithm converges to better results in comparison with ABC, MABC and GABC algorithms, in terms of the mean and standard deviation of the results.

Table 1. Studied benchmark functions.

Name	Formula	D (Problem Dimension)	Search Space	Global Minimum
Schaffer [39]	$f_1(x) = 0.5 + \frac{\sin^2\left(\sqrt{\sum_{i=1}^{n} x_i^2}\right) - 0.50}{\left(1 + 0.001 \times \left(\sum_{i=1}^{n} x_i^2\right)\right)^2}$	30	$[-100, 100]^n$	0
Rosenbrock [39]	$f_2(x) = \sum_{i=1}^{n-1} (100(x_{i+1} - x_i^2))^2 + (x^i - 1)^2$	30	$[-50, 50]^n$	0
Sphere [39]	$f_3(x) = \sum_{i=1}^{n} x_i^2$	30	$[-100, 100]^n$	0
Griewank [39]	$f_4(x) = 1 + \frac{1}{4000} \sum_{i=1}^{n} (x_i - 100)^2 - \prod_{i=1}^{n} \cos(\frac{x_i - 100}{\sqrt{i}})$	30	$[-600, 600]^n$	0
Rastrigin [40]	$f_5(x) = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10)^2$	30	$[-5.12, 5.12]^n$	0
Ackly [39]	$f_6(x) = -20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - \exp(\frac{1}{n}\sum_{i=1}^{n} \cos(2\pi x_i)) + 20 + e$	30	$[-32.768, 32.768]^n$	0

Table 2. Comparison of the obtained results for benchmark functions.

Benchmark Function		ABC [40]		GABC [40]		MABC [39]		Proposed	
Number	Name	Mean	SD	Mean	SD	Mean	SD	Mean	SD
f_1 [39]	Schaffer	4.47×10^{-1}	2.22×10^{-2}	2.81×10^{-1}	9.12×10^{-2}	2.56×10^{-1}	4.65×10^{-2}	2.12×10^{-1}	2.23×10^{-2}
f_2 [39]	Rosenbrock	3.65×10^{-1}	5.04×10^{-1}	7.93×10^{-1}	1.36×10^{0}	1.73×10^{-1}	1.61×10^{-1}	1.05×10^{-1}	1.45×10^{-1}
f_3 [39]	Sphere	6.38×10^{-16}	1.20×10^{-16}	4.17×10^{-16}	7.36×10^{-17}	9.43×10^{-32}	6.67×10^{-32}	3.21×10^{-35}	1.30×10^{-34}
f_4 [39]	Griewank	1.27×10^{-15}	1.46×10^{-15}	2.96×10^{-17}	4.99×10^{-17}	0.00	0.00	0.00	0.00
f_5 [40]	Rastrigin	1.35×10^{-13}	7.97×10^{-14}	1.32×10^{-14}	2.44×10^{-14}	0.00	0.00	0.00	0.00
f_6 [39]	Ackly	4.70×10^{-14}	5.95×10^{-15}	3.21×10^{-14}	3.25×10^{-15}	2.98×10^{-14}	2.26×10^{-15}	2.87×10^{-14}	3.65×10^{-15}

4.2. Study-Ii: Investigations on Large-Scale Benchmark Functions

To examine the capability of proposed IABC algorithm for solution of large-scale optimization problems, it is implemented on the 300 variables version of the benchmark functions f_4– f_6 [41]. Table 3 compares the mean value obtained by the proposed IABC algorithm with GSO [41], GA [41], PSO [41], EP [41], ES [41], ABC [36], and MABC [39]. It is evidently observed from this table that IABC algorithm outperforms the above existing approaches, since the obtained solution by IABC is very close to the global optimal solution in all considered benchmark functions.

Table 3. Comparison of the obtained results by IABC with other algorithms for large-scale benchmark functions.

#	GSO [41]	GA [41]	PSO [41]	EP [41]	ES [41]	ABC [36]	MABC [39]	Proposed
f_4	1.82×10^{-7}	3.70×10^{-1}	1.81×10^{0}	2.84×10^{-2}	1.58×10^{-1}	1.35×10^{-10}	8.35×10^{-15}	0
f_5	98.9	121.3	427.1	383.3	583.2	6.82×10^{-14}	1.04×10^{-5}	0
f_6	1.35×10^{-3}	6.24×10^{0}	3.95×10^{-6}	2.95×10^{-1}	9.62×10^{0}	7.52×10^{-4}	9.62×10^{-10}	8.93×10^{-11}

Besides, in Table 4 the performance of IABC algorithm is compared with the basic ABC [36] and MABC [39] in terms of the best and worst obtained solutions for the above three benchmarks. It is observed from this table that the proposed IABC approach gives smaller values for both the best and worst solutions. especially for f_4 and f_5 these values are both zero, which means the algorithm always converges to the global optimal point.

Table 4. Comparison of the obtained results for the variants of ABC algorithm in large-scale benchmark functions.

	ABC [36]		MABC [39]		Proposed	
#	Best	Worst	Best	Worst	Best	Worst
f_4	0	6.46×10^{-10}	7.55×10^{-15}	9.44×10^{-15}	0	0
f_5	0	1.14×10^{-13}	4.27×10^{-11}	5.20×10^{-5}	0	0
f_6	5.33×10^{-4}	1.10×10^{-3}	7.80×10^{-10}	1.08×10^{9}	6.82×10^{-11}	1.10×10^{-10}

5. Implementation of Iabc on the Chped Problem

To solve the CHPED problem by the IABC algorithm, the following steps are performed.

1. *Step-1:* The first stage in IABC algorithm is initialization of the employed bees. Every food source location is a candidate solution of CHPED problem. The position of each food source (X_i) is a vector of all real power and heat outputs of the units as presented in the following.

$$X_i = [X_i^{(1)}, X_i^{(2)}, X_i^{(3)}] \tag{22}$$

$$X_i^{(1)} = [P_{i,1}^{po}, \cdots, P_{i,N_{po}}^{po}] \tag{23}$$

$$X_i^{(2)} = [P_{i,1}^{chp}, \cdots, P_{i,N_{chp}}^{chp}] \tag{24}$$

$$X_i^{(3)} = [H_{i,1}^{chp}, \cdots, H_{i,N_{chp}}^{chp}, H_{i,1}^{ho}, \cdots, H_{i,N_{ho}}^{ho}] \tag{25}$$

The initial population of employed bees for power-only and heat-only units are determined from (26) and (27), respectively. Also, the population of employed bees for CHP units is determined from (28) and (29), respectively.

$$P_{i,j}^{po} = P_j^{po,min} + rand(0,1) \times (P_j^{po,max} - P_j^{po,min}) \tag{26}$$

$$H_{i,j}^{ho} = H_j^{ho,min} + rand(0,1) \times (H_j^{ho,max} - H_j^{ho,min}) \tag{27}$$

$$P_{i,j}^{chp} = P_j^{chp,min} + rand(0,1) \times (P_j^{chp,max} - P_j^{chp,min}) \tag{28}$$

$$H_{i,j}^{chp} = H_j^{chp,min} + rand(0,1) \times (H_j^{chp,max} - H_j^{chp,min}) \tag{29}$$

2. *Step-2:* By setting $Iter = 1$ (where $Iter$ is the iteration number of the algorithm), discover new food source locations by employed bees using Equation (21).
3. *Step-3:* In this step the objective function value for the population of bees are calculated at the current iteration. Since the CHPED is a constrained optimization problem, it is converted to an unconstrained problem using penalty coefficient (λ). λ is assumed to be 10000 for all test systems studied in the following section. Hence, the objective function will be as follows.

$$f(X_i) = \sum_{e=1}^{N_{po}} C_e(P_e^{po}) + \sum_{c=1}^{N_{chp}} C_c(P_c^{chp}, H_c^{chp}) + \sum_{h=1}^{N_{ho}} C_h(H_h^{ho}) \tag{30}$$
$$+ \lambda \times (\sum_{e=1}^{N_{po}} P_e^{po} + \sum_{c=1}^{N_{chp}} P_c^{chp} - P^D - P^L)^2 + \lambda \times (\sum_{c=1}^{N_{chp}} H_c^{chp} + \sum_{kh=1}^{N_{ho}} H_h^{ho} - H^D)^2$$

4. *Step-4:* Fitness of i-th food source is calculated from Equation (16). If the new food source fitness is better than the old, then the old food source is replaced with the new location (obtained in *Step-2*, and $B_{Scout,i} = 0$ ($B_{Scout,i}$ is a counter that determines limit value for converting i-th employed bee to scout bee), otherwise old location is preserved and $B_{Scout,i} = B_{Scout,i} + 1$.
5. *Step-5:* At this step onlooker bees select food source of employed bees by using the roulette wheel criterion given in (17). Based on the value of P_i for each food source, the onlooker bees modify the selected locations of employed bees by using (21), as follows: If $P_i > rand(0,1)$, then the fitness of new food source is calculated from (16). If it is better than old fitness value, the old food source is replaced with new location and $B_{Scout,i} = 0$, otherwise the old food location is kept and $B_{Scout,i} = B_{Scout,i} + 1$.
6. *Step-6:* After the completion of the food source update process for employed and onlooker bees, if $B_{Scout,i} > Limit$, then that food source is abandoned, and the employed bee is converted to a scout bee. The scout bee selects its new food source randomly in the space via (18).

7. *Step-7:* Check the stopping criterion. If the algorithm converged, then go to *Step-8*, else $Iter = Iter + 1$ and go to *Step-2* and repeat the above procedure. In this paper, the stopping criterion is reaching to the maximum number of iterations in each run. In other words, if $Iter = Iter_{max}$, then the algorithm stopped.
8. *Step-8:* Stop. To clarify the optimization process for energy engineers, the implemented method is presented in Figure 3.

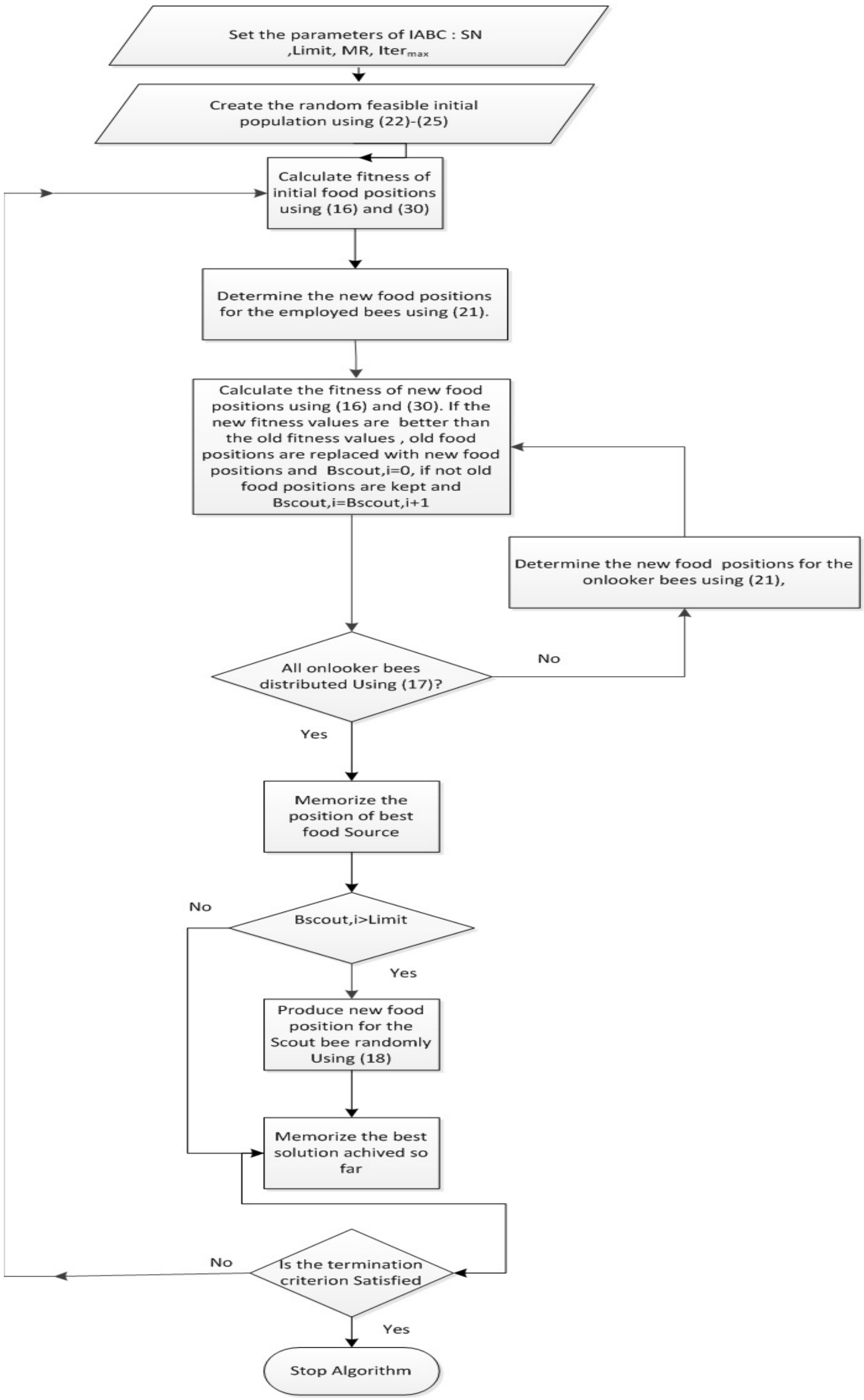

Figure 3. Flowchart of implementation process of proposed method on CHPED problem.

6. Case Studies

In this section, the effectiveness and validity of the proposed method is evaluated by implementing it on three different test systems. The numerical study is performed using MATLAB 7.5 software on a PC with an Intel Core i7, 2.93 GHz CPU and 8 GB of RAM. The obtained results using the proposed IABC are compared with the reported results in the literature. The parameters used in the algorithm for different case studies are presented in Table 5.

Table 5. Parameters of IABC for different test systems.

Test System #	SN	Limit	$Iter_{max}$
I	100	50	300
II	200	50	2000
III	200	100	5000

6.1. Test System I (7-Unit System)

The first test system consists of four power-only units (units 1–4), two CHP units (units 5–6) and a heat-only unit (unit 7). The valve-point effects and transmission losses are considered in this test system. The cost function parameters of this case along with the feasible region coordinates of CHP units are available in [23]. Data of test system I is provided in Table 6. By investigating the published papers, it was found that there were three different loss matrix data for this system. Hence, we have solved the problem for this system using the available data in three cases.

Table 6. Cost function parameters of test system I.

			Power Only Units				
Unit	a_e^{po}	b_e^{po}	c_e^{po}	d_e^{po}	f_e^{po}	$P_e^{po,min}$	$P_e^{po,max}$
1	0.008	2	25	100	0.042	10	75
2	0.003	1.8	60	140	0.04	20	125
3	0.0012	2.1	100	160	0.038	30	175
4	0.001	2	120	180	0.037	40	250
			CHP Units				
	a_c^{chp}	b_c^{chp}	c_c^{chp}	d_c^{chp}	e_c^{chp}	f_c^{chp}	feasible region coordinates $[P_c^{chp}, H_c^{chp}]$
5	0.0345	14.5	2650	0.03	4.2	0.031	[98.8,0], [81,104.8], [215,180], [247,0]
6	0.0435	36	1250	0.027	0.6	0.11	[44,0], [44,15.9],[40,75],[110.2,135.6], [125.8,32.4],[125.8,0]
			Heat Only Units				
	a_h^{ho}	b_h^{ho}	c_h^{ho}	$H_h^{ho,min}$	$H_h^{ho,max}$		
7	0.038	2.0109	950	0	2695.20		

6.1.1. Case I

In this case, the coefficients of the network loss matrix are provided as follows [15].

$$B = 10^{-7} \times \begin{pmatrix} 49 & 14 & 15 & 15 & 20 & 25 \\ 14 & 45 & 16 & 20 & 18 & 19 \\ 15 & 16 & 39 & 10 & 12 & 15 \\ 15 & 20 & 10 & 40 & 14 & 11 \\ 20 & 18 & 12 & 14 & 35 & 17 \\ 25 & 19 & 15 & 11 & 17 & 39 \end{pmatrix}$$

The optimal dispatches of the units in this case are provided in Table 7. The obtained results using the proposed IABC algorithm are compared with the results of line-up competition algorithm (LCA) [42], teaching learning-based optimization (TLBO) [42], oppositional teaching learning-based optimization (TLBO) [42], conventional PSO (CPSO) [15] and time varying acceleration coefficients PSO (TVAC-PSO) [15] in Table 7. The distribution of the total cost for 50 independent runs is presented in Figure 4. It is inferred from this figure that in 33 runs the obtained value for cost is less than the mean value, which means the algorithm is capable of attaining a solution better that than the mean value in 66% of trails. The minimum, average and maximum values of the obtained costs for these runs are also provided in Table 7. The convergence characteristics of the proposed method for this case is depicted in Figure 5. It is observed that the proposed IABC algorithm converges quickly in early iterations and hence, the number of maximum runs can be decreased to save the solution time.

Table 7. Comparison of the obtained results for 7-unit test system (Case I).

Control Variable	LCA [42]	OTLBO [43]	TLBO [43]	CPSO [15]	TVAC-PSO [15]	Proposed
P_1	44.2812	45.8860	45.2660	75.0000	47.3383	45.8514
P_2	98.5446	98.5398	98.5479	112.3800	98.5398	98.5388
P_3	112.7192	112.6741	112.6786	30.0000	112.6735	112.6734
P_4	211.4443	209.8141	209.8284	250.0000	209.8158	209.8169
P_5	93.7494	93.8249	94.4121	93.2701	92.3718	93.8594
P_6	40.0000	40.0002	40.0062	40.1585	40.0000	40.0000
H_5	29.7358	29.2914	25.8365	32.5655	37.8467	29.0616
H_6	74.5000	75.0002	74.9970	72.6738	74.9999	74.9839
H_7	45.2641	45.7084	49.1666	44.7606	37.1532	45.9542
Minimum Cost ($/h)	10,104.38	10,094.3529	10,094.8384	10,325.3339	10,100.3164	10,094.2718
Average Cost ($/h)	NA	10,099.4057	10,114.1539	NA	NA	10,095.4446
Maximum Cost ($/h)	NA	10,106.8314	10,133.6130	NA	NA	10,100.9445
CPU time (s)	NA	3.06	2.86	3.29	3.25	2.21

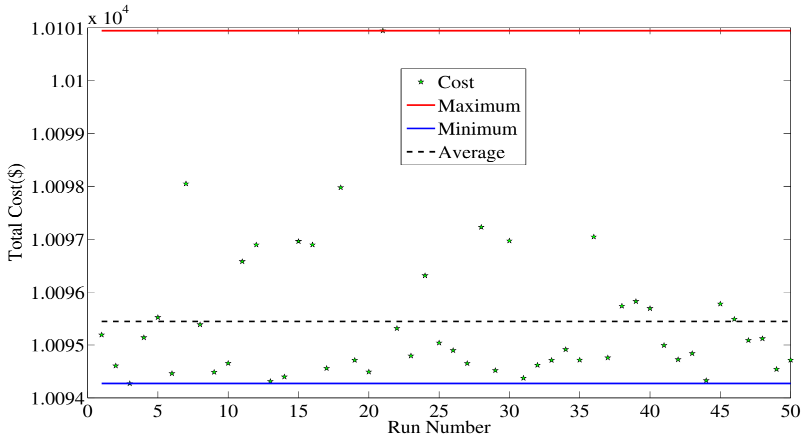

Figure 4. Distribution of total costs for 50 independent runs for 7-unit test system (Case I).

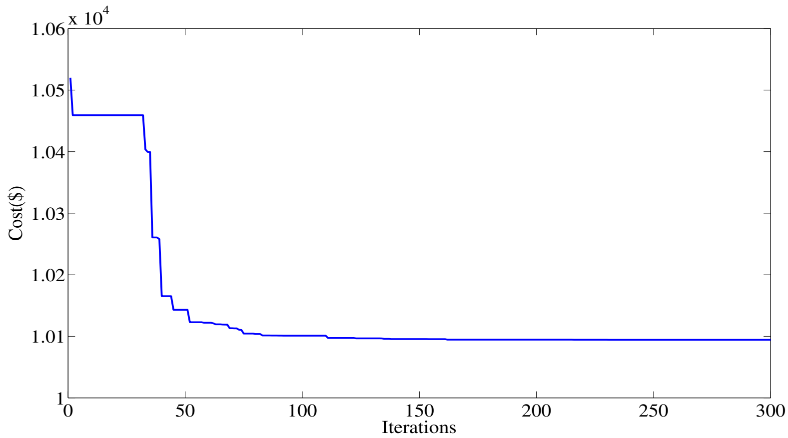

Figure 5. Convergence characteristics of the proposed IABC algorithm for 7-unit test system (Case I).

6.1.2. Case II

In this case, the coefficients of the network loss matrix are assumed to be as follows [44].

$$B = 10^{-6} \times \begin{pmatrix} 49 & 14 & 15 & 15 & 20 & 25 \\ 14 & 45 & 16 & 20 & 18 & 19 \\ 15 & 16 & 39 & 10 & 12 & 15 \\ 15 & 20 & 10 & 40 & 14 & 11 \\ 20 & 18 & 12 & 14 & 35 & 17 \\ 25 & 19 & 15 & 11 & 17 & 39 \end{pmatrix}$$
$$B_0 = 10^{-3} \times \begin{pmatrix} -0.3908 & -0.1297 & 0.7047 & 0.0591 & 0.2161 & -0.6635 \end{pmatrix}$$
$$B_{00} = 0.056$$

The optimal solution results using the proposed algorithm are presented in Table 8 and is compared with the real coded genetic algorithm (RCGA) [44] and Grey wolf optimization algorithm (GWO) [45]. It can be observed that the average of the obtained costs using the proposed algorithm for 50 independent runs is lower than the minimum cost obtained using RCGA.

Table 8. Comparison of the obtained results for 7-unit test system (Case II).

Control Variable	RCGA [44]	GWO [45]	Proposed (IABC)
P_1	74.5357	52.8074	45.2848
P_2	99.3518	98.5398	98.5507
P_3	174.7196	112.6735	112.6845
P_4	211.017	209.8158	209.8439
P_5	100.9363	93.8115	93.8194
P_6	44.1036	40	40.0000
H_5	24.3678	29.3704	29.3226
H_6	72.527	75	74.9944
H_7	53.1052	29.3704	45.6832
Minimum Cost ($/h)	10,712.86	10,111.24	10,092.9593
Average Cost ($/h)	NA	10,194.41	10,152.5012
Maximuum Cost ($/h)	NA	10,452.12	11,547.5437
CPU time (s)	20.3438	5.2618	2.21

6.1.3. Case III

In this case, the coefficients of the network loss matrix are provided as follows [22].

$$B = 10^{-6} \times \begin{pmatrix} 49 & 14 & 15 & 15 & 20 & 25 \\ 14 & 45 & 16 & 20 & 18 & 19 \\ 15 & 16 & 39 & 10 & 12 & 15 \\ 15 & 20 & 10 & 40 & 14 & 11 \\ 20 & 18 & 12 & 14 & 35 & 17 \\ 25 & 19 & 15 & 11 & 17 & 39 \end{pmatrix}$$

Using the above data, the problem is solved using the proposed algorithm and the results are presented in Table 9. The obtained results are compared with particle swarm optimization (PSO) [22], real-coded genetic algorithm (RCGA) [22], evolutionary programming (EP) [22], artificial immune system (AIS) [21], bee colony optimization (BCO) [22] and differential evolution (DE) [23] in Table 9. It can be found that the proposed algorithm outperforms all of the previously proposed algorithms in the literature in less time.

Table 9. Comparison of the obtained results for 7-unit test system (Case III).

Control Variable	EP [22]	BCO [22]	AIS [21]	PSO [22]	DE [23]	RCGA [22]	Proposed
P_1	61.361	43.9457	50.1325	18.4626	44.2118	74.6834	52.5848
P_2	95.1205	98.5888	95.5552	124.2602	98.5383	97.9578	98.5685
P_3	99.9427	112.9320	110.7515	112.7794	112.6913	167.2308	112.7003
P_4	208.7319	209.7719	208.7688	209.8158	209.7741	124.9079	209.8723
P_5	98.8000	98.8000	98.8000	98.8140	98.8217	98.8008	93.8212
P_6	44.0000	44.0000	44.0000	44.0107	44.0000	44.0001	40.0000
H_5	18.0713	12.0974	19.4242	57.9236	12.5379	58.0965	29.3057
H_6	77.5548	78.0236	77.0777	32.7603	78.3481	32.4116	74.9573
H_7	54.3739	59.879	53.4981	59.3161	59.1139	59.4919	45.7375
Minimum Cost ($/h)	10,390.0000	10,317.0000	10,355.0000	10,613.0000	10,317.0000	10,667.0000	10,111.8592
Average Cost ($/h)	NA	NA	NA	NA	NA	NA	10,656.4161
Maximum Cost ($/h)	NA	NA	NA	NA	NA	NA	13,638.7295
CPU time (s)	5.27	5.16	5.29	5.38	5.26	6.47	2.21

6.2. Test System Ii (24-Unit System)

This system is one of the large benchmarks for the CHPED problem, which is proposed in [15]. Data of cost functions of the second test system is presented in Table 10. The valve-point effects are considered in this test system. This test system consists of 13 power only units, 6 CHP units, and 5 heat-only units. The total electricity demand is 2350 MW and the total heat demand is 1250 MWth. The power only units' data are based on the 13-unit challenging standard economic dispatch test system [46]. Data of the test system can be accessed from [15].

Table 11 shows the obtained results using the proposed algorithm for this system. The obtained results in this table are compared with the recent algorithms such as TLBO [43], OTLBO [43], CPSO [15], TVAC-PSO [15], arithmetic crossover harmony search (ACHS) [47], group search optimization method (GSO) [48], improved GSO (IGSO) [48] and Grey wolf optimization algorithm (GWO) [45]. As it can be observed from this table, the proposed IABC algorithm obtains a better solution compared to other reported algorithms in the literature. It should be mentioned that the minimum, average, and maximum cost of 50 independent runs are also presented in Table 11. Distribution of total costs in these trail runs for this test system is depicted in Figure 6. It is inferred from this figure that in 27 runs the obtained cost by the proposed IABC algorithm is less than the average value of 50 trials. This means that the algorithm is able to find a solution better that than the mean value in 54% of trails. The convergence characteristics of the proposed IABC algorithm for 24-unit test system is provided in Figure 7. As it can be observed from this figure the proposed algorithm is converged to the optimal solution in earlier iterations.

Table 10. Cost function parameters of test system II.

Power Only Units							
Unit	a_e^{po}	b_e^{po}	c_e^{po}	d_e^{po}	f_e^{po}	$P_e^{po,min}$	$P_e^{po,max}$
1	0.00028	8.1	550	300	0.035	0	680
2	0.00056	8.1	309	200	0.042	0	360
3	0.00056	8.1	309	200	0.042	0	360
4	0.00324	7.74	240	150	0.063	60	180
5	0.00324	7.74	240	150	0.063	60	180
6	0.00324	7.74	240	150	0.063	60	180
7	0.00324	7.74	240	150	0.063	60	180
8	0.00324	7.74	240	150	0.063	60	180
9	0.00324	7.74	240	150	0.063	60	180
10	0.00284	8.6	126	100	0.084	40	120
11	0.00284	8.6	126	100	0.084	40	120
12	0.00284	8.6	126	100	0.084	55	120
13	0.00284	8.6	126	100	0.084	55	120
CHP Units							
	a_c^{chp}	b_c^{chp}	c_c^{chp}	d_c^{chp}	e_c^{chp}	f_c^{chp}	feasible region coordinates $[P_c^{chp}, H_c^{chp}]$
14	0.0345	14.5	2650	0.03	4.2	0.031	[98.8,0], [81,104.8], [215,180], [247,0]
15	0.0435	36	1250	0.027	0.6	0.011	[44,0], [44,15.9],[40,75],[110.2,135.6], [125.8,32.4],[125.8,0]
16	0.0345	14.5	2650	0.03	4.2	0.031	[98.8,0], [81,104.8], [215,180], [247,0]
17	0.0435	36	1250	0.027	0.6	0.011	[44,0], [44,15.9],[40,75],[110.2,135.6], [125.8,32.4],[125.8,0]
18	0.1035	34.5	2650	0.025	2.203	0.051	[20,0],[10,40], [45,55],[60,0]
19	0.072	20	1565	0.02	2.34	0.04	[35,0],[35,20],[90,45],[90,25], [105,0]
Heat Only Units							
	a_h^{ho}	b_h^{ho}	c_h^{ho}	$H_h^{ho,min}$	$H_h^{ho,max}$		
20	0.038	2.0109	950	0	2695.20		
21	0.038	2.0109	950	0	60		
22	0.038	2.0109	950	0	60		
23	0.052	3.0651	480	0	120		
24	0.052	3.0651	480	0	120		

Table 11. Comparison of the obtained results for 24-unit test system.

Control Variable	TLBO [43]	TVAC-PSO [15]	ACHS [47]	OTLBO [43]	CPSO [15]	GSO [48]	IGSO [48]	GWO [45]	Proposed (IABC)
P_1	538.5656	538.5656	628.3185	538.5656	680	627.7455	628.152	538.584	628.3185
P_2	299.2123	299.2123	299.1992	299.2123	0	76.2285	299.4778	299.3423	299.1993
P_3	299.122	299.122	299.199	299.122	0	299.5794	154.5535	299.3423	299.1993
P_4	109.992	109.992	109.8665	109.992	180	159.4386	60.846	109.9653	109.8665
P_5	109.9545	109.9545	109.8665	109.9545	180	61.2378	103.8538	109.9653	109.8666
P_6	110.4042	110.4042	60	110.4042	180	60	110.0552	109.9653	60
P_7	109.8045	109.8045	109.8665	109.8045	180	157.1503	159.0773	109.9653	109.8666
P_8	109.6862	109.6862	109.8665	109.6862	180	107.2654	109.8258	109.9653	109.8665
P_9	109.8992	109.8992	109.8665	109.8992	180	110.1816	159.992	109.9653	109.8665
P_{10}	77.3992	77.3992	40	77.3992	50.5304	113.9894	41.103	77.6223	40
P_{11}	77.8364	77.8364	76.9505	77.8364	50.5304	79.7755	77.7055	77.6223	76.9498
P_{12}	55.2225	55.2225	55	55.2225	55	91.1668	94.9768	55	55
P_{13}	55.0861	55.0861	55	55.0861	55	115.6511	55.7143	55	55
P_{14}	81.7524	81.7524	81	81.7524	117.4854	84.3133	83.9536	83.465	81
P_{15}	41.7615	41.7615	40	41.7615	45.9281	40	40	40	40
P_{16}	82.273	82.273	81	82.273	117.4854	81.1796	85.7133	82.7732	81
P_{17}	40.5599	40.5599	40	40.5599	45.9281	40	40	40	40
P_{18}	10.0002	10.0002	10	10.0002	10.0013	10	10	10	10
P_{19}	31.4679	31.4679	35	31.4679	42.1109	35.097	35	31.4568	35
H_{14}	105.2219	105.2219	104.8	105.2219	125.2754	106.6588	106.4569	106.0991	104.8
H_{15}	76.5205	76.5205	75	76.5205	80.1175	74.998	74.998	75	75
H_{16}	105.5142	105.5142	104.8	105.5142	125.2754	104.9002	107.4073	105.789	104.8
H_{17}	75.4833	75.4833	75	75.4833	80.1174	74.998	74.998	75	75
H_{18}	39.9999	39.9999	40	39.9999	40.0005	40	40	40	40
H_{19}	18.3944	18.3944	20	18.3944	23.2322	19.7385	20	18.3782	20
H_{20}	468.9043	468.9043	470.4	468.9043	415.9815	469.3368	466.2575	469.7337	470.3907
H_{21}	59.9994	59.9994	60	59.9994	60	60	60	60	60
H_{22}	59.9999	59.9999	60	59.9999	60	60	60	60	60
H_{23}	119.9854	119.9854	120	119.9854	120	119.6511	120	120	120
H_{24}	119.9768	119.9768	120	119.9768	120	119.7176	119.8823	120	120
Minimum cost	58,006.9992	58,122.746	57,825.4368	57,856.2676	59,736.2635	58,225.745	58,049.0197	57,846.84	57,825.2594
Maximum cost	58,038.5273	58,359.552	NA	57,913.7731	60,076.6903	58,318.8792	58,219.1413	57,910.98	57,857.1058
Mean cost	58,006.9992	58,198.3106	NA	59,853.478	59,853.478	58,295.9243	58,156.5192	57,873.86	57,836.9224
CPU time (s)	5.67	52.25	NA	5.82	53.36	35.54	35.54	5.48	49.98

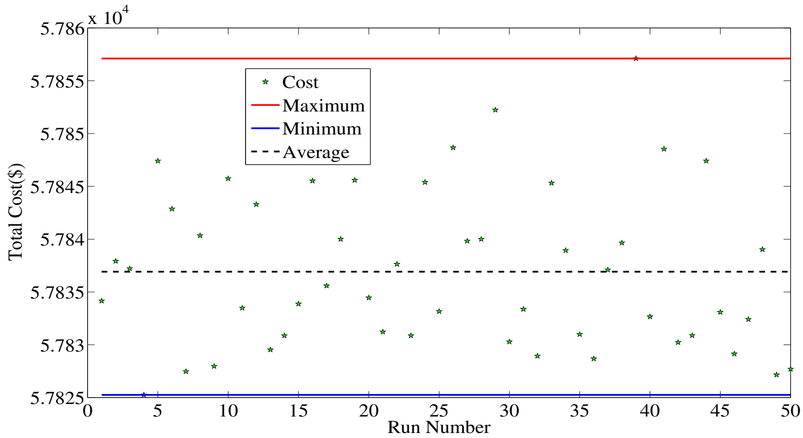

Figure 6. Distribution of total costs for 50 independent runs for 24-unit test system.

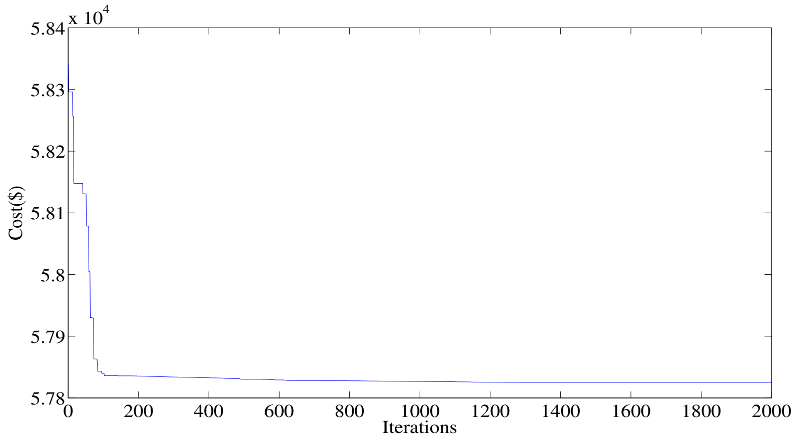

Figure 7. Convergence characteristics of the proposed IABC algorithm for 24-unit test system.

6.3. Test System Iii (48-Unit System)

This system consists of 48 units which includes 26 power-only, 12 CHP, and 10 heat-only units. The data of this system is given in [15]. The total electricity demand is 4700 MW and the total heat demand is 2500 MWth. Table 12 summarizes the optimal heat and power dispatches using the proposed method. The obtained results using the proposed algorithm are compared with the recent algorithms such as TLBO [43], OTLBO [43], CPSO [15], TVAC-PSO [15], group search optimization method (GSO) [48] and improved GSO (IGSO) [48].

It should be mentioned that the total costs presented in this table are directly quoted from the corresponding references. Some of the algorithms have obtained a lower cost, but with the expense of violating some of the technical constraints. Table 13 presents the feasibility analysis of the different algorithms. As it can be observed from this table, the solutions obtained by TLBO, OTLBO, GSO, and IGSO are not feasible. The CHP units which violate their corresponding feasible operation regions are indicate in Table 13 by $\times$ symbol. For example, for the TLBO method the CHP unit 38 violates its feasible region and the total power and heat mismatch is -69.9975. The minimum, mean and maximum obtained total costs are presented for 50 independent runs. Distribution of total costs for 50 independent runs for this test system is presented in Figure 8. It is observed from this figure that in 28 runs the obtained solution is better than the average solution, and if we ignore solution 34, the remaining solutions are also very close to the average solution. This shows good diversity of the obtained solutions by this algorithm. The convergence characteristics of the proposed algorithm for this system is depicted in Figure 9. As it can be observed from this figure the proposed algorithm is converged to the optimal solution in earlier iterations, similar to previous test cases, which confirms the capability of the algorithm in dealing with large scale CHPED problems. To calculate the annual cost-saving, the studied hourly load is considered to be the average load during a year. Therefore, in the 48-unit test system, as an example, the saving will be 6,082,861 ($/year) if the result of the proposed algorithm is compared with the result of the best available feasible algorithm (TVAC-PSO). This comparison shows that the proposed optimization algorithm reduces the system operation cost considerably.

Table 12. Comparison of the obtained results for 48-unit test system.

Control Variable	TVAC-PSO [15]	CPSO [15]	GSO [48]	IGSO [48]	TLBO [43]	OLTBO [43]	Proposed
P_1	538.5587	359.0392	627.5814	629.4952	538.5693	628.3199	628.3071
P_2	75.134	74.5831	302.5046	151.9991	225.3021	225.3313	224.5321
P_3	75.134	74.5831	225.3696	299.2996	229.9473	223.9653	224.6053
P_4	140.6146	139.3803	178.6488	159.2254	159.1352	159.8516	159.7442
P_5	140.6146	139.3803	178.2134	173.6004	160.0561	109.915	109.8049
P_6	140.6146	139.3803	159.8844	93.4383	109.7821	159.7795	159.7348
P_7	140.6146	139.3803	161.4173	160.773	159.6609	109.8946	109.9910
P_8	140.6146	139.3803	108.776	159.351	159.6492	109.9321	110.0123
P_9	140.6146	139.3803	109.0234	161.4184	109.966	159.9569	159.7589
P_{10}	112.1998	74.7998	115.1364	115.2927	40.3726	40.897	40.0033
P_{11}	112.1998	74.7998	114.2308	112.8994	77.5821	41.3115	40.0604
P_{12}	74.7999	74.7998	107.2839	97.5394	92.2489	55.1748	55.1632
P_{13}	74.7999	74.7998	93.0811	55	55.1755	92.4003	92.3016
P_{14}	269.2794	679.881	0	0	448.6854	448.8359	359.0530
P_{15}	299.1993	148.6585	223.7257	299.268	149.4238	225.7871	224.3763
P_{16}	299.1993	148.6585	356.9056	225.4102	224.7173	75.46	74.8094
P_{17}	140.3973	139.0809	109.2667	162.4605	109.9355	160.1192	159.6180
P_{18}	140.3973	139.0809	160.4169	160.9664	159.9052	110.3532	109.7450
P_{19}	140.3973	139.0809	109.6482	164.0177	159.7255	159.819	159.7410
P_{20}	140.3973	139.0809	160.0005	168.4149	159.782	159.7765	159.8047
P_{21}	140.3973	139.0809	174.5336	159.5402	60.0777	159.737	159.6745
P_{22}	140.3973	139.0809	118.6394	110.8099	110.0689	160.1751	159.6378
P_{23}	74.7998	74.7998	40.063	40.6399	77.6818	40.114	40.0053
P_{24}	74.7998	74.7998	41.2253	114.3701	40.2707	40.3042	40.0109
P_{25}	112.1997	112.1993	55	92.3275	92.4108	92.4149	92.1754
P_{26}	112.1997	112.1993	92.0406	55	55.0956	92.5012	92.4037
P_{27}	86.9119	92.8423	81.3512	82.1821	81.4882	85.9857	90.0393
P_{28}	56.1027	98.7199	40	40	44.5478	98.5005	81.0528
P_{29}	86.9119	92.8423	81.0383	81.089	81.056	81.7197	82.4319
P_{30}	56.1027	98.7199	40	40.4281	91.6819	48.9055	81
P_{31}	10.0031	10.0002	10	10.6913	10.548	10.0832	10
P_{32}	35	56.7153	35.2736	35.0696	52.718	39.311	38.8071
P_{33}	95.4799	109.1877	82.878	81	82.1522	82.0236	98.9499
P_{34}	54.9235	65.6006	40	40.1014	52.0606	40.1105	81.0677
P_{35}	95.4799	109.18	81	81.0922	82.7394	81.3039	98.9518
P_{36}	54.9235	65.6006	40.3336	40.1056	45.7398	45.67	47.3001
P_{37}	23.4981	10.6158	10.5087	10	10.0075	13.8709	10
P_{38}	54.0882	60.5994	35	35.6838	30.0332	30.3881	35.3241
H_{27}	108.1177	111.4458	104.9965	103.5903	105.0678	107.5951	109.8506
H_{28}	88.9006	125.6898	74.998	74.998	78.9162	125.4997	110.4369
H_{29}	108.1177	111.4458	104.8209	104.2548	104.827	105.1942	105.5403
H_{30}	88.9006	125.6898	74.998	75.3686	119.6006	82.6853	110.3925
H_{31}	40.0013	40.0001	40.001	40.0999	40.2345	40.0346	39.9999
H_{32}	20	29.8706	19.2636	19.2943	28.0508	21.9568	21.7102
H_{33}	112.926	120.6188	105.5564	104.8032	105.4339	105.3622	114.8715
H_{34}	87.8827	97.0997	74.998	75.0858	85.40864	75.0938	110.4267
H_{35}	112.926	120.6188	104.8032	104.8511	105.7694	104.9667	114.8542
H_{36}	87.8827	97.0997	332.3293	75.086	79.9447	79.8936	81.2985
H_{37}	45.7849	40.2639	39.5514	40	40.0001	41.6554	39.9999
H_{38}	28.6765	31.6361	20	20.3111	17.7401	17.9018	20.1409
H_{39}	433.9113	357.9456	486.4858	428.0157	394.616	445.0937	399.5313
H_{40}	60	59.9916	60	59.5061	59.93	59.9967	60
H_{41}	60	59.9916	60	59.9205	59.9578	59.9974	60
H_{42}	120	120	118.7549	114.8048	118.5797	119.8834	120
H_{43}	120	120	113.2371	117.9877	118.3425	119.5231	119.9999
H_{44}	415.9741	370.6214	212.5981	535.65	480.6566	428.7605	400.9482
H_{45}	60	59.9999	59.5362	60	59.9346	59.9957	60
H_{46}	60	59.9999	59.9138	60	59.981	59.9638	60
H_{47}	119.9989	119.9856	113.9272	107.7179	117.8207	119.5025	120
H_{48}	119.9989	119.9856	119.2305	118.6434	119.1898	119.444	119.9999
Minimum cost	117,824.8956	119,708.8818	117,824.896	112,320.4159	116,739.364	116,579.239	117,130.505
Maximum cost	NA	NA	NA	NA	116,756.0057	116,613.6505	117,182.5525
Mean cost	NA	NA	NA	NA	116,825.8223	116,649.4473	117,145.5397
CPU time (s)	89.63	93.32	70.65	70.65	10.38	10.93	89.51

Table 13. Feasibility analysis of the obtained results for 48-unit test system.

CHP Unit No.	TVAC-PSO [15]	CPSO [15]	OLTLBO [43]	TLBO [43]	GSO [48]	IGSO [48]	Proposed
27	3	3	3	3	3	3	3
28	3	3	3	3	3	3	3
29	3	3	3	3	×	3	3
30	3	3	3	3	3	3	3
31	3	3	3	3	3	3	3
32	3	3	3	3	3	3	3
33	3	3	3	3	3	3	3
34	3	3	3	3	3	3	3
35	3	3	3	3	3	3	3
36	3	3	3	3	3	×	3
37	3	3	3	3	3	3	3
38	3	3	×	×	3	3	3
Total Mismatch of Equations (6) and (8)	-2.00×10^{-4}	-0.0079	-3.00×10^{-4}	-69.9975	0.064	-1.00×10^{-4}	6.23×10^{-4}

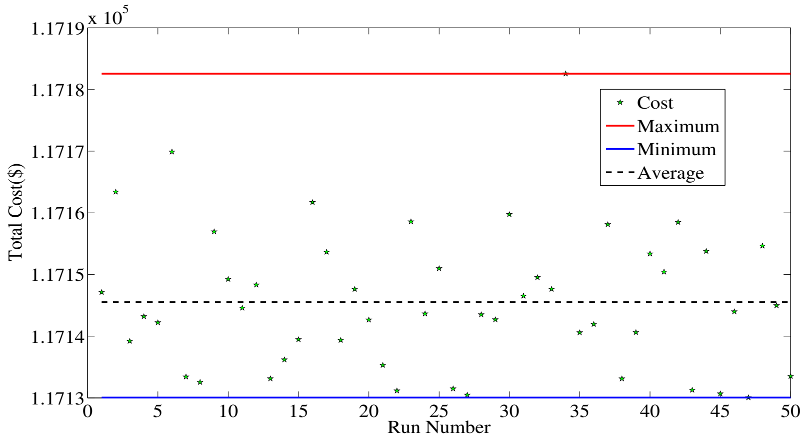

Figure 8. Distribution of total costs for 50 independent runs for 48-unit test system.

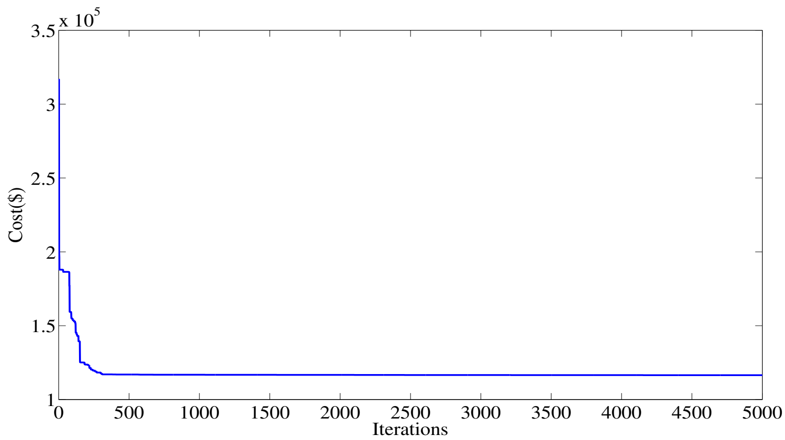

Figure 9. Convergence characteristics of the proposed IABC algorithm for 48-unit test system.

7. Conclusions

A new approach based on the improved artificial bee colony (IABC) algorithm is proposed in this paper for an efficient solution of CHPED problem. Different characteristics and constraints such as valve-point effect, power losses, feasible operation region of CHP units, and capacity limits of units are taken into account in the formulation. The effectiveness of the proposed IABC algorithm is verified using standard benchmark functions and statistical analysis. It is found that the proposed algorithm can find better solutions in terms of the objective function value, convergence speed and the number of solutions with lower objective function than the mean value, compared with other versions of ABC algorithm and other heuristic algorithms. Three test cases with different size and characteristics are used to evaluate the efficiency of the propose algorithm. The obtained results using the proposed IABC algorithm are compared with the most recent proposed algorithms and it is observed that the IABC converges to a feasible solution with the lower total cost in a reasonable time in comparison with the previously reported algorithms. The numerical results substantiate that:

- The obtained results by the proposed IABC algorithm has small diversity and in most cases the algorithm converges to optimal or near optimal solutions. In other words, the variance of the obtained solutions is small.
- The algorithm converges in relatively small number of iterations. This means that the algorithm has a good converge speed which enables it to be used in large systems.
- In test system I, the obtained value for the objective function is less than the average value in 66% of the trial runs. This is 54% for test system II and 56% for test system III, which means that the proposed algorithm is able to attain solutions lower than the mean value, in more than half of the trials.
- The obtained results are also feasible which indicates that the algorithm has the capability of attaining solutions which are both optimal and feasible.

The better solution results, especially in large test systems, confirms the applicability of the proposed algorithm for dealing with the real world systems. From the application perspective, the proposed method results in an hourly saving of $205.14 per hour which means $1,797,033 saving in each year for small scale 7-unit test system. The hourly saving of $694.4 is obtained for a 48-unit case, which equals more than $6 million saving annually. As a future work, the proposed method could be extended to solve unit commitment problem considering CHP units.

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Parameters

P^D	The electric power load of the system [MW].
H^D	The heat load of the system [$MWth$].
P^L	The electric power loss of the transmission system [MW].
$P_e^{po,max/min}$	Maximum/minimum generated electric power of eth power-only unit [MW].
$H_h^{ho,max/min}$	Maximum/minimum generated heat of hth boiler [$MWth$].
$P_c^{chp,max}(H_c^{chp})$	Maximum power output of cth CHP unit in MW when generating (H_c^{chp}) MWth heat.
$P_c^{chp,min}(H_c^{chp})$	Minimum power output of cth CHP unit in MW when generating (H_c^{chp}) MWth heat.
$H_c^{chp,max}(P_c^{chp})$	Maximum heat output of cth CHP unit in $MWth$ when generating (P_c^{chp}) MW power.
$H_c^{chp,min}(P_c^{chp})$	Minimum heat output of cth CHP unit in $MWth$ when generating (P_c^{chp}) MW power.
a_e^{po}	Quadratic cost coefficient of power-only unit e [$\$/(MW)^2h$].
b_e^{po}	Linear cost coefficient of power-only unit e [$/MWh].
c_e^{po}	No-load cost coefficient of power-only unit e [$/h].
d_e^{po}	Magnitude of sinusoidal term in cost function of power-only unit e [$/h].
f_e^{po}	Frequency of sinusoidal term in cost function of power-only unit e [rad/MWh].
a_h^{ho}	Quadratic cost coefficient of heat-only unit h $\$/((MWth)h)^2h$.
b_h^{ho}	Linear cost coefficient of heat-only unit h [$/(MWth)h].
c_h^{ho}	No-load cost coefficient of heat-only unit h [$/h].
a_c^{chp}	Quadratic cost coefficient of CHP unit c [$\$/(MW)^2h$].
b_c^{chp}	Linear cost coefficient of CHP unit c [$/MWh].

c_c^{chp} No-load cost coefficient of CHP unit c [$\$/h$].
d_c^{chp} Quadratic cost coefficient of CHP unit c [$\$/(MWth)^2h$].
e_c^{chp} Linear cost coefficient of CHP unit c [\$/(MWth)h].
f_c^{chp} Quadratic cost coefficient of CHP unit c [$\$/(MW)(MWth)h$].

Continuous Variables

P_e^{po} Power output of ith power-only unit at time t.
P_c^{chp} Power output of jth CHP unit at time t.
H_c^{chp} Heat output of jth CHP unit at time t.
H_h^{ho} Heat output of kth heat-only unit at time t.

Functions

$C_e(P_e^{po})$ Cost function of e-th power-only unit.
$C_c(P_c^{chp}, H_c^{chp})$ Cost function of c-th CHP unit.
$C_h(H_h^{ho})$ Cost function of h-th heat-only unit.

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