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The cooperative, reliable and responsive characteristics make smart grid more popular than traditional power grid. However, with the extensive employment of smart grid concepts, the traditional centralized control methods expose a lot of shortcomings, such as communication congestion, computing complexity in central management systems, and so on. The distributed control method with flexible characteristics can meet the timeliness and effectiveness of information management in smart grid and ensure the information collection timely and the power dispatch economically. This article presents a decentralized approach based on multi agent system (MAS) for solving data collection and economic dispatch problem of smart grid. First, considering the generators and loads are distributed on many nodes in the space, a flooding-based consensus algorithm is proposed to achieve generator and load information for each agent. Then, a suitable distributed algorithm called  $\lambda$ -consensus is used for solving the economic dispatch problem, eventually, all generators can automatically minimize the total cost in a collective sense. Simulation results in standard test cases are presented to demonstrate the effectiveness of the proposed control strategy.

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## Article

# A Fully Distributed Approach for Economic Dispatch Problem of Smart Grid

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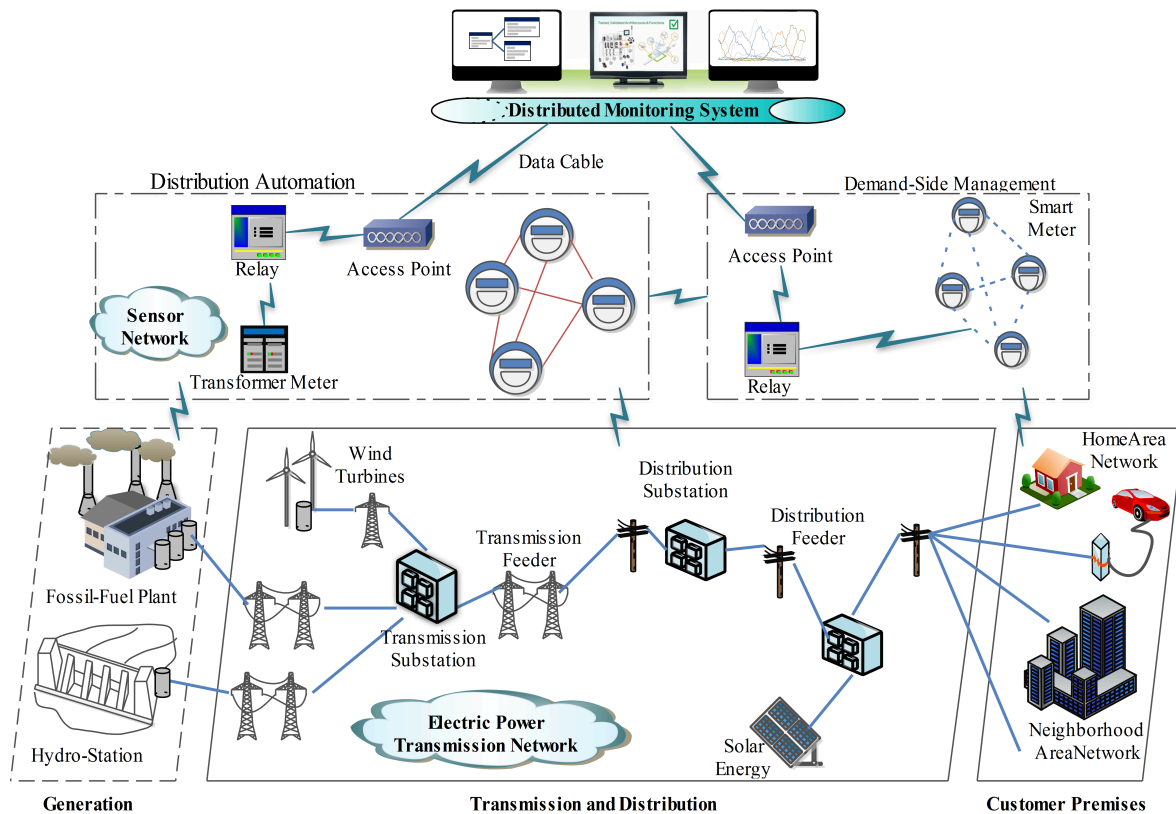
**Abstract:** The cooperative, reliable and responsive characteristics make smart grid more popular than traditional power grid. However, with the extensive employment of smart grid concepts, the traditional centralized control methods expose a lot of shortcomings, such as communication congestion, computing complexity in central management systems, and so on. The distributed control method with flexible characteristics can meet the timeliness and effectiveness of information management in smart grid and ensure the information collection timely and the power dispatch economically. This article presents a decentralized approach based on multi agent system (MAS) for solving data collection and economic dispatch problem of smart grid. First, considering the generators and loads are distributed on many nodes in the space, a flooding-based consensus algorithm is proposed to achieve generator and load information for each agent. Then, a suitable distributed algorithm called  $\lambda$ -consensus is used for solving the economic dispatch problem, eventually, all generators can automatically minimize the total cost in a collective sense. Simulation results in standard test cases are presented to demonstrate the effectiveness of the proposed control strategy.

**Keywords:** optimal resource management; consensus algorithm; distributed control; Sensor data collection; economic dispatch problem

## 1. Introduction

### 1.1. Motivation

In traditional power grid, power is carried from a few central generators (e.g., thermal power plant, hydroelectric station, etc.) to a large number of users, and traditional power grid concentrates on the electricity generation, transmission, distribution and control [1,2]. Recently, new renewable energy generators (e.g., photovoltaic systems, wind turbines, etc.) have been used to provide an alternate way for electricity production. In addition, the demand for electricity is diversified, and the traditional power grid is inadequate to overcome modern day challenges. An intelligent electrical network, known as smart grid (SG) [3–5], is used for improving reliability of the power system by enabling the system to be controllable and automated [6–8]. The deployment of sensor technology and smart meters has been accelerating the development of smart grid [9,10]. The data communication network infrastructures [3] include wide area measurement systems (WAMSs), sensor and actuator networks (SANETs), home area networks (HANs), neighborhood area network (NANs), and wide area networks (WANs). Different communications media, i.e., wired and wireless, can be used for data transmission between smart meters and electric utilities. SANETs can be grouped under a hierarchical structure on the basis of HANs, NANs, and WANs, as shown in Figure 1. For a detailed introduction to HAN and WAN, refer to Reference [1].



**Figure 1.** Smart grid communication network architecture for data sensing and collection.

As an alternative to traditional power grid, SG can relieve the burden caused by various energy demands. However, the increasing electricity demand, together with the complex and nonlinear nature of the electric power distribution network, have caused many challenges for the future grid, for example, serious network communication issues [11,12], and it is expensive and unreliable to solve economic dispatch problem (EDP) using traditional centralized methods. SG, with distributed energy sources accessing, require distributed approaches to deal with its energy management problems.

### 1.2. Literature Review

The conventional methods used to deal with EDP in power system require global information about total load demand and parameters about power cost, then compute optimal power output via a centralized controller, and receive minimum cost [13]. In addition, the centralized approaches, such as Lambda iteration method [14,15] and interior point method [16,17], require the cost function to be convex. Conventional approaches also include heuristic methods such as genetic algorithm (GA) [18,19], particle swarm optimization (PSO) [20–22], differential evolution [23,24], and other heuristic algorithms [25–27] that handle non-convex solution spaces and more stringent constraints. A computationally intelligent load forecasting system in smart energy management grid is discussed in [28], in which the single and the hybrid computational intelligence is mentioned. Despite excellent performance, majority of existing approaches for solving EDP are performed centrally [29], which require a high-bandwidth communication infrastructure. Furthermore, once the controller fails, the whole system will be in a state of collapse. It is difficult to ensure the reliability in centralized control [30]. Moreover, due to the widespread access of distributed power generation, both transmission grid and distribution network (including communication network) are likely to have a variable topology, which further undermines the efficacy of centralized mechanisms.

With the access of distributed power sources, the approach of control for smart grid is also transformed from centralized to distributed [31]. Distributed algorithm can provide better service

for SG; it is immune to topological variations and accommodate desired plug-and play features; and it enables real-time modeling and simulation of complex power systems. Furthermore, distributed decision-making can also create solution based on partial information from participating agents [29,30]. There exists plenty of studies on EDP of SG in terms of distributed algorithm. In [32], an approach based on a consensus term and innovations framework is presented, in which each agent only requires local information of cost, predicted load and neighborhood communication to solve EDP. Literature [33] elaborated on a mathematical formulation for the incremental cost consensus variable and gives some case studies to show the influence of difference network topologies on the convergence rate. In [34], a peer-to-peer communication architecture and a new analysis on loss penalty factors are proposed for solving EDP in distributed approach. In [35], a multi-agent system (MAS) based distributed control approach for optimal resource management in an islanded microgrid is proposed. In [36], an optimization model for minimizing total daily energy cost is proposed. The electricity costs and CO<sub>2</sub> emissions are considered in this model. A new decentralized approach for solving the EDP is proposed in [37], which offers the ability to share information among agents using a flooding-based consensus algorithm. An algorithm based on NSGA-II is presented in [38], in which the renewable energy sources and distributed generations are considered. Furthermore, a fluctuation model is discussed with different input load demands and renewable energy sources. In [39], an event-triggered communication-based distributed optimization is proposed for economic dispatch problem to reduce information exchange requirements in smart grids. In [40], a distributed solution to DC optimal power flow (DCOPF) considering dispatchable generations, responsive loads and line congestion cost is proposed.

Previous work on energy management, especially on EDP, take the predicted total power demand of load ( $P_D$ , a parameter on global information) and the number of generators ( $M$ , an essential parameter to construct communication matrix) as known information, and lack the specific process of data collection for load demand. In essence, it cannot be regarded as a real sense of distribution.

### 1.3. Contribution

In the article, a fully decentralized solution is provided to collect data from each agent and solve the EDP considering generation limits. The distributed approach contains two stages, both based on consensus algorithms. In the first stage, the flooding based consensus (FBC) algorithm is used to share information among agents, and a ring communication graph is proposed as agent can receive (or send) information from (to) its two neighbors. In the second stage, a algorithm called  $\lambda$ -consensus is presented, which is based on a consensus procedure. The parameter  $\lambda$  works as a correction term to satisfy the generation-demand equality constraint. Our contributions in this paper can be summarized as follows: Firstly, the algorithm in this paper is fully distributed. All agents (or nodes) in our algorithm have the same status, this is, no agent is different from other agents in their information and behavior. There is no leader agents, thus the algorithm in this paper not only achieves plug and play, but also has strong robustness. Secondly, in the first stage of the proposed algorithm, we give a method that can be used to collect global information by sharing information with neighbors, which can effectively alleviate communication congestion.

Briefly, the remainder of this paper is outlined as follows. Section 2 provides an overview of graph theory and consensus protocols. Section 3 discusses the EDP, including FBC algorithm for data information collection, and  $\lambda$ -consensus algorithm. Several case studies are presented in Section 4. Section 5 concludes the paper.

## 2. Preliminary

In this paper, the interconnection topologies of smart grids are represented schematically by graphs called interaction graphs. The aim is to control a group of generators connected through a sensor or communication network to meet the load demands cooperatively with the global objective of minimum cost and certain constraint conditions about generators and loads. In this Section,

the basic graph terminologies and consensus algorithm are introduced. We concentrate on the deeper connections between nonnegative matrices and undirected graphs. The main purpose of this section is to provide a mathematical foundation, based on the theory of graphs.

### 2.1. Graph Theory

In graph theory, graphs are composed of vertices (also called nodes or points) and edges (also called arcs or lines), denoting objects and some kind of relationship, respectively. Graph theory has been widely applied in many fields, such as computer science, operations research, practical engineering, and so on. According to the direction of edges, graphs can be divided into directed graphs and undirected graphs.

A directed graph [41] (or just digraph)  $G$  is an ordered triple:  $G = \{V(G), E(G), \psi_G\}$  where  $V(G)$  is a set of nodes, and  $V(G)$  is nonempty.  $E(G)$  is a set of lines, the two terminals of the line are in set  $V(G)$ .  $\psi_G$  is an function of line, an ordered pair of (not necessarily distinct) vertices of  $G$ . This is the symbol,  $\psi_G(e) = (u, v)$ , where  $e$  is said to join  $u$  to  $v$ ;  $u$  is the tail of  $e$ , and  $v$  is the head. An *undirected graph* [42],  $G$ , can be treated as a bidirectional diagram by replacing each line or edge  $(u, v)$  of  $G$  with the pair of arcs  $(u, v)$  and  $(v, u)$ .

Let  $A(G)$  be the adjacency matrix that associates with the directed or undirected graph  $G$ , and  $V(G)$  also be the index set representing the nodes. In digraphs corresponding to smart grid networks, a single node may be a pure power generation node, a pure load node or the node including power generation and load. A directed edge or arc  $(i, j) \in E$  means that node  $i$  can send message to node  $j$  and  $j$  can receive message from  $i$ . For the node  $i \in V(G)$ , its in-neighbor set is denoted by:

$$N_i^{in} = \{j \in V(G) | (j, i) \in E(G)\}$$

Similarly, its out-neighbor set is denoted by:

$$N_i^{out} = \{j \in V(G) | (i, j) \in E(G)\}$$

Physically, a node can receive message from its in-neighbors, and send message to its out-neighbors. We define the in-degree and out-degree of node  $i$  as:

$$d_i^{in} = |N_i^{in}|; \quad d_i^{out} = |N_i^{out}|$$

where  $|\cdot|$  denotes the cardinality of a set, i.e., the numbers of nodes in in-neighbor set and out-neighbor set. Since it is possible to suppose that each node can get messages from its in-neighbors or out-neighbors. Let  $(i, j)$  denotes a path from node  $i$  to  $j$ ; if  $i$  and  $j$  can represent any two nodes of  $G$  and the path,  $(i, j)$ , exists, and  $G$  is said to be strongly connected. Evidently, a graph associates with a connected smart grid is a strongly connected graph, and  $d_i^{in} + d_i^{out} \neq 0$  for each node  $i$  in a strongly connected graph. Obviously, in an undirected graph,  $d_i^{in} = d_i^{out} = |N_i^{neib}|$ , where  $N_i^{neib} = \{j \in V(G) | (i, j) \in E(G)\}$ , and  $d_i^{neib} = |N_i^{neib}|$  denotes the number of nodes that adjacent to node  $i$ .

### 2.2. Consensus Algorithm

Let us define a matrix  $Q \in \mathbb{R}^{N \times N}$  associated with a strongly connected graph  $G$  as follows:

$$Q = \{q_{ij}\} = \begin{cases} 1/(d_i^{neib} + 1) & \text{if } j \in N_i^{neib} \text{ or } j = i \\ 0 & \text{otherwise} \end{cases} \quad \forall i, j \in V \quad (1)$$

From the definition of  $Q$ ,  $q_{ij} > 0$  if  $j \in N_i^{neib}$ , otherwise,  $q_{ij} = 0$ , which means that  $Q$  is a nonnegative matrix, and it is not difficult to verify that  $Q$  is row stochastic, i.e.,  $\sum_{j=1}^N q_{ij} = 1$ . Nodes (including generating units and loads) in SG can be considered as a group of  $M$  participants or agents,

each of them can define his own subjective probability distribution, and they work together as a team to reach an target values under some constraint conditions. To solve such problems, DeGroot [43] proposed a model. Agents can reach consensus in this model and form a common subjective probability distribution for a certain parameter,  $\theta$ , by pooling their opinions. The value of  $\theta$  is supposed to locate in an abstract parameter space,  $\Omega$ . Let  $\phi_i (i = 1, 2, \dots, M)$  denote the subjective probability distribution that individual  $i$  assigns to the parameter  $\theta$ , and each individual maintains a distributions  $\phi = [\phi_1, \phi_2, \dots, \phi_M]^T$  on space,  $\Omega$ . Each agent have its own background and perspective to deal with problem, thus different individuals have different subjective distributions,  $\phi$ . Let  $Q_i = [q_{i1}, q_{i2}, \dots, q_{iM}]$  denote the subjective distributions on  $\phi$  and  $\sum_{j=1}^M q_{ij} = 1$ , where  $q_{ij} \geq 0$ . Let  $Q$  denote the  $M \times M$  matrix comprising the elements  $q_{ij}$ . It is reasonable to suppose individual  $i$  updates the subjective distribution from  $\phi(t)$  to  $\phi(t+1)$  after learning other members states. This process can be described by:

$$\phi(t+1) = Q\phi(t) = Q(Q\phi(t-1)) = Q^2\phi(t-1) = \dots = Q^t\phi(1) = Q^t(Q\phi(0)) = Q^{t+1}\phi(0) \quad (2)$$

where  $t$  denotes the number of iterations; it represents the number that individual,  $i$ , revises its subjective distributions after learning others' distributions. When the distributions of the  $M$  members converge to the same limit as  $t \rightarrow \infty$ , that is,

$$\lim_{t \rightarrow \infty} \phi(t) = \phi^* = [\phi_1^*, \phi_2^*, \dots, \phi_M^*]^T$$

and now, a consensus is reached. Then, it follows from Equation (2) that a consensus is reached if and only if there exists a vector,  $\Pi^* = [\pi_1^*, \pi_2^*, \dots, \pi_M^*]$ , where  $\pi_j^* = [\pi_{1j}, \pi_{2j}, \dots, \pi_{Mj}]^T = [\pi_j^*, \pi_j^*, \dots, \pi_j^*]^T$  such that,

$$\lim_{t \rightarrow \infty} q_{ij}(t) = \pi_j^* \quad (3)$$

In the following text, we take an example to describe the intuitive appeal exhibited in the iteration process. Consider the graph shown in Figure 2.

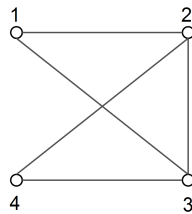


Figure 2. An undirected graph which includes four nodes.

According to Equation (1), we get the matrix,  $P$ , associating with the undirected graph.

$$Q = \{q_{ij}\} = \begin{bmatrix} 1/3 & 1/3 & 1/3 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 1/3 & 1/3 & 1/3 \end{bmatrix}$$

When positive integer,  $t \geq 8$ , and  $t$  is the number of iterations:

$$Q^t = Q^{t+1} = \begin{bmatrix} 0.2143 & 0.2857 & 0.2143 & 0.2857 \\ 0.2143 & 0.2857 & 0.2143 & 0.2857 \\ 0.2143 & 0.2857 & 0.2143 & 0.2857 \\ 0.2143 & 0.2857 & 0.2143 & 0.2857 \end{bmatrix} = \begin{bmatrix} \pi_1^* & \pi_2^* & \pi_3^* & \pi_4^* \end{bmatrix} = \Pi^*$$

where  $\pi_1^* = [0.2143 \ 0.2143 \ 0.2143 \ 0.2143]^T$ ,  $\pi_2^* = [0.2875 \ 0.2875 \ 0.2875 \ 0.2875]^T$ ,  $\pi_3^* = [0.2143 \ 0.2143 \ 0.2143 \ 0.2143]^T$ ,  $\pi_4^* = [0.2875 \ 0.2875 \ 0.2875 \ 0.2875]^T$ . The symbol  $A^T$  denotes the transposition of the matrix (or vector)  $A$  in this paper.

Let  $t$  and  $Q = \{q_{ij}\}$  denote the number of iterations and matrix associated with undirected graph  $G$  respectively. From Equations (2) and (3), we can get the following linear iterative formula,

$$\phi_i(t+1) = q_{ii}\phi_i(t) + \sum_{j \in N_i^{neib}} q_{ij}\phi_j(t) \quad (4)$$

where  $\phi_i(t)$  is the  $i$ th element in the column stack vectors,  $\phi(t)$ . Equation (4) can be rewritten as follow,

$$\phi(t+1) = Q\phi(t) \quad (5)$$

To investigate the asymptotic behavior of Equation (5), the following theorems are needed.

**Theorem 1.** Let  $Q = \{q_{ij}\}$  denote subjective distributions matrix, if there exists a positive integer number  $t$ , make every element in  $q_j^t$  positive, where  $q_j^t$  is a column of  $Q^t$ ; then, we can say a consensus can be reached for this MAS [43].

**Theorem 2.** If  $Q \in \mathbb{R}^{N \times N}$  is nonnegative and primitive, and if  $x$  and  $y$  are, respectively, the right and left Perron vectors of  $Q$ , then  $\lim_{t \rightarrow \infty} (\rho(Q)^{(-1)} Q)^t = xy^T$ , which is a positive rank-one matrix [44].

From Theorem 1, it is easy to determine whether a group can reach consensus, and it provides a simple condition.

From Theorem 2, there is a unique positive real vector  $x = \{x_i\}$ ,  $y = \{y_i\}$ , such that  $Ax = \rho(A)x$ ,  $y^T A = \rho(A)y^T$ , and  $x^T y = x_1 y_1 + \dots + x_t y_t = 1$ , where  $\rho(A)$  is an algebraically simple eigenvalue of  $A$  and  $\rho(A) > 0$ ,  $x_1 + x_2 + \dots + x_t = 1$ ,  $y_1 + y_2 + \dots + y_t = 1$ .

Based on the definition (as shown in Equation (1)),  $Q$  is nonnegative and stochastic, and spectral radius  $\rho(Q) = 1$ . Since  $Q$  is associated with a strongly connected graph, there exists a positive integer  $t$  such that every element in at least one column of the matrix  $Q^t$  is positive, and  $\lim_{t \rightarrow \infty} Q^t = \mathbf{1}\pi^T$ , where  $\pi > 0$  and  $\mathbf{1}^T \pi = 1$ .

From the above description, we can get  $\lim_{t \rightarrow \infty} \phi_i(t) = \pi^T \phi(0)$  in the system in Equation (4). This means that all state variables will converge to a common value after  $t$  iterations, which depends on the initial value,  $\phi(0)$ , and the communication topology,  $Q$ .

### 3. Problem Formulation

The classic economic dispatch problem of power systems, including its analytic solution and a fully decentralized approach proposed in this paper, is discussed in this section.

#### 3.1. Analytic Solution to EDP

##### 3.1.1. Modeling

In this paper, we consider power systems with  $M$  buses. The power system contains  $N$  generating units, ( $N \leq M$ ), and the received electrical load is denoted by  $P_{Di}$ ,  $i = 1, 2, \dots, M$ . In other words, the number of generation nodes and bus nodes in the network associated with the power system are  $N$  and  $M$ , respectively. Let  $F_i(P_{Gi})$  denote the cost of power generated by generator  $i$ . The total cost of this system is denoted by  $F$ . The essential constraint is the supply and demand balance, that is to say, the total power generated by generating power must equal the total load demand. We can describe this system by,



Objective function:

$$\min F = \sum_{i=1}^N F_i(P_{Gi}) \quad (6)$$

Essential constraint:

$$\sum_{j=1}^M P_{Dj} - \sum_{i=1}^N P_{Gi} = 0 \quad (7)$$

$$i = 1, 2, \dots, N; j = 1, 2, \dots, M$$

### 3.1.2. Solution

This is an optimization problem with equal constraint. We can solve the problem with Lagrange function. That is,

$$\begin{aligned} L(P_G, \lambda) &= F + \lambda \Delta P \\ \Delta P &= \sum_{j=1}^M P_{Dj} - \sum_{i=1}^N P_{Gi} \end{aligned} \quad (8)$$

where  $\lambda$  is the Lagrange multiplier associated with the equality constraint in Equation (7). There are  $N + 1$  variables in this case,  $\lambda$  and  $P_{Gi}, i = 1, 2, \dots, N$ . To make the solutions of Equations (6) and (8) consensus,  $\Delta P$  must be equal to 0, and  $\min F = L(P_G, \lambda)$ .

$$\frac{\partial L}{\partial P_{Gi}} = \frac{dF_i(P_{Gi})}{dP_{Gi}} - \lambda = 0 \quad (9)$$

or, equivalently

$$\lambda = \frac{dF_i(P_{Gi})}{dP_{Gi}} \quad (10)$$

Therefore, the incremental cost rates  $\frac{dF_i(P_{Gi})}{dP_{Gi}} (i = 1, 2, \dots, N)$  are equal to  $\lambda$ , and the power cost reaches the minimum value.

Besides the equal constraint condition, there are  $2N$  inequality constraints associate with each generating unit. That is, the generated power of each generation  $P_{Gi}, i = 1, 2, \dots, N$  cannot exceed the corresponding minimum,  $P_{Gi}^{\min}$ , and maximum,  $P_{Gi}^{\max}$ , bounds. These necessary conditions, equations and inequalities, are shown in Equation (11).

$$\begin{aligned} \frac{dF_i(P_{Gi})}{dP_{Gi}} &= \lambda \rightarrow N - \text{equations} \\ P_{Gi}^{\min} &\leq P_{Gi} \leq P_{Gi}^{\max} \rightarrow 2N - \text{inequalities} \\ \sum_{i=1}^N P_{Gi} &= \sum_{j=1}^M P_{Dj} \rightarrow 1 - \text{constraint} \end{aligned} \quad (11)$$

Based on the above inference, we get features about optimal solution,



$$\begin{aligned}
\frac{dF_i(P_{Gi})}{dP_{Gi}} &= \lambda & P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \\
\frac{dF_i(P_{Gi})}{dP_{Gi}} &\leq \lambda & P_{Gi} = P_{Gi}^{max} \\
\frac{dF_i(P_{Gi})}{dP_{Gi}} &\geq \lambda & P_{Gi} = P_{Gi}^{min}
\end{aligned} \tag{12}$$

The generation cost function  $F_i(P_{Gi})$  is usually approximated [45] by a quadratic function

$$F_i(P_{Gi}) = a_i P_{Gi}^2 + b_i P_{Gi} + c_i = \frac{(P_{Gi} + \frac{b_i}{2a_i})^2}{\frac{1}{a_i}} + c_i - \frac{b_i^2}{4a_i} \tag{13}$$

Let  $\alpha_i = -(b_i)/(2a_i)$ ,  $\beta_i = 1/(2a_i)$ , and  $\gamma_i = c_i - (b_i^2)/(4a_i)$ ; the quadratic function can be now written as

$$F_i(P_{Gi}) = \frac{(P_{Gi} - \alpha_i)^2}{2\beta_i} + \gamma_i \tag{14}$$

where  $\alpha_i \leq 0$  and  $\beta_i > 0$ . Using the result of Equation (14), the incremental cost for the generator  $G_i$  is

$$\frac{dF_i(P_{Gi})}{dP_{Gi}} = \frac{P_{Gi} - \alpha_i}{\beta_i} \tag{15}$$

From Equations (12) and (15), we get:

$$\begin{aligned}
\frac{P_{Gi}^* - \alpha_i}{\beta_i} &= \lambda^* & P_{Gi}^{min} \leq P_{Gi}^* \leq P_{Gi}^{max} \\
\frac{P_{Gi}^* - \alpha_i}{\beta_i} &\leq \lambda^* & P_{Gi}^* = P_{Gi}^{max} \\
\frac{P_{Gi}^* - \alpha_i}{\beta_i} &\geq \lambda^* & P_{Gi}^* = P_{Gi}^{min}
\end{aligned} \tag{16}$$

where  $\lambda^*$  is the optimal incremental cost. From Equation (16), considering the constraints of generated power of each generating units,  $\lambda_i \in [\lambda_i, \bar{\lambda}_i]$ , where  $\lambda_i = (P_{Gi}^{min} - \alpha_i)/(\beta_i)$ , and  $\bar{\lambda}_i = (P_{Gi}^{max} - \alpha_i)/(\beta_i)$ . Hence, the optimal power generation for each individual generator can be given by

$$\begin{aligned}
P_{Gi}^* &= \lambda^* \beta_i + \alpha_i & \text{for } \lambda_i \leq \lambda^* \leq \bar{\lambda}_i \\
P_{Gi}^* &= P_{Gi}^{max} & \text{for } \lambda^* > \bar{\lambda}_i \\
P_{Gi}^* &= P_{Gi}^{min} & \text{for } \lambda^* < \lambda_i
\end{aligned} \tag{17}$$

The solution shows  $\gamma_i$  does not affect the incremental cost. According to Equation (17), when the value of parameter,  $\lambda^*$ , is determined, the values of  $P_{Gi}^*$  will be fixed. Most of conventional approaches to gain the values of  $\lambda^*$  are centralized methods, such a Lambda-iteration [45] and gradient methods, where the total demand or total losses as global information must be known to the centralized computing center.

### 3.2. Fully Distributed Solution to EDP

The proposed approach for solving EDP is composed of two stages, data collection based on FBC algorithm and optimal solution based on  $\lambda$ -consensus algorithm.

### 3.2.1. Data Collection

In the stage of data collection, agents share information about their neighbors who connect to them in the power system, and parameters including load demand and power outputs that can be generated by generators in their buses. Agents share information and reach consensus about the power network based on FBC algorithm mentioned in [46]. In the FBC algorithm, a token is defined to permit agent to send its message to its neighbors in communication network. The token plays the role of transmission permits. Agent with a token can send message to a selected neighbor, and it is necessary to define the communication topology before agent communicating with each other. It is assumed that there is an agent in each bus of the power system; these agent are embedded in the power system and responsible for message sharing and EDP calculating. We choose an undirected ring graph as the communication topology in the data collecting stage. Furthermore, we assume each agent is marked with a unique identifier (ID). Let us suppose that the agent's ID is known for its neighbors in the ring topology graph. Thus, a message from node  $i$  or agent  $i$  can be represented by a multi-tuple  $msg_i = \{id_i, ID_i^{neib}, param_i\}$ , where  $id_i$  is the ID of agent  $i$  (for simplicity, we set the agent's ID equals to its bus number in this paper),  $ID_i^{neib}$  is set of IDs to which agent  $i$  is connected,  $param_i$  contains some parameters about bus  $i$ , and  $param_i = \langle P_{Di}, P_{Gi}^{min}, P_{Gi}^{max} \rangle$ . If bus  $i$  does not include power generator,  $P_{Gi}^{min} = P_{Gi}^{max} = 0$ , if there is no load demand in bus  $i$ ,  $P_{Di} = 0$ . Next, we illustrate the process of agent gathering information through an example.

Take the four-bus network, as shown in Figure 2, as an example. An undirected ring communication graph as shown in Figure 3 is assumed to connected each agents. We choose the undirected ring graph as the communication topology because it is easy to observe the stopping condition in the ring communication topology. With each iteration, each agent receives two messages from its two neighbors; meanwhile, each agent sends message to its two neighbors in per iteration. Based on the FBC algorithm, the total number of iterations required for a agent to gather global information can be computed as follows:

$$It = \begin{cases} M/2 & \text{if } M \text{ is even} \\ (M-1)/2 & \text{if } M \text{ is odd} \end{cases} \quad (18)$$

where  $M$  is the total number of buses in the system. After  $M$  iterations, agents' information about the power system reach consensus. As a fully decentralized algorithm, the global information about parameter  $M$  is an unknown variable, so we provide another iteration stopping criterion for information sharing among agents.

As shown in Figure 3, before the first iteration ( $It = 0$ ), each agent prepares its message,  $msg_i^0 = \{id_i, ID_i^{neib}, param_i\}$ , ( $i = 1, 2, 3, 4$ ). When agent in the network receives a message, it will get the contents of the message and updates its message,  $msg_i^{It} = msg_i^{It-1} \cup msg_{(neibs \text{ of } i)}^{It-1}$ , where  $msg_{(neibs \text{ of } i)}^{It-1}$  denotes message from the neighbors of agent  $i$ . After each iteration, each agent will check its message,  $msg_i^{It}$ , if the set  $\{id_{i-first}\} = \{ID_{i-second}^{neib}\}$ , it will stop the iteration, where  $\{id_{i-first}\}$  and  $\{ID_{i-second}^{neib}\}$  are sets of first and second terms, respectively, in  $msg_i^{It}$ . This means that each agent reaches consensus about the power system after  $M$  iterations.

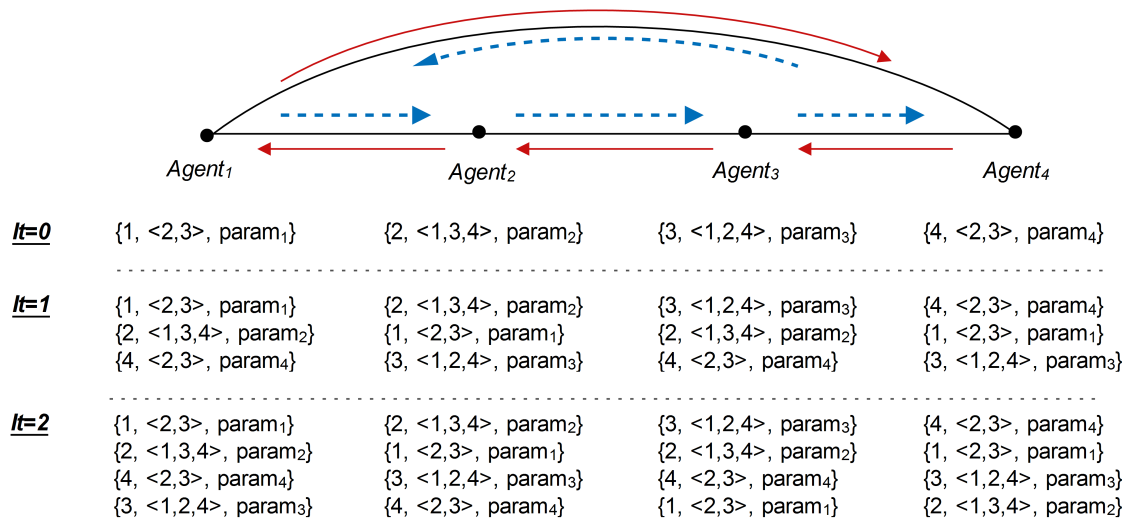


Figure 3. The process of collecting data for a four-bus network.

The pseudo code of algorithm for data collection based on FBC is given below (Algorithm 1):

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**Algorithm 1** Data Collection Algorithm Based on FBC

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**Input:**  $T_1$ : time threshold for receipt a new message;

$msg_i^0$ : prepare a message that contains the agent data;

**Output:**  $msg_i^{It}$ : stopping condition  $\{id\} = \{ID_i^{neib}\}$

1: initial  $It = 1$ , clock time  $t = 0$ ;

2: **repeat**

3: if new message receive from neighbors, get the content of the message and upload  $msg_i^{It}$ , check the new message whether meet stopping condition, if not  $It = It + 1$ , and set  $t = 0$ ;

4: check  $t$ , if  $t \geq T_1$

5: send the message  $msg_i$  to the neighbor agents  $\{ID_i^{neib}\}$ ;

6: **until**  $msg_i^{It}$  meets stopping condition

---

### 3.2.2. $\lambda$ -Consensus Algorithm

After finishing the data sharing among the agents, one agent is chosen randomly to get matrix  $Q$  according to its message. From Equation (1), we observe each row in  $Q$  is only related to the number of neighbors' nodes. After sharing information, each agent reaches consensus on its information, so we select an agent randomly and build matrix  $Q$  based on the information  $\{ID_i^{neib}\}$  and  $\{id_i\}$ .

Again taking the four-bus network as an example, if the chosen agent is  $agent_3$ , based on its message, we can get:

$$\left\{ \begin{array}{l} \{3, <1,2,4>, param_3\} \\ \{2, <1,3,4>, param_2\} \\ \{4, <2,3>, param_4\} \\ \{1, <2,3>, param_1\} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} Q = \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/3 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 1/3 \end{bmatrix} \\ Param = \begin{bmatrix} param_3 \\ param_2 \\ param_4 \\ param_1 \end{bmatrix} = \begin{bmatrix} P_{D3} & p_{G3}^{min} & p_{G3}^{max} \\ P_{D2} & p_{G2}^{min} & p_{G2}^{max} \\ P_{D4} & p_{G4}^{min} & p_{G4}^{max} \\ P_{D1} & p_{G1}^{min} & p_{G1}^{max} \end{bmatrix} \end{array} \right.$$

Note that choosing a different agent corresponds to different matrix  $Q$  and vector  $Param$ , however, due to the one-to-one match between  $Q$  and  $Param$ , the final result will not be affected.

If without power generation constraints on each generator, when all generators operate at the optimal configuration, incremental costs are equal to the optimal value, that is

$$\lambda^* = (x_i^* - \alpha_i) / \beta_i, \quad i = 1, 2, \dots, M \quad (19)$$

where  $x_i^*$  is the optimal power generated by generator  $i$ , and if the optimal incremental cost  $\lambda^*$  is known, i.e.,

$$x_i^* = \beta_i \lambda^* + \alpha_i, \quad i = 1, 2, \dots, M \quad (20)$$

Inspired by Equations (19) and (20), we propose the following algorithm:  $\lambda$ -consensus algorithm. Let  $\lambda_i$ ,  $x_i(t)$ , and  $y_i(t)$  denote the estimation of incremental cost by generating unit  $i$ , the corresponding power generation which is an estimation of optimal power generation, and the local estimation of the mismatch between demand and total power generation, respectively.

The rationale of the proposed approach is to reach consensus on the Lagrangian multiplier by running the following protocol

$$\begin{aligned} \lambda_i(t+1) &= q_{ii}\lambda_i(t) + \sum_{j \in N_i^{neib}} q_{ij}\lambda_j(t) + \zeta y_i(t) \\ x_i(t+1) &= \beta_i \lambda_i(t+1) + \alpha_i \\ y_i(t+1) &= \omega_{ii}y_i(t) + \sum_{j \in N_i^{neib}} \omega_{ij}y_j(t) - (x_i(t+1) - x_i(t)) \end{aligned} \quad (21)$$

where  $\zeta$  is a sufficiently small positive constant,  $\mathbf{W} = \{\omega_{ij}\} = \mathbf{Q}^T$ . The initial value of  $\lambda_i(0)$ ,  $x_i(0)$  and  $y_i(0)$  can be set to any admissible value in the case without power generation constraints.

To take account of power generation constraints, define the following projection operators:

$$\psi(\lambda_i) = \begin{cases} \bar{x}_i & \text{if } \lambda_i > \bar{\lambda}_i \\ \beta_i \lambda_i + \alpha_i & \text{if } \underline{\lambda}_i \leq \lambda_i \leq \bar{\lambda}_i \\ \underline{x}_i & \text{if } \lambda_i < \underline{\lambda}_i \end{cases} \quad i = 1, 2, \dots, M$$

where  $\underline{x}_i$  and  $\bar{x}_i$  denote the corresponding minimum and maximum bounds for bus  $i$ , if the bus  $i$  contains load only, then  $\underline{x}_i = \bar{x}_i = 0$ .  $\underline{\lambda}_i = (\underline{x}_i - \alpha_i) / \beta_i$  and  $\bar{\lambda}_i = (\bar{x}_i - \alpha_i) / \beta_i$ . Now, the distributed algorithm becomes

$$\begin{aligned} \lambda(t+1) &= \mathbf{Q}\lambda(t) + \zeta \mathbf{y}(t) \\ \mathbf{x}(t+1) &= \psi(\lambda(t+1)) \\ \mathbf{y}(t+1) &= \mathbf{W}\mathbf{y}(t) - (\mathbf{x}(t+1) - \mathbf{x}(t)) \end{aligned} \quad (22)$$

where  $\mathbf{x}(t+1) = [x_1(t+1), x_2(t+1), \dots, x_M(t+1)]^T$ ,  $\mathbf{y}(t+1) = [y_1(t+1), y_2(t+1), \dots, y_M(t+1)]^T$ , and  $\psi(\lambda(t+1)) = [\psi(\lambda_1(t+1)), \psi(\lambda_2(t+1)), \dots, \psi(\lambda_M(t+1))]^T = \mathbf{B}\lambda(t+1) + \boldsymbol{\alpha}$ , and  $\mathbf{B}$  is a diagonal matrix with  $B_{ii} = \beta_i$ ,  $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_M]^T$ . The initial value for the iteration in Equation (22) can be set as follows:

$$\begin{aligned} x_i(0) &= \begin{cases} \bar{x}_i & \text{if } \bar{x}_i < P_{Di} \\ P_{Di} & \text{if } \underline{x}_i \leq P_{Di} \leq \bar{x}_i \\ \underline{x}_i & \text{if } P_{Di} < \underline{x}_i \end{cases} \\ \lambda_i(0) &= \frac{x_i(0) - \alpha_i}{\beta_i} \\ y_i(0) &= P_{Di} - x_i(0) \end{aligned} \quad (23)$$

It is easy to calculate that  $\sum_{i=1}^M x_i(0) + \sum_{i=1}^M y_i(0) = P_D$ , where  $P_D = \sum_{i=1}^M P_{Di}$  is the global power demand.

Figure 4 shows the flowchart of the proposed algorithm in this paper.

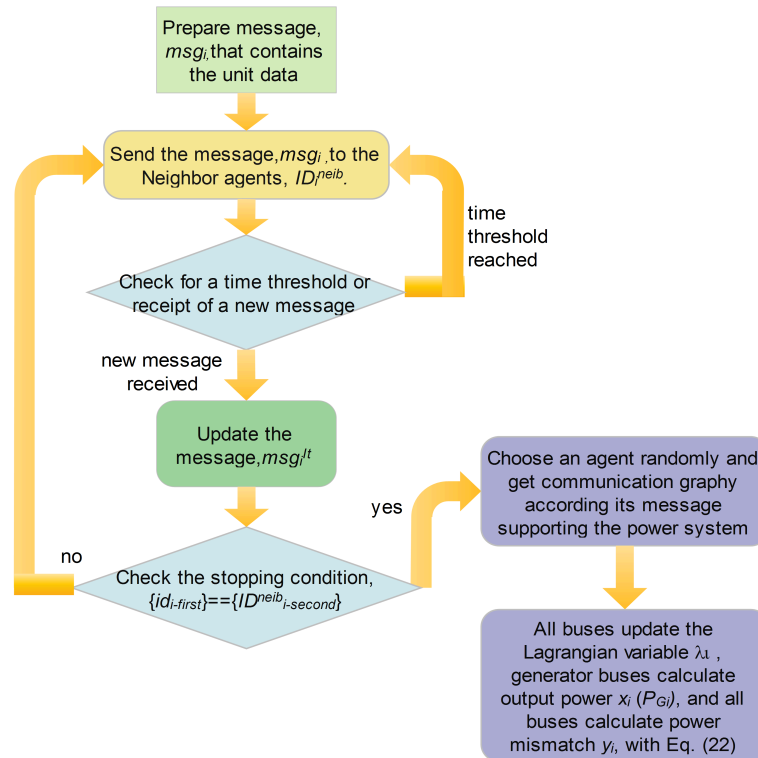


Figure 4. Flowchart of the proposed algorithm.

### 3.2.3. Algorithm Analysis

The convergence theorem of  $\lambda$ -consensus algorithm and its proof are as follows.

**Theorem 3.** If  $\zeta$  is positive and sufficiently small, then the algorithm in Equation (22) converges to a stable value, that is,  $\lim_{t \rightarrow \infty} \lambda_i(t) = \lambda^*$ ,  $\lim_{t \rightarrow \infty} y_i(t) = 0$ .

**Proof.** According to  $x(t+1) = B\lambda(t+1) + \alpha$  and  $\lambda(t+1) = Q\lambda(t) + \zeta y(t)$ , we can get,

$$y(t+1) = Wy(t) - (B\lambda(t+1) + \alpha - B\lambda(t) - \alpha) = B(I - Q)\lambda(t) + (W - \zeta B)y(t)$$

Algorithm in Equation (22) can be rewritten as,

$$\begin{bmatrix} \lambda(t+1) \\ y(t+1) \end{bmatrix} = \begin{bmatrix} Q & \zeta I \\ B(I - Q) & W - \zeta B \end{bmatrix} \begin{bmatrix} \lambda(t) \\ y(t) \end{bmatrix} \quad (24)$$

For simplification, let  $A = \begin{bmatrix} Q & 0 \\ B(I - Q) & W \end{bmatrix}$ ,  $p(k) = \begin{bmatrix} \lambda(k) \\ y(k) \end{bmatrix}$ ,  $\Delta = \begin{bmatrix} 0 & I \\ 0 & -B \end{bmatrix}$ , and gives the following notation,  $u(k) = \zeta \Delta p(k)$ . Above all, we have

$$p(k+1) = Ap(k) + u(k) \quad (25)$$

Since  $\zeta$  is a sufficient small number, Equation (25) can be seem as a system perturbed by  $u(k)$ . As  $A$  has a simple eigenvalue 1 and the remaining eigenvalues lie in the open unit disk, construct vectors  $H = [h_1, h_2] = \begin{bmatrix} 0 & \mathbf{1} \\ h & -\mu h \end{bmatrix}$ , where  $h_1$  and  $h_2$  are two independent right eigenvectors of  $A$ ,  $\mu = \sum_{i=1}^M \beta_i$ .

Construct vectors  $U^T = \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix} = \begin{bmatrix} \mathbf{1}^T B & \mathbf{1}^T \\ \mathbf{v}^T & \mathbf{0} \end{bmatrix}$ , where  $u_1$  and  $u_2$  are two independent left eigenvectors of  $A$ , and  $U^T \Delta H = \begin{bmatrix} 0 & 0 \\ \mathbf{v}^T & -\mu \mathbf{v}^T \mathbf{h} \end{bmatrix}$

The eigenvalues of  $U^T \Delta H$  are 0 and  $-\mu \mathbf{v}^T \mathbf{h} < 0$ . Thus,  $\lim_{k \rightarrow \infty} u(k) = 0$ , that is to say, the system in Equation (22) is stable.

From the definition of  $Q$  and  $W$ , it can be verified  $\begin{bmatrix} \lambda(t) \\ \mathbf{y}(t) \end{bmatrix}$  converges to  $\text{span } [\mathbf{0}, \mathbf{1}]^T$ , when  $t \rightarrow \infty$ . That is  $\lim_{t \rightarrow \infty} \lambda_i(t) = \lambda^*$ ,  $\lim_{t \rightarrow \infty} y_i(t) = 0$ . Hence, the theorem is proved.  $\square$

#### 4. Simulation Results

In this section, we analyze four cases to show the performance of the proposed algorithm. Firstly, we consider the cases with and without power generator constraints. Secondly, the plug and play properties are demonstrated through adding generator and load shedding. Lastly, the IEEE 57-bus is used to show the effectiveness of the proposed  $\lambda$ -consensus algorithm on large cases.

In our first three cases, we set the generator parameters the same as in the literature [45]. There are three types of generators, namely, Type A, Type B and Type C, they correspond to coal-fired steam unit, oil-fired steam unit and oil-fired\* steam unit, respectively. Besides the types and power ranges of generator, the cost function parameters  $a$ ,  $b$  and  $c$  are given in Table 1. Furthermore, the parameters,  $a - b - c$ , are converted to  $\alpha - \beta$ , where  $\alpha = -b/(2a)$ , and  $\beta = 1/(2a)$ .

Table 1. Generator parameters.

Generator Type	A	B	C
Range (MW)	[150, 600]	[100, 400]	[50, 200]
$a$ (\$/MW <sup>2</sup> h)	0.00142	0.00194	0.00482
$b$ (\$/MWh)	7.2	7.85	7.97
$c$ (\$/h)	510	310	78
$\alpha$ (MW)	−2535.2	−2023.2	−826.8
$\beta$ (MW <sup>2</sup> h)	352.1	257.7	103.7

The power system [45], which is shown in Figure 5a, contains six buses, denoted by thick line; eleven single lines; and three generators, denoted by circle. Disregarding transmission loss of line, the generators in this SG are labeled as A, B and C. They correspond to the generator types A, B and C in Table 1.

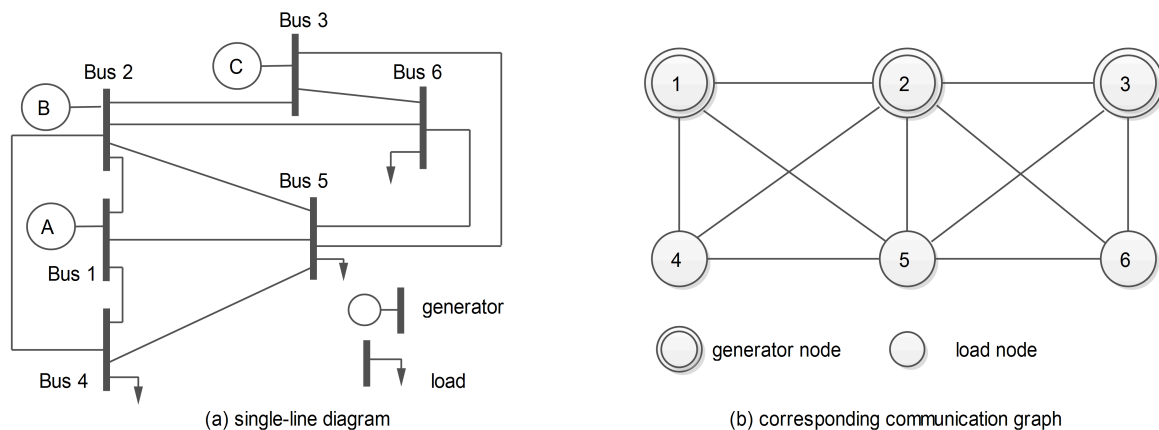


Figure 5. Six-bus system.

Create six agents using Java agent development framework (JADE) software. They correspond to the six bus, respectively, and each agent prepares its message,  $msg_i = \{id_i, ID_i^{neib}, param_i\}$ . In addition, construct ring topology based on the bus number to collect information from different bus. Figure 6 shows a screen shot of JADE GUI for the six-bus system. In the ring topology, we consider  $1 - 2 - 3 - 4 - 5 - 6 - 1$  as standard link order for nodes. That is, agent  $i$  in JADE, adds message receiver: agent  $id_{(i-)}$  and  $id_{(i+)}$ , where  $id_{(i-)}$  and  $id_{(i+)}$  are adjacent agents in ring topology. Meanwhile, agent  $i$  selects messages sent by  $id_{(i-)}$  and  $id_{(i+)}$  from message queue. Each agent in the platform is given the ability to detect the existence of its neighbor agents. When finishing data collection, we can build matrix  $Q$ , which must associate with the communication graph as shown in Figure 5b.

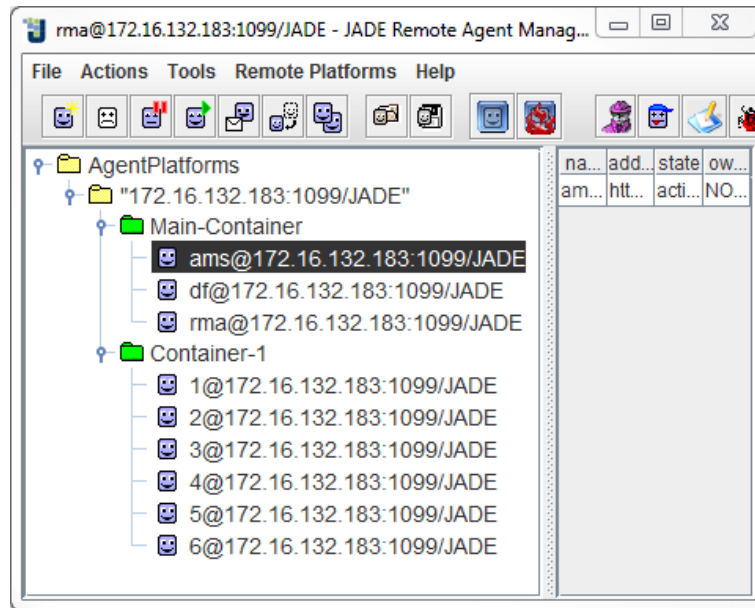


Figure 6. JADE GUI for six-bus system.

#### 4.1. Case Study 1: Without Generator Constraints

In our first case study, the power generator constraints are not considered. We set the initial load demands as 180 MW, 350 MW and 500 MW in bus 4, bus 5 and 6, respectively. The generators output  $x_i$ , estimated mismatch  $y_i$ , incremental cost  $\lambda_i$  and total power generated by generating units are displayed in Figure 7. The power outputs of each generator become stable gradually, and the mismatch between demand and total power generated,  $y_i$ , goes to zero after 18 iterations. Moreover, the estimated incremental costs  $\lambda_i$  of all generators trends toward a stable value. The results are  $\lambda^* = 8.99$  \$/MWh,  $x_1^* = 630.6$  MW,  $x_2^* = 293.8$  MW, and  $x_3^* = 105.6$  MW. From left top chart, we observe the first generator is beyond the limit on its maximum output power  $P_2^{max} = 600$  MW. In Case Study 2, we add the power generator constraints.



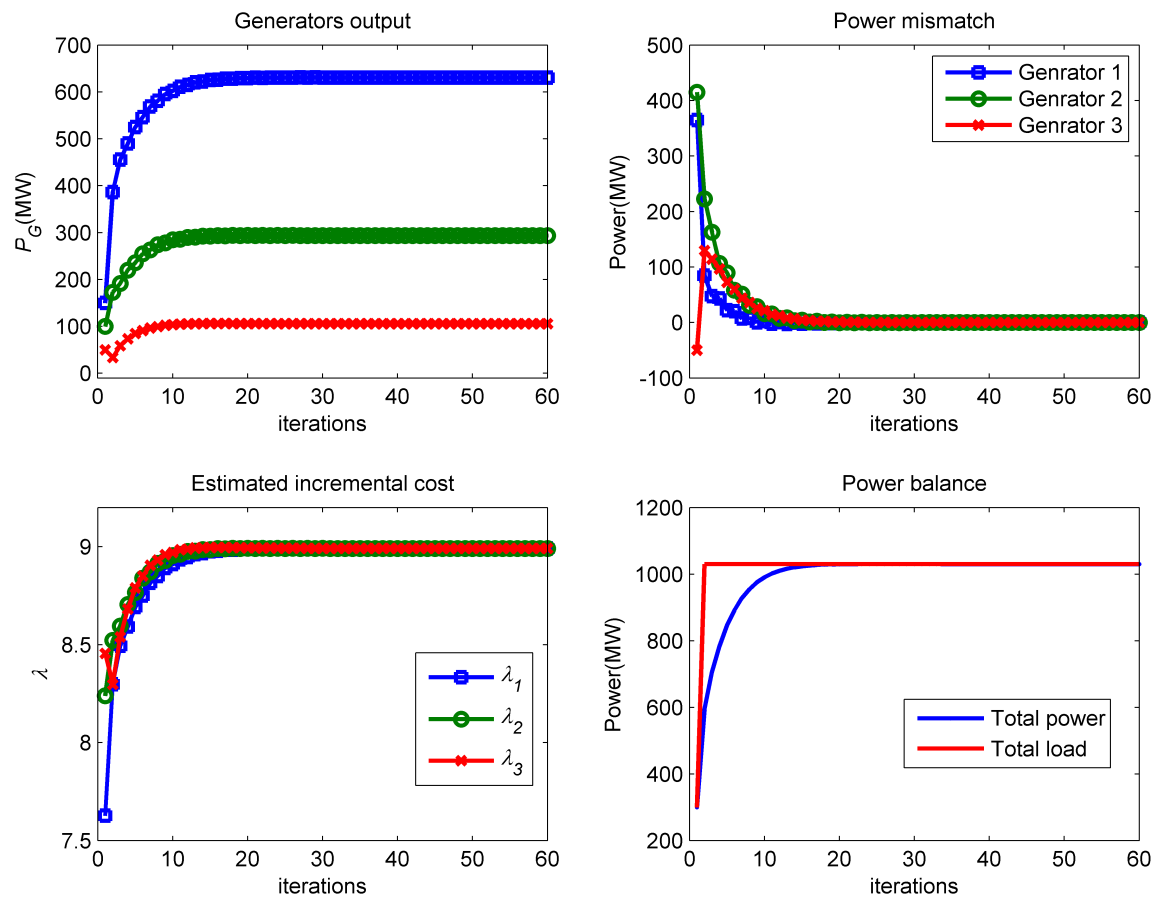


Figure 7. Case Study 1, without power generator constraints.

#### 4.2. Case Study 2: With Generator Constraints

In this case study, we add the power generator constraints on the basis of Case Study 1. The behavior of the proposed algorithm is shown in Figure 8; after 40 iterations, the system tends to be stable, and  $x_1^* = 600$  MW,  $x_2^* = 315.5$  MW, and  $x_3^* = 114.5$  MW, with  $\lambda^* = 9.05$  \$/MWh. In this case, no generator exceeds its maximum output power.

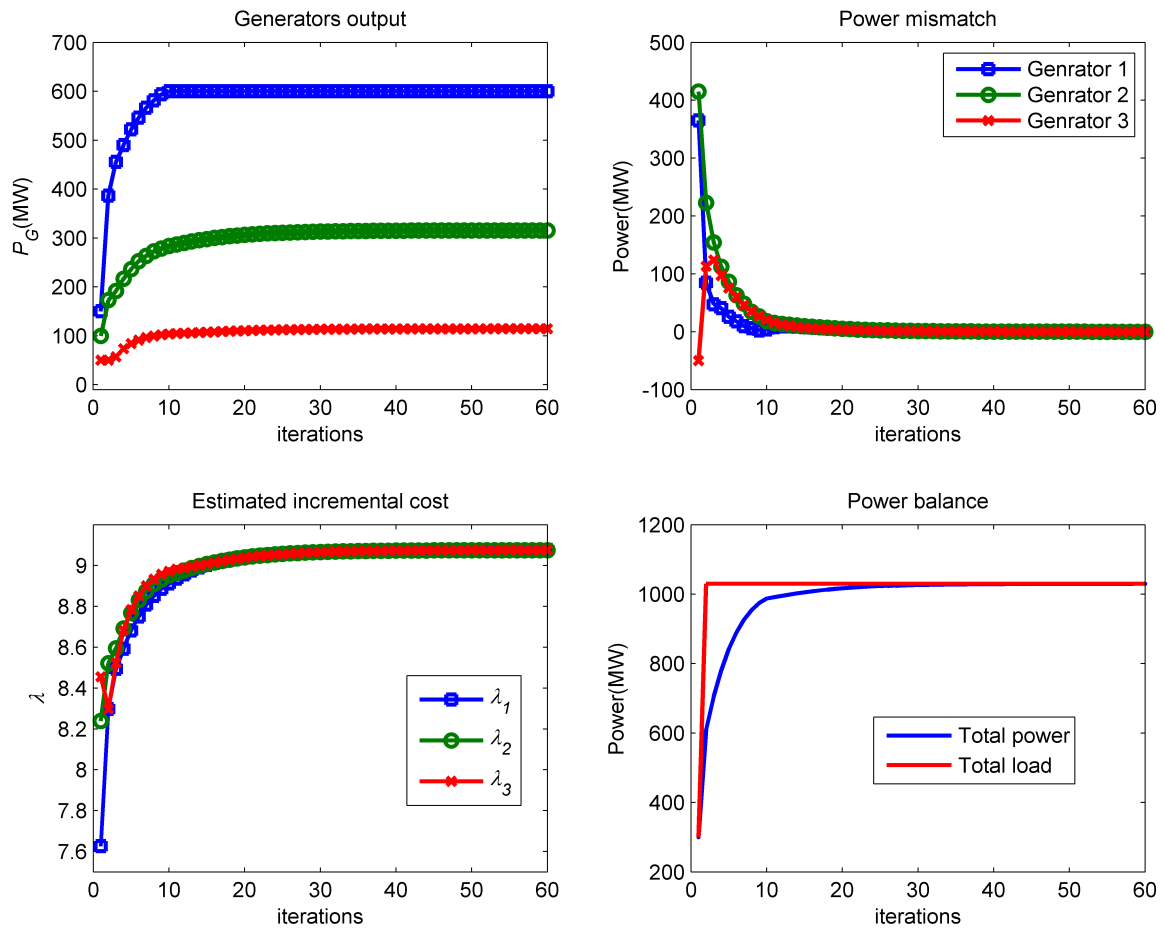


Figure 8. Case Study 2, with power generator constraints.

#### 4.3. Case Study 3: Plug and Play Capability

Since the plug and play characteristics of SG simplify the process of accessing to grid for new energy power generation, the plug and play capability is an important features of SG. In this case study, the plug and play adaptability of the proposed  $\lambda$ -consensus algorithm is discussed considering the same power system as in Case Study 2. At time step  $k = 60$ , the load demand at bus 6 drops from 500 MW to 300 MW. The fourth generator is connected in bus 4 at time step  $k = 120$ . We set the type of fourth generator as C. Figure 9 gives the result of this case. At the beginning, the three generators have already stabilized at the steady-state values before load shedding from bus 6. When load shedding at time step  $k = 60$ , total power mismatch increases from 0 MW to  $-200$  MW, and the system reaches a new balance of power generated and demand after a few iterations by using the proposed algorithm. At this point,  $\lambda' = 8.7108$  \$/MWh, and  $x'_1 = 531.9$  MW,  $x'_2 = 221.6$  MW, and  $x'_3 = 76.5$  MW. At time step  $k = 120$ , the buses detect the connection of a new generator in node 4, change the bus parameters  $P_4^{min} = 50$  MW and  $P_4^{max} = 200$  MW. Under the new conditions, the load demand is redistributed among these four generators, and converge to a new solution with  $\lambda^* = 8.6172$  \$/MWh, and  $x_1^* = 498.9$  MW,  $x_2^* = 197.5$  MW, and  $x_3^* = x_4^* = 66.8$  MW. The total power generated by generators is 830 MW.

The simulation results show that the total power mismatch, generator outputs, estimated incremental cost can reach new steady-state values after a few iterations under the conditions of changing generators and load demand. This suggests the proposed algorithm can adapt to the dynamic environment of SG, which demonstrates the effectiveness of the proposed  $\lambda$ -consensus algorithm.

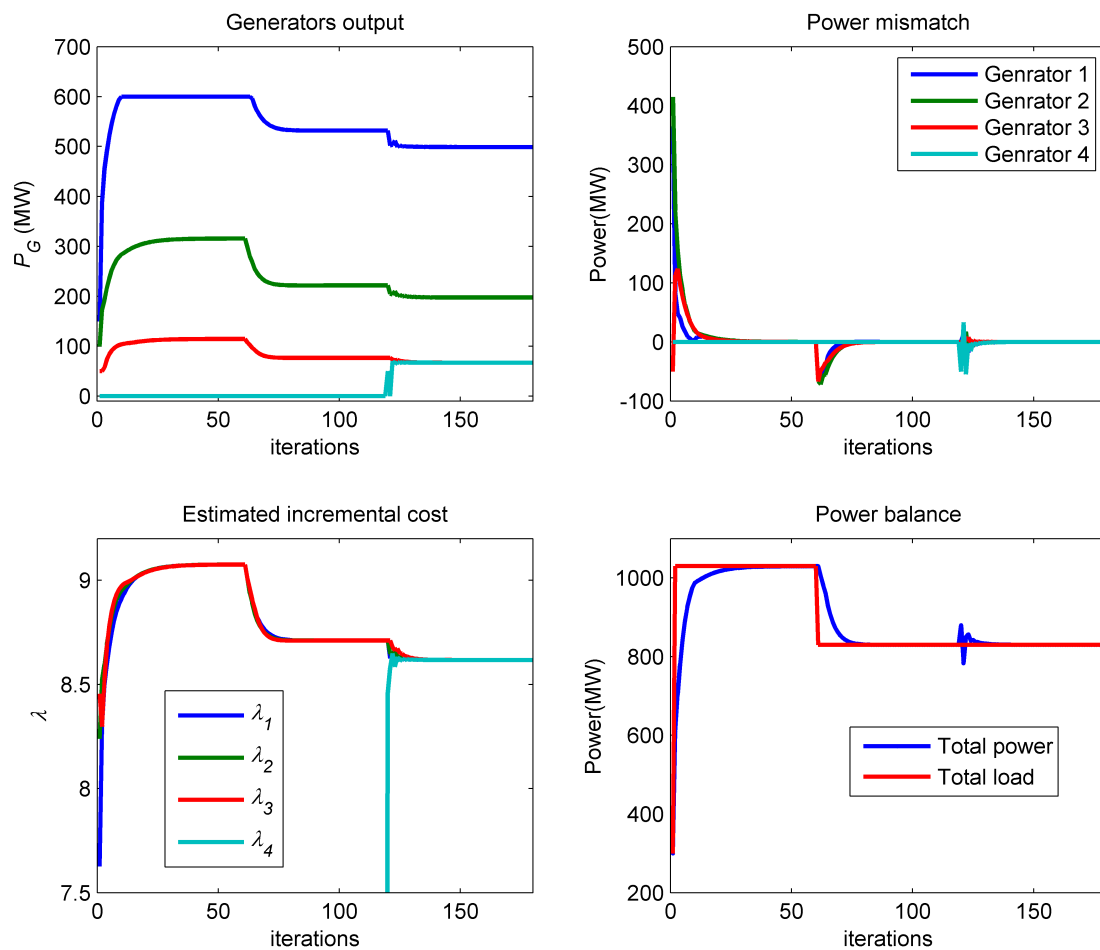


Figure 9. Case Study 3, plug and play capability.

#### 4.4. Case Study 4: Implementation on IEEE 57-Bus System

This power system of IEEE 57-bus contains seven generator buses. The generator parameters are shown in Table 2.

Table 2. IEEE 57-bus test system Generator parameters.

Bus	$a$ (\$/MW <sup>2</sup> h)	$b$ (\$/MWh)	$P^{min}$ (MW)	$P^{max}$ (MW)
1	0.078	20	0	575.88
2	0.01	40	0	100
3	0.25	20	0	140
6	0.01	40	0	100
8	0.022	20	0	550
9	0.01	40	0	100
12	0.003	20	0	410

The initial power demands at each buses are denoted as  $\{P_{Di}, i = 1, 2, \dots, 57\} = \{55, 3, 41, 0, 13, 75, 0, 150, 121, 5, 0, 377, 18, 10.5, 22, 43, 42, 27.2, 3.3, 2.3, 0, 0, 6.3, 0, 6.3, 9.3, 4.6, 17, 3.6, 5.8, 1.6, 3.8, 0, 6, 0, 0, 14, 0, 0, 6.3, 7.1, 2, 12, 0, 0, 29.7, 0, 18, 21, 18, 4.9, 20, 4.1, 6.8, 7.6, 6.7\}$ , and total demand is 1250.8 MW. We consider the case study with generator constraints. The numerical results are shown in Figure 10. The generators outputs are fulfilling the constraints on their maximum output power. After 760 iterations, the system reaches the balance of power generated and load demand, and the optimal incremental

cost is  $\lambda^* = 22.4461$  \$/MWh, and generator outputs are  $x_2^* = x_6^* = x_9^* = 100$  MW,  $x_1^* = 131.6$  MW,  $x_3^* = 40.8$  MW,  $x_8^* = 460.4$  MW, and  $x_{12}^* = 317.6$  MW.

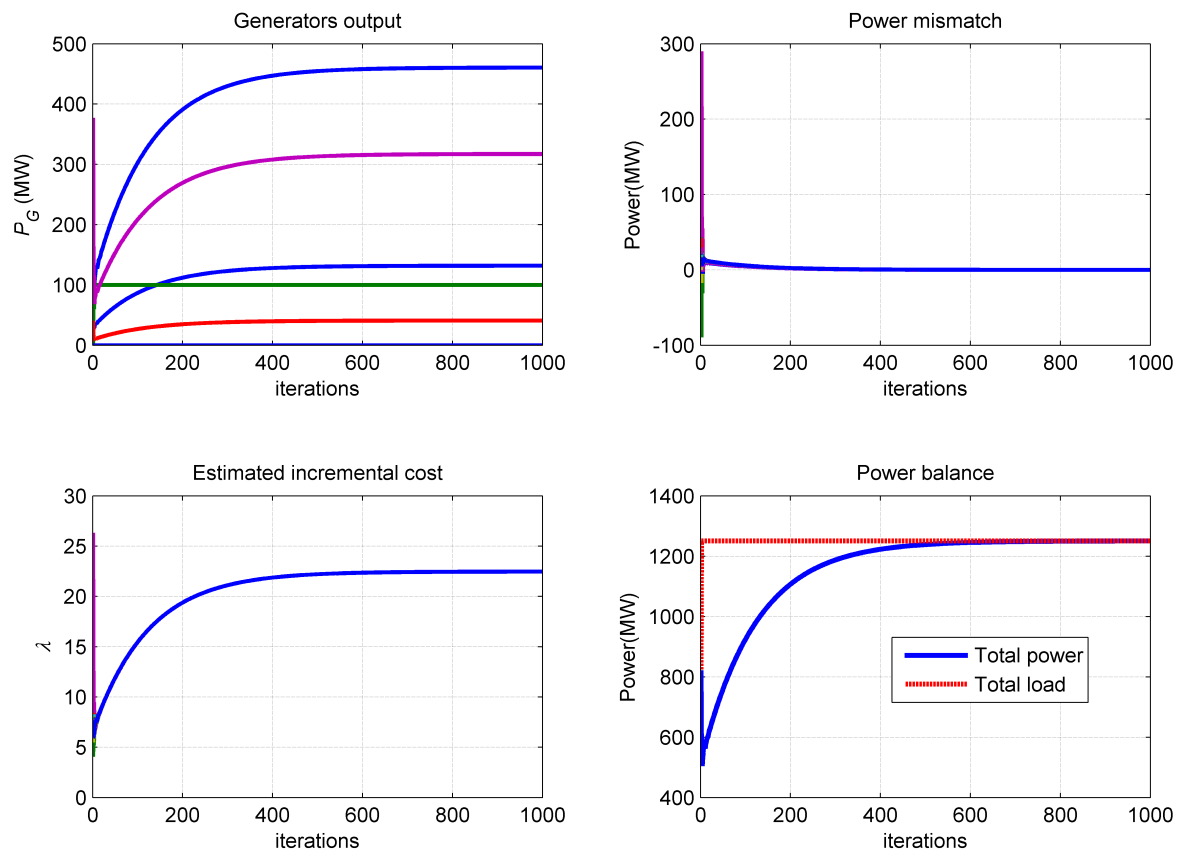


Figure 10. Case Study 4, IEEE 57-bus test system.

## 5. Conclusions

This paper presents a fully distributed approach for EDP of SG. The proposed approach contains two stages, collecting data from neighbors using FBC algorithm, and getting optimal power outputs for each generator using  $\lambda$ -consensus algorithm. In the first stage, a ring communication graph is considered; agents can get global information by communicating many times with its neighbors; and this data collecting mechanism provides a stopping condition for data gathering. In the second stage, the computing agent creates the communication matrix according to the information from its number, and adjusts the estimated cost parameter  $\lambda$  based on the mismatch power. Numerical simulation results show the effectiveness of the proposed algorithm. Our future work will concentrate on two aspects: extending the proposed algorithm to address EDP with transmission loss, and considering energy management problem of SG with renewable resources and energy storage units.

**Author Contributions:** B.L., J.L. and S.C. proposed and validated the main idea. B.L. and Y.W. developed the simulation and wrote the remaining manuscript. All authors together organized and refined the manuscript in the present form.

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## Abbreviations

The following abbreviations are used in this manuscript:

SG	Smart Grid
EDP	Economic Dispatch Problem
WAMSs	Wide Area Measurement Systems
SANETs	Sensor and Actuator Networks
WAN	Wide Area Network
NAN	Neighborhood Area Network
HAN	Home Area Network
MAS	Multi-Agent System
DCOPF	DC Optimal Power Flow
ID	Identifier
JADE	Java Agent DEvelopment Framework
FBC	Flooding Based Consensus
$N_i^{in}$	In-neighbor set for node $i$
$N_i^{out}$	Out-neighbor set for node $i$
$d_i^{in}$	In-degree of node $i$
$d_i^{out}$	Out-degree of node $i$
$ \cdot $	The cardinality of a set
$P_{Gi}$	Power generated by generator $i$
$P_{Dj}$	Load demand in bus $j$
$P_D$	Total load demand
$a_i, b_i, c_i$	Parameters about the cost of generator $i$
$p_{Gi}^{max}$	Maximum power generated by generator $i$
$p_{Gi}^{min}$	Minimum power generated by generator $i$
$msg_i^{It}$	The holding message of agent $i$ after $It$ iterations
$id_i$	unique identifier of agent $i$
$ID_i^{neib}$	Identifiers set about the neighbors of agent $i$
$param_i$	Parameters of agent $i$ or bus $i$

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