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Article

A Hybrid Multi-Step Model for Forecasting Day-Ahead Electricity Price Based on Optimization, Fuzzy Logic and Model Selection

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Abstract: The day-ahead electricity market is closely related to other commodity markets such as the fuel and emission markets and is increasingly playing a significant role in human life. Thus, in the electricity markets, accurate electricity price forecasting plays significant role for power producers and consumers. Although many studies developing and proposing highly accurate forecasting models exist in the literature, there have been few investigations on improving the forecasting effectiveness of electricity price from the perspective of reducing the volatility of data with satisfactory accuracy. Based on reducing the volatility of the electricity price and the forecasting nature of the radial basis function network (RBFN), this paper successfully develops a two-stage model to forecast the day-ahead electricity price, of which the first stage is particle swarm optimization (PSO)-core mapping (CM) with self-organizing-map and fuzzy set (PCMwSF), and the second stage is selection rule (SR). The PCMwSF stage applies CM, fuzzy set and optimized weights to obtain the future price, and the SR stage is inspired by the forecasting nature of RBFN and effectively selects the best forecast during the test period. The proposed model, i.e., CM-PCMwSF-SR, not only overcomes the difficulty of reducing the high volatility of the electricity price but also leads to a superior forecasting effectiveness than benchmarks.

Keywords: selection rule (SR); reducing volatility; self-organizing-map; fuzzy logic; particle swarm optimization (PSO); forecasting

1. Introduction

Electricity is one of the most essential energy inputs to the industry and has increasingly significant influences on modern industry. Meanwhile, the management of operation process is more sensitive and vulnerable to the electricity supply fluctuations and its cost changes more than ever before. This demands more stable and reliable energy supply, cost management, as well as risk management. There is rising demand for more accurate analysis and forecasting of the electricity price movement [1]. To obtain accurate estimated electricity prices, modeling and prediction techniques are frequently applied to bid or hedge against the volatility of electricity prices [2,3]. Overall, it is not difficult to find that the electricity price is not only related to the interests of market participants but also affects many aspects of society and the economy. Thus, it is necessary to explore its nature in order to aid participants of the electricity market.

To show the significance of this paper better, some effective forecasting approaches for the electricity price from previous research investigations will be introduced here. One forecast strategy is a new two-stage feature selection (FS) algorithm, which is proposed by Keynia [4] and is based on the mutual information (MI) criterion; it selects representative features of the composite neural network (CNN) among feature candidates. Yan et al. [5,6] applied a multiple support vector machine (SVM) to forecast mid-term electricity price and developed a hybrid mid-term electricity price forecasting model by combining SVM and auto-regressive moving average with external input (ARMAX) modules. The Markov-switching generalized autoregressive conditional heteroskedasticity (MS-GARCH) model was developed to forecast low and high volatility electricity prices by Cifter [7]. Anbazhagan and Kumarappan proposed feed-forward neural network (FFNN) featured by one-dimensional discrete cosine transforms (DCT) and day-ahead electricity price classification using three-layered FFNN, cascade-forward neural network (CFNN) and generalized regression neural network (GRNN) [8–10]. A novel grey model was proposed using particle swarm optimization (PSO) algorithm by Lei and Feng [11]. Based on panel co-integration and particle filter (PCPF), Li et al. [12] investigated a two-stage hybrid model to achieve two main goals: (1) to expand the dimension of the dataset; and (2) to consider the model parameters as a time-varying process. Zhang and Tan [13,14] proposed new hybrid methods based on wavelet transform (WT), autoregressive integrated moving average (ARIMA) and least squares support vector machine (LSSVM) optimized by PSO and WT, chaotic least squares support vector machine (CLSSVM) and exponential generalized autoregressive conditional heteroskedastic (EGARCH) to predict electricity prices. Liu et al. [2] applied various autoregressive moving average (ARMA) models with generalized autoregressive conditional heteroskedasticity (GARCH) processes, namely ARMA-GARCH models, along with their modified forms, ARMA-GARCH-in-mean (ARMA-GARCH-M), to model and forecast hourly-ahead electricity prices. Najeh Chaâbane, based on the idea of choosing forecasting models, proposed a model that exploited the feature and strength of the auto-regressive fractionally integrated moving average (ARFIMA) model, as well as the feedforward neural networks model [15]. A new hybrid ARIMA-ANN model for the prediction of time series data based on the linear ARIMA and nonlinear artificial neural network (ANN) models was proposed by Babu et al. [16]. Shrivastava et al. [17] investigated the performance of extreme learning machine (ELM) in the price forecasting problem. Shayeghi et al. [18] proposed a new combination of the FS technique based on the MI technique and WT in. The delta and bootstrap methods were employed for the construction of prediction intervals (PIs) for uncertainty quantification by Khosravi et al. [19–21]. Bordignon et al. [22] studied combined versus individual forecasts for the prediction of British electricity prices. Grimes et al. [23] showed that simply optimizing price forecasts based on classical regression error metrics did not work well for scheduling. Nowotarski et al. [24] applied seven averaging and one selection scheme and performed backtesting analysis on day-ahead electricity prices in three major markets. From a dynamical system perspective, Sharma and Srinivasan [25] proposed a hybrid model that employed a synergistic combination of recurrent neural network (RNN) and coupled excitable system for electricity price forecasting. Dev and Martin [26] proposed an approach for the predictive capacity of neural networks and applied Australian National Electricity Market data to test their model. Wang et al. [27] proposed a forecasting model of electricity price using chaotic sequences for forecasting short-term electricity prices. The forecasting performances of four ARMAX-GARCH models for five MISO pricing hubs (Cinergy, First Energy, Illinois, Michigan, and Minnesota) were analyzed by Hickey et al. [28]. Christensen et al. [29] focused on the prediction of price spikes using a nonlinear variant of the autoregressive conditional hazard model. Amjady and Keynia [30] proposed a strategy that included a new closed-loop prediction mechanism composed of probabilistic neural network (PNN) and hybrid neuro-evolutionary system (HNES) forecast engines to forecast Pennsylvania–New Jersey–Maryland (PJM) electricity prices. Dudek [31] applied Multilayer perceptron for GEFCom2014 probabilistic electricity price forecasting. Panapakidis and Dagoumas [32] reviewed recent literature related to electricity price forecasting and applied ANN to predict future electricity prices. The K-support vector regression (K-SVR), a hybrid model to combine clustering algorithms, SVM, and SVR to forecast electricity price of PJM, is presented by Feijoo et al. [33]. Abedinia et al. [34] proposed a Combinatorial Neural Network-based forecasting engine to forecast the electricity price. The curvelet denoising-based approach was proposed to improve the forecasting effectiveness of the electricity price by He et al. [35]. Ziel et al. [36] gave an introduction of an econometric model for the hourly time series of electricity prices that incorporated specific features such as renewable energy. Hong et al. [37] applied a principal component analysis (PCA) network cascaded with a multi-layer feedforward (MLF) network for forecasting locational marginal prices (LMPs). By combining statistical techniques for pre-processing data and a multi-layer neural network, a dynamic hybrid model was proposed by Cerjan et al. [38] for forecasting electricity prices and price spike detection. Monteiro et al. [39] showed comparisons of forecasts, which led to the identification of the most important variables for forecasting purposes. By relying on simple models, forecasting approaches were derived and analyzed by Jónsson et al. [40]. Weron [41] reviewed literature related to electricity price forecasting and speculated on the directions electricity price forecasting should take in the next decade or so.

In this paper, based on reducing the volatility of the electricity price and the forecasting nature of the radial basis function network (RBFN), we successfully develop a two-stage model to forecast the day-ahead electricity price, of which the first stage is PSO-core mapping (CM) with self-organizing-map and fuzzy set (PCMwSF) and the second stage is selection rule (SR). The PCMwSF stage aims to apply CM, fuzzy set and optimized weights to obtain the future price, and the SR stage is inspired by the forecasting nature of RBFN and effectively selects the best forecast during the test period. The highlights of this paper are as follows:

- ➢ We successfully overcome the volatility of the electricity price through the CM method.
- ➢ Improvement from reducing the volatility is obvious during the test period.
- ➢ Self-organizing map (SOM) is assigned to divide the original data into three parts: low, medium and high.
- ➢ Divided price is weighted by the PSO algorithm and performs well during forecasting.
- ➢ SR is based on three new defined criteria and effectively selects the forecasting model.

2. Self-Organizing-Map

Figure 1 shows an application of SOM.

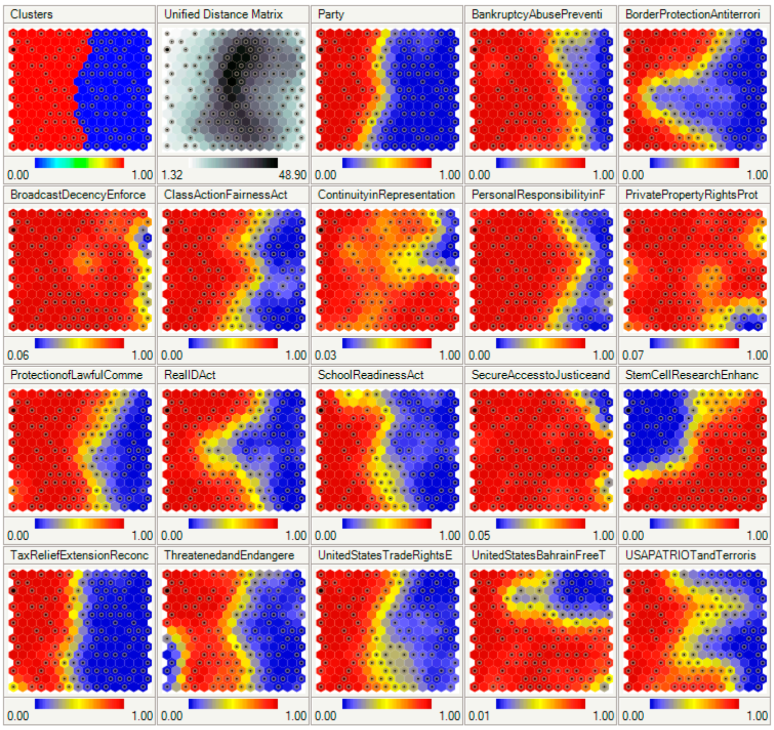


Figure 1. A self-organizing map showing U.S. Congress voting patterns visualized in Synapse. The first two boxes show clustering and distances, while the remaining ones show the component planes. Red means a yes vote, while blue means a no vote in the component planes (except the party component, where red is Republican and blue is Democratic) [42].

Because this paper focuses on the pre-process of forecasting, RBFN, i.e., the main forecasting tool, will not be introduced. Details of this method are described in [43], and the introduction of fuzzy logic and PSO can be found in [44–47]. As an ANN, SOM maps the training samples into low dimensional (typically two-dimensional), discretized representations in the input space using unsupervised learning. Unlike the other ANNs, SOM can preserve the topological properties of the input space by introducing a neighborhood function. Thus, SOM is able to visualize high-dimensional or multi-dimensional data as low-dimensional vectors. [48]. Besides, the ability of handling a high number of nodes makes SOM a powerful tool in clustering [49]. Details of the learning algorithm of SOM can be found in [50].

3. Core Mapping-Particle Swarm Optimization-Core Mapping with Self-Organizing-Map and Fuzzy Set-Selection Rule for Electricity Price Forecasting

To illustrate these approaches specifically, this section will give details of these models for forecasting electricity price.

3.1. Core Idea of This Paper

To demonstrate the core idea of this paper, the reason why high forecasting errors occur will be shown initially. In the process of forecasting, data firstly will be pre-processed to suit for model, which will be obtained by training through pre-processed data. Then, this trained model is utilized in the forecast. From research related to forecasting, it is apparent that the volatility of data has a huge effect on forecasting accuracy, which means that the volatility of data directly determines the accuracy level the model can reach. Thus, legitimately reducing volatility is an important problem in forecasting and is also the inspiration of this paper. However, from the above section, many researchers have concentrated on the promotion of algorithms, such as BP neuron network, LSSVM, ARIMA, GARCH and so on, rather than on the pre-processing of data or initial transformation of data. To improve this part of the entire forecasting process, mapping f is proposed in this paper:

$$f(x) = \int_0^x \ln(t+1)\,\mathrm{d}t \tag{1}$$

Thus, for discrete data, Equation (1) can be expressed by:

$$f(price(x)) = \sum_{i=1}^{x} \ln(price(i)+1) \tag{2}$$

that means:

$$f : price(x) \to \sum_{i=1}^{x} \ln(price(i)+1) \tag{3}$$

This mapping is also called **CM** in this paper.

Furthermore, to reduce the volatility of the electricity price, it is divided into high price, low price and medium price by a SOM. Then, a fuzzy logic is established:

- **IF** *price*(i) **IS** High price, **THEN** *price*(i) equals *price*(i) × Highweight;
- **IF** *price*(i) **IS** Medium price, **THEN** *price*(i) equals *price*(i) × Mediumweight; and
- **IF** *price*(i) **IS** Low price, **THEN** *price*(i) equals *price*(i) × Lowweight.

Thus, the CM will be changed to:

$$\begin{aligned} f : price \to & \sum_{High\_price} HIGHWeight \times \ln(price+1) + \\ & \sum_{Medium\_price} \ln(price+1) + \sum_{Low\_price} LOWWeight \times \ln(price+1) \end{aligned} \tag{4}$$

Finally, PSO is used to optimize Highweight and Lowweight to make sure that a greater forecasting accuracy can be obtained. In post-processing, the formula of post-processing is as follows (where n is the length of forecasting series):

$$price_{\text{forecast}}(i) = e^{price_{\text{forecast}}^{\text{pre-processed}}(i)} - e^{price_{\text{forecast}}^{\text{pre-processed}}(i-1)} - 1,\ i = 2, \ldots, n \tag{5}$$

Thus, the CM method and PSO-CM with SOM and fuzzy logic (PCMwSF) method are proposed and used to pre-process price data in this paper. The pre-processed data will be given to RBFN to forecast the day-ahead electricity price. Mean absolute percentage error (*MAPE*), mean absolute error (*MAE*) and root mean square error (*RMSE*) obtained from the forecasting results demonstrate that the proposed model can efficiently forecast the price.

Furthermore, to obtain excellent forecasting accuracy of electricity prices, a rule of model selection is proposed to choose which model should be used. The final forecasting model, named CM-PCMwSF-SR, outperforms the others in each season of 2002 in the PJM power market, which is commonly recognized as one of the most successful markets in the US.

3.2. Basic Pre-Process

Before introducing proposed methods, simple pre-processes of data need to be defined first. In this paper, basic pre-processes can be expressed by:

$$price(i) = \begin{cases} \max\limits_{0<i<N}(price(i)), & price(i) > 10 \times \frac{1}{N}\sum\limits_{i=1}^{N} price(i) \\ price(i), & \text{otherwise} \end{cases} \tag{6}$$

then:

$$price(i) = \begin{cases} \frac{price(i-1)+price(i+1)}{2}, & 0.8 < \frac{price(i)}{\frac{price(i-1)+price(i+1)}{2}} < 1 \\ \frac{price(i-1)+price(i+1)}{2}, & price(i) < 1 \\ price(i), & \text{otherwise} \end{cases} \tag{7}$$

where N is the length of the electricity price, which is prepared to train RBFN and $i = 1, 2, \ldots, N$. Equations (6) and (7) indicate that if the gap of $price(i)$ and mean of $price(i-1)$ and $price(i+1)$ are less than 20% or if $price(i)$ is too small, $price(i)$ will be changed to the mean value of $price(i-1)$ and $price(i+1)$. This can be observed in Figure 2. Obviously, the linearized line is smoother than the actual line.

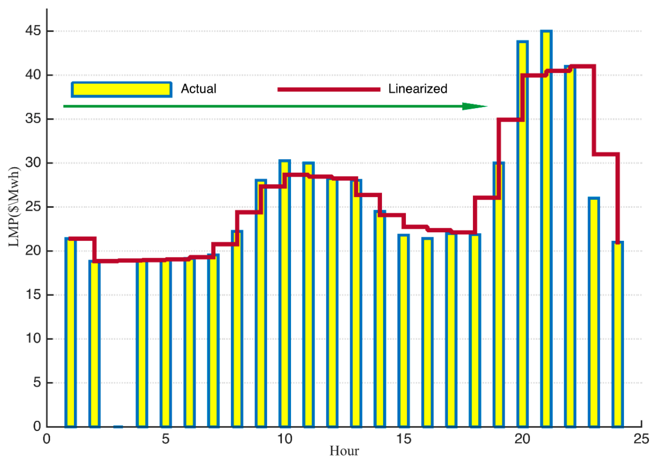


Figure 2. Actual and linearized one day Pennsylvania–New Jersey–Maryland (PJM) electricity price on 7 April 2002.

3.3. Core Mapping Method

In this section, the CM approach will be described using an actual example. Taking the electricity price of 26 June 2002 in the PJM electricity market as an example, the data are first linearized and then mapped by CM. The mapped data are shown in Figure 3.

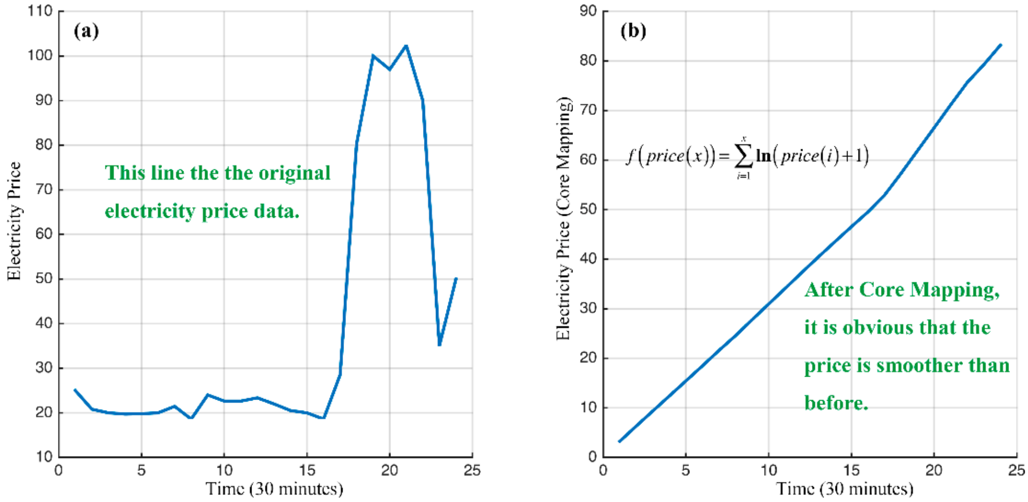


Figure 3. (**a**) Actual price; and (**b**) core-mapped price of 26 June 2001 in PJM electricity market.

It is obvious that the volatility of mapped data is smaller than the volatility of actual data. This means that the CM method can reduce the volatility of data and consequently makes the accuracy of the forecasted electricity price much higher than that of the original method, which is shown in the experiments in Section 4.

3.4. Swarm Optimization Algorithm-Core Mapping with Self-Organizing Map and Fuzzy (Particle Swarm Optimization-Core Mapping with Self-Organizing-Map and Fuzzy Set) Method

Although the CM method can reduce the volatility of the electricity price, there are always high prices or low prices, which increase this volatility of the electricity price. In this section, PCMwSF is proposed to address this problem.

3.4.1. Forecasting Rules

To evaluate the effectiveness of the methods, this paper uses three rules in the forecasting process:

(1) A previous month's data are used to forecast the price of the target day.
(2) There is only the historical electricity price considered in this paper (without data of demand or environmental data (for the environmental data, we do not find the corresponding dataset (24 h in one day))).
(3) All forecasting results are day-ahead forecasting, and the forecasting mode is shown in Figure 4.

Remark 1. *In some literatures related to electricity price forecasting, electricity demand is regarded as a feature to predict the electricity price. However, adding electricity demand as one of the features cannot help to improve forecasting effectiveness after the experiment (the final experiment shows that the forecasting results with electricity demand is similar to the results without it, which means that electricity demand is not a key factor to influence the forecasting effectiveness). Thus, this paper does not select the electricity demand as one of features in our paper, which is the reason why there is only the historical electricity price considered in this paper.*

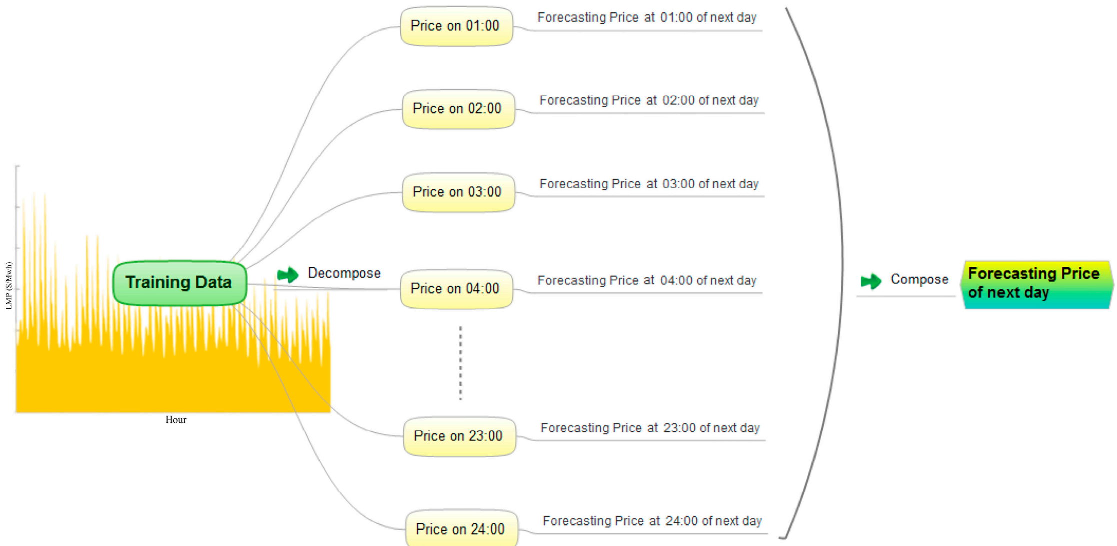


Figure 4. Forecasting mode.

3.4.2. Classification of Price with Self-Organizing Map and Fuzzy Logic

Before linearizing the price and applying CM, the PCMwSF method is used to divide the processed price into three categories: High price, Medium price and Low price by using SOM. The price mentioned above is the historical price prior to the price that needs to be forecasted. For example, if the price data on 26 April 2002 need to be forecasted, the PCMwSF method will divide the price data that are between 1 January and 25 April into three categories.

In the introduction section, a fuzzy logic was established to change CM to ensure good forecasting accuracy. When three classifications of the historical price are obtained, Highweight and Lowweight need to be determined to forecast the next spot price. How to determine both of them is a very important problem in the predication process, and the PSO algorithm, which is a powerful tool for optimizing parameters, is used to solve this problem.

3.4.3. Applying of Swarm Optimization Algorithm Algorithm

In the process of PSO, the fitness function is key to the optimization problem. Before identifying the fitness function, the index of measuring the degree of volatility needs to be established.

Definition 1. *The identity of volatility of price is defined as:*

$$vop\,(i) = \boldsymbol{var}\left(\left[\boldsymbol{var}\left(price\left(i,\,1:\tfrac{T}{4}\right)\right),\,\boldsymbol{var}\left(price\left(i,\,\tfrac{T}{4}+1:\tfrac{2T}{4}\right)\right),\,\ldots,\,\boldsymbol{var}\left(price\left(i,\,\tfrac{3T}{4}:T\right)\right)\right]\right) \tag{8}$$

where vop(i) is the volatility of price of the ith day, T represents the number of points observed in one day, and ***var*** *refers to the variance of a specific series in the ith day.*

Then, another index to evaluate the forecasting accuracy is proposed for PSO algorithm.

Definition 2. *The index to evaluate the in-sample forecasting effectiveness can be expressed as following:*

$$aob\,(i) = \frac{1}{T}\sqrt{\sum_{t=1}^{T}\left(price_{\text{forecast}}\,(i-1,\,t) - price_{\text{actual}}\,(i-1,\,t)\right)^2} \tag{9}$$

where T represents the number of points observed in one day, $price_{\text{forecast}}$ represents the forecasting value at time point t of the ith day, and $price_{\text{actual}}$ represents the observed value at time point t of the day.

This index is the forecasting accuracy of the previous day of the day the needs to be forecasted. Next, the fitness function of PSO is identified as follows.

Definition 3. *The fitness function* $\Phi(\cdot)$ *of PSO algorithm used in PCMwSF model is defined as:*

$$\Phi_i(\cdot) = aob(i) \times vop(i) \tag{10}$$

where i represents the ith day and this definition indicates that lower fitness values can represent lower values of vop and aob, indicating lower volatility and higher forecasting accuracy.

We assign **Ind** to represent the output values of $\Phi(\cdot)$. In the last step, Highweight and Lowweight are changed by the PSO algorithm to make sure **Ind** reaches a minimum. Then, the optimized HIGHWeight and Lowweight are used to forecast the next-day price with RBFN.

3.5. Selection Rule Based on Forecasting Nature of Radial Basis Function Network

The CM method and PCMwSF method have different merits when forecasting the electricity price. Thus, it is important to correctly select a method to pre-process the original data. To solve this problem, this paper studies the properties of the RBF network in forecasting.

- RBF Network in Forecasting

Initially, this paper applies the RBF network to forecast price with the CM method and compares results with the previous day's actual price. Then, it is observed that the forecasting values of RBFN have little changes compared with the former day's electricity prices (shown in Section 4.2). Thus, the index of changes of price (***ICP***) is proposed as a criterion to measure the magnitude of price changes.

Definition 4. ***ICP*** *is defined as follows:*

$$ICP(P_1, P_2; i, j) = \frac{1}{T}\sum_{t=1}^{T}\frac{|P_1(i, t) - P_2(j, t)|}{P_1(i, t)} \tag{11}$$

where $P_c(i, t)$ *is the ith day's price (actual or forecasted) at t hour (c = 1, 2).*

Based on Equation (11), we define a criterion to evaluate what extent the former day's electricity price changes.

Definition 5. *Index of changes of actual price (**ICP-P**) is defined as follows:*

$$ICP - P(i) = ICP(P_{\text{actual}}, P_{\text{actual}}; i-1, i) = \frac{1}{T}\sum_{t=1}^{T}\frac{|P_{\text{actual}}(i-1, t) - P_{\text{actual}}(i, t)|}{P_{\text{actual}}(i-1, t)} \tag{12}$$

where $P_{\text{actual}}(i, t)$ *is the ith day's actual price at t hour (c = 1, 2).*

Additionally, if we obtain the forecasting values of the electricity price, we can define another criterion to evaluate to what extent the forecasting electricity price changes from the former day.

Definition 6. *Index of changes of forecasting price (**ICP-F**) is expressed as follows:*

$$ICP - F(i; P_{\text{forecast}}) = ICP(P_{\text{actual}}, P_{\text{forecast}}; i-1, i) = \frac{1}{T}\sum_{t=1}^{T}\frac{|P_{\text{actual}}(i-1, t) - P_{\text{forecast}}(i, t)|}{P_{\text{actual}}(i-1, t)} \tag{13}$$

where $P_{\text{actual}}(i, t)$ *is the ith day's actual price at t hour (c = 1, 2) and* $P_{\text{forecast}}(i, t)$ *is the ith day's forecasting price at t hour (c = 1, 2).*

From Definition 4, it is obvious that different forecasting values have their own ***ICP-F***, meaning that this new criterion can help us select the best forecasting models under the condition that we do not know the actual electricity price of the *i*th day. Thus, this paper proposes a **SR** to choose the best forecasting model based on ***ICP-F***.

Definition 7. *When forecasting the electricity price of the ith day, the* ***SR*** *can be expressed as follows:*

$$\mathbf{SR}\,(i) = \{m|\boldsymbol{ICP} - \boldsymbol{F}(i; P_{forecast}^{(m)}) = \max_{m=1,\,2,\,\ldots,\,M}(\boldsymbol{ICP} - \boldsymbol{F}(i; P_{forecast}^{(m)}))\} \tag{14}$$

where M is the number of forecasting models and $P_{\text{forecast}}^{(m)}$ *is the forecasting values of the ith day obtained by the mth model.*

It is obvious that the **SR** is an integer series. Thus, for the *i*th day, we should select $P_{\text{forecast}}^{(\text{SR}(i))}$ as the forecasting values of this day. Algorithm 1 demonstrates the Pseudo code of forecasting the electricity price of the *i*th day using the CM-PCMwSF-SR model.

Algorithm 1 Pseudo code of forecasting the electricity price of the *i*th day using the CM-PCMwSF-SR model.

P: The electricity price series

T: Number of time points in one-day electricity price series.

Iter: Number of iterations.

$\boldsymbol{t} = 1$.

Assign Equations (6) and (7) to pre-process $\boldsymbol{P}$

According to CM method, map $\boldsymbol{P}$ to $\boldsymbol{P}_{\mathbf{CM}}$

Divide $\boldsymbol{P}_{\mathbf{CM}}$ into $\boldsymbol{T}$ subseries and denote them by $\boldsymbol{P}_{\mathbf{CM1}}, \boldsymbol{P}_{\mathbf{CM2}}, \ldots, \boldsymbol{P}_{\mathbf{CM}T}$

According to CM method, map $\boldsymbol{P}$ to $\boldsymbol{P}_{\mathbf{PCM}}$

Divide $\boldsymbol{P}_{\mathbf{PCM}}$ into $\boldsymbol{T}$ subseries and denote them by $\boldsymbol{P}_{\mathbf{PCM1}}, \boldsymbol{P}_{\mathbf{PCM2}}, \ldots, \boldsymbol{P}_{\mathbf{PCM}T}$
While $t < \boldsymbol{T} + 1$
  Assign $\boldsymbol{Pcm_t}$ and RBFN, forecast the time t of electricity price of *i*th day and denote it by $pfcm(i, t)$.
  Assign $\boldsymbol{Ppcm_t}$ and RBFN, forecast the time t of electricity price of *i*th day and denote it by $pfpcm(i, t)$.
  $t = t + 1$
End
Calculate $\boldsymbol{ICP\text{-}F}(i; pfcm)$ and $\boldsymbol{ICP\text{-}F}(i; pfpcm)$
IF $\boldsymbol{ICP\text{-}F}(i; pfcm) > \boldsymbol{ICP\text{-}F}(i; pfpcm)$
  $Pf = pfcm$
Else
  $Pf = pfpcm$
End
Return *Pf*

3.6. Forecasting Principle and Evaluation Criteria

Because the input of RBFN must be between 0 and 1, the processed data need to be changed by the following formula:

$$Price = \frac{Price - P_{\min}}{P_{\max} - P_{\min}} \tag{15}$$

where $P_{\min}$ is the minimum value of the training data of RBFN and $P_{\max}$ is the maximum value of the training data of RBFN. To evaluate the accuracy of the forecast, the *MAPE*, *MAE* and *RMSE* are all used. The *MAPE*, *MAE*, and *RMSE* are defined as:

$$MAPE = \frac{1}{T}\sum_{t=1}^{T} \frac{\left|P_t^{\text{actual}} - P_t^{\text{forecast}}\right|}{P_t^{\text{actual}}} \tag{16}$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T} \left|P_t^{\text{actual}} - P_t^{\text{forecast}}\right| \tag{17}$$

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T} \left|P_t^{\text{actual}} - P_t^{\text{forecast}}\right|^2} \tag{18}$$

where P_t^{actual} is the actual price at time t and P_t^{forecast} is the forecasted price at time t. The range of Highweight is 0.9–1.05 and the range of Lowweight is 0.9–1.05 in the PSO algorithm.

4. Data Analyses and Numerical Results

PJM electricity price is selected to test the proposed methods. In Case 1, the forecasting results show that the PCMwSF method is better than the CM method. In Case 2, we illustrate the forecasting natures of RBFN, ***ICP-P*** and ***ICP-F***, which lay a strong foundation for **SR**. In other cases, weeks in different seasons are selected to test models. The details of each case are shown in Table 1.

Table 1. Six cases to evaluate effectiveness of the forecasting models.

Case	Forecasted Data	Remarks
1	26 June 2002	Test data 1
2	28 June 2002	Test data 2
3	18–22 March 2002	Spring week
4	24–28 June 2002	Summer week
5	23–27 September 2002	Autumn week
6	23–27 December 2002	Winter week

4.1. Study of Case 1

Figure 5 shows the day-ahead price forecasting results of RBFN for Case 1. Figure 6 shows the day-ahead price forecasting (CM method) for Case 1. Figure 7 shows the day-ahead price forecasting (PCMwSF method) for Case 1. The forecasting results are compared with the actual LMP value.

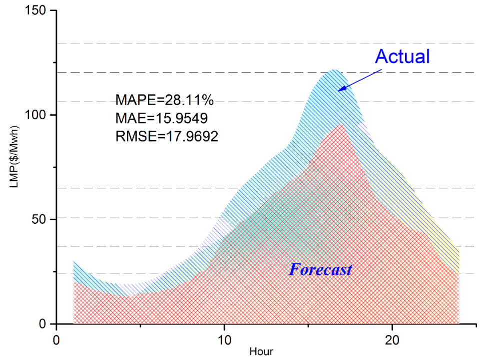


Figure 5. Actual PJM electricity price and forecasted values using radial basis function network (RBFN) in Case 1. *MAPE*: mean absolute percentage error; *MAE*: mean absolute error; and *RMSE*: root mean square error.

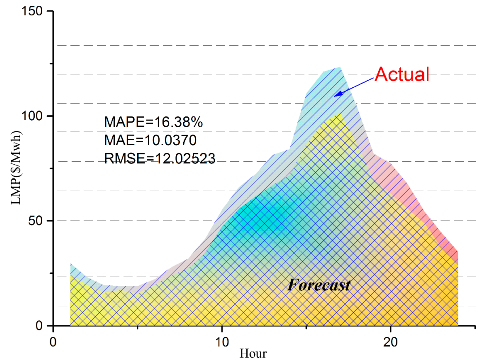


Figure 6. Actual PJM electricity price and forecasted values using CM in Case 1.

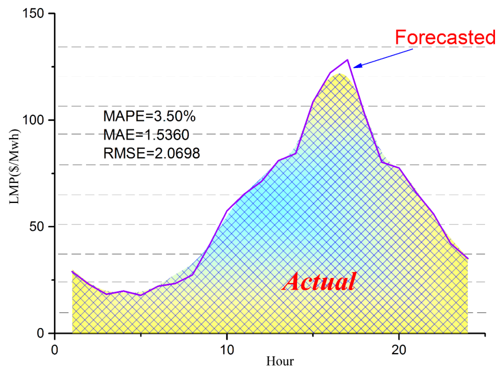
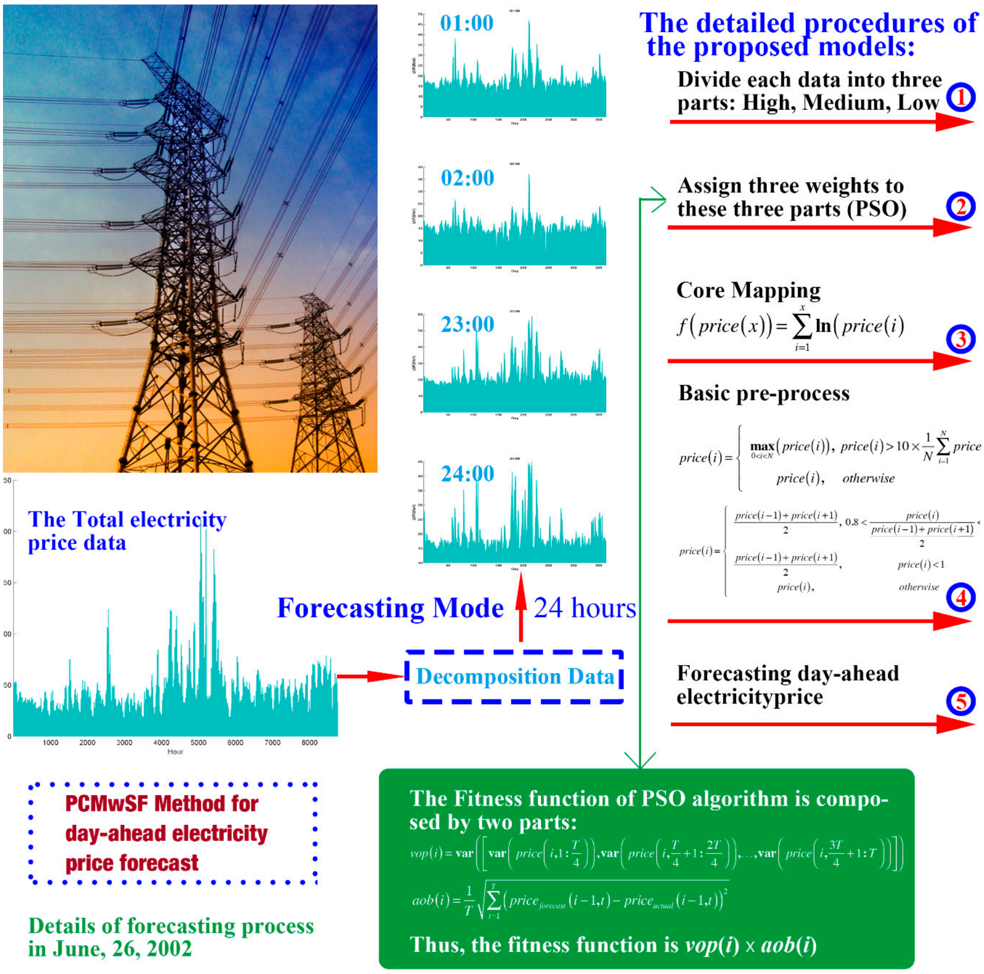


Figure 7. Actual PJM electricity price and forecasted values using particle swarm optimization (PSO)-core mapping (CM) with self-organizing-map and fuzzy set (PCMwSF) in Case 1.

Obviously, the forecasting result with the PCMwSF method is better than the others in Case 1. Details of the forecasting process are shown in Table 2. Table 2 collects data of the forecasting process with the PCMwSF method. The optimal Lowweight, optimal Highweight, optimal **Ind,** *vop*, accuracy of price forecasting on 25 June with optimized weight, actual price and forecasted price on 26 June, *MAPE* in the forecasting process and lower limit, upper limit of high price, medium price and low price are shown. It is obvious that **Ind** and *vop* are well optimized. The forecast on 25 June achieves desired results with the optimal Highweight and Lowweight, which means that the PCMwSF method has the ability to improve the forecasting effectiveness of the electricity price. The *MAPE* of the forecasted price in this model varies from a low of 0.01% at 20:00 to a high of 16% at 7:00. Figure 8 shows a flowchart of the PCMwSF method.



Hour	Optimized LOWWeight	Optimized HIGHWeight	Optimized Ind	*vop*	The accuracy of price forecasting in June,25,2002 with optimized	The actual price	The forecasting price	The accuracy of price forecasting	Lowprice		Mediumprice		Highprice	
									Lower limit	Upper limit	Lower limit	Upper limit	Lower limit	Upper limit
1	1.05	1.05	3.00E-07	3.03585E-05	0.009871573	30.042584	28.88083544	0.038670061	13.35	16.38	16.47	19.99	20.33	21.77
2	0.9	1.05	1.17E-06	0.001745776	0.000669503	23.227286	22.82923041	0.017137413	6.21	11.06	12.04	15.06	15.18	15.19
3	0.9	1.05	0.00120567	0.006251109	0.192873029	19.417743	18.27777973	0.0587073	5.00	8.95	10.57	14.55	14.64	14.85
4	0.9	1.05	2.93E-05	0.007200137	0.004063085	19.021642	19.81686622	0.041806287	3.54	5.77	8.17	14.21	14.38	14.43
5	1.05	1.05	0.000339681	0.001800231	0.188687235	19.093959	17.87229801	0.063981545	4.01	6.89	8.77	15.26	15.29	15.43
6	1.008359785	1.040921332	9.99E-08	0.000246223	0.000405769	22.249014	22.09561104	0.006894821	4.41	16.51	16.67	22.09	22.59	23.07
7	1.040584125	0.994075966	9.68E-07	0.000315899	0.003064402	28.075555	23.42978837	0.165473724	6.00	21.77	22.34	31.85	32.00	33.01
8	1.032145082	1.000301928	2.97E-07	0.000115513	0.002570408	32.14557	27.37297416	0.148468229	17.61	24.62	24.90	32.37	32.80	35.30
9	0.9	1.05	4.64E-05	0.002140017	0.02169611	41.589847	41.75262301	0.00391384	18.89	25.44	25.91	31.62	32.38	33.14
10	1.05	1.05	7.69E-07	0.000197496	0.003894983	55.481059	57.52763013	0.036887745	20.34	27.72	27.85	36.03	37.15	37.15
11	1.001952171	1.038700537	2.96E-07	0.000596362	0.000496936	66.185004	65.30751722	0.013258091	21.59	27.90	28.32	37.55	38.21	40.35
12	0.905717746	1.034068082	1.29E-05	0.004175504	0.003094359	72.513762	71.32707075	0.016365049	20.56	27.00	27.26	39.80	43.01	50.29
13	0.9	1.044438258	3.01E-06	0.00517251	0.000581067	81.512989	80.9840073	0.006489539	19.09	25.91	26.73	39.84	41.21	50.82
14	1.05	1.0372643	8.39E-07	0.001232575	0.000680358	84.263068	84.35103019	0.0010439	18.09	25.96	26.52	41.51	42.30	55.67
15	0.9	1.05	0.000256599	0.005783927	0.044364121	111.008751	108.5720506	0.021950525	17.09	23.93	24.70	40.01	42.29	55.44
16	1.05	1.05	1.78E-05	0.004554014	0.003898985	121.142204	122.1447286	0.008275601	17.00	24.62	24.88	42.53	45.56	57.86
17	0.9	1.05	1.26E-05	0.003470063	0.00364351	123.608716	128.2363494	0.03743776	17.53	27.01	27.54	44.36	46.35	58.49
18	1.04960601	1.037940309	3.66E-06	0.002068125	0.001768166	104.870005	102.8891348	0.018888816	20.34	29.15	29.68	43.58	44.09	44.09
19	0.964008496	1.03333071	5.44E-06	0.003207458	0.001697436	82.145528	80.21448758	0.023507554	20.96	29.12	29.45	39.76	40.66	42.57
20	1.05	1.05	8.91E-07	0.00080401	0.001107933	77.622871	77.63220279	0.00012022	20.64	27.92	28.19	36.50	37.18	37.52
21	0.914156095	1.04028055	2.80E-07	0.000868856	0.000322583	67.682837	65.96782212	0.025338992	19.61	27.96	28.21	44.06	46.67	58.77
22	0.941927055	1.034638089	1.52E-07	0.000707583	0.000214754	54.653942	55.93324169	0.023407272	17.17	23.54	23.60	30.80	31.32	32.11
23	1.0092341	1.031291967	9.25E-07	0.000321861	0.002874501	44.597215	42.1100053	0.055770516	15.33	19.96	20.35	25.52	27.83	29.15
24	0.9	1.05	3.35E-05	0.0011804	0.028342292	35.380436	35.16495484	0.006090404	14.92	18.27	18.50	22.92	24.78	25.22

Figure 8. The flowchart of PCMwSF method. The "forecasting model" part illustrates how to predict 24-h eletricity prices for the next day. The "detailed procedures of the proposed models" demonstrates procedures of PCMwSF model and provides fitness function of PSO algorithm. The table in this figure demonstrates details of forecasting process on 26 June 2002.

Table 2. Details of forecasting process on 26 June 2002.

Hour	Optimized Lowweight	Optimized Highweight	Optimized Ind	*vop*	Accuracy of Price Forecasting in 25 June with Optimized Weight	Actual Price	The Forecasting Price	The *MAPE* in Forecasting	Lowprice		Mediumprice		Highprice	
									Lower Limit	Upper Limit	Lower Limit	Upper Limit	Lower Limit	Upper Limit
1	1.05	1.05	3.00×10^{-7}	3×10^{-5}	0.009871573	30.042584	28.88083544	0.038670061	13.35	16.38	16.47	19.99	20.33	21.77
2	0.9	1.05	1.17×10^{-6}	0.00175	0.000669503	23.227286	22.82923041	0.017137413	6.21	11.06	12.04	15.06	15.18	15.19
3	0.9	1.05	0.0012057	0.00625	0.192873029	19.417743	18.27777973	0.0587073	5.00	8.95	10.57	14.55	14.64	14.85
4	0.9	1.05	2.93×10^{-5}	0.0072	0.004063085	19.021642	19.81686622	0.041806287	3.54	5.77	8.17	14.21	14.38	14.43
5	1.05	1.05	0.0003397	0.0018	0.188687235	19.093959	17.87229801	0.063981545	4.01	6.89	8.77	15.26	15.29	15.43
6	1.00835979	1.04092133	9.99×10^{-8}	0.00025	0.000405769	22.249014	22.09561104	0.006894821	4.41	16.51	16.67	22.09	22.59	23.07
7	1.04058413	0.99407597	9.68×10^{-7}	0.00032	0.003064402	28.075555	23.42978837	0.165473724	6.00	21.77	22.34	31.85	32.00	33.01
8	1.03214508	1.00030193	2.97×10^{-7}	0.00012	0.002570408	32.14557	27.37297416	0.148468229	17.61	24.62	24.90	32.37	32.80	35.30
9	0.9	1.05	4.64×10^{-5}	0.00214	0.02169611	41.589847	41.75262301	0.00391384	18.89	25.44	25.91	31.62	32.38	33.14
10	1.05	1.05	7.69×10^{-7}	0.0002	0.003894983	55.481059	57.52763013	0.036887745	20.34	27.72	27.85	36.03	37.15	37.15
11	1.00195217	1.03870054	2.96×10^{-7}	0.0006	0.000496936	66.185004	65.30751722	0.013258091	21.59	27.90	28.32	37.55	38.21	40.35
12	0.90571775	1.03406808	1.29×10^{-5}	0.00418	0.003094359	72.513762	71.32707075	0.016365049	20.56	27.00	27.26	39.80	43.01	50.29
13	0.9	1.04443826	3.01×10^{-6}	0.00517	0.000581067	81.512989	80.9840073	0.006489539	19.09	25.91	26.73	39.84	41.21	50.82
14	1.05	1.0372643	8.39×10^{-7}	0.00123	0.000680358	84.263068	84.35103019	0.0010439	18.09	25.96	26.52	41.51	42.30	55.67
15	0.9	1.05	0.0002566	0.00578	0.044364121	111.008751	108.5720506	0.021950525	17.09	23.93	24.70	40.01	42.29	55.44
16	1.05	1.05	1.78×10^{-5}	0.00455	0.003898985	121.142204	122.1447286	0.008275601	17.00	24.62	24.88	42.53	45.56	57.86
17	0.9	1.05	1.26×10^{-5}	0.00347	0.00364351	123.608716	128.2363494	0.03743776	17.53	27.01	27.54	44.36	46.35	58.49
18	1.04960601	1.03794031	3.66×10^{-6}	0.00207	0.001768166	104.870005	102.8891348	0.018888816	20.34	29.15	29.68	43.58	44.09	44.09
19	0.9640085	1.03333071	5.44×10^{-6}	0.00321	0.001697436	82.145528	80.21448758	0.023507554	20.96	29.12	29.45	39.76	40.66	42.57
20	1.05	1.05	8.91×10^{-7}	0.0008	0.001107933	77.622871	77.63220279	0.00012022	20.64	27.92	28.19	36.50	37.18	37.52
21	0.91415609	1.04028055	2.80×10^{-7}	0.00087	0.000322583	67.682837	65.96782212	0.025338992	19.61	27.96	28.21	44.06	46.67	58.77
22	0.94192706	1.03463809	1.52×10^{-7}	0.00071	0.000214754	54.653942	55.93324169	0.023407272	17.17	23.54	23.60	30.80	31.32	32.11
23	1.0092341	1.03129197	9.25×10^{-7}	0.00032	0.002874501	44.597215	42.1100053	0.055770516	15.33	19.96	20.35	25.52	27.83	29.15
24	0.9	1.05	3.35×10^{-5}	0.00118	0.028342292	35.380436	35.16495484	0.006090404	14.92	18.27	18.50	22.92	24.78	25.22

4.2. Study of Case 2

In this case, we will illustrate ***ICP-P***, ***ICP-F*** and the forecasting results of CM and PCMwSF and show that the **SR** is an effective tool to select the best model to forecast the next-day electricity price.

Figure 9 shows the day-ahead price forecasting from the CM method for Case 2. Figure 10 shows the day-ahead price forecasting from PCMwSF for Case 2. The forecasted results are compared with the actual LMP value, including the price on 27 June. It is obvious that the CM method is better than the PCMwSF method and that both methods are able to forecast the price changing trend. Thus, it is important to select a method of pre-processing correctly. Based on the **SR** defined in Section 3, the ***ICP-F*** of CM is more than that of PCMwSF; thus, CM is the selected model, indicating that the **SR** correctly selects the model with higher precision.

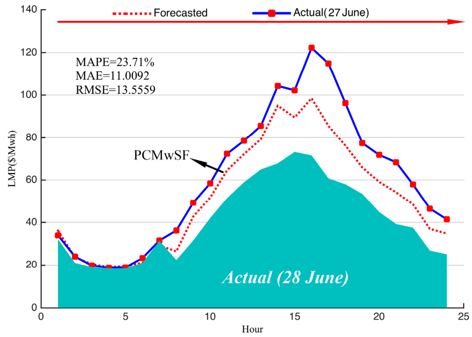


Figure 9. Actual PJM electricity price and forecasted values using CM in Case 2.

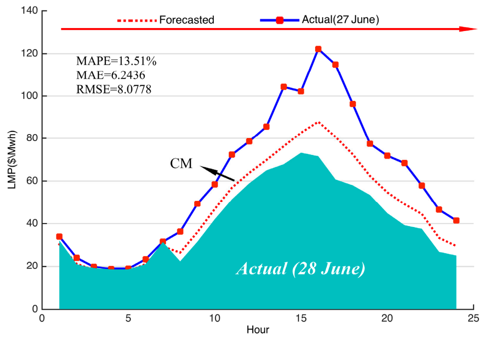


Figure 10. Actual PJM electricity price and forecasted values using PCMwSF in Case 2.

From Figure 11, it is obvious that the RBF network is conservative in the forecasting process. It makes little change in the forecasting process, and the change in price is observed to be relatively larger than that of the forecasted price in this figure.

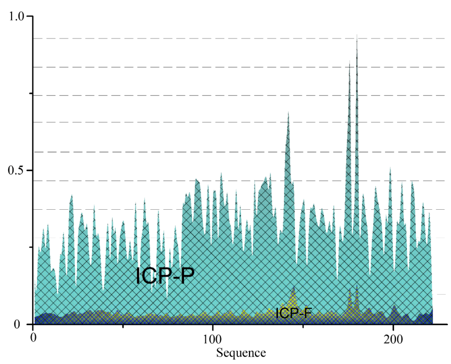


Figure 11. Illustration of ***ICP-F*** (index of changes of forecasting price) and ***ICP-P*** (index of changes of actual price).

4.3. Study of Case 3

In this section, the forecasting effectiveness of each model is highlighted. Figure 12 shows the day-ahead price forecasting for 18 March using both forecasting methods. It is apparent that the forecasted values of PCMwSF change more significantly than that of CM. Thus, PCMwSF is chosen to forecast the electricity price, and the *MAPE*, *MSE* and *RMSE* are 9.71%, 2.8821 and 3.6112, respectively. Figure 13 shows the day-ahead price forecasting for 19 March using both forecasting methods. The price from PCMwSF is selected as the final forecasted price because CM's forecasted price changes within a small range. The *MAPE*, *MSE* and *RMSE* are 6.25%, 1.6700 and 1.9214, respectively.

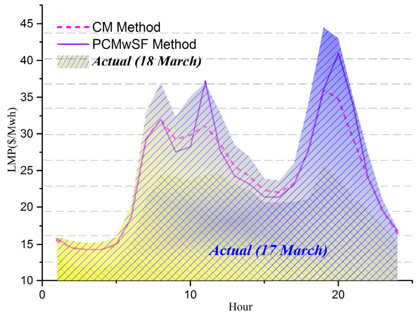


Figure 12. Actual PJM electricity price and forecasted values of 18 March.

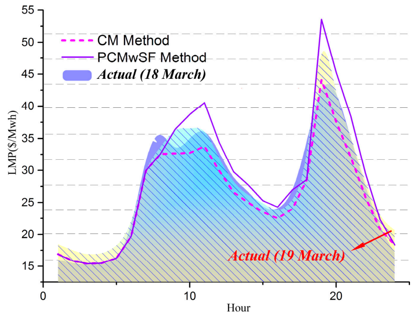


Figure 13. Actual PJM electricity price and forecasted values of 19 March.

In Figure 14, the forecasted price from the CM method is chosen because it changes more significantly than the forecasted price from the other method, and the *MAPE*, *MSE* and *RMSE* are 3.67%, 1.0212% and 1.1589%, respectively. The forecasted price from PCMwSF is selected as the final chosen price because the ***ICP-F*** of PCMwSF is larger than the index of the CM method. The *MAPE*, *MSE* and *RMSE* are 2.53%, 0.8096% and 1.1965%, respectively (Figure 15). By using the **SR** in Figure 16, the forecasted price from PCMwSF is regarded as the final result of forecasting, and the *MAPE*, *MSE* and *RMSE* are 13.40%, 4.9012% and 5.3977%, respectively.

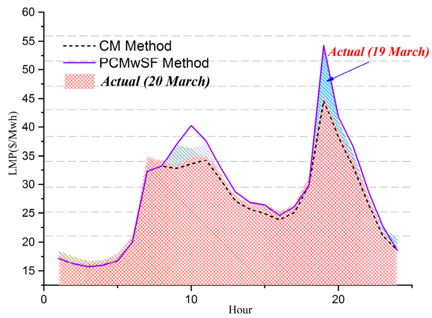


Figure 14. Actual PJM electricity price and forecasted values of 20 March. The blue area represents the actual electricity prices of 19 March.

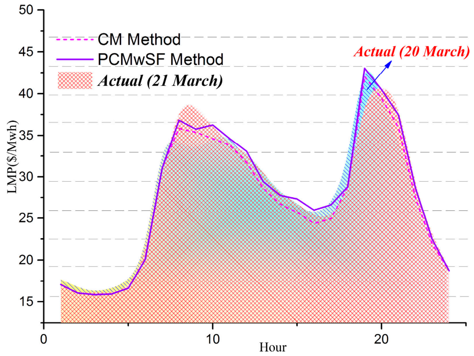


Figure 15. Actual PJM electricity price and forecasted values of 21 March. The blue area represents the actual electricity prices of 20 March.

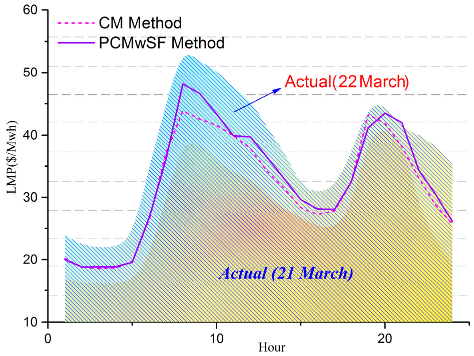


Figure 16. Actual PJM electricity price and forecasted values of 22 March. It is obvious that actual electricity prices of 21 March are less than those of 22 March.

The details of the forecasting results of Case 3 are shown in Table 3 and Figure 17 illustrates the forecasting results. In Table 3, the forecasting details from 18 March to 22 March are demonstrated. It is clearly seen that four days are forecasted using the PCMwSF method. For 18 March, the *MAPE* ranges from 1.6% at 1:00 to 19.7% at 10:00. The average *MAPE* is 9.70%. The *MAPE* of the PCMwSF forecasting model on 19 March varies from a low of 0.6% at 23:00 to a high of 11.2% at 24:00, and the average *MAPE* of this day is 6.25%. The *MAPE* varies from 0.2% at 1:00 to 7.5% at 9:00. The average *MAPE* on 21 March is 2.53%. Similarly, the lowest *MAPE* on 22 March is 2.7% at 21:00, and this day's highest *MAPE* is 26.1% at 24:00. The average *MAPE* of this day is 13.40%. The CM method is chosen to forecast the price on 20 March, and the *MAPE* of this day varies from a low of 1.2% at 9:00 to a high of 8.0% at 7:00. The average *MAPE* on 20 March is 3.67%. Thus, the lowest *MAPE* of Case 3 is 0.2% at 2:00 on 21 March, and the highest *MAPE* of this case is 26.1% at 24:00 on 22 March.

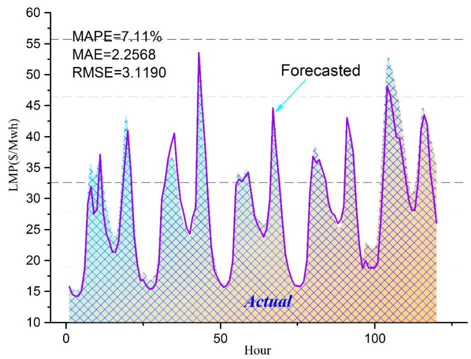


Figure 17. Actual PJM electricity price and forecasted values of Case 3. In this figure, the area represents the actual electricity prices of this week and the line is the forecasted values.

Table 3. Details of forecasting process for Case 3.

Hour	18 March (PCMwSF)			19 March (PCMwSF)			20 March (CM)			21 March (PCMwSF)			22 March (PCMwSF)		
	Actual	**Forecasted**	***MAPE***	**Actual**	**Forecasted**	***MAPE***	**Actual**	**Forecasted**	***MAPE***	**Actual**	**Forecasted**	***MAPE***	**Actual**	**Forecasted**	***MAPE***
1	16.03	15.774	0.016	18.51	16.898	0.087	17.72	17.227	0.028	17	17.038	0.002	24	19.967	0.168
2	15.45	14.524	0.06	17.51	15.878	0.093	16.72	16.222	0.03	16.01	16.046	0.002	22.33	18.79	0.159
3	15.207	14.256	0.063	16.79	15.403	0.083	16.15	15.703	0.028	16.16	15.861	0.018	21.9	18.856	0.139
4	15.312	14.254	0.069	16.94	15.472	0.087	16.58	15.946	0.038	16	15.904	0.006	22	18.852	0.143
5	16	15.082	0.057	18.01	16.295	0.095	17.43	16.778	0.037	16.58	16.606	0.002	23.5	19.555	0.168
6	20.27	18.624	0.081	21.01	19.687	0.063	20.51	20.001	0.025	20.356	20.084	0.013	35	26.786	0.235
7	33	29.261	0.113	31	29.958	0.034	35	32.211	0.08	30.177	31.02	0.028	45	36.19	0.196
8	37	32.009	0.135	33.51	32.574	0.028	34.164	33.182	0.029	39.109	36.824	0.058	54.255	48.145	0.113
9	32.42	27.554	0.15	36.917	36.381	0.015	33.144	32.734	0.012	38.668	35.768	0.075	51.7	46.633	0.098
10	35.146	28.222	0.197	36.345	38.725	0.065	34.75	33.562	0.034	36.114	36.264	0.004	50.447	43.354	0.141
11	36.94	37.145	0.006	37.09	40.525	0.093	35.1	34.246	0.024	33.785	34.628	0.025	47.568	39.927	0.161
12	33.005	27.715	0.16	31.724	34.423	0.085	31.889	30.771	0.035	33.101	33.054	0.001	45.617	39.734	0.129
13	28.522	24.293	0.148	28	29.776	0.063	27.981	27.152	0.03	30.465	29.432	0.034	41.289	36.414	0.118
14	26.891	23.162	0.139	26.033	27.753	0.066	26.677	25.695	0.037	28.066	27.746	0.011	37.516	33.067	0.119
15	23.95	21.386	0.107	25.03	25.276	0.01	26.565	24.861	0.064	26.811	27.337	0.02	31.726	29.668	0.065
16	23.685	21.376	0.097	23.233	24.273	0.045	25.415	23.799	0.064	25.212	26.002	0.031	30.707	28.166	0.083
17	26.071	23.105	0.114	25.011	26.981	0.079	26.54	25.115	0.054	24.998	26.637	0.066	31.55	28.129	0.108
18	34.44	29.583	0.141	29.573	28.589	0.033	31.429	29.819	0.051	28.076	28.791	0.025	37	32.474	0.122
19	44.535	36.006	0.192	54.635	53.567	0.02	45.681	44.629	0.023	40.057	43.06	0.075	45	41.186	0.085
20	43.06	40.996	0.048	41	45.297	0.105	39.436	38.278	0.029	41.064	40.447	0.015	45	43.544	0.032
21	34.96	33.898	0.03	35.51	38.372	0.081	34.421	33.04	0.04	39.486	37.466	0.051	40.9	41.984	0.027
22	27.41	24.028	0.123	28	29.499	0.054	28	26.745	0.045	28.537	28.673	0.005	40.242	34.35	0.146
23	21	19.499	0.071	22.44	22.312	0.006	21.65	21.129	0.024	23	22.493	0.022	38.26	30.501	0.203
24	17.03	16.82	0.012	20.7	18.375	0.112	19	18.599	0.021	19	18.713	0.015	35.31	26.084	0.261

4.4. Study of Cases 4–6

By applying two forecasting models and **SR**, Cases 4–6 can be solved. Figures 18–20 separately illustrate the forecasting results of CM-PCMwSF-SR in the three cases.

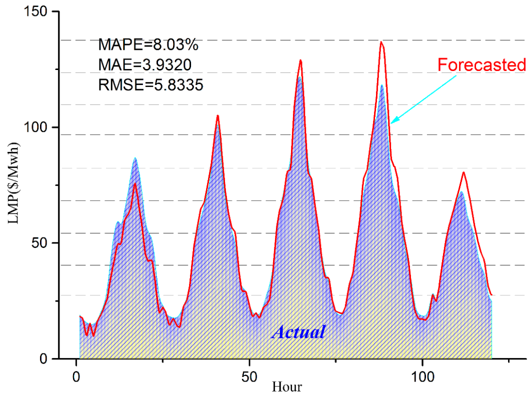


Figure 18. Actual PJM electricity price and forecasted values of Case 4. In this figure, the area represents the actual electricity prices of this week and the line is the forecasted values.

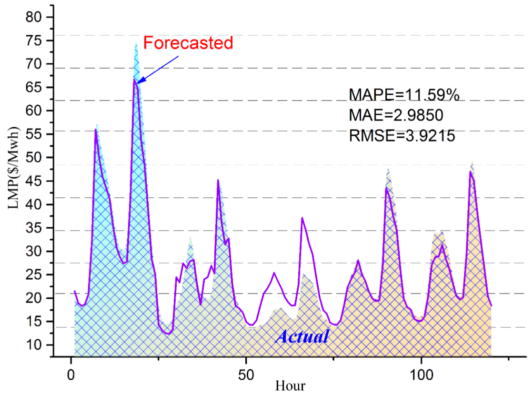


Figure 19. Actual PJM electricity price and forecasted values of Case 5. In this figure, the area represents the actual electricity prices of this week and the line is the forecasted values.

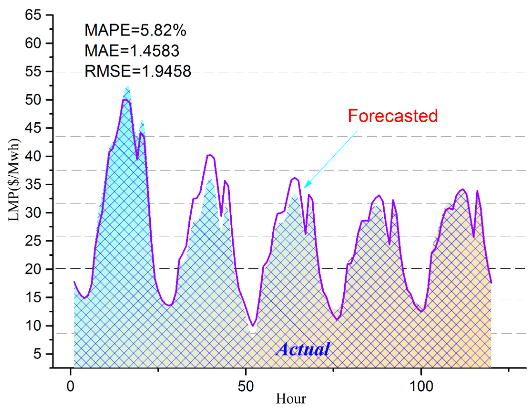


Figure 20. Actual PJM electricity price and forecasted values of Case 6. In this figure, the area represents the actual electricity prices of this week and the line is the forecasted values.

Details of the forecasting results of Cases 4–6 are shown in Tables 4–6. Table 4 shows the forecasting details from 24 June to 28 June. It is clearly seen that three days are forecasted using the PCMwSF method. For 25 June, the *MAPE* ranges from 0.1% at 2:00 to 30.6% at 6:00. The average *MAPE* is 6.0%. The *MAPE* of the PCMwSF forecasting model on 26 June varies from a low of 0.012% at 20:00 to a high of 19.8% at 7:00, and the average *MAPE* of this day is 3.78%. Similarly, the lowest *MAPE* on 27 June is 0.8% at 1:00, and this day's highest *MAPE* is 13.4% at 20:00. The average *MAPE* of this day is 6.32%. The CM method is chosen to forecast the prices on 24 June and 28 June. The *MAPE* of the previous day varies from a low of 1.5% at 2:00 to a high of 22.80% at 23:00. The average *MAPE* on 24 June is 15.30%. The other day's *MAPE* ranges from 0.1% at 12:00 to 18.10% at 17:00, and the average *MAPE* is 8.48%. Thus, the lowest *MAPE* of Case 4 is 0.012% at 20:00 on 26 June, and the highest *MAPE* of this case is 30.6% at 6:00 on 25 June.

The forecasting details from 23 September to 27 September are shown in Table 5. It is obvious that three days are forecasted using the CM method. For 24 September, the *MAPE* ranges from 0.019% at 6:00 to 20.9% at 19:00. The average *MAPE* is 8.74%. The *MAPE* of the CM forecasting model on 25 September varies from a low of 0.6% at 24:00 to a high of 35.7% at 4:00, and the average *MAPE* of this day is 8.30%. The *MAPE* varies from 0.009% at 15:00 to 8.4% at 4:00. The average *MAPE* on 27 September is 3.10%. The PCMwSF method is chosen to forecast the prices on 23 September and 26 September. The *MAPE* of 23 September varies from a low of 0.016% at 13:00 to a high of 12.4% at 22:00. The average *MAPE* is 4.61%. The *MAPE* of 26 September ranges from 0.011% at 12:00 to 13.1% at 4:00, and the average *MAPE* of this day is 4.34%. Thus, the lowest *MAPE* of Case 5 is 0.009% at 15:00 on 27 September, and the highest *MAPE* of this case is 35.7% at 4:00 on 25 September.

Table 6 lists the details of the forecasting result from 23 December to 27 December. It is easy to see that three days are forecasted using the CM method. For 24 December, the *MAPE* ranges from 2.6% at 24:00 to 37.1% at 16:00. The average *MAPE* is 14.04%. The *MAPE* on 25 December varies from a low of 6.0% at 2:00 to a high of 47.1% at 7:00, and the average *MAPE* of this day is 24.43%. Similarly, the lowest *MAPE* on 26 December is 0.3% at 24:00, and this day's highest *MAPE* is 17.3% at 7:00. The average *MAPE* of this day is 4.94%. 23 December and 27 December use the PCMwSF method as their forecasting method. The *MAPE* of 23 December varies from a low of 0.5% at 3:00 to a high of 14.9% at 20:00. This day's *MAPE* is 7.60%. The last column of this table shows that the *MAPE* of 27 December ranges from 1.7% at 22:00 to 17.3% at 7:00, and the average *MAPE* is 6.98%. Thus, the lowest *MAPE* of Case 6 is 0.3% at 24:00 on 26 December, and the highest *MAPE* of this case is 47.1% at 7:00 on 25 December.

Table 4. Details of forecasting process for Case 4.

Hour	24 June (CM)			25 June (PCMwSF)			26 June (PCMwSF)			27 June (PCMwSF)			28 June (CM)		
	Actual	**Forecast**	***MAPE***	**Actual**	**Forecast**	***MAPE***	**Actual**	**Forecast**	***MAPE***	**Actual**	**Forecast**	***MAPE***	**Actual**	**Forecast**	***MAPE***
1	18.901	18.338	0.03	23.573	22.374	0.054	30.043	28.881	0.04	33.965	34.238	0.008	32.087	28.339	0.132
2	16.62	16.365	0.015	19.062	19.075	0.001	23.227	22.829	0.017	24.03	25.41	0.054	21.035	19.954	0.054
3	15.51	10.128	0.347	18.238	14.721	0.239	19.418	18.278	0.062	19.814	20.565	0.037	19.06	17.323	0.1
4	14.8	15.295	0.033	17.64	17.48	0.009	19.022	19.817	0.04	18.8	19.973	0.059	18.43	17.126	0.076
5	14.92	9.917	0.335	17.651	14.32	0.233	19.094	17.872	0.068	18.96	19.873	0.046	18.43	16.809	0.096
6	16.17	16.076	0.006	19.268	14.759	0.306	22.249	22.096	0.007	23.35	24.628	0.052	21.14	20.013	0.056
7	19.966	18.865	0.055	23.186	22.684	0.022	28.076	23.43	0.198	31.55	32.574	0.031	31.41	27.63	0.137
8	23.219	22.491	0.031	25.918	25.693	0.009	32.146	27.373	0.174	36.25	37.572	0.035	22.376	24.931	0.102
9	29.326	26.367	0.101	34.028	34.768	0.021	41.59	41.753	0.004	49.29	49.998	0.014	31.538	33.637	0.062
10	44.447	35.531	0.201	49.044	48.853	0.004	55.481	57.528	0.036	58.442	61.623	0.052	41.975	43.397	0.033
11	55.216	43.013	0.221	54.909	57.439	0.044	66.185	65.308	0.013	72.358	77.943	0.072	51.399	52.636	0.023
12	61.1	49.62	0.188	60.652	64.9	0.065	72.514	71.327	0.017	78.544	81.822	0.04	58.915	58.952	0.001
13	56.153	50.403	0.102	66.413	67.902	0.022	81.513	80.984	0.007	85.559	91.998	0.07	64.995	64.462	0.008
14	64.393	59.737	0.072	71.894	75.777	0.051	84.263	84.351	0.001	104.294	102.081	0.022	67.832	70.257	0.035
15	71.692	62.255	0.132	80.659	84.246	0.043	111.009	108.572	0.022	102.227	118.042	0.134	73.239	75.898	0.035
16	81.962	67.378	0.178	97.686	97.306	0.004	121.142	122.145	0.008	122.228	136.956	0.108	71.587	80.658	0.112
17	89.208	75.863	0.15	104.657	105.039	0.004	123.609	128.236	0.036	114.878	130.954	0.123	60.741	74.127	0.181
18	82.122	66.567	0.189	84.824	89.533	0.053	104.87	102.889	0.019	96.25	110.481	0.129	57.986	66.78	0.132
19	72.609	56.682	0.219	67.768	72.747	0.068	82.146	80.214	0.024	77.466	86.297	0.102	53.425	57.771	0.075
20	57.966	44.994	0.224	62.682	62.609	0.001	77.623	77.632	1.16E	71.847	82.991	0.134	44.856	50.84	0.118
21	53.88	42.471	0.212	54.379	56.513	0.038	67.683	65.968	0.026	68.396	74.936	0.087	39.225	45.684	0.141
22	52.403	41.59	0.206	49.969	53.021	0.058	54.654	55.933	0.023	57.934	58.991	0.018	37.543	41.408	0.093
23	38.681	29.851	0.228	35.46	37.805	0.062	44.597	42.11	0.059	46.488	50.42	0.078	26.801	31.185	0.141
24	24.805	19.889	0.198	30.043	29.191	0.029	35.38	35.165	0.006	41.45	41.919	0.011	25.081	27.661	0.093

Table 5. Details of forecasting process for Case 5.

Hour	23 September (PCMwSF)			24 September (CM)			25 September (CM)			26 September (PCMwSF)			27 September (CM)		
	Actual	Forecast	*MAPE*	Actual	Forecast	*MAPE*	Actual	Forecast	*MAPE*	Actual	Forecast	*MAPE*	Actual	Forecast	*MAPE*
1	17.93	17.77	0.01	16.05	16.15	0.01	14.12	14.93	0.06	15.59	14.97	0.04	15.72	15.59	0.01
2	16.39	16.26	0.01	14.41	14.64	0.02	12.06	13.15	0.09	13.52	13.09	0.03	14.17	13.84	0.02
3	15.50	15.29	0.01	13.79	13.85	0.01	9.16	11.19	0.22	13.08	11.88	0.09	13.84	13.03	0.06
4	15.00	14.90	0.01	13.55	13.56	0.00	7.33	9.94	0.36	12.71	11.04	0.13	13.59	12.44	0.08
5	15.73	15.33	0.03	13.80	13.89	0.01	9.00	11.11	0.24	13.14	11.87	0.10	13.74	12.97	0.06
6	17.83	17.34	0.03	15.90	15.91	0.00	15.11	15.31	0.01	16.33	15.51	0.05	16.96	16.49	0.03
7	25.75	23.80	0.08	21.76	21.66	0.00	19.87	20.50	0.03	22.13	20.86	0.06	24.37	22.95	0.06
8	30.90	27.53	0.11	20.81	22.77	0.09	20.47	21.30	0.04	21.68	21.03	0.03	25.58	23.64	0.08
9	32.88	29.91	0.09	21.59	24.14	0.12	22.23	22.86	0.03	23.61	22.72	0.04	27.51	25.49	0.07
10	37.70	35.46	0.06	26.34	28.96	0.10	26.47	27.29	0.03	26.60	26.34	0.01	29.85	28.60	0.04
11	42.22	40.73	0.04	28.96	32.51	0.12	28.20	29.85	0.06	28.48	28.50	0.00	31.27	30.46	0.03
12	42.24	41.36	0.02	28.54	32.55	0.14	28.32	29.93	0.06	28.59	28.59	0.00	31.98	30.85	0.04
13	43.55	43.56	0.00	28.91	33.66	0.16	28.16	30.36	0.08	28.15	28.59	0.02	31.48	30.59	0.03
14	46.80	46.07	0.02	33.04	37.01	0.12	31.19	33.48	0.07	31.32	31.65	0.01	33.16	33.04	0.00
15	51.12	49.93	0.02	35.99	40.15	0.12	32.83	35.79	0.09	31.01	32.57	0.05	33.89	33.89	0.00
16	52.72	50.01	0.05	36.09	40.21	0.11	33.49	36.18	0.08	31.71	33.11	0.04	33.96	34.21	0.01
17	52.62	49.39	0.06	35.58	39.70	0.12	33.07	35.72	0.08	30.09	32.06	0.07	33.27	33.31	0.00
18	45.78	43.83	0.04	30.70	34.84	0.14	29.15	31.43	0.08	26.01	27.99	0.08	29.86	29.46	0.01
19	42.10	39.39	0.07	24.37	29.45	0.21	23.98	26.24	0.09	23.69	24.41	0.03	26.12	25.72	0.02
20	46.69	44.22	0.05	32.30	35.68	0.10	31.72	33.17	0.05	32.94	32.27	0.02	34.17	33.90	0.01
21	46.91	43.38	0.08	31.12	34.75	0.12	30.93	32.31	0.05	28.67	29.75	0.04	30.96	30.97	0.00
22	38.92	34.11	0.12	23.18	26.72	0.15	23.02	24.47	0.06	22.74	23.09	0.02	25.28	24.62	0.03
23	28.09	25.11	0.11	18.24	20.44	0.12	18.54	19.20	0.04	20.07	19.23	0.04	21.57	20.74	0.04
24	18.48	18.33	0.01	16.19	16.47	0.02	16.24	16.15	0.01	17.38	16.42	0.06	18.27	17.62	0.04

Table 6. Details of forecasting process for Case 6.

Hour	23 December (PCMwSF)			24 December (CM)			25 December (CM)			26 December (CM)			27 December (PCMwSF)		
	Actual	**Forecast**	***MAPE***	**Actual**	**Forecast**	***MAPE***	**Actual**	**Forecast**	***MAPE***	**Actual**	**Forecast**	***MAPE***	**Actual**	**Forecast**	***MAPE***
1	20.129	21.535	0.07	17.91	14.333	0.2	15.723	16.953	0.078	15.8	16.669	0.055	18.516	17.551	0.052
2	18.492	18.993	0.027	15.245	13.179	0.136	14.128	14.972	0.06	14.117	14.8	0.048	16.394	15.562	0.051
3	18.295	18.391	0.005	14.528	12.476	0.141	13.517	14.35	0.062	13.859	14.358	0.036	15.983	15.135	0.053
4	18.99	18.707	0.015	14.426	12.426	0.139	13.41	14.325	0.068	13.858	14.348	0.035	16.076	15.174	0.056
5	20.425	20.885	0.023	15.632	13.304	0.149	13.884	15.284	0.101	14.789	15.314	0.035	16.932	16.087	0.05
6	33.275	32.466	0.024	19.035	24.49	0.287	14.386	18.253	0.269	18.242	18.603	0.02	22.397	20.394	0.089
7	61.526	56.038	0.089	19.331	23.353	0.208	14.234	20.942	0.471	22.959	22.372	0.026	32.67	27.019	0.173
8	53.876	50.023	0.072	23.866	27.451	0.15	15.085	22.029	0.46	25.22	24.063	0.046	34.544	28.811	0.166
9	51.101	45.891	0.102	28.265	26.419	0.065	16.427	23.424	0.426	26.332	25.36	0.037	33.155	28.97	0.126
10	47.329	43.639	0.078	34.867	27.883	0.2	17.952	25.459	0.418	29.655	28.061	0.054	35.156	31.376	0.108
11	43.324	41.221	0.049	29.488	28.192	0.044	17.395	23.729	0.364	25.718	25.205	0.02	32.18	28.453	0.116
12	38.111	35.248	0.075	23.901	22.755	0.048	18.277	22.203	0.215	24.337	23.696	0.026	29.215	26.286	0.1
13	32.935	30.741	0.067	21.015	18.606	0.115	17.323	20.269	0.17	22.202	21.599	0.027	24.851	23.144	0.069
14	30.412	28.817	0.052	20.133	24.026	0.193	16.185	19.104	0.18	20.307	20.038	0.013	20.983	20.483	0.024
15	30.398	27.43	0.098	19.868	24.496	0.233	15.47	18.407	0.19	19.698	19.362	0.017	20.201	19.756	0.022
16	30.43	27.764	0.088	19.642	26.931	0.371	15.573	18.471	0.186	20.017	19.552	0.023	20.769	20.13	0.031
17	47.471	41.881	0.118	27.353	25.298	0.075	16.743	23.011	0.374	31.276	27.336	0.126	32.265	29.666	0.081
18	76.514	66.692	0.128	47.185	45.225	0.042	26.72	37.133	0.39	48.81	43.524	0.108	50.867	46.992	0.076
19	74.707	64.847	0.132	43.588	36.176	0.17	24.514	34.632	0.413	47.498	41.472	0.127	49.224	45.126	0.083
20	63.244	53.816	0.149	37.947	31.452	0.171	24.05	31.629	0.315	42.577	37.508	0.119	39.31	38.351	0.024
21	55.39	48.473	0.125	31.732	32.804	0.034	23.802	29.33	0.232	38.793	34.46	0.112	30.677	32.477	0.059
22	43.07	38.821	0.099	25.028	22.954	0.083	20.914	24.556	0.174	26.971	26.261	0.026	27.121	26.657	0.017
23	31.186	28.374	0.09	20.14	18.346	0.089	18.203	20.089	0.104	19.084	19.963	0.046	21.076	20.49	0.028
24	26.393	25.116	0.048	18.361	17.881	0.026	15.329	17.503	0.142	18.057	18.112	0.003	18.844	18.455	0.021

4.5. Comparison Study

In this section, a comparison study will be provided to present the forecasting effectiveness of the proposed model. In detail, genetic algorithm (GA) will be applied to optimize weights of low and high prices, and backward propagation neural network (BPNN), elman neural network (ENN) and GRNN are selected as benchmarks. For the GA-based method, we use CM-GCMwSF-SR to present it in Table 7, which shows the forecasting results of models in Cases 3–6. In addition, we provide an experiment to show the forecasting effectiveness when electricity demand is considered as one of features, which is represented as CM-PCMwSF-SR (with demand) in Table 7.

Table 7. Comparison with other algorithms in Cases 3–6. SR: selection rule; BPNN: backward propagation neural network; ENN: elman neural network; GRNN: generalized regression neural network. MAE: mean absolute error; RMSE: root mean square error.

Season	Criteria	CM-PCMwSF-SR	CM-GCMwSF-SR	CM-PCMwSF-SR (with Demand)	PCMwSF	CM	BPNN	ENN	GRNN
Spring	*MAPE*	7.11%	7.01%	7.21%	7.51%	10.08%	20.90%	21.75%	21.90%
	MAE	2.2568	2.1948	2.3761	2.5682	3.5268	6.2178	6.4994	6.3687
	RMSE	3.119	3.098	3.202	3.9865	5.1268	7.8962	7.9463	8.2122
Summer	*MAPE*	8.03%	9.28%	8.01%	11.21%	15.58%	26.60%	27.81%	26.79%
	MAE	3.932	4.8329	3.917	6.1025	8.0256	14.8875	15.3582	15.2014
	RMSE	5.8335	6.9726	5.8017	8.1564	10.1526	19.9902	20.0877	20.9055
Autumn	*MAPE*	5.82%	7.20%	5.72%	10.54%	8.25%	16.30%	16.53%	16.95%
	MAE	1.4583	1.9872	1.2918	2.8658	2.2139	4.2477	4.3638	4.4515
	RMSE	1.9458	2.8977	1.3681	4.0213	2.9684	5.8511	6.1312	6.0429
Winter	*MAPE*	11.59%	12.29%	12.33%	15.68%	12.86%	29.16%	30.57%	29.21%
	MAE	2.985	3.6298	3.7288	4.2681	3.0254	7.29	7.3474	7.5995
	RMSE	3.9215	4.7892	4.4025	5.9812	4.1285	9.6061	10.0723	10.0547
Average	*MAPE*	8.14%	8.95%	8.32%	11.24%	11.69%	23.24%	24.16%	23.71%
	MAE	2.66	3.16	2.83	3.95	4.20	8.16	8.39	8.41
	RMSE	3.70	4.44	3.69	5.54	5.59	10.84	11.06	11.30

In Table 7, it is obvious that the proposed model has better performance than benchmarks and the following conclusions can be made:

(a) PSO is a better selection to optimize the weights of low and high electricity prices than GA because CM-PCMwSF-SR has better overall forecasting effectiveness than CM-GCMwSF-SR.
(b) PCMwSF and CM have ability to improve the forecasting accuracy.
(c) The electricity price of autumn can be predicted more precisely.
(d) Although some literature regard the electricity demand as features to predict electricity price, adding the electricity demand data as a feature cannot help to improve forecasting effectiveness of prices in this paper (forecasting results are similar in Table 7).

As a demonstration of (d), regarding electricity demand as a feature cannot improve the forecasting effectiveness, which is different with some electricity price forecasting methods. The main reasons are demonstrated as following:

(1) The proposed model mostly concentrates on reducing the volatility of electricity price for a higher accuracy, which means the electricity demand is not important compared to the pre-processed electricity price.
(2) Model performance under specific conditions should be analyzed and understood and incremental improvements made based on knowledge gained. Moghram and Rahman review five short-term load forecasting methods:

    (i) multiple linear regression;
    (ii) time series;

(iii) general exponential smoothing;
(iv) state space and Kallman filter; and
(v) knowledge-based approach.

The forecasting results show that no one method was determined to be superior. The transfer function approach was the second worst predictor over the winter months but was the best method over the summer months. The authors conclude that because of its strong dependency on historical data, the transfer function approach did not respond well to abrupt changes as did the knowledge based approaches. The conclusion reached is that there is no one best approach, which means that it is possible that regarding electricity demand as a feature cannot improve the forecasting effectiveness [51].

Thus, the proposed method combining PCMwSF method, CM method and SR is better than traditional approaches according to the numerical calculating results. Concretely, the CM method is helpful to reduce the volatility of the electricity price and, consequently, to improve the forecasting effectiveness. For other techniques presented in this paper, PSO aims to obtain the best weights of high and low electricity prices, and SOM and Fuzzy logic are effective tools to confirm three levels of electricity prices (high, medium and low), and the purpose of SR is to select the best model for each day based on the nature of RBFN.

5. Conclusions

Forecasting electricity is a key problem for generators and consumers in a deregulated electricity market, and the difficulty of an accurate forecast is due to the high volatility of the electricity price. The reduction of this volatility is the key to improving prediction accuracy. In this paper, based on SOM, Fuzzy logic, PSO and the forecasting nature of the RBF network, the PCMwSF method, CM method and SR were developed to reduce the volatility of the electric price and to improve the accuracy of the forecast. The final model, CM-PCMwSF-SR, successfully reduced the volatility of the electricity price and was able to obtain a higher accuracy compared to other benchmarks. In the numerical simulation of four seasons, the proposed model exhibited the best performance, where the *MAPE*s are 7.11%, 8.03%, 5.82%, and 11.59% for each season (spring, summer, autumn and winter respectively). The PCMwSF method and CM method were the best models (except when using the SR approach) for two different seasons. The BP network, i.e., a classical neuron network method for forecasting the electricity price, did not have a good performance compared to the other models in these four seasons. The experimental results showed that reducing the volatility and effectively selecting forecasting models not only improve the forecasting effectiveness of the electricity price but also obtained a satisfactory forecasting accuracy.

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Abbreviations

RBFN	Radial basis function network
PCMwSF	Particle swarm optimization-core mapping with self-organizing-map and fuzzy set
CM	Core mapping
SR	Selection rule
MI	Mutual information
CNN	Composite neural network
SVM	Support vector machine
ARMAX	Auto-regressive moving average with external input
MS-GARCH	Markov-switching generalized autoregressive conditional heteroskedasticity

DCT	Discrete cosine transforms
FFNN	Feed-forward neural network
CFNN	Cascade-forward neural network
GRNN	Generalized regression neural network
PSO	Particle swarm optimization
PCPF	Panel cointegration and particle filter
WT	Wavelet transform
ARIMA	Autoregressive integrated moving average
LSSVM	Least squares support vector machine
CLSSVM	Chaotic least squares support vector machine
EGARCH	Exponential generalized autoregressive conditional heteroskedastic
ARMA	Autoregressive moving average
GARCH	Generalized autoregressive conditional heteroskedasticity
ARMA-GARCH-M	ARMA-GARCH-in-mean
ARFIMA	Auto-regressive fractionally integrated moving average
ANN	Artificial neural network
ELM	Extreme learning machine
PIs	Prediction intervals
RNN	Recurrent neural network
PNN	Probabilistic neural network
HNES	Hybrid neuro-evolutionary system
PCA	Principal component analysis
MLF	Multi-layer feedforward
BPNN	Backward propagation neural network
ENN	Elman neural network
GA	Genetic algorithm

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