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Keywords: soil oscillation law, oscillation velocity, blasting, seismic effect, working environment

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Article

Some Models for Determination of Parameters of the Soil Oscillation Law during Blasting Operations

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Abstract: In order to evaluate and control the seismic effect of blasting, as well as its planning, it is required to determine the soil oscillation law, with the strike/mining facilities to be protected. One of the most commonly used equations is that of M.A. Sadovskii, defining the law of alteration in the oscillation velocity of the soil depending on distance, the explosive amount, and conditions of blasting and geologic characteristics of the soil; all of this being determined on the basis of test blasting for the specific work environment. In the Sadovskii equation two parameters, K and n appear and they are conditioned both by rock mass characteristics and blasting conditions. The practical part of this study includes experimental investigations performed in the Veliki Krivelj Open Pit in the Bor District located in Eastern Serbia and investigations carried out during mass mining in the Kovilovača Open Pit near Despotovac, Eastern Serbia. Thus this paper offers several modes for determination of parameters K and n in the Sadovskii equation. To determine the parameters in the Sadovskii formula, in addition to the usual least square method, two more new models were applied. In the models the parameters K and n were determined by applying the quotient of the relative growth of oscillation velocities and reduced distances for Model 2. The link between the parameters K and n is determined by applying the trapezoidal formula for finding the value of definite integral for Model 3. In doing so, it was noted that all three models can be used to calculate the oscillation velocity of the rock mass.

Keywords: working environment; blasting; seismic effect; oscillation velocity; soil oscillation law

1. Introduction

As the relation between the soil oscillation velocity and basic parameters affecting its magnitude, being: the amount of explosive, the distance from the blast site, characteristics of the rock material, and the type of blasting, the equation of M.A. Sadovskii, where the oscillation velocity v is given in the form of the function, is most frequently used:

$$v = K \cdot R^{-n} \tag{1}$$

where R represents reduced distance, meaning the distance from the blast site to the monitoring point r, reduced to the used explosive amount Q. K and n parameters conditioned by soil characteristics and blasting conditions, thereby v is the decreasing convex function of the variable R.

By applying the law of rock mass oscillation while blasting, the determination of the soil oscillation velocity is enabled for each blast operation in advance; thus, blasting is, as regards seismic effect, under control, and that gives an opportunity to plan the magnitude of shock waves for each future blast operation [1]. In this way adverse blasting effects are reduced. Adverse effects of blasting imply, in addition to the seismic ones, those of air blast waves, fly rock, etc. Thus, production efficiency is increased and, at the same time, construction and mining facilities, as well as the environment in the vicinity of the blast site, are protected.

2. Soil Oscillation Law

To establish the correlation between the oscillation velocity and three basic parameters affecting its size, the explosive quantity, properties of rock material, and the distance, several mathematical models have been developed. One of the most frequently used models is the equation of Sadovskii defining the law on velocity alteration of soil oscillation depending on the distance, the explosive quantity, and the mode of blasting [2]. The law defined in this way offers the possibility to determine the seismic effect of blasting towards a facility or a settlement, whereby the connection, between the velocity of soil oscillation and consequences that can affect facilities, is used. The equation of M.A. Sadovskii is given in the form:

$$v = K \cdot R^{-n} = K \cdot \left(\frac{r}{\sqrt[3]{Q}} \right)^{-n} \tag{2}$$

where we have:

v—velocity of soil oscillation (m/s);
K—coefficient conditioned by soil characteristics, as well as blasting conditions, where the explosive amount is given by way of the volume. K is being determined by terrain surveying;
n—exponent, conditioned by soil properties and mining conditions and determined by field measurements as well;
r—distance from the blast site to the monitoring point (m);
Q—amount of explosive (kg); and
R—reduced distance, expressed as $R = \frac{r}{\sqrt[3]{Q}}$.

2.1. Derivation of Equation of Rock Mass Oscillation Law

2.1.1. Derivation of Equation of Rock Mass Oscillation—First Mode

The equation of Sadovskii has been derived from the condition that if the radius of charge and the distance from the blast site to the monitoring point increase in the same, or approximately the same ratio, the soil oscillation velocity remains the same [3], i.e.:

$$v = K_v \cdot \left(\frac{r_0}{r} \right)^{n} \tag{3}$$

where:

v—velocity of soil oscillation (cm/s);
K_v—coefficient conditioned by soil characteristics, as well as blasting conditions, where the explosive amount is given through radius of the explosive charge. K_v is being determined by terrain surveying;
r—distance from the blast site to the monitoring point (m);
r_0—radius of the explosive charge; and
n—exponent, conditioned by soil properties and mining conditions and determined by field measurements as well.

The radius of the explosive charge r_0 and the amount of explosive Q are related by the equation:

$$Q = \frac{4}{3} \cdot \pi \cdot r_0^3, \text{ from this formula we get: } r_0 = \sqrt[3]{\frac{3 \cdot Q}{4 \cdot \pi}} \tag{4}$$

By replacing the value r_0 from the Equation (4) into Equation (3), we obtain:

$$v = K_v \left(\frac{\sqrt[3]{\frac{3 \cdot Q}{4 \cdot \pi}}}{r} \right)^n = K_v \cdot \left(\sqrt[3]{\frac{3}{4 \cdot \pi}} \right)^n \cdot \left(\frac{\sqrt[3]{Q}}{r} \right)^n = K_v \cdot K_1 \cdot \left(\frac{r}{\sqrt[3]{Q}} \right)^{-n} = K \cdot \left(\frac{r}{\sqrt[3]{Q}} \right)^{-n} = K \cdot R^{-n} \quad (5)$$

where as:

$$\left(\sqrt[3]{\frac{3}{4 \cdot \pi}} \right)^n = K_1; \; K_v = \frac{K}{K_1}; \; \frac{r}{\sqrt[3]{Q}} = R \quad (6)$$

where:

R—reduced distance is the distance from blasting point to observation point reduced to a quantity of explosive and given in the following form: $R = \frac{r}{r_0}$.

Thus, we have obtained the oscillation law of rock mass, i.e., the equation of Sadovskii.

2.1.2. Derivation of Equation of Rock Mass Oscillation—Second Mode

Here we give one more mode to derive the equation for the rock mass oscillation. If, by blasting in the specific environment, the relative increase in the oscillation velocity of the rock mass and the relative increase of the reduced distance are monitored, then it can be seen that their relations at various levels have approximately the same value [4], which will be marked $-n$, meaning that:

$$\frac{\frac{\Delta v}{v}}{\frac{\Delta R}{R}} \approx -n \quad (7)$$

Thereby it can be considered that:

$$lim_{\Delta R \to 0} \frac{\frac{\Delta v}{v}}{\frac{\Delta R}{R}} = -n, \text{ which means: } \frac{\frac{dv}{v}}{\frac{dR}{R}} = -n \quad (8)$$

Equation (8) can be written in the form: $\frac{dv}{v} = -n \cdot \frac{dR}{R}$, where by integration is obtained:

$$\int \frac{dv}{v} = -n \cdot \int \frac{dR}{R}, \text{ namely: } logv = logR^{-n} + logC \quad (9)$$

C—represents constant of integration.

Equation (9) can be written in the form:

$$logv = logC \cdot R^{-n} \quad (10)$$

If in the previous equation C is replaced by K, $C = K$, then we get the rock mass oscillation equation given by M.A. Sadovskii.

The equation of Sadovskii is commonly shown in the form:

$$v = K \cdot Q_{red}^n \quad (11)$$

There is a reduced amount of explosive Q_{red}:

$$Q_{red} = \frac{\sqrt[3]{Q}}{r} \quad (12)$$

R—distance from the blast site to the monitoring point (m); and

Q—overall amount of explosive in a mine series (kg).

2.2. Models of Determination of Soil Oscillation Law Parameters

There are two parameters, K and n, in Equation (1) which should be determined for the specific work environment and by particular blasting conditions.

2.2.1. Model 1—Determining the Parameters by Applying the Least Square Method

The least square method is mainly used to obtain the parameters K and n representing a common model [5,6].

2.2.2. Model 2—Determining the Parameters by Applying the Quotient of the Relative Growth of Oscillation Velocities and Reduced Distances

Beginning with the rock mass oscillation law from Equation (1), which is derived in a different way (Section 2.1.2), whereby the parameter K, occurring as a constant of integration [7], can be determined from conditions (initial condition) that, for $R = R_1$, is $v = v_1$.

Parameters K and n will be determined by using experimental data of pairs (R_i, v_i), $i = 1, 2, ..., N$, provided that the curve of the oscillation velocity of rock mass passes through the point $M_1(R_1, v_1)$. In that case from (1) for $R = R_1$ and $v = v_1$ we obtain:

$$v_1 = K \cdot R_1^{-n}, \text{ where is: } K = v_1 \cdot R_1^n \tag{13}$$

By replacing values for K from Equation (13) into Equation (1) we obtain the equation:

$$v = v_1 \cdot \left(\frac{R_1}{R}\right)^n \tag{14}$$

From Equation (14), for $R = R_1$ there is obtained $v = v_1$ for any n, in our case $n > 0$. For $R = R_i$, $i = 2, 3, ..., N$, from Equation (14), we can take that: $v_i = v_1 \cdot \left(\frac{R_1}{R_i}\right)^n$, i = 2, 3, ..., N, from there the relation is obtained:

$$v_1 \cdot v_2 \cdot \ldots \cdot v_N = v_1^N \cdot \left(\frac{R_1^N}{R_1 \cdot R_2 \cdot \ldots \cdot R_N}\right)^n \tag{15}$$

From Equation (15), we can determine parameter n. By the logarithm operation of Equation (15) we obtain:

$$n \, log\left(\frac{R_1^N}{R_1 \cdot R_2 \cdot \ldots \cdot R_N}\right) = log\left(\frac{v_1 \cdot v_2 \cdot \ldots \cdot v_N}{v_1^N}\right), \quad n = \frac{log\left(\frac{v_1 \cdot v_2 \cdot \ldots \cdot v_N}{v_1^N}\right)}{log\left(\frac{R_1^N}{R_1 \cdot R_2 \cdot \ldots \cdot R_N}\right)} \tag{16}$$

Replacing the value for the parameter n in Equation (14), found in this way, we obtain the relation for the oscillation velocity of rock mass in the monitored environment $v = v_1 \cdot \left(\frac{R_1}{R}\right)^n$. Thus, to determine the parameter n, all experimental data were taken into account.

2.2.3. Model 3—Determining the Parameters between Parameters K and n is Determined by Applying the Trapezoidal Formula for Finding the Value of Definite Integral

In this model we will use the trapezoidal formula [8] to find the approximate value of the definite integral of the observed function.

Surface S confined by continuous curve $y = f(x)$, x-axis, and vertical lines $x = a$ and $x = b$, if $f(x) \geqslant 0$ for $a \leqslant x \leqslant b$ (Figure 1), as the definite integral of the function $f(x)$, obtains the following:

$$S = \int_a^b f(x)\, dx \tag{17}$$

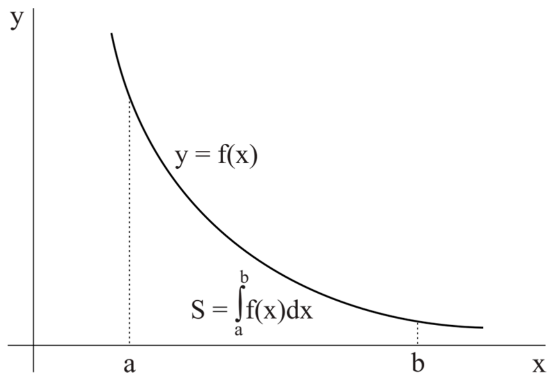

Figure 1. Surface *S* confined by continuous curve $y = f(x)$, x-axis, and vertical lines $x = a$ and $x = b$.

When carrying out blasting operations, we register (measure) at suitable measuring points the values of oscillation velocities of the rock mass v_i for corresponding values of reduced distances R_i (i = 1, 2, ..., N). If we connect points (R_i, v_i), (i = 1, 2, ..., N) with straight lines, then we will obtain a polygonal line (Figure 2). The surface confined by this polygonal line, R-axis, and the lines $R = R_1$ and $R = R_N$, we mark as S_T. In this way, we obtained *N-1* of the trapezoid. The sum of their surfaces is:

$$S_T = \sum_{m=1}^{N-1} S_m, \tag{18}$$

where:

S_m—surfaces of some trapezoids.

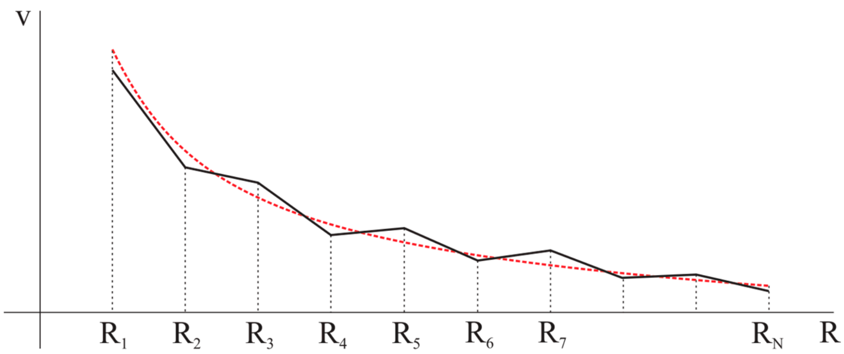

Figure 2. Graphic view of the polygonal line, *N-1* of the trapezoid and curve $v = K \cdot R^{-n}$.

If:

$$R_{i+1} - R_i = h = \frac{R_N - R_1}{N - 1} \tag{19}$$

then, for S_T, the following value is obtained:

$$S_T = \frac{h}{2}\left[v_1 + 2\left(v_2 + v_3 + \ldots + v_{N-1}\right) + v_N\right] \tag{20}$$

If points R_i are not equally spaced, namely if:

$$R_{i+1} - R_i = h_i \left(i = 1, 2, \ldots, N - 1\right) \tag{21}$$

then the value of S_T is obtained according to formula:

$$S_T = \frac{(v_1 + v_2) \cdot h_1 + (v_2 + v_3) \cdot h_2 + \ldots + (v_{N-1} + v_N) \cdot h_{N-1}}{2} \tag{22}$$

If we presume that certain values of registered oscillation velocities of the rock mass v_i represent the approximate value of the function $v = K \cdot R^{-n}$, then we can presume that:

$$\int_{R_1}^{R_N} v dR = K \int_{R_1}^{R_N} R^{-n} dR = \frac{K}{n-1}\left(\frac{1}{R_1^{n-1}} - \frac{1}{R_N^{n-1}}\right) = \frac{K}{n-1}\left[\frac{R_N^{n-1} - R_1^{n-1}}{(R_1 R_N)^{n-1}}\right] = S_T = \sum_{m=1}^{N-1} S_m \quad \text{(trapezoidal formula)} \tag{23}$$

From Equation (23) we find that:

$$K = \frac{S_T\,(n-1)\,(R_1 R_N)^{n-1}}{R_N^{n-1} - R_1^{n-1}} \tag{24}$$

By substituting K from Equation (23) into Equation (1) we obtain the formula:

$$v = \frac{S_T\,(n-1) \cdot (R_1 R_N)^{n-1}}{R_N^{n-1} - R_1^{n-1}} \cdot R^{-n},\ n \neq 1 \tag{25}$$

For $n = 1$, Equation (25) is reduced to:

$$v = \frac{S_T}{logR_N - logR_1} \cdot R^{-1} \tag{26}$$

Taking different values from Equation (25) for parameter n, the appropriate formulas are obtained.

Previous investigations have shown that the value of parameter n generally ranges in an interval from 1–3, so that in Equation (25) we may take that $n = 1.5$.

3. Defining Statistical Criteria

For the above mentioned models 1, 2, and 3, based on experimental data, we have obtained equations which make possible the determination of the oscillation velocities of the rock mass v depending on the reduced distance R. In order to assess the degree of connection between v and R, we have used the curved line dependency index ρ [9].

The evaluation of the relationship degree of two variables [5] to values of the curved line dependency index ρ is given in the following survey:

- $0.0 < \rho < 0.2$—none or highly poor correlation;
- $0.2 < \rho < 0.4$—poor correlation;
- $0.4 < \rho < 0.7$—significant correlation; and
- $0.7 < \rho < 1.0$—strong or highly strong correlation.

As a convenience measure of the obtained functional relationship for the given experimental data, the criterion "3S" was also used [10]. This criterion uses squares of differences between the obtained experimental data and the calculated ones for oscillation velocities of v. If those differences are one after another ε_1, ε_2, ... ε_N, then it is:

$$S = \sqrt{\frac{\varepsilon_1^2 + \varepsilon_2^2 + \ \ldots\ + \varepsilon_N^2}{N}} \tag{27}$$

According to this criterion, for the evaluation of convenience of the obtained functional correlation, the following relations are valid:

- if $\mid \varepsilon_{max} \mid \geqslant 3S$, the obtained functional correlation is rejected as unfavorable; and
- if $\mid \varepsilon_{max} \mid < 3S$, the functional correlation is accepted as a good one.

4. Methodology of Research

The practical part of this study includes:

- experimental investigations performed in the Veliki Krivelj Open Pit in the Bor District, and
- investigation carried out during mass mining in the Kovilovača Open Pit near Despotovac.

4.1. Experimental Investigation in the VelikiKrivelj Open Pit

4.1.1. General Characteristics of the Work Environment in the Veliki Krivelj Open Pit

In the Veliki Krivelj Open Pit we performed experimental investigations on copper ore [7]. The basic physical and mechanical properties of the rock mass were determined on samples in the laboratory. By examining the physical and mechanical properties of the work environment, the following values were obtained:

- compressive strength, σ_p = 31 MPa;
- bending strength, σ_s = 5.1 MPa;
- tensile strength, σ_I = 3.4 MPa;
- cohesion, C = 7.5 MPa;
- strength coefficient, f = 3;
- volume density, γ = 24 kN/m^3;
- angle of internal friction, ϕ = 52°;
- porosity, p = 3.5%;
- velocity of longitudinal waves, c_p = 2400.00 m/s;
- velocity of transverse waves, c_s = 1400.00 m/s.

4.1.2. Method of Blasting in the Veliki Krivelj Open Pit

The measurements of shocks resulting from blasting in the Veliki Krivelj Open Pit were performed during blasting using half-second electric detonators. The delay time of initiation between boreholes was 0.5 s, which led to ten explosions and appropriate soil oscillation velocities.

The explosive used was powdered ammonium nitrate for general purposes (Amoneks-I, manufactured by Trayal Corporation AD, Kruševac, Serbia). The holes were arranged in a single line while one cartridge of explosive was placed in each hole. The diameter of the explosive cartridge was 28 mm, with a cartridge length of 0.15 m, and a cartridge weight of 0.1 kg. An electric capacitor was used for initiation of the explosive.

During experimental investigations performed in the Veliki Krivelj Open Pit, the following blasting parameters were specified:

- depth of borehole: 0.5 m;
- weight of explosive charge per borehole: 0.10 kg;
- number of boreholes: 10;
- distance between boreholes: 1.0 m;
- distance between measuring point and the first borehole: 5.0 m; and
- delay time between initiation of boreholes: 0.5 s.

Figure 3 shows a record of the soil oscillation velocity for blasting in the Veliki Krivelj Open Pit.

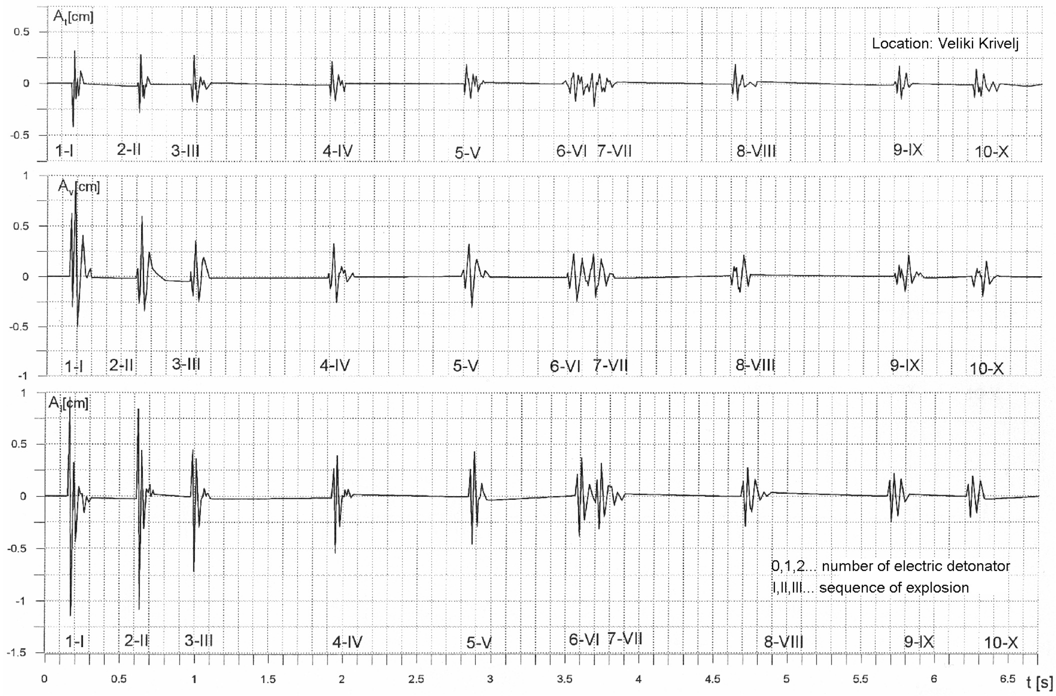

Figure 3. Record of soil oscillation velocity for blasting in the Veliki Krivelj Open Pit.

4.1.3. Calculation for Parameters of the Soil Oscillation Law in the Veliki Krivelj Open Pit

The values of distance from the blasting point to the point of observation r, quantity of explosive Q, calculated values of reduced distances R, registered values of soil oscillation velocities by components v_t, v_v, and v_l, and the resulting soil oscillation velocities v_{rez} for a total of ten explosions, are given in Table 1.

Table 1. Survey of blasting parameters and measurement results in the Veliki Krivelj Open Pit.

No.	Blasting	r (m)	Q (kg)	R	v_t (cm/s)	v_v (cm/s)	v_l (cm/s)	v_{rez} (cm/s)
1	I	5.0	0.1	10.7722	0.4200	0.9300	1.1400	1.5300
2	I	6.0	0.1	12.9266	0.3000	0.6000	1.0500	1.2460
3	I	7.0	0.1	15.0810	0.2400	0.4000	0.6450	0.7960
4	I	8. 0	0.1	17.2355	0.2000	0.3750	0.5000	0.6562
5	I	9.0	0.1	19.3899	0.1800	0.4000	0.4750	0.6465
6	I	10.0	0.1	21.5443	0.1500	0.3000	0.4000	0.5220
7	I	11.0	0.1	23.6988	0.1900	0.2700	0.3300	0.4668
8	I	12.0	0.1	25.8532	0.1900	0.2100	0.2900	0.4053
9	I	13.0	0.1	28.0077	0.1500	0.2150	0.2500	0.3622
10	I	14.0	0.1	30.1621	0.1400	0.2250	0.2000	0.3320

On the basis of the data given in Table 1, the soil oscillation law is calculated by Equation (1)—by the models 1, 2, and 3. The calculation of the curve was carried out for values of reduced distances from R = 10.7722 to R = 30.1621. Thus, curve parameters were calculated enabling us to determine the equation of soil oscillation in the form of:

- Model 1

$$v_1 = 49.4477 \cdot R^{-1.478} \tag{28}$$

A graphic survey of the soil oscillation law is shown in Figure 4.

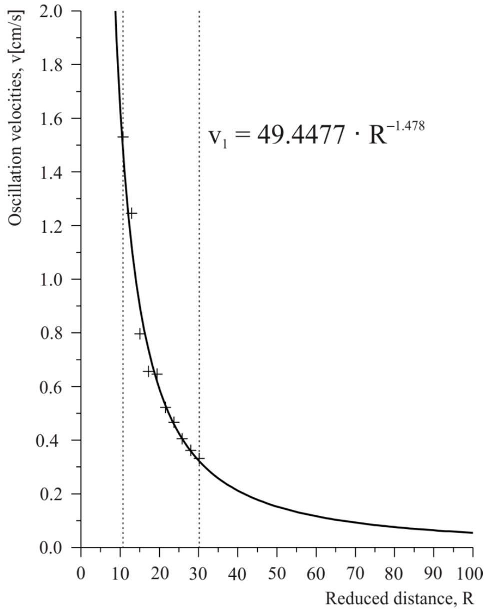

Figure 4. Graphic survey of soil oscillation law curve in the Veliki Krivelj Open Pit.

- Model 2

$$v_2 = 59.7588 \cdot R^{-1.5419} \tag{29}$$

- Model 3

By using experimental data given in Table 1, where:

$R_1 = 10.7722$; $R_N = R_{10} = 30.1621$; $R_{i+1} - R_i = h = 2.1544$,

according to the Equation (20) we obtain:

$S_T = 12.9953$

In this case for $n = 1.5$, according to Equation (25) we obtained the equation of the soil oscillation velocity law in the following form:

$$v_3 = 52.9989 \cdot R^{-1.5} \tag{30}$$

On the basis of the obtained equations of soil oscillation (28)–(30), it is possible to calculate values of soil oscillation velocities for corresponding reduced distances for models 1, 2 and 3.

The survey of reduced distances R, recorded oscillation velocities v_r, calculated oscillation velocities v_{i1}, v_{i2}, and v_{i3}, as well as the difference between the recorded and calculated soil oscillation velocities for models 1, 2, and 3, are presented in Table 2.

Table 2. Survey of recorded and calculated soil oscillation velocities for models 1, 2, and 3.

No.	R	v_r (cm/s)	v_{i1} (cm/s)	v_{i2} (cm/s)	v_{i3} (cm/s)	$v_r - v_{i1}$	$v_r - v_{i2}$	$v_r - v_{i3}$
1	10.7722	1.5300	1.4737	1.5300	1.4990	0.0563	0.0000	0.0310
2	12.9266	1.2460	1.1256	1.1551	1.1404	0.1204	0.0909	0.1056
3	15.0810	0.7960	0.8962	0.9107	0.9049	−0.1002	−0.1147	−0.1089
4	17.2355	0.6562	0.7357	0.7412	0.7407	−0.0795	−0.0850	−0.0845
5	19.3899	0.6465	0.6182	0.6181	0.6207	0.0283	0.0284	0.0258
6	21.5443	0.5220	0.5290	0.5255	0.5300	−0.0070	−0.0035	−0.0080
7	23.6988	0.4668	0.4595	0.4536	0.4594	0.0073	0.0132	0.0074
8	25.8532	0.4053	0.4041	0.3967	0.4032	0.0012	0.0086	0.0024
9	28.0077	0.3622	0.3590	0.3506	0.3576	0.0032	0.0116	0.0046
10	30.1621	0.3320	0.3217	0.3128	0.3199	0.0103	0.0192	0.0121

Based on the data in Table 2, a statistical analysis was carried out and the following values were obtained:

- For model 1:

  The curved line dependency index ρ_1 between the reduced distance R and soil oscillation velocity v is:

  ρ_1 = 0.9880 (there is a strong correlation between R and v, given in Equation (28)).

  The maximum difference between the recorded and calculated oscillation velocities of the soil, $(\varepsilon_{max}) = max\ |\varepsilon_i|$, amounts:

  ε_{max1} = 0.1204, S_1 = 0.0592, $3S_1$ = 0.1776.
  As $\varepsilon_{max1} < 3S_1$, the supposed functional relationship is accepted as a good one.

- For model 2:

  ρ_2 = 0.9893 (there is a strong correlation between R and v, given in Equation (29)).
  ε_{max2} = 0.1147, S_2 = 0.0550, $3S_2$ = 0.1650.
  $\varepsilon_{max2} < 3S_2$ (the supposed functional relationship is accepted as a good one).

- For model 3:

  ρ_3 = 0.9887 (there is a strong correlation between R and v, given in Equation (30)).
  ε_{max3} = 0.1089, S_3 = 0.0566, $3S_3$ = 0.1698.
  $\varepsilon_{max3} < 3S_3$ (supposed functional relationship is accepted as a good one).

4.2. Investigation during Mass Mining in the Kovilovača Open Pit

In order to check the results of experimental investigations, we have also carried out investigations during mass mining, carried out for the purpose of exploitation of mineral deposits. Measurements were carried out in the Kovilovača Open Pit near Despotovac.

4.2.1. General Characteristics of the Kovilovača Open Pit

Kovilovača limestone deposits have an exceptionally simple geological structure. Limestone deposits of this area are massive or layered with layer thickness ranging from 0.20–0.80 m, the direction of propagation is NE–SW and a slope of about 42° towards the southwest. These rocks, in engineering geological terms, belong to a group of associated rocks, which are cracked and karstified [11]. During previous investigation and exploitation works, no significant burst deformations, which would significantly influence the process of exploration and exploitation, were noted in the deposit. Only after blasting do blocks of rock of 0.50 m^3 occur, which could be attributed to the effect of small faults and karstified cracks in the deposit.

By examination of physical and mechanical properties of the working environment, the following values are obtained:

- angle of internal friction: $\phi = 31°35'$
- cohesion: C = 138.33 kPa
- volume weight: γ = 26.26 kN/m^3
- compressive strength: σ_p = 80.808 MPa
- tensile strength: σ_z = 7.59 MPa
- velocity of longitudinal elastic waves: c_p = 6661.00 m/s
- velocity of transverse elastic waves: c_s = 2852.67 m/s

- dynamic elasticity modulus: E_{din} = 62.46 GPa
- dynamic Poisson's ratio: μ_{din} = 0.39 GPa

4.2.2. Method of Blasting in the Kovilovača Open Pit

Measurements of seismic shocks at Kovilovača Open Pit were performed during blasting, conducted for the purpose of deposit exploitation. Two blasting operations were performed.

Balkanit 60/1500, detonex 65/1500 and ANFO 70/1500 were used as explosives. Activation of explosives in the borehole was performed using nonel detonators with retardations of 500 ms in the borehole, while the retardation between boreholes on the surface was 25 ms and 42 ms. Activation of the nonel tube was performed using an electric detonator.

Basic data related to the number of boreholes N_b, the overall explosive amount Q_{uk}, the maximal explosive amount by deceleration interval Q_i, overall borehole depth L_{uk}, and average stemming length $L_{p\check{c}}$, are presented in Table 3.

Table 3. Survey of blasting parameters in the Kovilovača Open Pit.

Blasting	N_b	Q_{uk} (kg)	Q_i (kg)	L_{uk} (m)	$L_{p\check{c}}$ (m)
I	34	3623.0	108.0	842.0	2.2
II	36	3264.0	92.0	918.0	2.2

The record of soil oscillation velocity for blasting number II—measuring point 3 is shown in Figure 5.

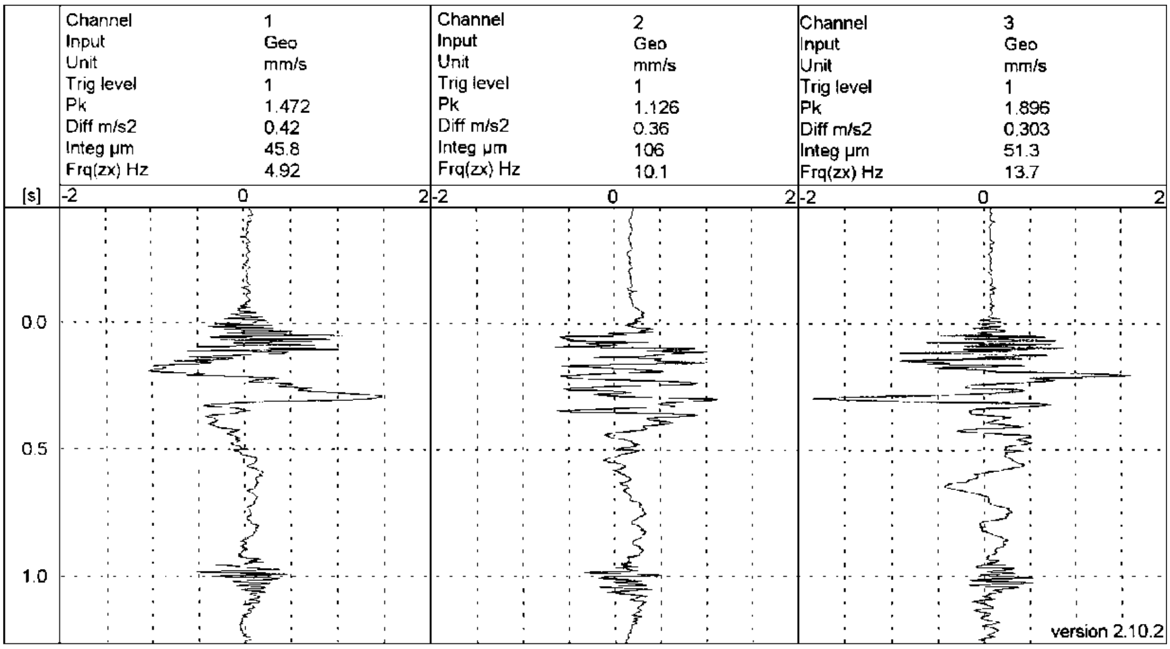

Figure 5. Image of soil oscillation velocity for blasting II-MM3 in the Kovilovača Open Pit.

4.2.3. Calculation of Soil Oscillation Law Parameters in the Kovilovača Open Pit

Values of distances from blast sites to monitoring points r, the amount of explosive Q, calculated values of reduced distances R, recorded values of soil oscillation velocities by components v_t, v_v, and v_l, and the resulting oscillation velocities v_{rez} for blasting from I to II of a total of ten measuring points MM are given in Table 4.

Table 4. Survey of blasting parameters and measurement results in the Kovilovača Open Pit Pit.

No.	MM	r (m)	Q (kg)	R	v_t (cm/s)	v_v (cm/s)	v_l (cm/s)	v_{rez} (cm/s)
1	MM–1	335.0	3.623.0	21.8117	0.1460	0.0517	1.2000	1.2100
2	MM–3	850.0	3.623.0	55.3430	0.0138	0.0270	0.0617	0.0687
3	MM–5	830.0	3.264.0	55.9536	0.0206	0.0133	0.0407	0.0475
4	MM–9	855.0	3.264.0	57.6390	0.0200	0.1070	0.0245	0.1116
5	MM–3	860.0	3.264.0	57.9760	0.1400	0.1010	0.1830	0.2516
6	MM–10	880.0	3.264.0	59.3234	0.0729	0.0936	0.0156	0.1197
7	MM–10a	885.0	3.264.0	59.6614	0.0134	0.1530	0.0426	0.1594
8	MM–7	950.0	3.623.0	61.8540	0.0222	0.0422	0.0623	0.0785
9	MM–7	960.0	3.264.0	64.7174	0.0174	0.0477	0.0708	0.0871
10	MM–6	965.0	3.264.0	65.0545	0.0045	0.0337	0.0555	0.0651
11	MM–6a	970.0	3.264.0	65.3916	0.0202	0.0568	0.0711	0.0932
12	MM–8	1,155.0	3.623.0	75.2014	0.0327	0.0317	0.0246	0.0518

On the basis of data given in Table 4, the soil oscillation law is calculated by Equation (1)—by the models 1, 2, and 3. The calculation of the curve was carried out for values of reduced distances from $R = 21.8117$ to $R = 75.2014$. Thus, the curve parameters were calculated enabling us to determine the equation of soil oscillation in the form of:

- Model 1

$$v_1 = 2131.52 \cdot R^{-2.4410} \tag{31}$$

Graphic survey of soil oscillation law is shown in Figure 6.

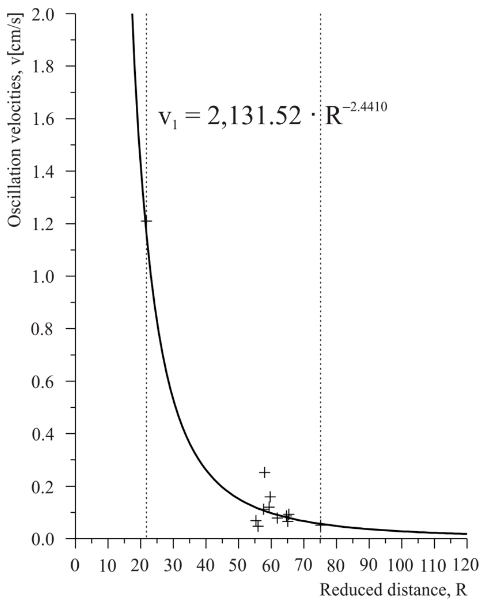

Figure 6. Graphic survey of soil oscillation law curve in the Kovilovača Open Pit.

- Model 2

$$v_2 = 2571.7694 \cdot R^{-2.4856} \tag{32}$$

- Model 3By using experimental data given in Table 4, where:

$R_1 = 21.8117$, $R_N = R_{12} = 75.2014$,

and where:

S_T = 22.9538,

Calculating according to Equation (22), then for n = 1.5, according to Equation (25) we obtain the equation of soil oscillation velocity law in the following form:

$$v_3 = 116.1585 \cdot R^{-1.5} \tag{33}$$

On the basis of the obtained equations of soil oscillation Equations (31)–(33), it is possible to calculate values of soil oscillation velocities for corresponding reduced distances for models 1, 2, and 3.

The survey of reduced distances R, recorded oscillation velocities v_r, calculated oscillation velocities v_{i1}, v_{i2}, and v_{i3}, as well as the difference between recorded and calculated soil oscillation velocities for models 1, 2, and 3 is presented in Table 5.

Table 5. Survey of recorded and calculated soil oscillation velocities for models 1, 2, and 3.

No.	R	v_r (cm/s)	v_{i1} (cm/s)	v_{i2} (cm/s)	v_{i3} (cm/s)	$v_r - v_{i1}$	$v_r - v_{i2}$	$v_r - v_{i3}$
1	21.8117	1.2100	1.1507	1.2100	1.1403	0.0593	0.0000	0.0697
2	55.3430	0.0687	0.1185	0.1196	0.2821	−0.0498	−0.0509	−0.2134
3	55.9536	0.0475	0.1154	0.1164	0.2775	−0.0679	−0.0689	−0.2300
4	57.6390	0.1116	0.1073	0.1081	0.2654	0.0043	0.0035	−0.1538
5	57.9760	0.2516	0.1058	0.1065	0.2631	0.1458	0.1451	−0.0115
6	59.3234	0.1197	0.1001	0.1006	0.2542	0.0196	0.0191	−0.1345
7	59.6614	0.1594	0.0987	0.0992	0.2521	0.0607	0.0602	−0.0927
8	61.8540	0.0785	0.0904	0.0907	0.2388	−0.0119	−0.0122	−0.1603
9	64.7174	0.0871	0.0809	0.0811	0.2231	0.0062	0.0060	−0.1360
10	65.0545	0.0651	0.0799	0.0800	0.2214	−0.0148	−0.0149	−0.1563
11	65.3916	0.0932	0.0789	0.0790	0.2197	0.0143	0.0142	−0.1805
12	75.2014	0.0518	0.0561	0.0558	0.1781	−0.0043	−0.0040	−0.1263

Based on the data in Table 5, a statistical analysis was carried out and the following values were obtained:

- For model 1

  The curved line dependency index ρ_1 between the reduced distance R and soil oscillation velocity v is:

  ρ_1 = 0.9843 (there is a strong correlation between R an v, given in Equation (31)).

  The maximum difference between the recorded and calculated oscillation velocities of the soil $(\varepsilon_{max}) = max \mid \varepsilon_i \mid$, amounts:

  ε_{max1} = 0.1458, S_1 = 0.0552, $3S_1$ = 0.1656.
  $\varepsilon_{max1} < 3S_1$, the (the supposed functional relationship is accepted as a good one).

- For model 2:

  ρ_2 = 0.9852 (there is a strong correlation between R and v, given in Equation (32)).
  ε_{max2} = 0.1541, S_2 = 0.0535, $3S_2$ = 0.1606.
  $\varepsilon_{max2} < 3S_2$ (the supposed functional relationship is accepted as a good one).

- For model 3:

  ρ_3 = 0.8768 (there is a strong correlation between R and v, given in Equation (33)).
  ε_{max3} = 0.2300, S_3 = 0.1503, $3S_3$ = 0.4509.
  $\varepsilon_{max3} < 3S_3$ (the supposed functional relationship is accepted as a good one).

5. Conclusions

To establish the relationship between the oscillation velocity of the rock mass and basic parameters affecting its magnitude, being the amount of explosive, the distance from the blast site, characteristics of the rock mass and the type of blasting, is the equation of M. A. Sadovskii that is used most commonly.

In the paper, the law of Sadovskii is also derived in another way by using the quotient of the relationship between the relative increase in oscillation velocities of the rock mass and the relative increase of reduced distances. It turned out that this quotient has approximately the similar value for any values whatsoever of reduced distance and corresponding velocity of rock mass oscillation. Thereby, in a marginal case, a differential equation, whose general integral overlaps with the law of Sadovskii, is obtained. In this case, for the parameter K as the integration constant, we may take values determined by the starting conditions, and that has been done in model 2.

The relation between parameters K and n has been obtained in this study. This relation made it possible to find the value of the second parameter for the given determined value of one parameter. In practice it is simpler to determine the value of the parameter K in advance for an adopted value of the parameter n in the interval from 1–3, as has been applied in model 3.

Thus, parameters n and K in Sadovskii's law have been determined by three modes—models in the specific work environment. Their corresponding functions have been obtained presenting oscillation velocities of the rock mass depending on a reduced distance. The calculated corresponding indexes of the curved line correlation point out that there is a rather strong curved line relationship between a reduced distance and the oscillation velocity of the rock mass expressed in the obtained functions.

Comparing values of the recorded oscillation velocities of the rock mass with the corresponding calculated ones, it can be seen that they are approximately the same. On the basis of the obtained values of the curved line dependency, we can conclude that all three models can be successfully used for calculating the oscillation velocity of the rock mass.

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References

1. Savić, M. *Blasting at of the Open Pits*; Monograph, RTB Bor; Cooper Institute Bor, Indok Center: Bor, Serbia, 2000; pp. 317–319.
2. Trajković, S.; Slimak, Š.; Lutovac, S. *Technique of Blasting and Shock Waves*; Faculty of Mining and Geology: Belgrade, Serbia, 2005; p. 199.
3. Medvedev, S.V. *Seismic of Mountainous Explosions*; Nedra Publisher: Moscow, Russia, 1964; pp. 42–43.
4. Lutovac, S.; Gligorić, Z.; Beljić, Č.; Ravilić, M. Derivation the soil oscillation law and determination of its parameters. *Min. Metall. Eng. Bor* **2014**, *3*, 41–51. [CrossRef]
5. Simeunović, D. *Mathematics*; Faculty of Mining and Geology, Mining Department: Belgrade, Serbia, 1985; pp. 101–103.
6. Lutovac, S.; Trajković, S.; Katona, O.; Savić, L.J.; Leković, B. Parameter determination of soil oscillation in limestone. *Tech. Technol. Educ. Manag. (DRUNPP Sarajevo)* **2013**, *7*, 376–383.
7. Lutovac, S. Model for Determining the Parameters of the Law of Oscillation of the Rock Mass in Blasting. Ph.D. Thesis, Faculty of Mining and Geology, University of Belgrade, Belgrade, Serbia, 2010; pp. 37–38, 48–55.
8. Kudryavtsev, V.A.; Demidovich, B.P. *A Brief Course of Higher Mathematics*, 2nd ed.; Mir Publishers: Moscow, Russia, 1984; pp. 293–295.
9. Petrović, Ž. *Business Statistics, Megatrend*; University of Applied Sciences: Belgrade, Serbia; Faculty for Management: Zaječar, Serbia, 2004; p. 182.

10. Vukadinović, S. *Elementsof Probability and Mathematical Statistics*; Technique for Copying Bureau of Construction: Belgrade, Serbia, 1970; pp. 329–331.
11. Trajković, S.; Tokalić, R.; Lutovac, S. *Study of Seismic Impacts on Adjacent Building due to Blasting Operations at Kovilovača Open Pit*; Faculty of Mining and Geology: Belgrade, Serbia, 2004; pp. 2–43.