

Article

Mathematical Model Derivation and Experimental Verification of Novel Consequent-Pole Adjustable Speed PM Motor[†]

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Abstract: This paper proposes a novel consequent-pole-type PM motor having a structure different from that of conventional consequent-pole-type PM motors. The proposed rotor structure is composed of a magnetic pole pair using a permanent magnet and an image-pole pair using a high permeability core. The windings facing the magnetic pole pair and the image-pole pair are connected in series in the rotor structure, the three-phase synchronous impedance is balanced, and the *d*-axis inductance is increased. Therefore, compared with the conventional consequent-pole type, the field weakening operation can be performed efficiently with a lower *d*-axis current. These advantages make it possible to expand the operating range during field weakening. Furthermore, to fix the driving control method of the proposed consequent-pole PM motor, the voltage equation of the proposed motor is derived and verified by analysis and experiment. In addition, the essential characteristics of the proposed motor were compared with that of a standard surface permanent magnet (SPM) motor and a conventional consequent-pole PM motor.



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Keywords: consequent-pole; adjustable speed; permanent magnet motor; synchronous motor; field weakening; voltage equation

1. Introduction

Recently, hybrid electric vehicles (HEV) and battery electric vehicles (BEV) have become popular. As a result, adjustable speed drive technology for the traction motor is being actively developed to improve the electric power consumption of HEV and BEVs. In addition, progress in the electrification of various devices in automotive and small motors is occurring. Therefore, the technological development of motors with adjustable speed characteristics, having both high speed and high torque, was studied to achieve high responsiveness and an expansive driving range [1–11]. As an example of automotive and small motors, electric power steering (EPS) motors are required to have high torque for turning the steering wheel when the car is stopped, and high rotation to cope with the reaction force of the steering wheel when avoiding an emergency or when climbing up a step. Therefore, as shown in Figure 1, an adjustable speed motor with a relationship between the rotating speed *N* and torque *T* characteristics (*N*-*T* characteristics) is required [12].

To achieve high torque, PM motors using permanent magnets are applied in many cases, and research and development aimed at expanding the operating range of permanent magnet (PM) motors are actively proceeding [13–16]. The PM motor can be designed to generate higher torque at low speed by increasing the electromotive force (e.m.f.). However, in a PM motor using high energy density PMs, such as NdFeB or SmCo, it is theoretically impossible to simultaneously realize high-speed operation at low torque and high torque generation at low speed. For this reason, interior permanent magnet (IPM) motors that uses reluctance torque, in addition to magnet torque generated by magnet flux and technology

for weakening the magnetic field, are widely introduced to suppress the back e.m.f. within the high-speed operating range [17–24]. However, in automotive and small motors, the outer diameter of the motor is small, and the reluctance torque cannot be effectively used. Therefore, surface permanent magnet (SPM) motors that can make the most of magnet torque have become mainstream. Field weakening control generally suppresses the back e.m.f. by reducing the PM flux using a negative d -axis current. However, in the high-speed region, copper loss due to the d -axis current cannot be avoided, and the efficiency of the PM motor decreases. In addition, in automotive and small motors that require high torque, since a high energy density PM is used, the magnetic flux cannot be weakened by field weakening, and high rotation cannot be achieved. Therefore, there is a strong need to develop a new configuration for PM motors that can achieve efficient field weakening with less d -axis current. Furthermore, to generate more cancellation flux efficiently to the PM flux with less d -axis current, a new PM motor that can increase d -axis inductance is essential.

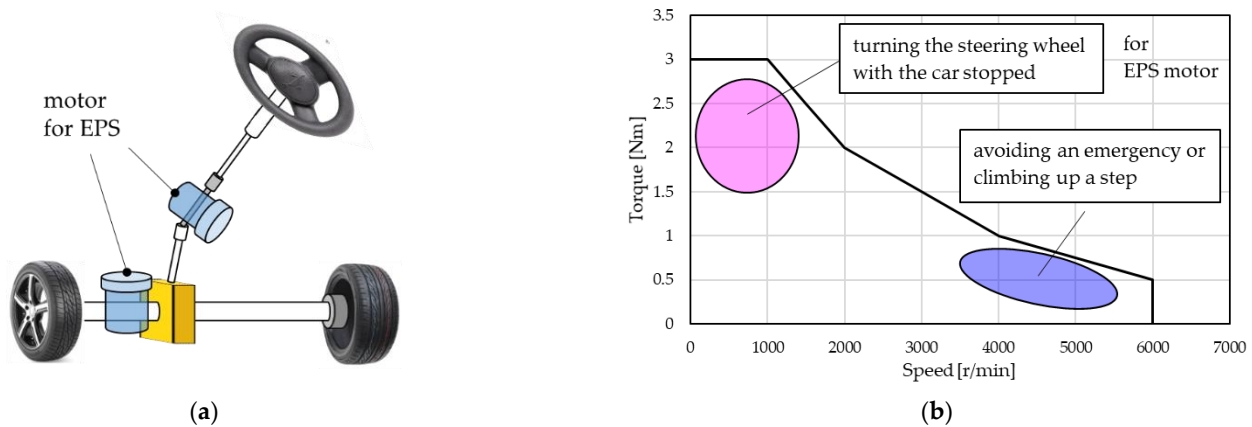


Figure 1. Motor operation of EPS system: (a) steering system diagram; (b) N-T characteristics.

The authors have investigated a novel consequent-pole-type PM motor with a high-speed operation capability with less field weakening current [25,26]. The proposed PM motor has a pair of image poles of a core using high permeability material for a pair of N and S magnet poles on the rotor. Figure 2 shows the motor cross-section of a conventional consequent-pole PM motor and a novel consequent-pole-type PM motor and the reluctance on the rotor surface viewed from each phase. However, the winding connection method is a three-phase star configuration in which the U1-phase coil and U2-phase coil are connected in series, and the reluctance that changes for the rotation of the rotor is represented by approximating a sinusoidal waveform. From Figure 2, it can be seen that the reluctance of each phase of the conventional consequent-pole PM motor is unbalanced. This unbalance increases the harmonic content in the air gap, causing motor vibration and noise. As a countermeasure, it is possible to improve the impedance imbalance by making the rotor a tandem structure or reducing the outer diameter of the core of the image pole. However, the performance of the motor is degraded due to a decrease in the effective magnetic flux. In addition, by high-frequency component injection on the input power, the harmonics in the air gap can be canceled. However, since the harmonics are even-numbered, it is necessary to devise the neutral point, and advanced motor drive technology is required [27–30].

On the other hand, the reluctance of each phase of the proposed motor is balanced, as shown in Figure 2b. Therefore, there is no need to take countermeasures against the harmful effects of unbalance, which were necessary for the conventional type. Furthermore, since a pair of image poles of a core using high permeability material for a pair of N and S magnet poles on the rotor is arranged, the proposed motor increased the d -axis inductance.

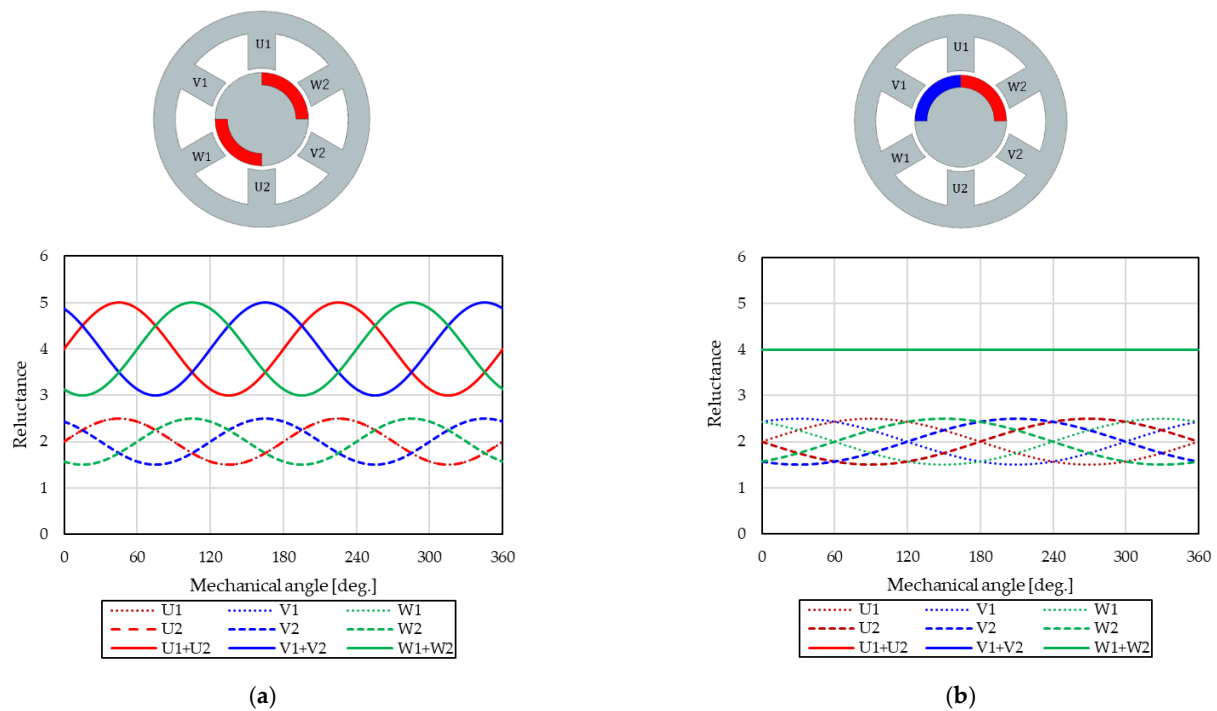


Figure 2. Motor cross-section and the reluctance on the rotor surface viewed from each phase: (a) conventional consequent-pole-type PM motors; (b) novel consequent-pole-type PM motor.

However, the detailed magnetic flux weakening mechanism of the novel consequent-pole-type motor with unique rotor structure has not yet been clarified by analysis and experiment. In addition, it is not theoretically clear whether the proposed motor can be driven by the same theory as before because the magnet arrangement in the proposed rotor is unique. The purpose of this study is to construct a control theory of the motor drive for a novel consequent-pole-type PM motor with a unique rotor structure and to clarify the potential of the new proposed motor by analysis and experiment. In this paper, after explaining the detailed flux-weakening mechanism caused by a novel consequent-pole-type PM motor with an unusual rotor structure, a mathematical model (voltage and torque equation) of the proposed motor with a unique magnetic circuit structure is derived for the construction of a control theory for the motor drive. Then, the results are verified through analysis and experiment. In addition, the essential operating characteristics are compared with those of a standard SPM motor and a conventional consequent-pole PM motor.

2. Overview of Novel Consequent-Pole PM Motor

2.1. Motor Structure and Driving Principle

Figure 3a shows the proposed consequent-pole motor structure having the minimum number of poles and slots. Figure 3b shows the winding connection diagram of the proposed motor. The novel consequent-pole motor employs a four-pole (two real magnetic poles and two image poles) and six-slot concentrated winding structure. The magnet arrangement of the rotor is SPM type. The winding connection method is a three-phase star configuration in which a coil facing a magnetic pole and a winding facing an image pole in the same phase are connected in series. Figure 4 shows the flux linkage waveforms of the U1-phase winding facing the magnet pole and the U2-phase winding facing the image pole at no load and field weakening. At field weakening, the linkage magnetic flux between the U1-phase coil facing the magnetic pole and the U2-phase coil facing the image pole has an antiphase relationship and the e.m.f. in each winding is canceled because the U1-phase coil and U2-phase coil are connected in series. This feature is the same in the V-phase and the W-phase. This cancellation of the e.m.f. would not be expected in conventional consequent-pole-type PM motors. Therefore, the field weakening effect is

increased by the e.m.f. cancellation in addition to the cancellation of the magnetic flux by the d -axis armature flux, so that high-speed drive is possible. The motor's slots per pole per phase (SPP) in Figure 3 are 0.5. Even if SPP are one (four-pole and twelve-slot) or two (four-pole and twenty-four-slot), the principle of this proposed motor holds. As a configurable condition, the stator configuration is such that the windings facing the magnet poles and the windings facing the image poles in each phase can be connected in series, and the ratio of these windings is 1:1.

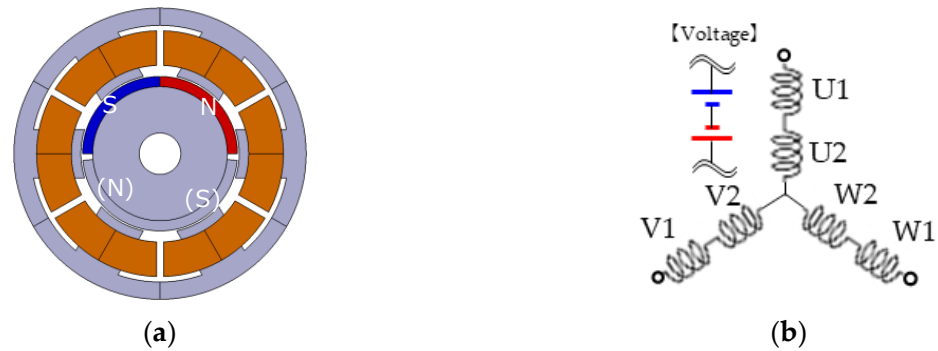


Figure 3. Proposed consequent-pole motor model: (a) Motor cross section; (b) Winding connection.

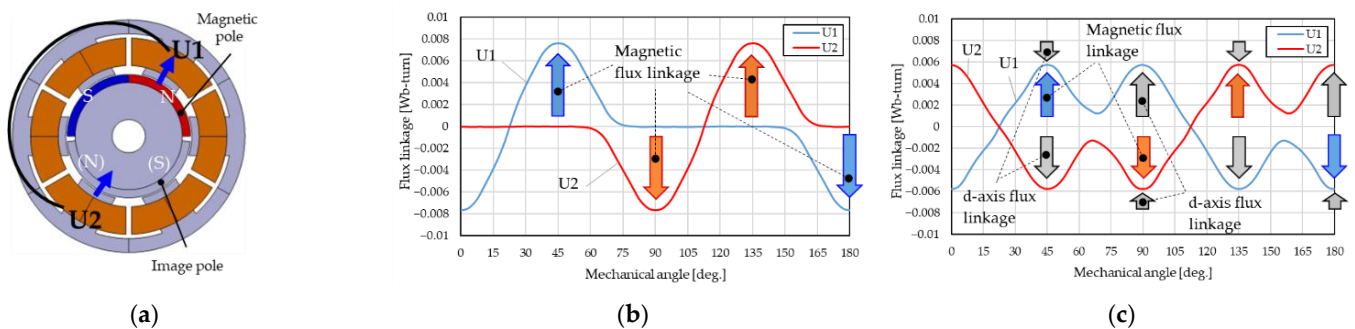


Figure 4. Outline of flux cancellation at field weakening: (a) Motor cross-section; (b) Flux linkage at no load; (c) Flux linkage at field weakening.

Furthermore, although the magnetic circuit of the rotor is necessarily unbalanced, the impedance of the stator winding can be balanced if the windings facing the magnet pole pair and the windings facing the image pole pair are connected in series. Therefore, the three-phase balanced winding does not affect the motor current control despite the unbalanced rotor magnetic circuit, and the magnetic circuit of the image pole part on the rotor can be designed freely.

2.2. Effect of Adjustable Speed

Figure 5 shows the U-phase flux linkage waveform at no load and field weakening ($I_d = -30$ A) and the FFT analysis results. From Figure 5, it can be seen that the U-phase flux linkage can be varied by field weakening ($I_d = -30$ A) because the primary component of the flux linkage at field weakening is 12% of the primary component of the flux linkage at no load. Figure 6 shows the N-T characteristics of the proposed motor at $I_d = 0$ control and $I_d = -30$ A. It can be seen from Figure 6 that the rotational speed at low load is increased by lowering the back e.m.f. due to field weakening.

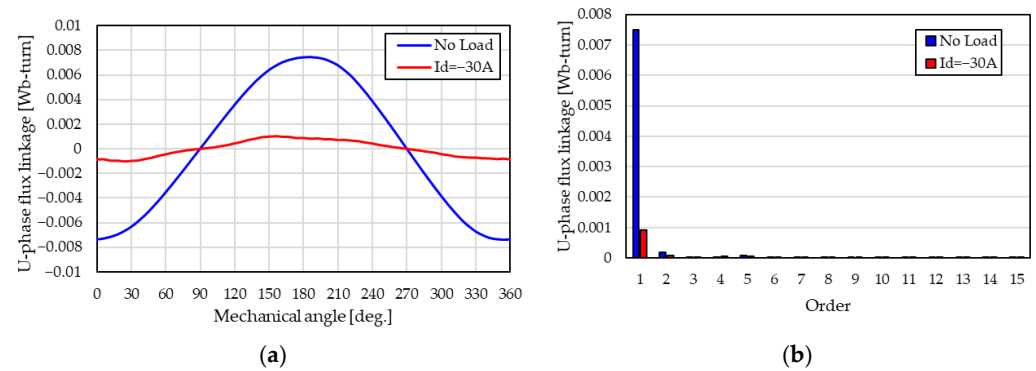


Figure 5. Outline of adjustable flux linkage at field weakening: (a) flux linkage waveform; (b) FFT analysis result.

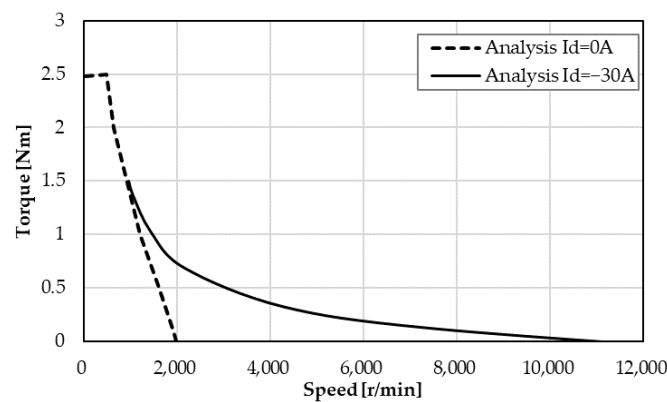


Figure 6. N-T characteristics at $I_d = 0$ A and field weakening.

2.3. Mathematical Model Derivation

2.3.1. Magnetic Circuit of the Novel Consequent-Pole PM Motor

The basic structure of a conventional SPM motor is two poles and three slots. The mathematical expression is carried out by solving the three-phase magnetic circuit and deriving the voltage equation. However, the rotor structure of the novel consequent-pole motor has magnetic poles and image poles for each NS pole pair. Therefore, in the configuration of the proposed motor, the minimum number of poles is four poles (magnetic pole: two poles, image pole: two poles). Furthermore, the minimum number of windings is six since the stator must have two windings that can be connected in series for each phase. Therefore, in this study, the basic design of the proposed motor has six slots. However, since the winding coefficient is not considered in deriving the mathematical model, it can be applied to any SPP, winding method, and pole-slot number combination, as long as the motor configuration is valid in principle. The mathematical model of the novel consequent-pole motor is derived by solving the six-phase magnetic circuit and deriving the voltage equation [31,32].

Figures 7 and 8 show the magnetic circuit structure diagram and the six-phase magnetic equivalent circuit of the novel consequent-pole motor. In Figure 7, R_s is the reluctance of the stator back yoke, R_t is the reluctance of the stator teeth, R_g is the reluctance of the air gap between stator and rotor, R_r is the reluctance of the rotor, and R_{pu} , R_{pv} , R_{pw} , R'_{pu} , R'_{pv} , and R'_{pw} are the reluctances of the rotor core surface. N is the number of coil turns in each phase. R_{pu} , R_{pv} , R_{pw} , R'_{pu} , R'_{pv} , and R'_{pw} are variable depending on the rotation position because the reluctance of the magnetic pole part and the image pole part have a different value. Since the reluctance of the rotor core surface in each phase is a square wave with respect to rotor rotation, it is expressed by Equations (1)–(6) using the Fourier series. When the U-phase axis coincides with the d -axis, $\theta = 0$ (electric angle) and R_{ac} is the square wave amplitude. The reason why Equations (4)–(6) have the opposite sign of Equations (1)–(3) is

that Equations (4)–(6) have the opposite phase of Equations (1)–(3). However, the model is expressed using the Fourier series to make the model similar to the real reluctance. In this way, the validity of the mathematical model can be judged by comparing it with the analysis results for each detailed part.

$$R_{pu} = R_{ac} \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{1}{(2n-1)} \sin \left((2n-1) \left(\frac{\theta}{2} + \frac{\pi}{2} + \frac{\pi}{4} \right) \right) = R_{ac} \Sigma_u \quad (1)$$

$$R_{pv} = R_{ac} \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{1}{(2n-1)} \sin \left((2n-1) \left(\frac{\theta}{2} + \frac{\pi}{2} + \frac{\pi}{4} - \frac{\pi}{3} \right) \right) = R_{ac} \Sigma_v \quad (2)$$

$$R_{pw} = R_{ac} \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{1}{(2n-1)} \sin \left((2n-1) \left(\frac{\theta}{2} + \frac{\pi}{2} + \frac{\pi}{4} - \frac{2\pi}{3} \right) \right) = R_{ac} \Sigma_w \quad (3)$$

$$R'_{pu} = -R_{ac} \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{1}{(2n-1)} \sin \left((2n-1) \left(\frac{\theta}{2} + \frac{\pi}{2} + \frac{\pi}{4} \right) \right) = -R_{ac} \Sigma_u \quad (4)$$

$$R'_{pv} = -R_{ac} \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{1}{(2n-1)} \sin \left((2n-1) \left(\frac{\theta}{2} + \frac{\pi}{2} + \frac{\pi}{4} - \frac{\pi}{3} \right) \right) = -R_{ac} \Sigma_v \quad (5)$$

$$R'_{pw} = -R_{ac} \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{1}{(2n-1)} \sin \left((2n-1) \left(\frac{\theta}{2} + \frac{\pi}{2} + \frac{\pi}{4} - \frac{2\pi}{3} \right) \right) = -R_{ac} \Sigma_w \quad (6)$$

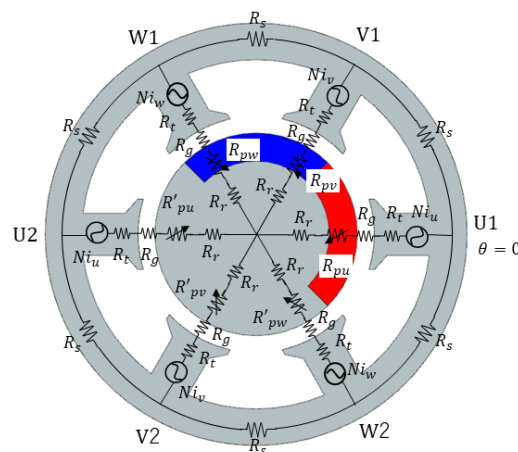


Figure 7. Magnetic circuit structure diagram of the novel consequent-pole PM motor.

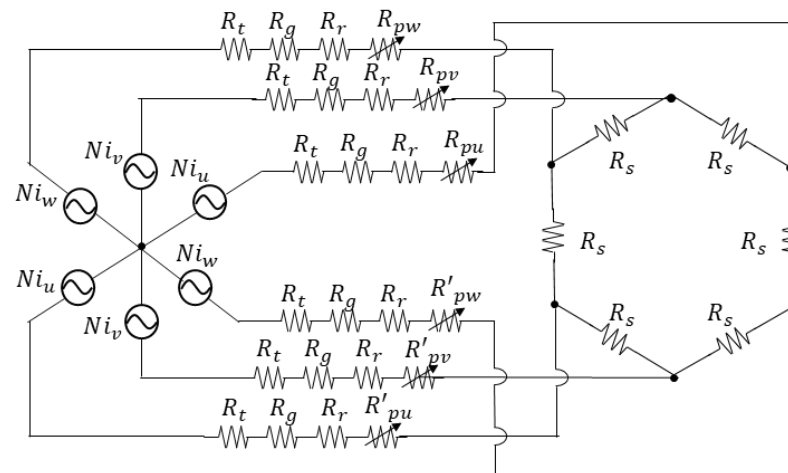


Figure 8. Six-phase magnetic equivalent circuit of the novel consequent-pole PM motor.

2.3.2. Six-Phase Hexagon-Star Transformation

In the six-phase magnetic circuit of Figure 9, the load side connection is a six-phase hexagon configuration. Therefore, a six-phase hexagon-star transformation is performed to solve the magnetic circuit. Figure 9 shows the magnetic circuit diagram of the six-phase hexagon and star configurations. However, the value of each of the six reluctances (R_1 – R_6 , r_1 – r_6) are equivalent (R , r), and the magnetomotive force is a symmetric six-phase alternating current with an amplitude value of E . In each magnetic circuit, the currents (i , i') flowing in one phase are calculated using the superposition theorem. Then, the equivalent conversion of the reluctance is considered from its current value. Figure 10 shows each magnetic circuit when only E_1 is supplied and E_2 to E_6 are shorted.

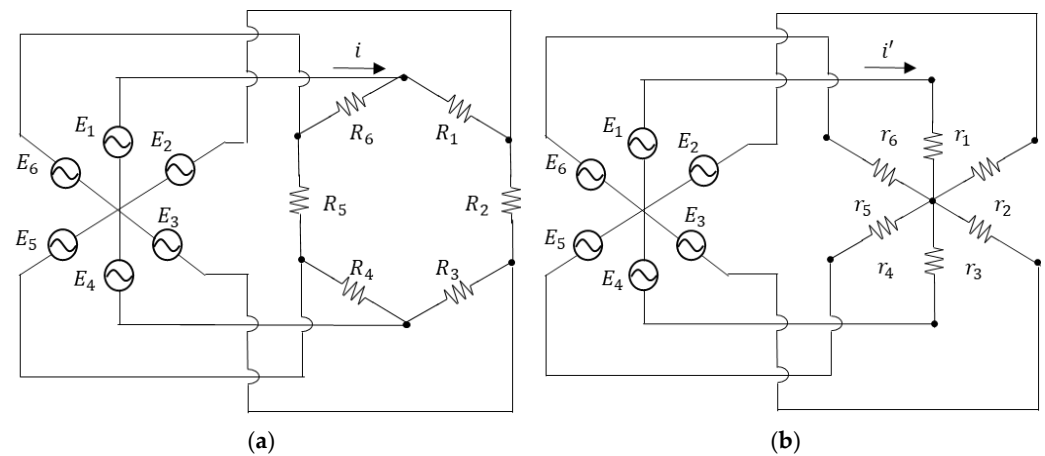


Figure 9. Six-phase magnetic circuit: (a) the load side connection is a hexagon configuration; (b) the load side connection is a star configuration.

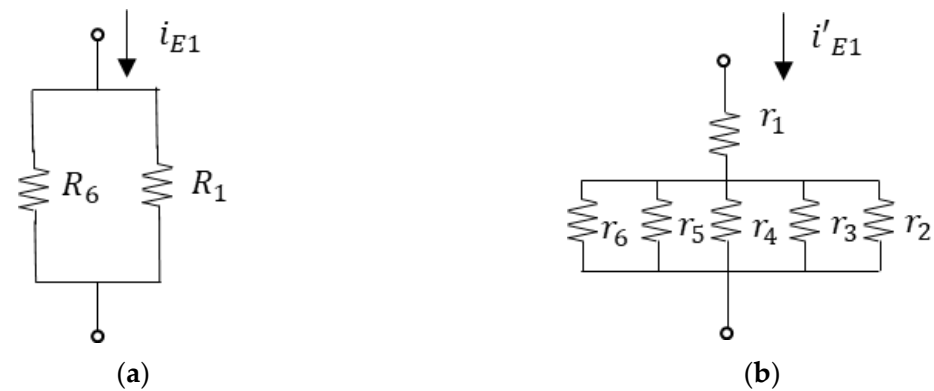


Figure 10. Load side magnetic circuit when only E_1 is supplied: (a) hexagon configuration; (b) star configuration.

The current i_{E1} in Figure 10a can be calculated as follows:

$$i_{E1} = \frac{2E}{R} \sin \theta \quad (7)$$

When only E_2 is supplied, and E_1 and E_3 to E_6 are shorted, the current i_{E2} is as follows:

$$i_{E2} = \frac{2E}{R} \sin(\theta - 60) \quad (8)$$

Therefore, the current i_{R1} flowing in R_1 is expressed by:

$$i_{R1} = \frac{1}{2}i_{E1} - \frac{1}{2}i_{E2} = \frac{E}{R} \{\sin \theta - \sin(\theta - 60)\} = \frac{E}{R} \sin(\theta + 60) \quad (9)$$

Similarly, the calculation for the current i_{R6} flowing in R_6 is as follows:

$$i_{R6} = \frac{E}{R} \sin(\theta + 120) \quad (10)$$

Therefore, the current i flowing in one phase is as follows:

$$i = \frac{E}{R} \sin \theta \quad (11)$$

On the other hand, the current i' in the star configuration in Figure 9b can be calculated using the principle of the superposition theorem. The current i'_{E1} in Figure 10b can be calculated as follows:

$$i'_{E1} = \frac{5E}{6r} \sin \theta \quad (12)$$

When only E_2 is supplied, and E_1 and E_3 to E_6 are shorted, the current i'_{E2} is as follows:

$$i'_{E2} = \frac{5E}{6r} \sin(\theta - 60) \quad (13)$$

Similarly, the currents $i'_{E3} \sim i'_{E6}$ can be calculated. Therefore, the current i' flowing in one phase is as follows:

$$\begin{aligned} i' &= i'_{E1} - \frac{1}{5}(i'_{E2} + i'_{E3} + i'_{E4} + i'_{E5} + i'_{E6}) \\ &= \frac{5E}{6r} \sin \theta - \frac{E}{6r} \{\sin(\theta - 60) + \sin(\theta - 120) + \sin(\theta - 180) \\ &\quad + \sin(\theta - 240) + \sin(\theta - 300)\} \\ &= \frac{E}{r} \sin \theta \end{aligned} \quad (14)$$

Therefore, when r is equal to R , the magnetic circuits of the hexagonal and star-connected configuration in Figure 9a,b can be treated as equivalent.

In the six-phase hexagon-star conversion, the resistance value on the load side in hexagon configuration is unchanged, and the connection can be converted from hexagon to star of the six-phase. Figure 11 shows a six-phase magnetic circuit in which the load side connection is a star configuration.

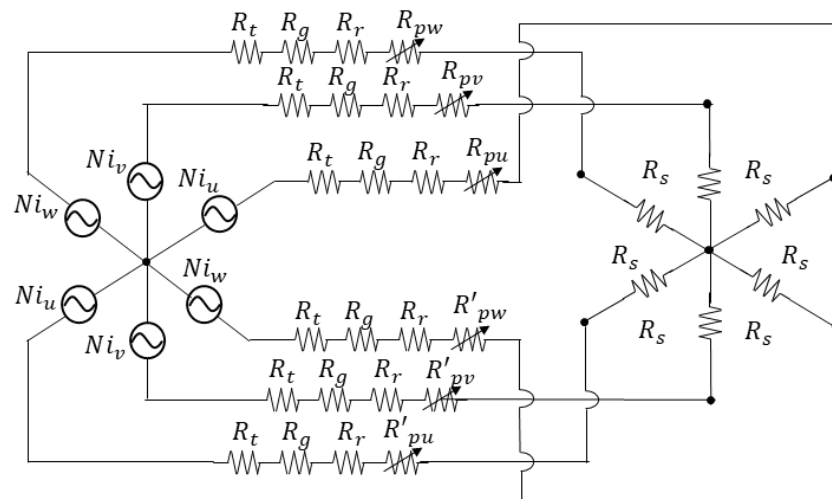


Figure 11. Six-phase star configuration magnetic equivalent circuit of the novel consequent-pole PM motor.

2.3.3. Derivation of the Theoretical Equation for the Armature Flux

The armature flux of the U1-phase coil is derived from the six-phase magnetic circuit shown in Figure 11. The reluctance of each phase is expressed by:

$$R_u = R'_{dc} + R_{ac} \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{1}{(2n-1)} \sin((2n-1)\theta') = R'_{dc} + R_{ac}\Sigma_u \quad (15)$$

$$R_v = R'_{dc} + R_{ac} \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{1}{(2n-1)} \sin\left((2n-1)\left(\theta' - \frac{\pi}{3}\right)\right) = R'_{dc} + R_{ac}\Sigma_v \quad (16)$$

$$R_w = R'_{dc} + R_{ac} \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{1}{(2n-1)} \sin\left((2n-1)\left(\theta' - \frac{2\pi}{3}\right)\right) = R'_{dc} + R_{ac}\Sigma_w \quad (17)$$

$$R'_u = R'_{dc} - R_{ac} \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{1}{(2n-1)} \sin((2n-1)(\theta')) = R'_{dc} - R_{ac}\Sigma_u \quad (18)$$

$$R'_v = R'_{dc} - R_{ac} \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{1}{(2n-1)} \sin\left((2n-1)\left(\theta' - \frac{\pi}{3}\right)\right) = R'_{dc} - R_{ac}\Sigma_v \quad (19)$$

$$R'_w = R'_{dc} - R_{ac} \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{1}{(2n-1)} \sin\left((2n-1)\left(\theta' - \frac{2\pi}{3}\right)\right) = R'_{dc} - R_{ac}\Sigma_w \quad (20)$$

where,

$$\theta' = \frac{\theta}{2} + \frac{\pi}{2} + \frac{\pi}{4}, \quad R'_{dc} = R_t + R_g + R_r + R_s$$

This reluctance is used to solve the six-phase magnetic circuit. R'_d is the DC component term, and R_{ac} is the amplitude of the AC component. The Fourier series is simply expressed as Σ_u , Σ_v , and Σ_w for each phase.

When the superposition theorem is used for a six-phase magnetic circuit, the armature flux (Φ_{uu1}) flowing from the U1-phase is generated by the magnetomotive force of the U1-phase, and the armature fluxes (Φ_{uv1} , Φ_{uw1} , Φ_{uu2} , Φ_{uv2} , Φ_{uw2}) flowing into the U1-phase are generated by the magnetomotive force of the V1, W1, U2, V2, and W2-phases. Therefore, the armature magnetic flux (Φ_{U1}) flowing in the U1-phase can be expressed by:

$$\begin{aligned} \Phi_{U1} &= \phi_{uu1} - (\phi_{uu2} + \phi_{uv1} + \phi_{uw1} + \phi_{uv2} + \phi_{uw2}) \\ &= \frac{Ni_u}{A} (R_w R'_u R'_w (R'_v + R_v) + R_v R'_u R'_v (R'_w + R_w)) \\ &\quad - \frac{Ni_v}{A} (R_w R'_u R'_w (R'_v + R_v)) - \frac{Ni_w}{A} (R_v R'_u R'_v (R'_w + R_w)) \\ &= \frac{Ni_u}{A} \left(4R'^2_{dc} (R'^2_{dc} - R^2_{ac}) - 4R'_{dc} R_{ac} (R'^2_{dc} - R^2_{ac}) \Sigma_u \right) \\ &\quad - \frac{Ni_v}{A} \left(2R'^2_{dc} (R'^2_{dc} - R^2_{ac}) - 2R'_{dc} R_{ac} (R'^2_{dc} - R^2_{ac}) \Sigma_u \right) \\ &\quad - \frac{Ni_w}{A} \left(2R'^2_{dc} (R'^2_{dc} - R^2_{ac}) - 2R'_{dc} R_{ac} (R'^2_{dc} - R^2_{ac}) \Sigma_u \right) \\ &= L_{u1} i_u + M_{uv1} i_v + M_{uw1} i_w \end{aligned} \quad (21)$$

where,

$$\begin{aligned} A &= R_v R_w R'_u R'_v R'_w + R_u R_w R'_u R'_v R'_w + R_u R_v R'_u R'_v R'_w \\ &\quad + R_u R_v R_w R'_u R'_v + R_u R_v R_w R'_v R'_w + R_u R_v R_w R'_u R'_w = 6R'^2_{dc} (R'^2_{dc} - R^2_{ac})^2 \end{aligned}$$

The self-inductance of the U1-phase coil is L_{u1} , the mutual inductance between the U-phase and V-phase is M_{uv1} , and the mutual inductance between the U-phase and W-phase is M_{uw1} .

The armature flux (Φ_{V1} , Φ_{W1} , Φ_{U2} , Φ_{V2} , Φ_{W2}) flowing in the V1, W1, U2, V2, and W2-phases can be derived in a similar way to the derivation of the armature magnetic flux (Φ_{U1}) flowing in the U1-phase. The winding connection of the novel consequence-pole

motor is a three-phase configuration connecting in-phase windings in series. Therefore, the theoretical value of the armature flux flowing in the U-phase is the sum of the armature flux flowing in the U1 and U2 phases. Here, only the theoretical equation of the armature flux linkage Φ_U flowing in the U-phase is shown as:

$$\begin{aligned}\phi_U &= N(\phi_{U1} + \phi_{U2}) \\ &= \left(\frac{8R_{dc}'^2(R_{dc}'^2 - R_{ac}^2)}{A} \right) N^2 i_u - \left(\frac{4R_{dc}'^2(R_{dc}'^2 - R_{ac}^2)}{A} \right) N^2 i_v - \left(\frac{4R_{dc}'^2(R_{dc}'^2 - R_{ac}^2)}{A} \right) N^2 i_w \\ &= L_u i_u + M_{uv} i_v + M_{uw} i_w\end{aligned}\quad (22)$$

The coefficient of the U-phase current is the self-inductance L_u of the U-phase, the coefficient of the V-phase current is the mutual inductance M_{uv} between the U-phase and the V-phase, and the coefficient of the W-phase current is the mutual inductance M_{uw} between the U-phase and the W-phase.

When the theoretical equation of the armature flux linkage of the V and W phases are calculated as well as the U-phase, the armature flux linkage flowing in each phase is as follows:

$$\begin{aligned}\begin{bmatrix} \phi_u \\ \phi_v \\ \phi_w \end{bmatrix} &= \begin{bmatrix} L_u & M_{uv} & M_{uw} \\ M_{uv} & L_v & M_{vw} \\ M_{uw} & M_{vw} & L_w \end{bmatrix} \begin{bmatrix} i_u \\ i_v \\ i_w \end{bmatrix} \\ &= \begin{bmatrix} \left(\frac{8R_{dc}'^2(R_{dc}'^2 - R_{ac}^2)}{A} \right) & -\left(\frac{4R_{dc}'^2(R_{dc}'^2 - R_{ac}^2)}{A} \right) & -\left(\frac{4R_{dc}'^2(R_{dc}'^2 - R_{ac}^2)}{A} \right) \\ -\left(\frac{4R_{dc}'^2(R_{dc}'^2 - R_{ac}^2)}{A} \right) & \left(\frac{8R_{dc}'^2(R_{dc}'^2 - R_{ac}^2)}{A} \right) & -\left(\frac{4R_{dc}'^2(R_{dc}'^2 - R_{ac}^2)}{A} \right) \\ -\left(\frac{4R_{dc}'^2(R_{dc}'^2 - R_{ac}^2)}{A} \right) & -\left(\frac{4R_{dc}'^2(R_{dc}'^2 - R_{ac}^2)}{A} \right) & \left(\frac{8R_{dc}'^2(R_{dc}'^2 - R_{ac}^2)}{A} \right) \end{bmatrix} \begin{bmatrix} i_u \\ i_v \\ i_w \end{bmatrix} \\ &= \begin{bmatrix} L_{dc} & -\frac{L_{dc}}{2} & -\frac{L_{dc}}{2} \\ -\frac{L_{dc}}{2} & L_{dc} & -\frac{L_{dc}}{2} \\ -\frac{L_{dc}}{2} & -\frac{L_{dc}}{2} & L_{dc} \end{bmatrix} \begin{bmatrix} i_u \\ i_v \\ i_w \end{bmatrix}\end{aligned}\quad (23)$$

It is found that the self-inductance and the mutual inductance of each phase are only the DC component.

2.3.4. Derivation of the Voltage Equation of the Novel Consequent-Pole PM Motor

The three-phase voltage equation is derived from the armature flux linkage and the magnetic flux linkage. The coil connection of the novel consequent-pole motor is a three-phase configuration connecting in-phase coils in series. Therefore, the e.m.f. of the in-phase winding can be calculated by addition. To add the e.m.f. of the winding facing the magnet pole and that of the winding facing the image-pole together, the theoretical equations of the magnetic flux linkage of each phase (Ψ_{u-mag} , Ψ_{v-mag} , Ψ_{w-mag}) and the flux linkage of the three-phases (Ψ_u , Ψ_v , Ψ_w) can be expressed by:

$$\psi_{u-mag} = \psi_{u1-mag} + \psi_{u2-mag} = \psi_f \cos \theta \quad (24)$$

$$\psi_{v-mag} = \psi_{v1-mag} + \psi_{v2-mag} = \psi_f \cos \left(\theta - \frac{2}{3} \pi \right) \quad (25)$$

$$\psi_{w-mag} = \psi_{w1-mag} + \psi_{w2-mag} = \psi_f \cos \left(\theta + \frac{2}{3} \pi \right) \quad (26)$$

$$\begin{aligned} \begin{bmatrix} \psi_u \\ \psi_v \\ \psi_w \end{bmatrix} &= \begin{bmatrix} \phi_u \\ \phi_v \\ \phi_w \end{bmatrix} + \begin{bmatrix} \psi_{u-mag} \\ \psi_{v-mag} \\ \psi_{w-mag} \end{bmatrix} \\ &= \begin{bmatrix} L_u & M_{uv} & M_{uw} \\ M_{uv} & L_v & M_{vw} \\ M_{uw} & M_{vw} & L_w \end{bmatrix} \begin{bmatrix} i_u \\ i_v \\ i_w \end{bmatrix} + \begin{bmatrix} \psi_f \cos \theta \\ \psi_f \cos(\theta - \frac{2}{3}\pi) \\ \psi_f \cos(\theta + \frac{2}{3}\pi) \end{bmatrix} \end{aligned} \quad (27)$$

Ψ_f is the amplitude value of the magnetic flux linkage to each phase, L_u , L_v , and L_w are the self-inductance of each phase coil, and M_{uv} , M_{uv} , and M_{uw} are the mutual inductances.

The voltage of each phase can be obtained as the sum of the voltage drops due to the winding resistance and the e.m.f. obtained by differentiating the armature flux linkage of each phase with time, and is expressed by the following equation:

$$\begin{bmatrix} V_u \\ V_v \\ V_w \end{bmatrix} = Ri + \frac{d}{dt} \left\{ \begin{bmatrix} L_u & M_{uv} & M_{uw} \\ M_{uv} & L_v & M_{vw} \\ M_{uw} & M_{vw} & L_w \end{bmatrix} \begin{bmatrix} i_u \\ i_v \\ i_w \end{bmatrix} + \begin{bmatrix} \psi_{u-mag} \\ \psi_{v-mag} \\ \psi_{w-mag} \end{bmatrix} \right\} \quad (28)$$

$$\vec{v} = R\vec{i} + \frac{d}{dt} (L\vec{i} + \vec{\psi}) \quad (29)$$

The voltage equation of the three-phase coordinate system is transformed into that of a two-phase static coordinate system ($\alpha\beta$ coordinate system) using the transformation matrix C . However, C^T is a transposed matrix of C , and $C^T C$ is a unit matrix, and absolute transformation in which the instantaneous power is invariant before and after the coordinate transformation was used. The transformation equation and transformation matrix are shown as:

$$\begin{aligned} \vec{v} &= R\vec{i} + \frac{d}{dt} (L\vec{i}) + L \frac{d}{dt} \left(\frac{\vec{i}}{\omega} \right) + \frac{d}{dt} \vec{\psi} \\ C\vec{v} &= CR\vec{i} + C \frac{dL}{dt} \vec{i} + CL \frac{d}{dt} \left(\frac{\vec{i}}{\omega} \right) + \omega C \frac{d}{d\theta} \vec{\psi} \\ C\vec{v} &= RC\vec{i} + CLC^T \frac{d}{dt} C \left(\frac{\vec{i}}{\omega} \right) + \omega C \frac{d}{d\theta} \vec{\psi} \end{aligned} \quad (30)$$

$$\vec{v}_{\alpha\beta} = Ri_{\alpha\beta} + L_{\alpha\beta} \frac{d}{dt} \left(\frac{\vec{i}_{\alpha\beta}}{\omega} \right) + \omega \vec{\psi}_{\alpha\beta} \quad (31)$$

$$C = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \quad (32)$$

The voltage equation of the two-phase static coordinate system ($\alpha\beta$ coordinate system) is transformed into that of the dq coordinate system using the transformation matrix D . However, D^T is the transposed matrix of D , and $D^T D$ is the unit matrix. However, the transformation equation, transformation matrix, and the voltage equation of the dq coordinate system can be shown as:

$$\begin{aligned} D\vec{v}_{\alpha\beta} &= DR\vec{i}_{\alpha\beta} + DL_{\alpha\beta} \frac{d}{dt} \left(\frac{\vec{i}_{\alpha\beta}}{\omega} \right) + D\omega \vec{\psi}_{\alpha\beta} \\ D\vec{v}_{\alpha\beta} &= DR\vec{i}_{\alpha\beta} + DL_{\alpha\beta} \frac{d}{dt} D^T D \left(\frac{\vec{i}_{\alpha\beta}}{\omega} \right) + D\omega \vec{\psi}_{\alpha\beta} \\ D\vec{v}_{\alpha\beta} &= DR\vec{i}_{\alpha\beta} + DL_{\alpha\beta} \left(\frac{d}{dt} D^T \vec{i}_{dq} + D^T \frac{d}{dt} \vec{i}_{dq} \right) + \omega D \vec{\psi}_{\alpha\beta} \\ \vec{v}_{dq} &= Ri_{dq} + DL_{\alpha\beta} \frac{d}{dt} D^T \vec{i}_{dq} + DL_{\alpha\beta} D^T \frac{d}{dt} \vec{i}_{dq} + \omega \vec{\psi}_{dq} \end{aligned} \quad (33)$$

$$D = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \quad (34)$$

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R_a & 0 \\ 0 & R_a \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega \begin{bmatrix} 0 & -\frac{3}{2}L_{dc} \\ \frac{3}{2}L_{dc} & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{3}{2}L_{dc} & 0 \\ 0 & \frac{3}{2}L_{dc} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega \vec{\psi}_a \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (35)$$

Equation (35) shows that both the d -axis inductance and the q -axis inductance are $3 L_{dc}/2$, which is a DC component only. Therefore, the voltage equation of the novel consequent-pole PM motor is the same as that of the general SPM motor.

The torque equation is also derived from the voltage equation in Equation (35). From Equation (35), the power equation is as follows:

$$\begin{aligned} \begin{bmatrix} v_d \\ v_q \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} &= \begin{bmatrix} R_a & 0 \\ 0 & R_a \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega \begin{bmatrix} 0 & -L_q \\ L_d & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} \\ &+ \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ \omega \psi_a \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} \\ &= R_a (i_d^2 + i_q^2) + \{ \omega (-L_q i_d i_q + L_d i_d i_q) + \omega \psi_a i_q \} \end{aligned} \quad (36)$$

The first term in Equation (36) is the copper loss, and the second is the motor's output. Therefore, the torque T is obtained from the second term in Equation (36) as follows:

$$T = \frac{\omega (-L_q i_d i_q + L_d i_d i_q) + \omega \psi_a i_q}{\omega} = \psi_a i_q + (L_d - L_q) i_d i_q \quad (37)$$

From the above, the voltage and torque equation of the novel consequent-pole PM motor could be derived, and the control theory for the proposed motor drive could be constructed.

2.3.5. Mathematical Model Validation by Analysis

Figure 12 shows a cross-section of the verification motor structure of the novel consequence-pole PM motor, and Table 1 shows the specification of the proposed motor. In the mathematical model, leakage flux and magnetic saturation are not considered. In order to minimize the leakage flux, the structure of the novel consequence-pole motor is straight teeth in the analysis model. To avoid being affected by magnetic saturation, the relative permeability of the stator core and the rotor core is made constant in the analysis model. The motor is operated at 1000 r/min. θ is zero when the center of the U1-phase coil coincides with the d -axis of the N-pole magnet. The number of coil turns in each phase is 32.

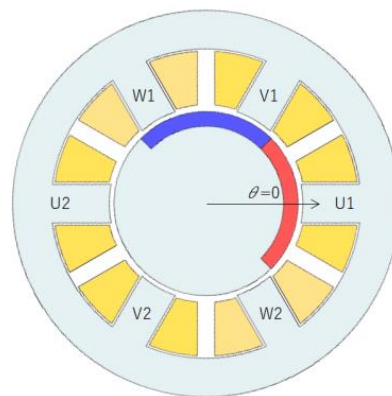
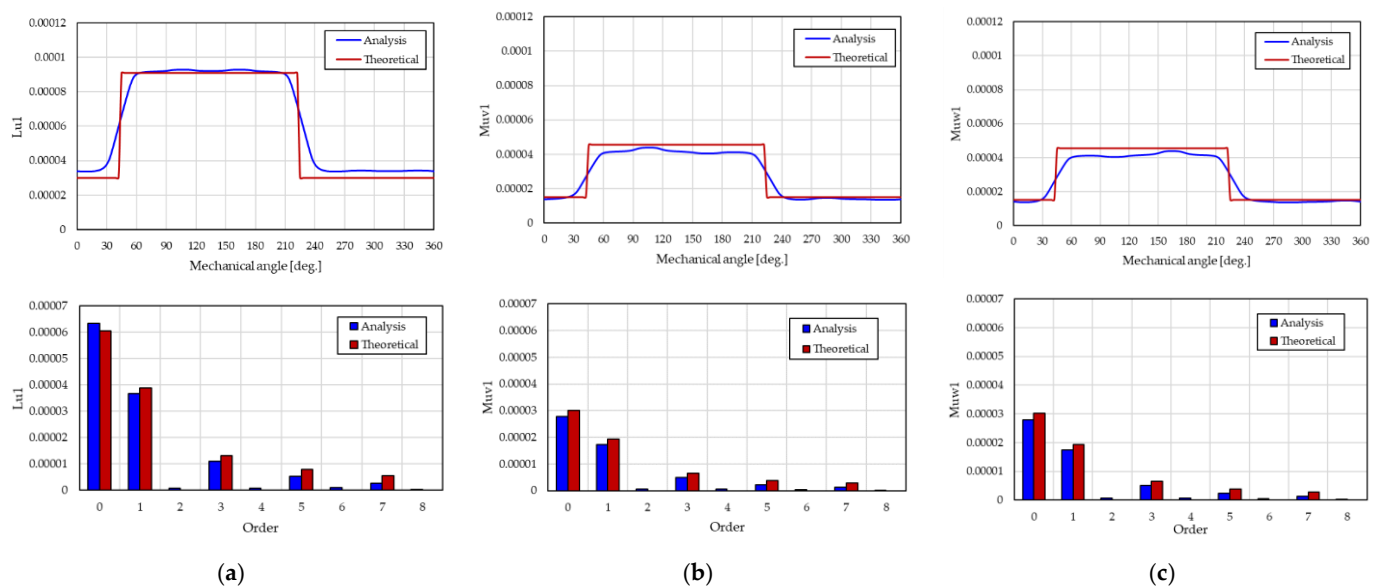


Figure 12. Cross-section of the verification motor structure of the novel consequence-pole PM motor.

First, the inductance in the derivation process of the equation is compared with the analytical value. In order to analyze the inductance, the magnet parts in the analysis model were replaced with air, DC of 1 A was applied, and the inductance was calculated from the armature flux linkage when the motor was operated at 1000 r/min and compared with the inductance of the mathematical model equation. Figure 13 compares the inductance (L_{u1} , M_{uv1} , M_{vw1}) of the analytical results and that of Equation (21) and their FFT results. However, the reluctance of the mathematical model has been calculated from the dimensions of the verification motor model. The relative permeability of the stator and rotor core of the analytical and mathematical model is 5000 to exclude the effect of magnetic saturation.

Table 1. Specifications of a novel consequent-pole PM motor.

Items		Value
number of poles and slots		4 poles, 6 slots
stack length		37 mm
stator	stator outer diameter	90 mm
	stator inner diameter	45 mm
	width of tooth	9 mm
	width of backyoke	9 mm
	relative permeability of core	5000
	winding method	Concentrated windings
	winding connection method	Star configuration
rotor	rotor outer diameter	42.85 mm
	thickness of magnets	3.425 mm
	coercive force of PM	985 kA/m
	remanent flux density of PM	1.29 T
	maximum energy product of PM	322 kJ/m ³

**Figure 13.** Comparison of inductance between analytical and theoretical value in U1-phase: (a) L_{u1} ; (b) M_{uv1} ; (c) M_{uw1} .

From Figure 13, it can be confirmed that the inductance waveform and each order component of the analytical value and the mathematical value in the U1-phase are fairly consistent.

Figure 14 compares the inductance (L_u , M_{uv} , M_{vw}) of the analytical results and that of Equation (22) and their FFT results. Figure 15 shows the armature flux linkage waveform (Φ_u , Φ_{u1} , and Φ_{u2}) when a DC 1 A is applied in the analysis and FFT results. Φ_u is the sum of Φ_{u1} and Φ_{u2} . From Figure 14, it can be confirmed that the DC component of the mathematical inductance in the U-phase was in good agreement with that of the analytical inductance. Moreover, it can be confirmed in Figure 14 that the inductance of the analytical results has only a few even-order harmonic components. This is because in the analysis, as shown in Figure 15, the change of the armature flux linkage when the reluctance of the rotor surface increases with rotor rotation, that is, when the rotor rotation position changes from the image-pole to the magnet pole, and the change of the armature flux linkage when

the reluctance of the rotor surface decreases with rotor rotation, that is, when the rotor rotation position changes from the magnet pole to the image-pole, are not symmetrical. In Figure 15, it can be confirmed that the waveform of Φ_U , which is the sum of Φ_{U1} and Φ_{U2} , has not only a DC component but also an even-order harmonic component.

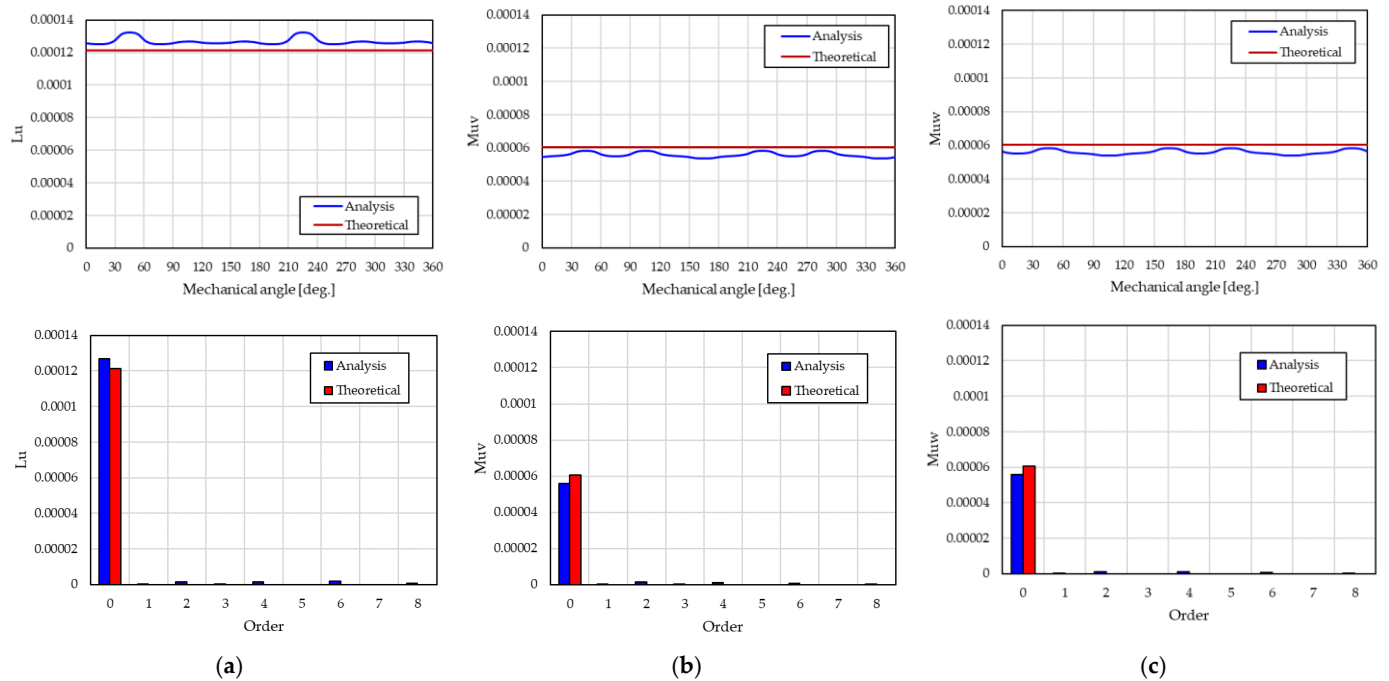


Figure 14. Comparison of inductance between analytical and theoretical value in U-phase: (a) L_u ; (b) M_{uv} ; (c) M_{uw} .

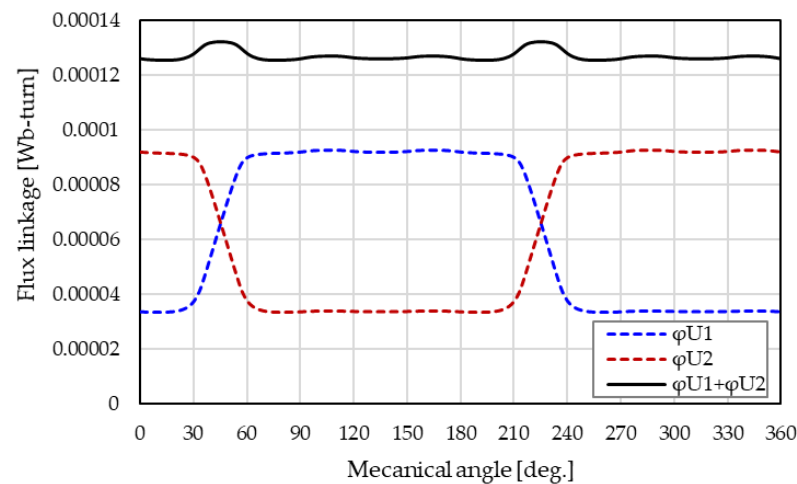


Figure 15. Analytical armature flux linkage waveform (Φ_U , Φ_{U1} , and Φ_{U2}) at 1 A (DC) and FFT results.

On the other hand, in the mathematical model, since the reluctance changes into an ideal rectangular wave shape with rotor rotation, there is no difference between the change in armature flux linkage when the rotor rotation position changes from the image-pole to the magnet pole and the change in armature flux linkage when the rotor rotation position changes from the image-pole to the magnet pole. Therefore, the mathematical model does not generate an even order component in the U-phase inductance. From the above, it was confirmed that, although there was a slight difference in some harmonic components, the inductance in the equation was in good agreement with the analytical value.

In the theoretical formula, the waveform of the number of flux linkages at no-load is defined as a sinusoidal wave. However, it is possible to calculate the amplitude value of the flux linkage using the permeance method. Furthermore, since the proposed PM motor has a pair of image poles of a core for a pair of N and S magnet poles on the rotor, the magnetic flux flows almost between the N and S poles in the rotor core. Therefore, it is possible to calculate the amplitude value of the flux linkage at no-load using a simple design method similar to the conventional motor [32,33].

Figure 16 shows an analysis and theoretical value of the magnetic flux linkage of the U1-phase coil and the U2-phase coil and the magnetic flux linkage of the U-phase, which is the sum of the U1-phase and U2-phase at no load. Figure 16 shows a discrepancy between the analytical and theoretical magnet flux linkage at the pole switching section. The factor is considered to be magnetic leakage flux at the end of the magnet. Furthermore, it can be seen that the magnetic flux linkage of the U-phase has a sinusoidal waveform by the sum of the U1-phase and the U2-phase magnetic flux linkages. It was confirmed that the mathematical magnetic flux approximated a sinusoidal form in each phase was valid. From the above, it is considered that the mathematical model is correctly derived.

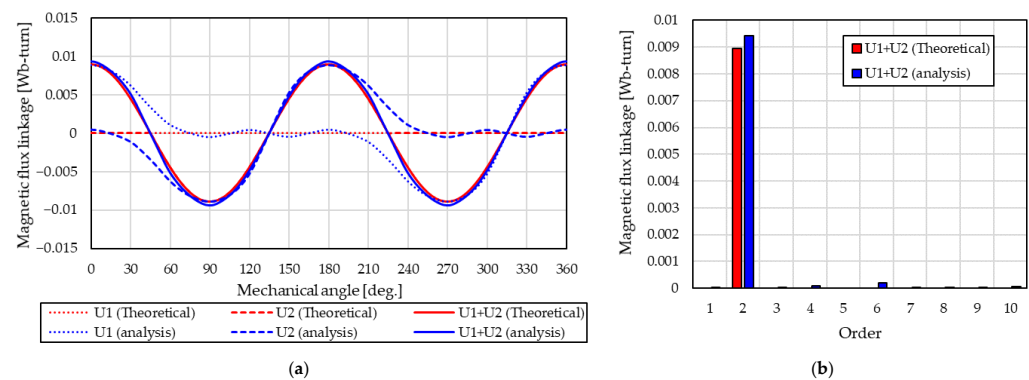


Figure 16. Theoretical and analytical magnetic flux linkage at no load: (a) flux linkage waveform of the U-phase, U1-phase, and the U2-phase; (b) FFT analysis result of the U-phase.

2.3.6. Induced Voltage Cancellation Verification by Analysis

The induced voltage cancellation relation is verified by the flux linkage of the winding facing the image pole and the winding facing the magnet pole. The winding facing the image pole and the winding facing the magnet pole are in series. Figure 17 shows the analytical and theoretical flux linkage of the U1 and U2-phases when only the d -axis current ($I_d = -36.74$ A) is supplied. However, the theoretical value is the sum of the d -axis magnetic flux calculated from Equation (21) and the magnetic flux linkage at no-load derived in the previous section. Figure 17 shows a discrepancy between the analytical and theoretical flux linkage at the pole switching section. It is considered that the factor is magnetic leakage flux at the end of the magnet. Therefore, it was confirmed that the analytical and theoretical values were almost identical. Furthermore, it was confirmed that the flux linkage of the winding facing the magnet pole and that facing the image pole were in opposite phases, and thus the induced voltage was canceled.

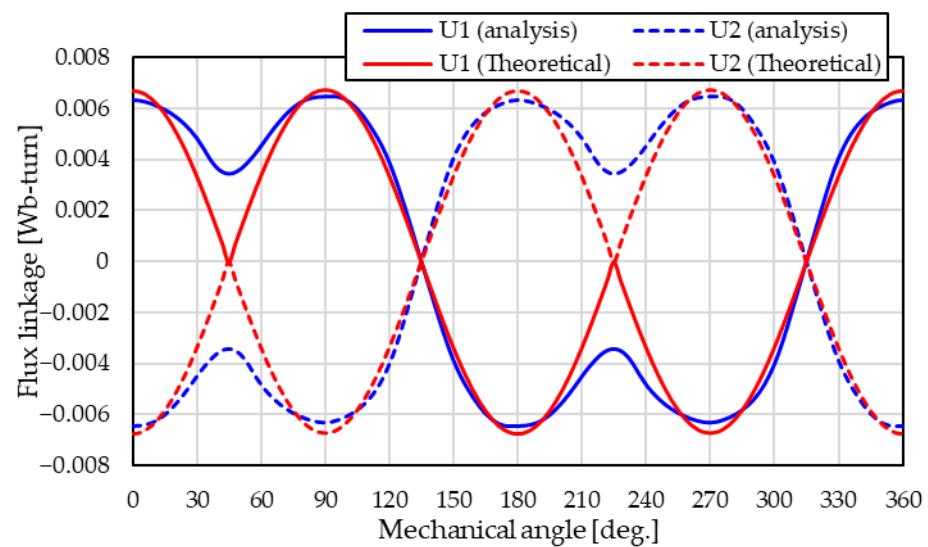


Figure 17. Analytical and theoretical flux linkage of the U1 and U2-phases at $I_d = -36.74$ A.

3. Mathematical Model Validation by Prototype Motor Drive

3.1. Prototype Motor

Figure 18 shows the prototype structure of the novel consequence-pole PM motor, and Table 2 shows the specifications of the prototype motor. The number of poles and slots is eight poles and twelve slots. The outer motor diameter is 81 mm, the motor stack length is 37 mm, and the winding method is a three-phase concentrated winding. The magnet arrangement of the rotor is SPM type, a neodymium sintered magnet is adopted in magnet material, and the stator winding has a connection structure of two series and two parallel lines. In series winding, the winding facing the magnet and the winding facing the image pole are connected in series. A resolver is arranged on the upper part of the motor to detect the rotor rotation position. Figure 19 shows the configuration of the prototype motor of the novel consequence-pole PM motor. The SUS tube covering the magnets and the rotor core are used to prevent scattering of the magnets. Figure 20 shows the measurement system configuration of the prototype motor.

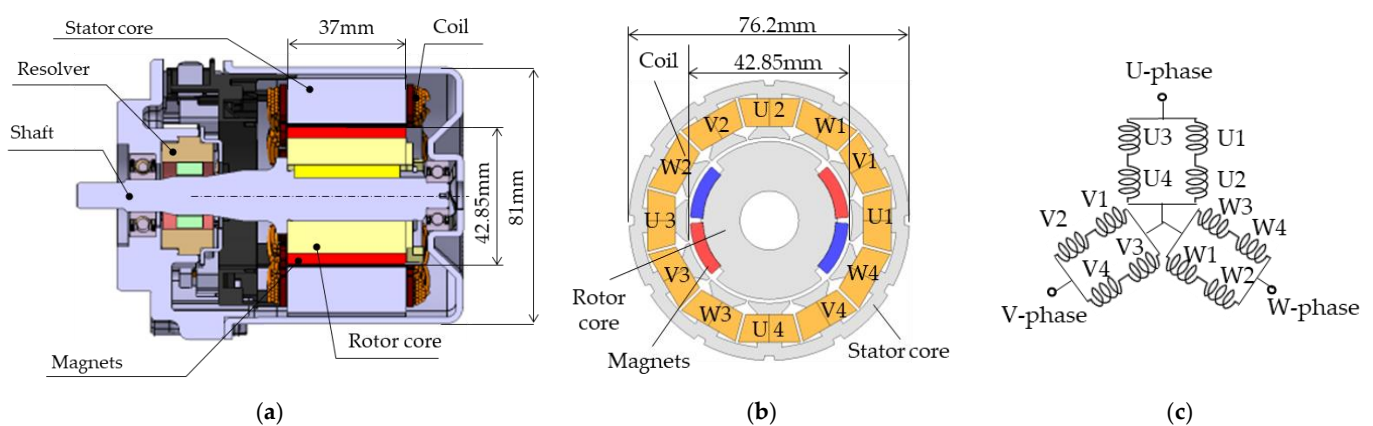
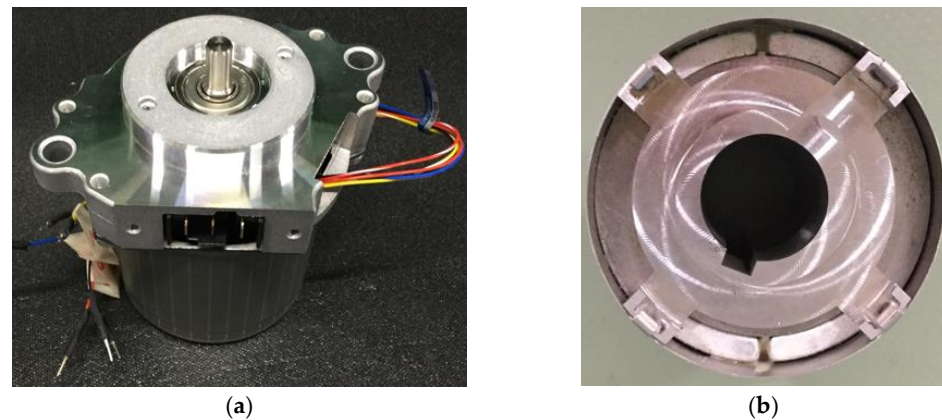
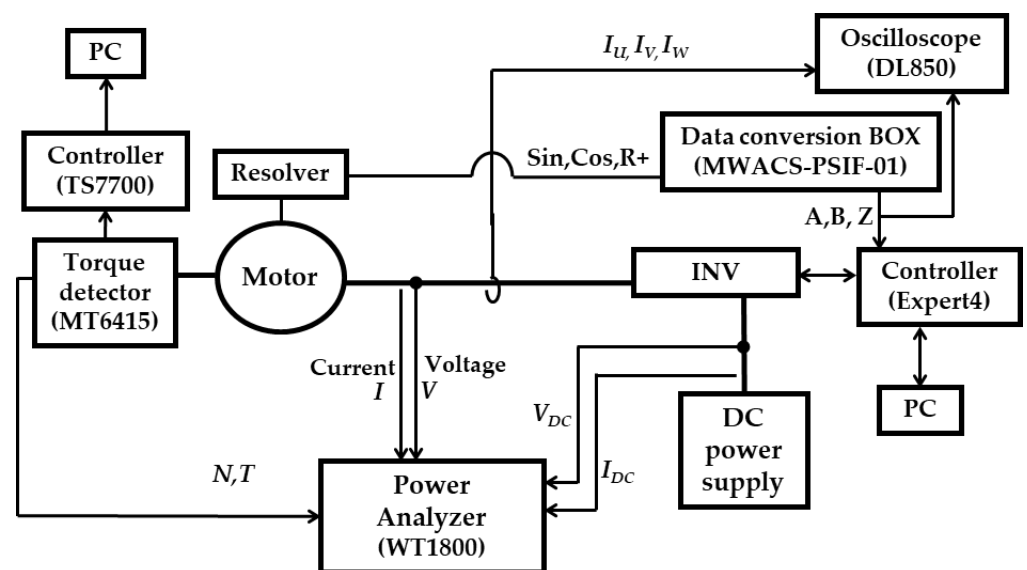


Figure 18. Configuration of prototype motor: (a) Cross-section of the prototype motor from the axial direction; (b) Cross-section of the prototype motor from the radial direction; (c) Winding diagram.

Table 2. Specifications of novel consequent-pole prototype motor.

Items		Value
number of poles and slots		8 poles, 12 slots
stack length		37 mm
stator	stator outer diameter	81 mm
	stator inner diameter	76.2 mm
	material of core	50 JNE300
	winding method	Concentrated windings
	winding connection method	Star configuration
rotor	rotor outer diameter	42.8 mm
	material of magnets	NMX-43SH
	magnet arrangement	SPM type
	thickness of magnets	5.7 mm

**Figure 19.** Configuration of prototype motor: (a) Photograph of the motor; (b) Photograph of the rotor.**Figure 20.** Measurement system configuration.

3.2. Validation of Effect of Adjustable Speed by Prototype Motor

From the voltage equation, it is possible to calculate the amount of magnetic flux linkage of the prototype motor backward from the rotational speed, the voltage, the current,

winding resistance, and the inductance measured when the current is applied. Figure 21 shows the adjustable flux by d -axis current in the prototype motor and analysis model. Figure 21a shows the relationship between the rotating speed with respect to the d -axis current in the prototype motor, and Figure 21b shows the estimated U-phase flux linkage (fundamental wave component) in the prototype motor when no d -axis current is supplied ($I_d = 0$ A) and when the d -axis current is supplied ($I_d = -30$ A). Although the q -axis current is applied to drive the motor, it is the minimum value to counter the windage loss, mechanical loss of the bearing, and core iron loss at each current condition. From Figure 21, it can be seen that the fundamental wave component of the estimated U-phase flux linkage at $I_d = -30$ A is about 3% of the fundamental wave component of the flux linkage at $I_d = 0$ A. Similar to the analysis results shown in Section 2.2, it was found that a wide range of magnetic flux variation was possible in the prototype motor. If the flux linkage is infinitely close to zero, the rotating speed should theoretically approach infinity. However, the reason why the rotating speed does not reach infinity when the flux linkage is close to zero is that only the fundamental wave component is used for the back-calculation and that the harmonic component is included in the magnetic flux and the inductance in the prototype motor.

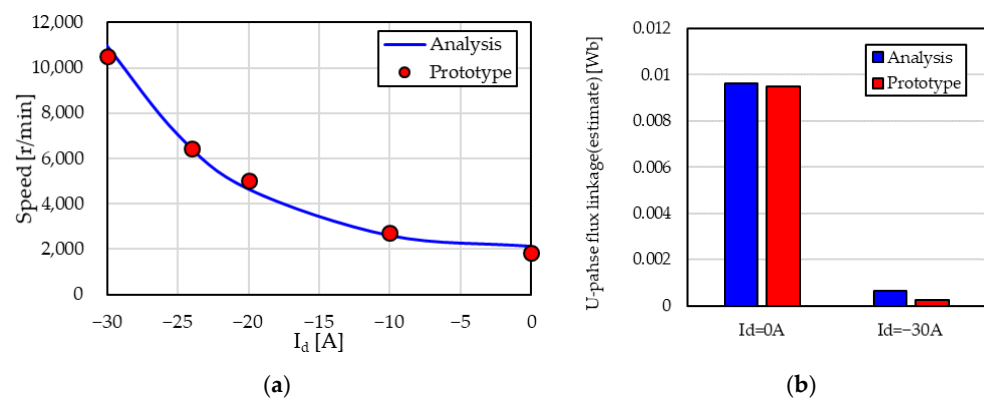


Figure 21. Adjustable flux by d -axis current: (a) Relationship between the rotating speed with respect to the d -axis current; (b) Estimated U-phase flux linkage in the prototype motor at no load and $I_d = -30$ A.

3.3. Back e.m.f. Characteristics

Figure 22 shows the waveform and FFT results of the U-phase back e.m.f. when the motor is operated at 1000 r/min. It can be seen from Figure 22 that the U-phase back e.m.f. waveform of both the analytical and experimental values is close to sinusoidal because the winding facing the magnet and the winding facing the image pole are connected in series. In addition, the first-order component of the analytical value and the experimental value is almost the same.

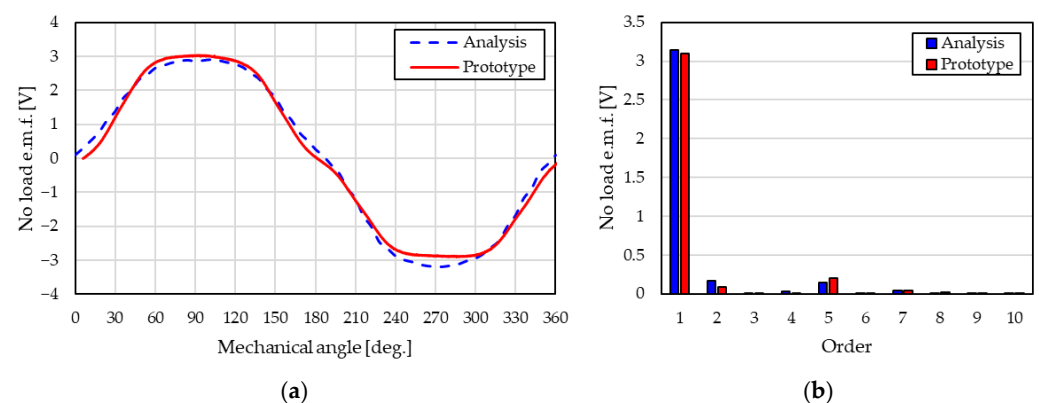


Figure 22. U-phase back e.m.f. at 1000 r/min: (a) waveforms; (b) FFT results.

3.4. N-T Characteristics

Figure 23 shows the relationships between the rotating speed N and torque T characteristic (N-T characteristic) at $I_d = 0$ A and $I_d = 30$ A. However, the results were measured by driving an ordinary three-phase inverter using the voltage equation derived in Section 2 as the motor control theory. It can be seen from Figure 23 that the rotating speed is increased in the low load region by applying field weakening. When no d -axis current is supplied (0 A), the experimental value is almost the same as the analysis value, but when a d -axis current of -30 A is provided, the rotating speed of the experimental value is smaller than that of the analysis value at low load. It is considered that the required supply voltage of the prototype motor was higher than that of the analysis motor due to the magnet sticking error and assembling error at the time of manufacturing and due to the harmonic component included in the magnetic flux and the inductance in the prototype motor. However, since the N-T characteristics of the analytical and experimental value were almost the same, it was confirmed that the driving control principle based on the derived voltage equation was established in the prototype motor.

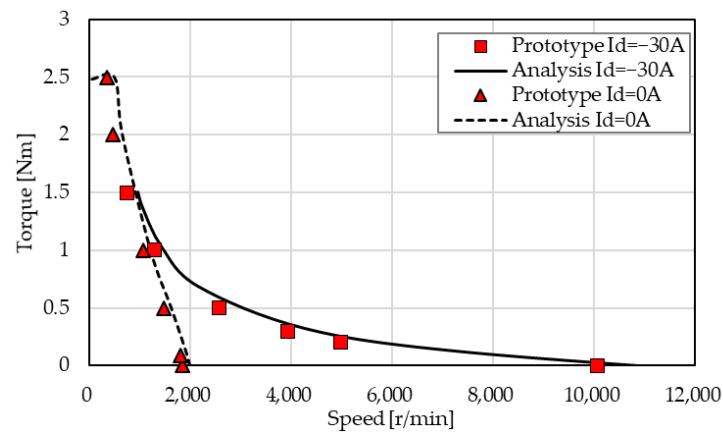


Figure 23. N-T characteristics of analysis model and prototype motor at $I_d = 0$ A, -30 A.

3.5. Demagnetizing Characteristics

Irreversible demagnetization may occur even in an automotive and small motor [34]. Figure 24 shows a contour diagram of the demagnetizing factor of the PM after demagnetization analysis. Under analytical conditions, only the d -axis current is supplied in forward and reverse rotation for an electrical angle of 360° . The d -axis current value was set to 1.5 times the maximum d -axis current during the experiment, considering the safety factor and the surge voltage that may occur unexpectedly. The maximum temperature of the magnet was set at 180°C . From the analysis result, the volume of the magnet that is demagnetized even slightly is 20.1% of the total magnet volume. Therefore, the maximum demagnetizing factor is 57.8%. The demagnetized area of the magnet was the corner on the outer diameter side. However, Figure 24a shows that the area where the demagnetizing factor is high is small.

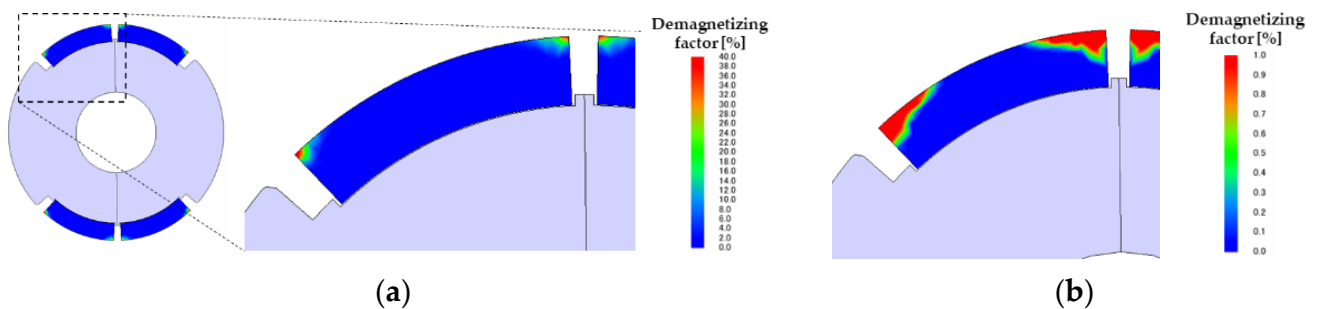


Figure 24. Contour diagram of the demagnetizing factor: (a) maximum range is 40%; (b) maximum range is 1%.

Figure 25 shows the analytical waveforms and FFT results of the U-phase back e.m.f. before and after demagnetization analysis. The fundamental component of the U-phase back e.m.f. after demagnetization analysis was -0.3% compared to that before. Therefore, even under the worst-case conditions, there should be little degradation of properties due to irreversible demagnetization.

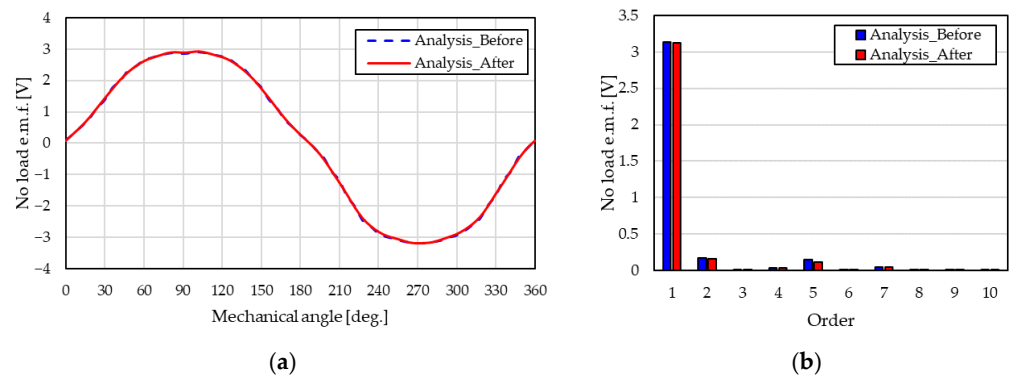


Figure 25. Analytical U-phase back e.m.f. before and after irreversible demagnetization at 1000 r/min: (a) waveforms; (b) FFT results.

The prototype motor determines whether the magnet is demagnetized or not by comparing the back e.m.f. before and after the characteristic measurement. Figure 26 shows the experimental waveforms and FFT results of the U-phase back e.m.f. before and after measurement. Figure 26 shows that the fundamental wave components are almost identical before and after the experiment. Therefore, it was confirmed that there was virtually no significant degradation of motor characteristics due to irreversible demagnetization.

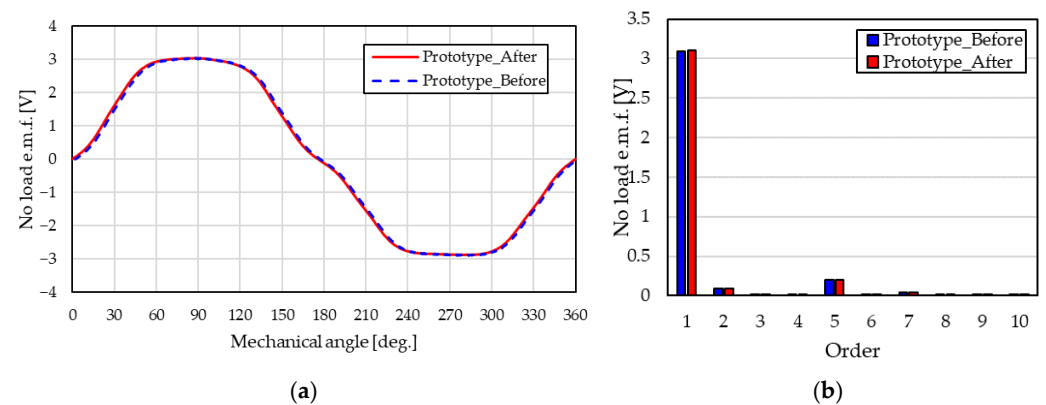


Figure 26. U-phase back e.m.f. of the prototype motor before and after the experiment at 1000 r/min: (a) waveforms; (b) FFT results.

4. Comparison of Characteristics of Novel Consequent-Pole PM Motor

4.1. Motor Specifications

The characteristics of the proposed novel consequent-pole motor, the conventional consequent-pole PM motor, and the standard surface permanent magnet (SPM) motor were compared in the analysis. Figure 27 shows a cross-section of the proposed novel consequent-pole motor, the conventional consequent-pole motor, and the SPM motor. The magnets in the SPM motor are arranged at all poles. Table 3 shows the specifications of these three models. Although the motor volumes of all three motors are the same, only the rotor structures of the three motors are different from each other. The proposed motor and the conventional consequent-pole motor employ equivalent eight poles (four real PM poles and four image poles), whereas the SPM motor has eight real PM poles. Hence the PM volume is double that of the consequent pole PM motors.

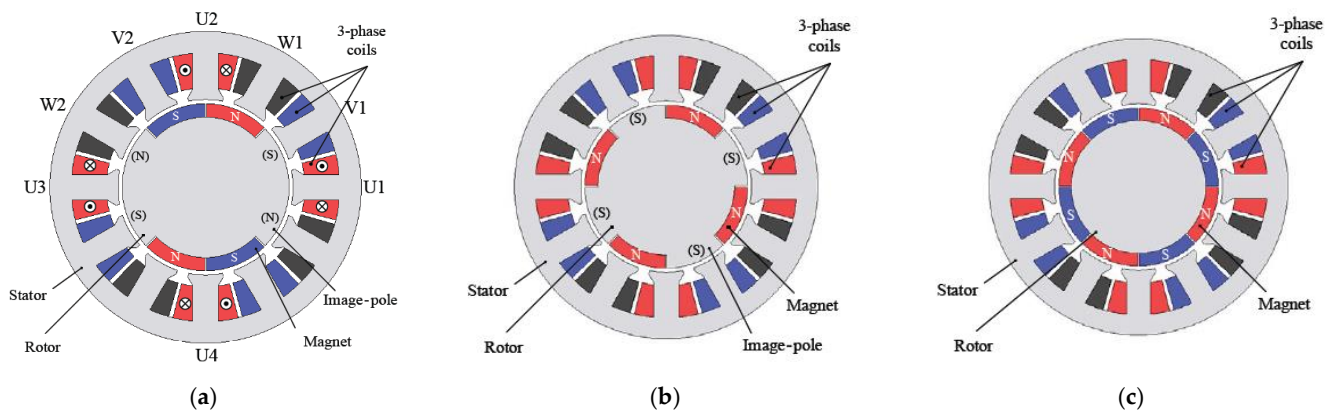


Figure 27. Cross-section of motor model: (a) proposed motor; (b) consequent pole motor; (c) SPM motor.

Table 3. Motor specifications of a novel consequent-pole prototype.

Items	Proposed PM Motor	Consequent Pole PM Motor	SPM Motor
Stator diameter	80 mm	80mm	80mm
Rotor diameter	42.85 mm	42.85 mm	42.85 mm
Stack length	37 mm	37 mm	37 mm
Air gap length	1.045 mm	1.045 mm	1.045 mm
Number of poles	8 (PM:4, Image pole:4)	8 (PM:4, Image pole:4)	8
Number of slots	12	12	12
Number of turns	16	16	16
PM volume	7.672 cc	7.672 cc	15.344 cc
Armature winding connection	4-series star connection	4-series star connection	4-series star connection
PM type	NMX-43SH	NMX-43SH	NMX-43SH

4.2. Comparison of No-Load Electromotive Force

Figure 28 shows the no-load e.m.f. waveforms of the U-phase and their FFT analysis results when the three motors are operated at 600 r/min. The e.m.f. of the motor is expressed as:

$$V_o = \omega \sqrt{(\psi_a + L_d i_d)^2 + (L_q i_q)^2} \quad (38)$$

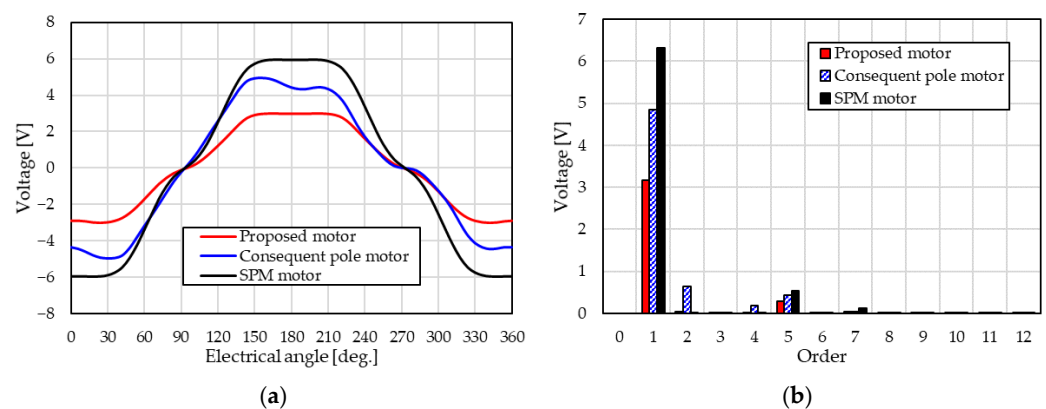


Figure 28. Comparison of back e.m.f. and FFT results: (a) back e.m.f. waveform; (b) FFT results.

The no-load e.m.f. is generated by only the magnetic flux (ψ_a). At field weakening control, the negative d -axis current is supplied. Therefore, it can be seen using Equation (38) that a motor with a larger d -axis inductance can reduce the back e.m.f. with a smaller negative d -axis current. In other words, efficient high-speed operation can be expected.

From Figure 28, the waveforms of the SPM motor and proposed motor are close to sinusoidal. On the other hand, the conventional consequent pole PM motor's waveform is distorted due to the deviation of the real magnet arrangement in the rotor. The harmonic component, especially the second component, of the proposed motor in the e.m.f. waveform is lower than the conventional consequent pole PM motor; thus, the proposed motor is suitable for high-speed operation with field weakening control.

4.3. Comparison of Inductance

Figure 29 shows variations of the d -axis and the q -axis inductances of the three models, where the current phase angle β is changed under the constant stator current amplitude. However, the inductance is calculated by the voltage equation. The magnet flux linkage at no-load, the number of the flux linkage at each current, the phase current, and the current phase angle are used in the voltage equation. As the β value increases, the d -axis inductance, as well as the q -axis inductance, is increased because the amounts of the magnetic fluxes to both axes are reduced.

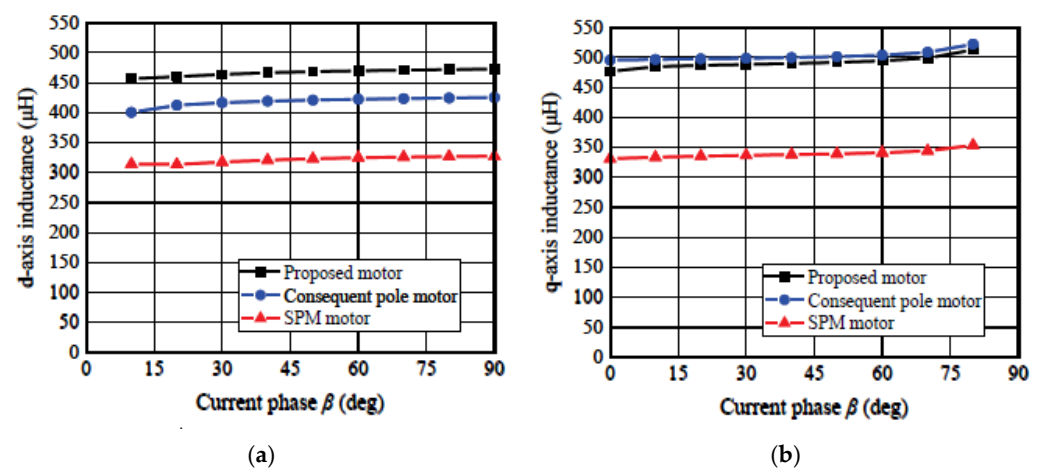


Figure 29. Comparison of inductance: (a) d -axis inductance; (b) q -axis inductance.

The d -axis inductance of the proposed motor is the highest among the three motors. It is larger by approximately 45% than the SPM motor, which means that the reluctance of the magnetic circuit in the proposed motor is the least. The SPM motor has the minimum inductance values on both the d -axis and the q -axis. However, the conventional consequent pole PM motor has the highest q -axis inductance. As described above, any types of the consequent pole PM motors have more iron in the rotor cores and higher inductance values compared with the standard SPM motor in which the rotor is entirely surrounded by the PMs. As can be expected, the proposed motor is capable of realizing the field weakening with less d -axis current owing to the higher d -axis inductance.

4.4. Comparison of N-T Characteristics

Figure 30 shows the N-T characteristics of the three motors, where the maximum output power is delivered to the loads. The test conditions are 12-V DC bus voltage, with a maximum current amplitude of 20 A.

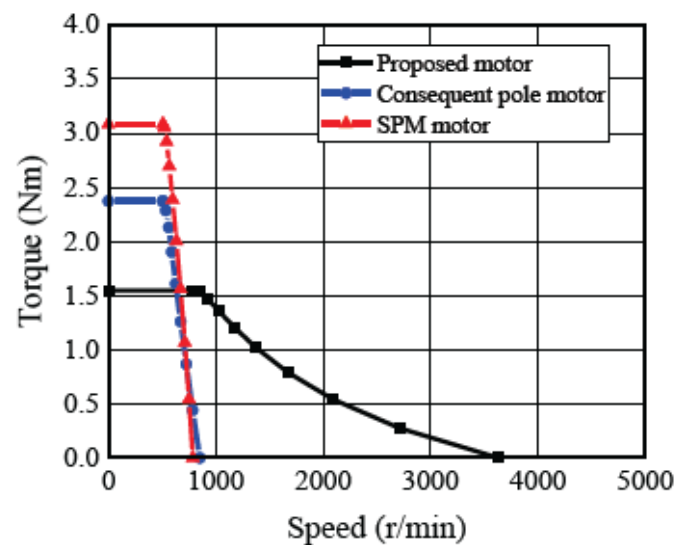


Figure 30. Comparison of N-T characteristics.

The SPM motor can deliver the higher torque in the low-speed range owing to the high e.m.f., compared with the other motors. On the other hand, the proposed motor can expand the higher-speed range because the motor has a low coefficient of the e.m.f. and the counter e.m.f. is high enough to cancel the PMs' e.m.f. Therefore, the higher-speed range can effectively be expanded using the field weakening. On the other hand, the conventional consequent pole PM motor has almost the same N-T characteristic as the SPM motor, and it is hard to expand the high-speed operation range. This is due to the even-order harmonic components in the e.m.f. when the field weakening is applied to the motor.

As described above, it has been found that the proposed motor has the most extensive high-speed operation range with the least negative d -axis current; thus, the field weakening operation can efficiently be achieved among the three motors.

5. Conclusions

The proposed motor has a unique rotor configuration where the real PM pole pairs and the image pole pairs are placed in every other pole pair. The image pole pairs are on the iron core parts next to the real PMs on the rotor. Since the windings facing the PM pole pair and the windings facing the image pole pair are connected in series, the proposed motor has a mechanism for canceling the induced voltages to increase the field weakening effect, in addition to a conventional field weakening mechanism for canceling the magnetic flux by the armature d -axis flux. In addition, because the three-phase impedance is balanced, the proposed motor does not require the conventional countermeasures due to the unbalanced impedance of a traditional consequent-pole PM motor. Furthermore, since a pair of image poles of a core using high permeability material for a pair of N and S magnet poles on the rotor is arranged, the proposed motor increases d -axis inductance.

This paper sought to construct a control theory of the motor drive for a novel consequent-pole-type PM motor with an unusual rotor structure and to clarify the proposed motor's potential. A three-phase voltage equation was derived using the equivalent magnetic circuit of six phases in the basic structure employing four poles and six slots, and the torque equation was derived. The validity of the derived mathematical model was confirmed by analysis. The validity of the control theory for the motor drive using the derived mathematical model was verified on the prototype motor. Furthermore, the amount of the adjustable flux linkage and the demagnetization characteristics of the proposed motor during field weakening were confirmed by analysis and measurement. In addition, the essential operating characteristics were compared with those of a standard SPM motor and a conventional consequent-pole PM motor.

The results demonstrated the following:

- The derived voltage equation is equivalent to that of the general SPM type.
- The derived voltage equation was shown to be valid, with the same trend as the analytical results.
- The control theory for the motor drive based on the derived mathematical model worked on the prototype motor and was shown to be valid by measurement.
- The proposed motor can vary the fundamental component of the number of the flux linkage in the range of 3–100% by field weakening in the experiment.
- The demagnetization resistance of the proposed motor was high. There was virtually no significant degradation of motor characteristics due to irreversible demagnetization in analysis and measurement.
- Analysis clarified that the harmonic component of the proposed motor was less than that of the conventional consequent-pole type.
- The proposed motor has the most extensive high-speed operation range with the least negative d -axis current; thus, the field weakening operation can be achieved efficiently through the three motors.

Based on these results, we have developed a novel consequent-pole PM motor with a wide operating range with less d -axis current during field weakening, using the same control theory for the motor drive as before, without the conventional countermeasures associated with the unbalanced impedance of a traditional consequent-pole PM motor.

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