Systems & Control Transactions

Research Article - Peer Reviewed Conference Proceeding
ESCAPE 35 - European Symposium on Computer Aided Process Engineering
Ghent, Belgium. 6-9 July 2025
Jan F.M. Van Impe, Grégoire Léonard, Satyajeet S. Bhonsale, Monika E. Polańska, Filip Logist (Eds.)

PSE PRESS

Updated-Absolute Expected Value Solution Approach for multistage stochastic programming problems

Yasuhiro Shoji and Selen Cremaschi*

Department of Chemical Engineering, Auburn University, Auburn, AL 36849, USA
* Corresponding Author: szc0113@auburn.edu.

ABSTRACT

This paper introduces the Updated Absolute Expected Value Solution, U-AEEV, a heuristic for solving multi-stage stochastic programming (MSSP) problems with type 2 endogenous uncertainty. U-AEEV is an evolution of the Absolute Expected Value Solution, AEEV [1]. This paper aims to show how U-AEEV overcomes the drawbacks of AEEV and performs better than AEEV. To demonstrate the performance of U-AEEV, we solve 6 MSSP problems with type 2 endogenous uncertainty and compare the solutions and computational resource requirements.

Keywords: Stochastic Optimization, endogenous uncertainty, heuristics

1. INTRODUCTION

Multi-stage stochastic programming (MSSP) is a common mathematical framework used to solve decision-making problems, such as planning and process design problems, under uncertainty often encountered in the process industry [2-3]. In MSSP problems, the outcomes of uncertain events materialize over a given time horizon, and the realized outcomes distinguish given scenarios, allowing the decision-maker to take corrective action in distinguished scenarios. Taxonomically [4], there are two categories of uncertainty in stochastic programming: decision-independent, also known as exogenous uncertainty, and decision-dependent, also known as endogenous uncertainty. The endogenous uncertainty can be further categorized into three sub-classes: type 1, where the probability distribution is decision-dependent; type 2, where the information structure is decision-dependent; and type 3, where both probability distribution and the information structure are decision-dependent. As for type 2 endogenous uncertainty, which U-AEEV mainly deals with, decisions affect the resolution of uncertainty, i.e., information structure (the shape of a scenario tree) is decision-dependent. The non-anticipativity constraints (NACs) used to ensure the indistinguishability among scenarios before knowing the outcomes of future events cause significant complexities and computational intractability in solving real-life-sized problems. This challenge has motivated researchers to develop various heuristic and decomposition-based solution approaches, such as the sample average approximation algorithm [5], and Benders decomposition [6], to obtain solutions.

A heuristic, Absolute Expected Value Solution (AEEV), which our group developed [1], yields feasible solutions to MSSP problems with type 2 endogenous uncertainty. AEEV circumvents the computational intractability by decomposing scenarios of an original MSSP problem into multiple deterministic sub-problems with expected uncertain parameter values. In each period from the initial to the end of the time horizon, AEEV iterates to solve a sub-problem, fixes the here-and-now decision variable values, judges uncertainty realization, updates the expected uncertain parameter values based on the realized uncertainty, solves deterministic recourse sub-problems to determine corrective action, and fixes wait-and-see decision variable values. A drawback of AEEV is that it may fail to generate a feasible solution if the original MSSP problem does not have complete recourse. More precisely, the expected uncertain parameter for a recourse sub-problem does not always ensure feasible corrective action following uncertainty realization; accordingly, the sub-problem may be infeasible if the original MSSP problem lacks complete recourse.

In this paper, we introduce the Updated Absolute Expected Value Solution (U-AEEV), which expands the applicability of the AEEV to MSSP problems without complete recourse by eliminating the potential infeasibility issues. The U-AEEV iteratively solves scenario-decomposed deterministic sub-problems using the outcomes of each scenario instead of the expected values of the uncertain parameters, evaluates the solutions to the sub-problems in terms of infeasibility and quality, and selects one. The U-AEEV yields tight feasible solutions without significant growth in computational time.

2. THE FRAMEWORK OF MSSP WITH TYPE 2 ENDOGENOUS UNCERTAINTY

MSSP problems with type 2 endogenous uncertainty consider multiple scenarios and a discretized time horizon, where scenarios gradually become distinguishable over the time horizon. Equations (1)-(15) represent a common formulation of MSSP with type 2 endogenous uncertainty (this is an extended formulation from [1]).

$$\min z = \sum_{s\in\mathcal{S}} p_s \sum_{i\in\mathcal{I},t\in\mathcal{T}} G_{i,t,s}(V_{i,t},\theta_{i,s},b_{i,t,s},x_{i,t,s},y_{i,t,s},w_{i,t,s}) \quad (1)$$

subject to

$$g\left(V_{i,t},\theta_{i,s},b_{i,t,s},x_{i,t,s},y_{i,t,s},w_{i,t,s}\right) \le 0 \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T},\quad \forall s\in\mathcal{S} \quad (2)$$

$$h\left(V_{i,t},\theta_{i,s},b_{i,t,s},x_{i,t,s},y_{i,t,s},w_{i,t,s}\right) = 0 \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T},\quad \forall s\in\mathcal{S} \quad (3)$$

$$b_{i,1,s} = b_{i,1,s'} \quad \forall i\in\mathcal{I},\quad \forall s,s'\in\mathcal{S} \quad (4)$$

$$x_{i,1,s} = x_{i,1,s'} \quad \forall i\in\mathcal{I},\quad \forall s,s'\in\mathcal{S} \quad (5)$$

$$b_{i,t,s}^{\text{dif}} = b_{i,t,s} \Leftrightarrow Z_{t,s,s'} = H\left(b_{i,t,s}\right) \quad \forall i\in\mathcal{D}(s,s'),\quad \forall t\in\mathcal{T},\quad \forall s,s'\in\mathcal{S} \quad (6)$$

$$w_{i,t,s}^{\text{dif}} = w_{i,t,s} \Leftrightarrow Z_{t,s,s'} = H\left(w_{i,t,s}\right) \quad \forall i\in\mathcal{D}(s,s'),\quad \forall t\in\mathcal{T},\quad \forall s,s'\in\mathcal{S} \quad (7)$$

$$Z_{t,s,s'} = H\left(b_{i,t,s}^{\text{dif}}, w_{i,t,s}^{\text{dif}}\right) \quad \forall i\in\mathcal{D}(s,s'),\quad \forall t\in\mathcal{T},\quad \forall s,s'\in\mathcal{S} \quad (8)$$

$$\begin{bmatrix} Z_{t,s,s'} \\ b_{i,t+1,s} = b_{i,t+1,s'} \\ x_{i,t+1,s} = x_{i,t+1,s'} \end{bmatrix} \vee \left[\neg Z_{t,s,s'}\right] \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T}|t\le T-1,\quad \forall s,s'\in\mathcal{S} \quad (9)$$

$$\begin{bmatrix} Z_{t,s,s'} \\ y_{i,t,s} = y_{i,t,s'} \end{bmatrix} \vee \left[\neg Z_{t,s,s'}\right] \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T},\quad \forall s,s'\in\mathcal{S} \quad (10)$$

$$b_{i,t,s}\in\{0,1\} \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T},\quad \forall s\in\mathcal{S} \quad (11)$$

$$x_{i,t,s}\ge 0 \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T},\quad \forall s\in\mathcal{S} \quad (12)$$

$$y_{i,t,s}\ge 0 \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T},\quad \forall s\in\mathcal{S} \quad (13)$$

$$w_{i,t,s}\ge 0 \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T},\quad \forall s\in\mathcal{S} \quad (14)$$

$$Z_{t,s,s'}\in\{0,1\} \quad \forall t\in\mathcal{T},\quad \forall s,s'\in\mathcal{S} \quad (15)$$

The formulation includes two types of decision variables: here-and-now decision variables corresponding to $b_{i,t,s}$ (binary) and/or $x_{i,t,s}$ (continuous or integer), and wait-and-see decision variables corresponding to $y_{i,t,s}$ (continuous or integer), with the problem-specific index $i\in\mathcal{I}=\{1,2,\dots,I\}$, time index $t\in\mathcal{T}=\{1,2,\dots,T\}$, and scenario index $s\in\mathcal{S}=\{1,2,\dots,S\}$. Variable $w_{i,t,s}$ notates non-decision variables that are entirely dependent on here-and-now and wait-and-see decision variables. If a scenario pair s and s' becomes distinguishable due to uncertainty realization at time t, here-and-now decision variables can take different values at $t+1$ and after, as conditional NACs (9) imply. In other words, here-and-now decision variables must take the same values at t because the decisions are made before realization at t. Since conditional NACs (9) do not ensure the scenario indistinguishability at $t=1$, here-and-now decision variables need initial NACs by separate equations (4)-(5). In contrast to here-and-now decision variables, wait-and-see decision variables can take different values at t when scenario realization occurs to determine the corrective action after realization, as conditional NACs (10) imply. Equation (10) includes initial NACs for wait-and-see decision variables, i.e., wait-and-see decision variables can omit separate equations for the initial NACs. The binary variable $Z_{t,s,s'}$ in conditional NACs (9)-(10) is called the indicator variable, which is 1 if a scenario pair s and s' is indistinguishable at the end of time t and 0 otherwise. The value of the indicator variable switches based on the values of particular binary decision variables or non-decision variables. We define the particular variables as differentiator variables, $b_{i,t,s}^{\text{dif}}$ and $w_{i,t,s}^{\text{dif}}$, in (6) and (7), i.e., a binary decision variable or a non-decision variable is a differentiator variable if the indicator variable is bounded using these variables. The differentiator variables switch the value of the indicator variable through the expression $H(b_{i,t,s}^{\text{dif}}, w_{i,t,s}^{\text{dif}})$ in (8). $\mathcal{D}(s,s')$ in (6)-(8) is a differentiator set, a set of i that has different outcomes in an uncertain parameter pair $\theta_{i,s}$ and $\theta_{i,s'}$. The objective function (1) provides the optimum of expression z based on the probability, p_s, of scenario s and the contribution of the scenario, $G_{i,t,s}(V_{i,t},\theta_{i,s},b_{i,t,s},x_{i,t,s},y_{i,t,s},w_{i,t,s})$. The two given parameters, $V_{i,t}$ and $\theta_{i,s}$, are deterministic and uncertain parameters, respectively. Equations (2)-(3) express scenario-specific inequality and equality constraints, such as demand constraints, capacity constraints, and material balances.

3. UPDATED ABSOLUTE EXPECTED VALUE SOLUTION APPROACH (U-AEEV)

The U-AEEV, similar to AEEV, is a scenario-decomposition approach to mitigating the computational intractability of an MSSP problem with type 2 endogenous uncertainty. The U-AEEV solves deterministic sub-problems using each scenario outcome for the uncertain parameters and enumerates multiple 'decision candidates' in a stage-wise (in the sense of MSSP) manner. It evaluates the 'decision candidates' and adopts a feasible and promising candidate for the current stage. After finding the end of the stage by detecting uncertainty realization, U-AEEV stores the stage's decision variable values to ensure the indistinguishability of scenarios by fixing them when it solves sub-problems in subsequent stages.

Figure 1 is an example (a process network synthesis problem [2]) to illustrate how U-AEEV works. Raw materials C and D are used to produce intermediate B through new Process 1 and Process 2, and intermediate B is used to produce product A through existing Process 3. The decision maker can purchase B and must satisfy the demand for product A by production or purchase while maximizing the expected net present value.

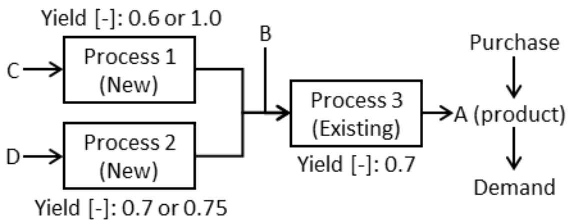

Figure 1. An example problem [2] to illustrate U-AEEV.

The here-and-now decisions of the example are the installation and capacity of each process, as well as the flow rates of raw materials and purchased intermediate B. The wait-and-see decision is the purchase rate of Product A. The source of endogenous uncertainty is the yield of Process 1 and Process 2, which materializes by installing and operating the process. The decision maker determines here-and-now decisions, such as installation, capacity, and flow rates, at the start of a time, waits until the yield of processes becomes clear, and then determines the rate of A purchased. **Table 1** lists scenarios and the yield and probability of each scenario.

Table 1: Scenarios, yields, and probabilities of the example problem in Figure 1.

Scenario	Yield [-]		Probability
	Process 1	Process 2	
$s1$	0.6	0.7	0.4
$s2$	0.6	0.75	0.3
$s3$	1.0	0.7	0.2
$s4$	1.0	0.75	0.1
Expected	0.72	0.72	-

The U-AEEV starts by solving four deterministic sub-problems with each scenario outcome and enumerates the decision candidates suggested by each scenario, as shown in **Figure 2**. In the case of this example, Scenario 1 (s_1) suggests Candidate 1, in which neither Process 1 nor 2 is installed, Scenario 2 (s_2) suggests installing Process 2 at Time 1 (t_1) in Candidate 2, and Scenario 3 (s_3) and 4 (s_4) suggest installing Process 1 at Time 1 (t_1) in Candidate 3. Note that the time horizon of the example has three time periods.

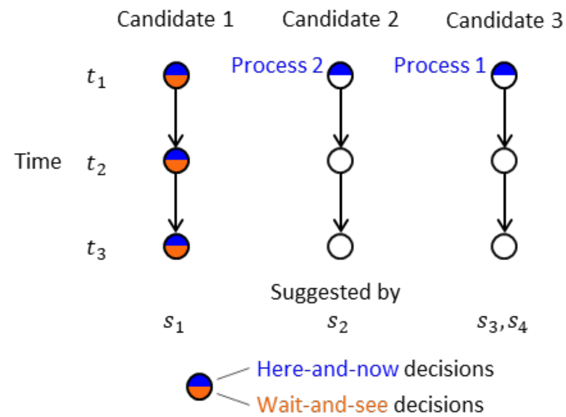

Figure 2. Decision candidates of the first stage (up to the first realization)

Since U-AEEV solves deterministic sub-problems stage-wise, decision candidates at this step refer to the decisions of the first stage (up to the first realization). In other words, the decision candidates of Candidate 1 are the here-and-now and wait-and-see decisions from Time 1 (t_1) to Time 3 (t_3) because Candidate 1 sees no uncertainty realization throughout the time horizon. Similarly, the decision candidates of Candidate 2 and 3 are the here-and-now decisions at Time 1 (t_1) because uncertainty partially materializes by installing and operating Process 2 or 1. For example, Scenarios 1 and 3 or Scenarios 2 and 4 become distinguishable through operating Process 2 in Candidate 2.

The U-AEEV picks up candidates to be evaluated based on the total probability of a candidate, p^{c}, in (16) and weighted objective value, $\mathrm{obj}^{\mathrm{c}}$, in (17), where p_s is the probability of the scenario s, obj_s is the objective value of the sub-problem solved with the outcome of scenario s, and S^{c} is a set of scenarios that suggest candidate c.

$$p^{\mathrm{c}} = \sum_{s \in S^{\mathrm{c}}} p_s \tag{16}$$

$$\mathrm{obj}^{\mathrm{c}} = \frac{\sum_{s \in S^{\mathrm{c}}} p_s \, \mathrm{obj}_s}{\sum_{s \in S^{\mathrm{c}}} p_s} \tag{17}$$

The U-AEEV only evaluates the candidates on the Pareto front of the p^{c}-$\mathrm{obj}^{\mathrm{c}}$ plane (**Figure 3**), i.e., the candidates with high probabilities and high objective values (in a maximization problem) are evaluated. In a minimization problem, the Pareto front is formed based on high probabilities and low objective values. The selection using a Pareto front helps to reduce computational time by focusing only on promising candidates. Since the example is a maximization problem, Candidates 1 and 3 on the Pareto front in Figure 3 are evaluated.

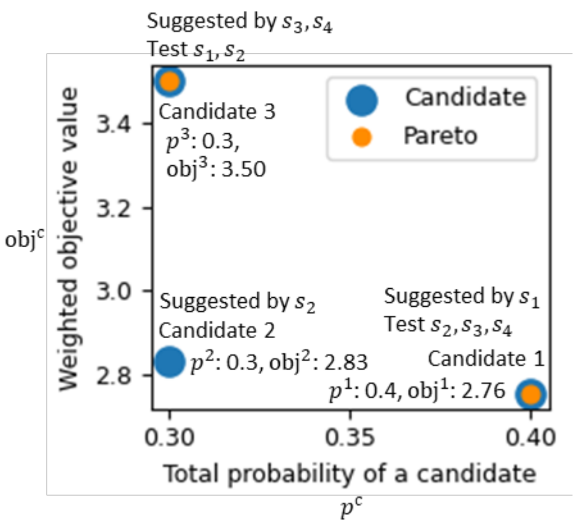

Figure 3. The total probability of a candidate (p^c) and weighted objective value (obj^c) plane for the evaluation of decision candidates

The U-AEEV checks feasibility and compares the expected objective values of candidates, $\underset{s\in S^{\text{InD}}}{\text{E}}[\text{obj}_s]$, in (18) on the Pareto front by testing each candidate with all other scenario outcomes that did not suggest the candidate. In the example, the U-AEEV solves sub-problems with Scenario 2 (s_2), 3 (s_3), and 4 (s_4) outcomes for Candidate 1, and Scenario 1 (s_1) and 2 (s_2) outcomes for Candidate 3, and adopts the best candidate regarding the expected objective value calculated with (18). S^{InD} is a set of indistinguishable scenarios associated with the sub-problems, i.e., Scenarios 1, 2, 3, and 4 in the example, since none of the scenarios are distinguishable at the first stage.

$$\underset{s\in S^{\text{InD}}}{\text{E}}[\text{obj}_s] = \sum_{s\in S^{\text{InD}}} p_s\, \text{obj}_s \tag{18}$$

The candidate with the largest expected objective value is adopted in a maximization problem, and the candidate with the smallest expected objective value is adopted in a minimization problem. If a candidate turns out to be infeasible when solving sub-problems due to a lack of complete recourse, the U-AEEV excludes the candidate from the evaluation process. Then, it forms a new Pareto front and evaluates candidates located on the updated Pareto front. For example, the U-AEEV removes Candidate 3 from the evaluation process and evaluates Candidate 2, located on the updated Pareto front, if Candidate 3 is infeasible.

Finally, the U-AEEV finds the end of the stage, i.e., the time uncertainty realization occurs and distinguishable scenarios at the time, and stores the decision variable values of the stage. The stored decision variable values are used to fix the decision variables of the stage to ensure scenario indistinguishability when the U-AEEV repeats the same procedure in the following stage. The U-AEEV continues the procedure until the final stage, i.e., until no uncertainty realization is found.

Equations (19)-(27) are the formulation of sub-problems solved in U-AEEV.

$$\min z^{\text{UAEEV}} = \sum_{i\in\mathcal{I},t\in\mathcal{T}} G_{i,t}\left(V_{i,t},\theta_i^n,b_{i,t},x_{i,t},y_{i,t},w_{i,t}\right) \tag{19}$$

subject to

$$g\left(V_{i,t},\theta_i^n,b_{i,t},x_{i,t},y_{i,t},w_{i,t}\right) \le 0 \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T} \tag{20}$$

$$h\left(V_{i,t},\theta_i^n,b_{i,t},x_{i,t},y_{i,t},w_{i,t}\right) = 0 \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T} \tag{21}$$

$$\begin{bmatrix} b_{i,t} = b_{i,t}^{\text{str},n} \\ x_{i,t} = x_{i,t}^{\text{str},n} \\ w_{i,t}^{\text{dif}} = w_{i,t}^{\text{difstr},n} \end{bmatrix} \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T}\,|\,t \le t^{\text{PRend},n} \tag{22}$$

$$y_{i,t} = y_{i,t}^{\text{str},n} \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T}\,|\,t < t^{\text{PRend},n} \tag{23}$$

$$b_{i,t} \in \{0,1\} \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T} \tag{24}$$

$$x_{i,t} \ge 0 \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T} \tag{25}$$

$$y_{i,t} \ge 0 \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T} \tag{26}$$

$$w_{i,t} \ge 0 \quad \forall i\in\mathcal{I},\quad \forall t\in\mathcal{T} \tag{27}$$

Equations (19), (20), and (21) are the scenario-decomposed objective function and scenario-specific constraints corresponding to (1), (2), and (3). The uncertain parameter $\theta_{i,s}$ in (1), (2), and (3) is replaced with a scenario outcome θ_i^n in (19), (20), and (21), where n denotes a sub-problem. Since the U-AEEV decomposes scenarios, it does not include NACs and indicator variable $Z_{t,s,s'}$ in its formulation. Instead, Equation (22) fixes all here-and-now decision variables and differentiator variables at and until the end time of the previous stage, $t^{\text{PRend},n}$, to ensure scenario indistinguishability using stored variables $b_{i,t}^{\text{str},n}$, $x_{i,t}^{\text{str},n}$, and $w_{i,t}^{\text{difstr},n}$. Similarly, Equation (23) fixes all wait-and-see decision variables using stored variable $y_{i,t}^{\text{str},n}$ until the previous time of the previous stage's end time.

The U-AEEV never fails to generate a solution even if a problem lacks complete recourse, as long as one of the decision candidates assumes the extremist (most pessimistic, optimistic, best, or worst) scenario. Also, it may locate tighter feasible solutions and yield smaller optimality gaps compared to the AEEV because the U-AEEV evaluates decision candidates suggested by all scenario outcomes and selects a good one, whereas decisions are offered only by the expected scenario outcomes (for example, the expected yields of Process 1 and 2 shown in Table 1 is 0.72) in AEEV. On the other hand, the U-AEEV may increase the computational time compared to the AEEV because it solves sub-problems with all scenario outcomes instead of solving sub-problems using only expected scenario outcomes. The U-AEEV offsets the increase in computational time by solving sub-problems stage-wise instead of AEEV's time-wise approach.

4. COMPUTATIONAL PERFORMANCE

We examine the U-AEEV performance using 6 published MSSP problems with type 2 endogenous uncertainty. **Table 2** lists the problem name, programming type (MILP or MINLP), the presence of complete recourse, the MSSP instance size, and the relative termination tolerance in solving the instances. The instance sizes are large, and the MSSP models cannot be solved to optimality within 48 hours. We implemented U-AEEV with Python 3.9.13, built optimization models using Pyomo 6.4.0, and solved all instances with CPLEX 20.10 for MILP or BARON 17.4.1 for MINLP problems, employing 48 processors (Intel(R) Xeon(R) Gold 6248R) and 380 GB of RAM on a node from Auburn University Easley Cluster. **Table 3** compares the computational performance of MSSP, U-AEEV, and AEEV. The second column from the left represents the optimality gaps reached at 48 hours for the MSSP models. The definition of the optimality gap, $\mathrm{gap}^{\mathrm{MSSP}}$, is the relative error between the feasible (upper/lower) and infeasible (lower/upper) bounds, FB^{MSSP} and IB^{MSSP}, as defined in (28). The third and fourth columns from the left are the relative error, $\mathrm{gap}^{\mathrm{heu}}$, between the feasible bounds obtained as the MSSP solution and our heuristics (FB^{heu}), as defined in (29) and (30) for minimization and maximization problems, respectively. Negative gaps indicate that the feasible bounds yielded by our heuristics are better than those by the MSSP models. 'No sol.' in the AEEV column indicates that AEEV fails to generate a feasible solution because of a lack of complete recourse. The fifth and sixth columns from the left are the orders of magnitude, MAG, in computational speed defined in (31), where t^{MSSP} and t^{heu} are the computational time of MSSP and heuristic models, respectively. Equation (31) implies the computational speed is 10, 100, 1,000, and 10,000 times faster if the orders of magnitude are 1, 2, 3, and 4, respectively. Note that t^{MSSP} is 48 hours in all instances.

$$\mathrm{gap}^{\mathrm{MSSP}} = \frac{\left|FB^{\mathrm{MSSP}} - IB^{\mathrm{MSSP}}\right|}{FB^{\mathrm{MSSP}}}100 \tag{28}$$

$$\mathrm{gap}^{\mathrm{heu}} = \frac{FB^{\mathrm{heu}} - FB^{\mathrm{MSSP}}}{FB^{\mathrm{MSSP}}}100 \tag{29}$$

$$\mathrm{gap}^{\mathrm{heu}} = \frac{FB^{\mathrm{MSSP}} - FB^{\mathrm{heu}}}{FB^{\mathrm{MSSP}}}100 \tag{30}$$

$$\mathrm{MAG} = \log_{10}\left(\frac{t^{\mathrm{MSSP}}}{t^{\mathrm{heu}}}\right) \tag{31}$$

The U-AEEV yielded feasible solutions for all problem instances. However, the gaps in U-AEEV are large depending on the instance, as the gap of the demand-side response scheme planning problem shows 12.3 %. For this instance, we observed that only the candidate suggested by the worst scenario successfully generated a solution, while all other candidates failed due to the lack of complete recourse. This result implies the decision candidates suggested by scenario outcomes may not always include a good one. All failed candidates are infeasible in their corrective actions, which is caused by solving scenario-decomposed sub-problems without considering the information of other scenarios, unlike MSSP. Utilizing outcomes of other scenarios when solving sub-problems remains a topic for future work.

The gaps among comparable instances between U-AEEV and AEEV are smaller by 5.7 % in U-AEEV on average. However, U-AEEV yielded larger gaps than AEEV for some problems, as the gap of the offshore oilfield infrastructure planning problem is 2.43 % for U-AEEV, while 1.78 % for AEEV. We observed that only one extreme decision candidate was evaluated and selected in the evaluation process, which caused a larger gap in U-AEEV. The problem has many combinations of decisions based on three binary, one integer, and six continuous decision variables. Accordingly, each scenario outcome generates a different candidate. Since the probability of candidates is equal and the problem is a maximization problem, only one extreme candidate with the largest objective value forms the Pareto front in the evaluation process and is selected. This result implies the decision candidate suggested by the extremist (most pessimistic, optimistic, best, or worst) scenario is not always a better decision than a moderate decision suggested by an expected scenario outcome in AEEV. When an extreme candidate is selected, i.e., when the number of candidates and the number of scenarios suggesting candidates are the same and only one candidate is evaluated on the Pareto front, a moderate candidate suggested by the expected uncertain parameter may result in a tighter optimality gap. In other words, incorporating the idea of AEEV into U-AEEV

Table 2: MSSP problems with type 2 endogenous uncertainty to examine U-AEEV performance.

Problem name	Programming type	Complete recourse	MSSP size: Variables	MSSP size: Constraints	Relative termination tolerance
Clinical trial planning [7]	MILP	Yes	556,033	1,806,849	0.01 %
Process network synthesis [2]	MILP	No	8,577	57,489	0.01 %
R&D project portfolio management [5]	MILP	Yes	20,097	298,881	0.01 %
Offshore oilfield infrastructure planning [3]	MINLP	Yes	31,154	66,178	0.1 %
Vehicle routing [8]	MILP	No	20,711	59,778	0.01 %
Demand-side response scheme planning [6]	MILP	No	2,939,317	15,682,701	0.01 %

Table 3: Computational performance of MSSP, U-AEEV, and AEEV.

Problem name	Optimality gap [%]	Gap from MSSP feasible bound [%]		Orders of magnitude in computational speed [-]	
	MSSP	U-AEEV	AEEV	U-AEEV	AEEV
Clinical trial planning	4.86	-1.33	8.96	1.4	4.5
Process network synthesis	16.0	3.69	No sol.	2.9	-
R&D project portfolio management	6.29	-1.53	5.93	2.7	4.0
Offshore oilfield infrastructure planning	10.6	2.43	1.78	0.8	1.3
Vehicle routing	6.99	0.50	No sol.	3.7	-
Demand-side response scheme planning	27.3	12.3	No sol.	1.7	-

may be a promising idea for future work.

The computational speed of U-AEEV is 0.8 to 3.7 orders faster than MSSP, while it is 1.6 orders slower than AEEV. The difference in computational speed between U-AEEV and AEEV mainly depends on the number of scenarios, stages, and time periods. The U-AEEV needs to solve more sub-problems as the number of scenarios and stages is larger because it tests each scenario outcome stage-wise. On the other hand, the AEEV needs to solve more sub-problems as the number of time periods is larger because it solves sub-problems time-wise. For example, the number of maximum stages of the clinical trial planning problem (1024 scenarios and 12 time periods) is 10 in U-AEEV and 1 in AEEV, and U-AEEV solved 16789 sub-problems, while AEEV solved only 12 sub-problems. Note that the number of stages is not constant in MSSP problems with type 2 endogenous uncertainty because the shape of a scenario tree is decision-dependent.

5. CONCLUSIONS AND FUTURE DIRECTIONS

We introduced the Updated Absolute Expected Value Solution, U-AEEV, a heuristic for solving MSSP problems with type 2 endogenous uncertainty. We examined 6 MSSP problems involving Type 2 endogenous uncertainty with U-AEEV. The results showed that the U-AEEV generates a tight feasible solution even if the problem lacks complete recourse. The gap from the MSSP feasible bound was smaller by 5.7 % for U-AEEV than AEEV on average. The computational speed of U-AEEV is 0.8 to 3.7 orders faster than MSSP, while 1.6 orders slower than AEEV. Future studies will focus on publishing a Python package of U-AEEV. The package will automate the procedure of U-AEEV following converting the MSSP model into the U-AEEV sub-problem model utilizing input information, such as the model, here-and-now and wait-and-see decision variables, differentiator variables, uncertain parameters, and NACs of the original MSSP problem.

REFERENCES

1. Zeng, Z. & Cremaschi, S. A general primal bounding framework for large-scale multistage stochastic programs under endogenous uncertainties. *Chem. Eng. Res. Des*. 141, 464-480 (2019).
2. Tarhan, B. & Grossmann, I. E. A multistage stochastic programming approach with strategies for uncertainty reduction in the synthesis of process networks with uncertain yields. *Comput. & Chem. Eng.* 32(4-5), 766-788 (2008).
3. Gupta, V. & Grossmann, I. E. Multistage stochastic programming approach for offshore oilfield infrastructure planning under production sharing agreements and endogenous uncertainties. *J. Pet. Sci. Eng.* 124, 180-197 (2014).
4. Hellemo, L., Barton, P. I., & Tomasgard, A. Decision-dependent probabilities in stochastic programs with recourse. *Comput. Manag. Sci.* 15, 369-395 (2018).
5. Solak, S., Clarke, J. P. B., Johnson, E. L., & Barnes, E. R. Optimization of R&D project portfolios under endogenous uncertainty. *Eur. J. Oper. Res.* 207(1), 420-433 (2010).
6. Giannelos, S., Konstantelos, I., & Strbac, G. Option Value of Demand-Side Response Schemes Under Decision-Dependent Uncertainty. *IEEE Trans.*, 33(5), 5103-5113 (2018).
7. Colvin, M. & Maravelias, C. T. A stochastic programming approach for clinical trial planning in new drug development. *Comput. & Chem. Eng.* 32, 2626-2642 (2008).
8. Khaligh, F. H. & MirHassani, S. A. A mathematical model for vehicle routing problem under endogenous uncertainty. *Int. J. Prod. Res.* 54:2, 579-590 (2016).