Supplementary Information for: The Value of Multi-Stage Stochastic Programming in Power Grid Capacity Expansion Planning - ESCAPE36

Formulation for the deterministic multi-stage power grid capacity expansion planning program

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1 Formulation

Objective Function

$$
\begin{aligned}
\text{StageCost}_k = \Bigg[&\sum_{n \in N_{th}} \sum_{b \in B} C_{n,k}^{inv} P_n^{\max} y_{n,b,k} + \sum_{r \in R} \sum_{b \in B} C_{r,k}^{inv} y_{r,b,k} + \sum_{j \in J} C_j^{inv} y_{j,k} \\
&+ \sum_{s \in S} \sum_{b \in B} C_{s,k}^{inv} y_{s,b,k} + \sum_{n \in N_{th}} \sum_{b \in B} C_n^{fom} P_n^{\max} x_{n,b,k} + \sum_{r \in R} \sum_{b \in B} C_r^{fom} x_{r,b,k} \\
&+ \sum_{s \in S} \sum_{b \in B} C_{s,k}^{fom} x_{s,b,k} + \sum_{\omega \in \Omega} \pi_\omega \sum_{g \in G} \sum_{b \in B} \sum_{t \in T} (C_{g,k}^{fuel} + C_g^{vom}) \, p_{g,b,t,k,\omega} \\
&+ \sum_{\omega \in \Omega} \pi_\omega \sum_{s \in S} \sum_{b \in B} \sum_{t \in T} (C_{s,k}^{vom} + C_s^-) \, p_{s,b,t,k,\omega}^- + \sum_{\omega \in \Omega} \pi_\omega \sum_{b \in B} \sum_{t \in T} C^{ls} \, p_{b,t,k,\omega}^{ue} \\
&+ \sum_{\omega \in \Omega} \pi_\omega C^{nc} p_{k,\omega}^{nc} \Bigg], \quad \forall \, k \in K
\end{aligned}
$$

$$
\min \quad Z = \sum_{k \in K} \frac{\text{StageCost}_k}{(1+d)^{\text{year}_k}} \tag{1s}
$$

Linking Constraints

$$
\begin{aligned}
x_{n,b,k} &= x_{n,b,k-1} + y_{n,b,k} & &\forall \, n \in \mathcal{N}_{th}, \, b \in B, \, k = 2, \ldots, K & &(2s) \\
x_{r,b,k} &= x_{r,b,k-1} + y_{r,b,k} & &\forall \, r \in \mathcal{R}, \, b \in B, \, k = 2, \ldots, K & &(3s) \\
x_{s,b,k} &= x_{s,b,k-1} + y_{s,b,k} & &\forall \, s \in \mathcal{S}, \, b \in B, \, k = 2, \ldots, K & &(4s) \\
x_{j,k} &= x_{j,k-1} + y_{j,k} & &\forall \, j \in \mathcal{J}, \, k = 2, \ldots, K & &(5s)
\end{aligned}
$$

Dispatch Limits

$$p_{r,b,t,k,\omega} \leq x_{r,b,k-1}\,P^{\max}_{r,b,t,\omega} \qquad\qquad \forall\, r \in R,\ b \in B,\ t \in T,\ k = 2,\ldots,K,\ \omega \in \Omega \tag{6s}$$

$$p_{n,b,t,k,\omega} \leq x_{n,b,k-1}\,P^{\max}_{n} \qquad\qquad \forall\, n \in N_{th},\ b \in B,\ t \in T,\ k = 2,\ldots,K,\ \omega \in \Omega \tag{7s}$$

$$p_{e,b,t,k,\omega} \leq P^{\max}_{e} \qquad\qquad \forall\, e \in E,\ b \in B,\ t \in T,\ k \in K,\ \omega \in \Omega \tag{8s}$$

Flow Limits for all lines

$$-F^{\max}_{l} \leq f_{l,t,k,\omega} \leq F^{\max}_{l} \qquad\qquad \forall\, l \in \mathcal{L}_{E},\ t \in T,\ k \in K,\ \omega \in \Omega \tag{9s}$$

$$f_{j,t,k,\omega} \leq F^{\max}_{j}\,x_{j,k-1} \qquad\qquad \forall\, j \in \mathcal{J},\ t \in T,\ k = 2,\ldots,K,\ \omega \in \Omega \tag{10s}$$

$$-f_{j,t,k,\omega} \leq F^{\max}_{j}\,x_{j,k-1} \qquad\qquad \forall\, j \in \mathcal{J},\ t \in T,\ k = 2,\ldots,K,\ \omega \in \Omega \tag{11s}$$

DC Power Flow for existing lines

$$f_{l,t,k,\omega} = B_l\Big(\theta_{b,t,k,\omega} - \theta_{b',t,k,\omega}\Big) \qquad\qquad \forall\, l(b,b') \in \mathcal{L}_{E},\ t \in T,\ k \in K,\ \omega \in \Omega \tag{12s}$$

DC Power Flow for candidate lines

$$f_{j,t,k,\omega} - B_j\big(\theta_{b,t,k,\omega} - \theta_{b',t,k,\omega}\big) \leq M_j\left(1 - x_{j,k-1}\right) \quad \forall\, j \in \mathcal{J}(b,b'),\ t \in T,\ k = 2,\ldots,K,\ \omega \in \Omega \tag{13s}$$

$$-\Big[f_{j,t,k,\omega} - B_j\big(\theta_{b,t,k,\omega} - \theta_{b',t,k,\omega}\big)\Big] \leq M_j\left(1 - x_{j,k-1}\right) \quad \forall\, j \in \mathcal{J}(b,b'),\ t \in T,\ k = 2,\ldots,K,\ \omega \in \Omega \tag{14s}$$

Energy Storage Systems (ESS)

$$p_{s,b,t,k,\omega} \leq \lambda_s\,x_{s,b,k} \qquad\qquad \forall\, s \in S,\ b \in B,\ t \in T,\ k \in K,\ \omega \in \Omega \tag{15s}$$

$$p^{+}_{s,b,t,k,\omega} \leq x_{s,b,k} \qquad\qquad \forall\, s \in S,\ b \in B,\ t \in T,\ k \in K,\ \omega \in \Omega \tag{16s}$$

$$p^{-}_{s,b,t,k,\omega} \leq x_{s,b,k} \qquad\qquad \forall\, s \in S,\ b \in B,\ t \in T,\ k \in K,\ \omega \in \Omega \tag{17s}$$

$$p_{s,b,t,k,\omega} = p_{s,b,t-1,k,\omega} + \tau(\eta^{+}_s p^{+}_{s,b,t,k,\omega} - p^{-}_{s,b,t,k,\omega}) \qquad \forall\, s \in S,\ b \in B,\ t = 2,\ldots,T,\ k \in K,\ \omega \in \Omega \tag{18s}$$

$$p_{s,b,1,k,\omega} = p_{s,b,T,k,\omega} + \tau(\eta^{+}_s p^{+}_{s,b,1,k,\omega} - p^{-}_{s,b,1,k,\omega}) \qquad \forall\, s \in S,\ b \in B,\ t = 2,\ldots,T,\ k \in K,\ \omega \in \Omega \tag{19s}$$

Nodal Balance

$$\sum_{l(:,b)\in L} f_{l,t,k,\omega} - \sum_{l(b,:)\in L} f_{l,t,k,\omega} + \sum_{g\in G} p_{g,b,t,k,\omega}$$
$$+ \sum_{s\in S}\eta^{-}_s p^{-}_{s,b,t,k,\omega} + p^{ue}_{b,t,k,\omega} = P^{d}_{b,t,k,\omega} + \sum_{s\in S} p^{+}_{s,b,t,k,\omega} \quad \forall\, b \in B,\ t \in T,\ k \in K,\ \omega \in \Omega \tag{20s}$$

Soft RPS

$$\sum_{r\in\mathcal{R}}\sum_{b\in B}\sum_{t\in T} p_{r,b,t,k,\omega} + p^{nc}_{k,\omega} \geq \mathrm{RPS}_k\left(\sum_{g\in G}\sum_{b\in B}\sum_{t\in T} p_{g,b,t,k,\omega}\right) \qquad\qquad \forall\, k \in K,\ \omega \in \Omega \tag{21s}$$

Limit number of new generator builds and ESS per bus

$$P_g^{\max} x_{g,b,k} \ \leq \ N_g^{\max} \quad \forall b \in B, g \in G, k \in K \tag{22s}$$

$$x_{s,b,k} \ \leq \ N_s^{\max} \quad \forall b \in B, s \in S, k \in K \tag{23s}$$

Table 1: Notation for The Optimization Formulation

<u>Sets</u>

K	Number of investment stages, indexed by k
T	Hours in a day, indexed by t
B	Buses, indexed by b,b'
G	All generators, indexed by g
E	Existing generators, indexed by e
N_{th}	Candidate thermal generators, indexed by n
R	Candidate renewable generators, indexed by r
L	All transmission lines, indexed by l
J	Candidate transmission lines, indexed by j
L_E	Existing transmission lines, indexed by l
S	Candidate ESS, indexed by s
Ω	Representative days in stage k, indexed by ω

<u>Parameters</u>

$year_k$	Number of years elapsed from base year to the beginning of stage k (with $year_1 = 0$)
$C_{g,k}^{inv}$	Generator investment cost [\$/MW-y]
C_g^{fom}	Generator fixed O&M cost [\$/MW-y]
C_g^{vom}	Generator variable O&M cost [\$/MWh]
$C_{g,k}^{fuel}$	Fuel cost [\$/MWh]
P_g^{max}	Generator capacity upper bound [MW]
$P_{r,b,t,\omega}^{max}$	Hourly renewable resource production upper bound [%]
C_j^{inv}	Transmission line investment cost [\$/y]
N_g^{max}	Maximum capacity allowed per bus for each new generator [MW]
F_l^{max}	Maximum long term rating [MW]
B_l	Line susceptance [p.u.]
$C_{s,k}^{inv}$	ESS investment cost [\$/MW-y]
$C_{s,k}^{fom}$	ESS fixed O&M cost [\$/MW-y]
$C_{s,k}^{vom}$	ESS variable O&M cost [\$/MWh]
C_s^{-}	ESS discharge cost [\$/MWh]
λ_s	Hours of storage capacity for ESS [h]
η_s^{+}	Efficiency when charging ESS
η_s^{-}	Efficiency when discharging ESS
$P_{b,t,k,\omega}^d$	Electricity demand [MW]
τ	Length of each time period in the representative day [h]
$P_{b,t,k,\omega}^d$	Electricity demand [MW]
C^{voll}	Value of lost load [\$/MWh]
RPS	Renewable Portfolio Standard [%]
C^{nc}	RPS non-compliance penalty [\$/MWh]
M	Big-M parameter for disjunctive constraints
π_ω	Probability of representative day ω
d	Discount factor

<u>Investment variables</u>

$x_{n,b,k}$, $y_{n,b,k}$	Integer number of cumulative and new thermal generators
$x_{r,b,k}$, $y_{r,b,k}$	Continuous number of cumulative and new renewable generation capacity [MW]
$x_{s,b,k}$, $y_{s,b,k}$	Continuous number of cumulative and new ESS capacity [MW]
$x_{j,k}$, $y_{j,k}$	Binary variables indicating whether transmission line j is available by stage k and constructed in stage k, respectively

<u>Operational variables</u>

$p_{g,b,t,k,\omega}$	Generation dispatch level [MW]
$p_{s,b,t,k,\omega}$	Energy storage level of ESS [MWh]
$p_{s,b,t,k,\omega}^{+}$	Power input to ESS [MW]
$p_{s,b,t,k,\omega}^{-}$	Power output from ESS [MW]
$p_{b,t,k,\omega}^{ue}$	Load shed [MW]
$f_{l,t,k,\omega}$	Line flow [MW]
$\theta_{b,t,k,\omega}$	Phase angle [radians]
$p_{k,\omega}^{nc}$	RPS non-compliance [MWh]