Supplementary Information for: The Value of Multi-Stage Stochastic Programming in Power Grid Capacity Expansion Planning - ESCAPE36

Formulation for the deterministic multi-stage stochastic power grid capacity expansion planning program

Sergio Bugosen[1], Tomas Valencia[2], Jean-Paul Watson[2], Chrysanthos E. Gounaris[1], and Carl D. Laird[1]

[1]Department of Chemical Engineering, Carnegie Mellon University, Pittsburgh, PA, USA
[2]Lawrence Livermore National Laboratory, Livermore, CA, USA

1 Formulation

Objective Function

$$
\begin{aligned}
\text{StageCost}_k = \Bigg[& \sum_{n \in N_{th}} \sum_{b \in B} C_{n,k}^{inv} P_n^{\max} y_{n,b,k} + \sum_{r \in R} \sum_{b \in B} C_{r,k}^{inv} y_{r,b,k} + \sum_{j \in J} C_j^{inv} y_{j,k} \\
& + \sum_{s \in S} \sum_{b \in B} C_{s,k}^{inv} y_{s,b,k} + \sum_{n \in N_{th}} \sum_{b \in B} C_n^{fom} P_n^{\max} x_{n,b,k} + \sum_{r \in R} \sum_{b \in B} C_r^{fom} x_{r,b,k} \\
& + \sum_{s \in S} \sum_{b \in B} C_{s,k}^{fom} x_{s,b,k} + \sum_{\omega \in \Omega} \pi_\omega \sum_{g \in G} \sum_{b \in B} \sum_{t \in T} \left(C_{g,k}^{fuel} + C_g^{vom} \right) p_{g,b,t,k,\omega} \\
& + \sum_{\omega \in \Omega} \pi_\omega \sum_{s \in S} \sum_{b \in B} \sum_{t \in T} \left(C_{s,k}^{vom} + C_s^{-} \right) p_{s,b,t,k,\omega}^{-} + \sum_{\omega \in \Omega} \pi_\omega \sum_{b \in B} \sum_{t \in T} C^{ls} p_{b,t,k,\omega}^{ue} \\
& + \sum_{\omega \in \Omega} \pi_\omega C^{nc} p_{k,\omega}^{nc} \Bigg], \quad \forall k \in K
\end{aligned}
$$

$$
\min \quad Z = \sum_{k \in K} \frac{\text{StageCost}_k}{(1+d)^{\text{year}_k}} \tag{1s}
$$

Linking Constraints

$$x_{n,b,k} = x_{n,b,k-1} + y_{n,b,k} \qquad \forall\, n \in \mathcal{N}_{th},\, b \in B,\, k = 2, \ldots, K \tag{2s}$$

$$x_{r,b,k} = x_{r,b,k-1} + y_{r,b,k} \qquad \forall\, r \in \mathcal{R},\, b \in B,\, k = 2, \ldots, K \tag{3s}$$

$$x_{s,b,k} = x_{s,b,k-1} + y_{s,b,k} \qquad \forall\, s \in \mathcal{S},\, b \in B,\, k = 2, \ldots, K \tag{4s}$$

$$x_{j,k} = x_{j,k-1} + y_{j,k} \qquad \forall\, j \in \mathcal{J},\, k = 2, \ldots, K \tag{5s}$$

Dispatch Limits

$$p_{r,b,t,k,\omega} \;\leq\; x_{r,b,k-1}\, P^{\max}_{r,b,t,\omega} \qquad \forall\, r \in R,\; b \in B,\; t \in T,\; k = 2,\ldots,K,\; \omega \in \Omega \tag{6s}$$

$$p_{n,b,t,k,\omega} \;\leq\; x_{n,b,k-1}\, P^{\max}_{n} \qquad \forall\, n \in N_{th},\; b \in B,\; t \in T,\; k = 2,\ldots,K,\; \omega \in \Omega \tag{7s}$$

$$p_{e,b,t,k,\omega} \;\leq\; P^{\max}_{e} \qquad \forall\, e \in E,\; b \in B,\; t \in T,\; k \in K,\; \omega \in \Omega \tag{8s}$$

Flow Limits for all lines

$$-F^{\max}_{l} \;\leq\; f_{l,t,k,\omega} \;\leq\; F^{\max}_{l} \qquad \forall\, l \in \mathcal{L}_{\mathrm{E}},\, t \in T,\, k \in K, \omega \in \Omega \tag{9s}$$

$$f_{j,t,k,\omega} \;\leq\; F^{\max}_{j}\, x_{j,k-1} \qquad \forall\, j \in \mathcal{J},\, t \in T,\, k = 2,\ldots,K, \omega \in \Omega \tag{10s}$$

$$-\,f_{j,t,k,\omega} \;\leq\; F^{\max}_{j}\, x_{j,k-1} \qquad \forall\, j \in \mathcal{J},\, t \in T,\, k = 2,\ldots,K, \omega \in \Omega \tag{11s}$$

DC Power Flow for existing lines

$$f_{l,t,k,\omega} = B_l \Big(\theta_{\,b,t,k,\omega} - \theta_{\,b',t,k,\omega} \Big) \qquad \forall\, l(b,b') \in \mathcal{L}_{\mathrm{E}},\, t \in T,\, k \in K, \omega \in \Omega \tag{12s}$$

DC Power Flow for candidate lines

$$f_{j,t,k,\omega} - B_j \big(\theta_{b,t,k,\omega} - \theta_{b',t,k,\omega} \big) \;\leq\; M_j \,(1 - x_{j,k-1}) \quad \forall\, j \in \mathcal{J}(b,b'),\, t \in T,\, k = 2,\ldots,K, \omega \in \Omega \tag{13s}$$

$$-\Big[f_{j,t,k,\omega} - B_j \big(\theta_{b,t,k,\omega} - \theta_{b',t,k,\omega} \big) \Big] \;\leq\; M_j \,(1 - x_{j,k-1}) \quad \forall\, j \in \mathcal{J}(b,b'),\, t \in T,\, k = 2,\ldots,K, \omega \in \Omega \tag{14s}$$

Energy Storage Systems (ESS)

$$p_{s,b,t,k,\omega} \;\leq\; \lambda_s\, x_{s,b,k} \qquad \forall\, s \in S,\; b \in B,\; t \in T,\; k \in K,\; \omega \in \Omega \tag{15s}$$

$$p^{+}_{s,b,t,k,\omega} \;\leq\; x_{s,b,k} \qquad \forall\, s \in S,\; b \in B,\; t \in T,\; k \in K,\; \omega \in \Omega \tag{16s}$$

$$p^{-}_{s,b,t,k,\omega} \;\leq\; x_{s,b,k} \qquad \forall\, s \in S,\; b \in B,\; t \in T,\; k \in K,\; \omega \in \Omega \tag{17s}$$

$$p_{s,b,t,k,\omega} = p_{s,b,t-1,k,\omega} + \tau(\eta^{+}_{s} p^{+}_{s,b,t,k,\omega} - p^{-}_{s,b,t,k,\omega}) \qquad \forall\, s \in S,\; b \in B,\; t = 2,...,T,\; k \in K,\; \omega \in \Omega \tag{18s}$$

$$p_{s,b,1,k,\omega} = p_{s,b,T,k,\omega} + \tau(\eta^{+}_{s} p^{+}_{s,b,1,k,\omega} - p^{-}_{s,b,1,k,\omega}) \qquad \forall\, s \in S,\; b \in B,\; t = 2,...,T,\; k \in K,\; \omega \in \Omega \tag{19s}$$

Nodal Balance

$$\sum_{l(:,b) \in L} f_{l,t,k,\omega} - \sum_{l(b,:) \in L} f_{l,t,k,\omega} + \sum_{g \in G} p_{g,b,t,k,\omega} + \sum_{s \in S} \eta^{-}_{s} p^{-}_{s,b,t,k,\omega} + p^{ue}_{b,t,k,\omega} = \; P^{d}_{b,t,k,\omega} + \sum_{s \in S} p^{+}_{s,b,t,k,\omega} \quad \forall\, b \in B,\, t \in T,\, k \in K, \omega \in \Omega \tag{20s}$$

Soft RPS

$$\sum_{r \in \mathcal{R}} \sum_{b \in B} \sum_{t \in T} p_{r,b,t,k,\omega} \;+\; p^{nc}_{k,\omega} \;\geq\; \mathrm{RPS}_k \left(\sum_{g \in G} \sum_{b \in B} \sum_{t \in T} p_{g,b,t,k,\omega} \right) \qquad \forall\, k \in K, \omega \in \Omega \tag{21s}$$

Limit number of new generator builds and ESS per bus

$$P_g^{\max} x_{g,b,k} \ \leq \ N_g^{\max} \quad \forall\, b \in B, g \in G, k \in K \tag{22s}$$

$$x_{s,b,k} \ \leq \ N_s^{\max} \quad \forall\, b \in B, s \in S, k \in K \tag{23s}$$

Table 1: Notation for The Optimization Formulation

Sets	
K	Number of investment stages, indexed by k
T	Hours in a day, indexed by t
B	Buses, indexed by b,b'
G	All generators, indexed by g
E	Existing generators, indexed by e
N_{th}	Candidate thermal generators, indexed by n
R	Candidate renewable generators, indexed by r
L	All transmission lines, indexed by l
J	Candidate transmission lines, indexed by j
L_E	Existing transmission lines, indexed by l
S	Candidate ESS, indexed by s
Ω	Representative days in stage k, indexed by ω
Parameters	
$year_k$	Number of years elapsed from base year to the beginning of stage k (with $year_1 = 0$)
$C^{inv}_{g,k}$	Generator investment cost [$/MW-y]
C^{fom}_{g}	Generator fixed O&M cost [$/MW-y]
C^{vom}_{g}	Generator variable O&M cost [$/MWh]
$C^{fuel}_{g,k}$	Fuel cost [$/MWh]
P^{max}_{g}	Generator capacity upper bound [MW]
$P^{max}_{r,b,t,\omega}$	Hourly renewable resource production upper bound [%]
C^{inv}_{j}	Transmission line investment cost [$/y]
$N^{\max}_{g}$	Maximum capacity allowed per bus for each new generator [MW]
F^{max}_{l}	Maximum long term rating [MW]
B_l	Line susceptance [p.u.]
$C^{inv}_{s,k}$	ESS investment cost [$/MW-y]
$C^{fom}_{s,k}$	ESS fixed O&M cost [$/MW-y]
$C^{vom}_{s,k}$	ESS variable O&M cost [$/MWh]
C^{-}_{s}	ESS discharge cost [$/MWh]
λ_s	Hours of storage capacity for ESS [h]
η^{+}_{s}	Efficiency when charging ESS
η^{-}_{s}	Efficiency when discharging ESS
$P^{d}_{b,t,k,\omega}$	Electricity demand [MW]
τ	Length of each time period in the representative day [h]
$P^{d}_{b,t,k,\omega}$	Electricity demand [MW]
C^{voll}	Value of lost load [$/MWh]
RPS	Renewable Portfolio Standard [%]
C^{nc}	RPS non-compliance penalty [$/MWh]
M	Big-M parameter for disjunctive constraints
π_ω	Probability of representative day ω
d	Discount factor
Investment variables	
$x_{n,b,k}$, $y_{n,b,k}$	Integer number of cumulative and new thermal generators
$x_{r,b,k}$, $y_{r,b,k}$	Continuous number of cumulative and new renewable generation capacity [MW]
$x_{s,b,k}$, $y_{s,b,k}$	Continuous number of cumulative and new ESS capacity [MW]
$x_{j,k}$, $y_{j,k}$	Binary variables indicating whether transmission line j is available by stage k and constructed in stage k, respectively
Operational variables	
$p_{g,b,t,k,\omega}$	Generation dispatch level [MW]
$p_{s,b,t,k,\omega}$	Energy storage level of ESS [MWh]
$p^{+}_{s,b,t,k,\omega}$	Power input to ESS [MW]
$p^{-}_{s,b,t,k,\omega}$	Power output from ESS [MW]
$p^{ue}_{b,t,k,\omega}$	Load shed [MW]
$f_{l,t,k,\omega}$	Line flow [MW]
$\theta_{b,t,k,\omega}$	Phase angle [radians]
$p^{nc}_{k,\omega}$	RPS non-compliance [MWh]