

Article

Research on Prediction Method of Volcanic Rock Shear Wave Velocity Based on Improved Xu–White Model

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Abstract: Volcanic rock reservoirs have received extensive attention from scholars all over the world because of their geothermal, mineral, and oil and gas resources. Shear wave velocity is the essential information for AVO (amplitude variation with offset) analysis and the reservoir description of volcanic rocks. However, due to factors such as cost, technical reasons, and so on, shear wave velocity is not provided in many logging data. This paper proposes a shear wave velocity prediction method suitable for the conventional logging of volcanic rocks. Firstly, the Xu–White model is improved. The probability distributions formed by the prior information of the logging area are used to initialize the key petrophysical parameters in the model instead of the fixed parameter value to establish the statistical petrophysical model between the logging curve and shear wave velocity. Then, based on the Bayesian inversion method, the simulated P-wave velocity is matched with the actual P-wave logging data to calculate the key petrophysical parameters, and is then used for S-wave velocity prediction. The method is applied to the actual logging data of the No. 5 structure in Nanpu Sag, eastern China. The prediction effect of shear wave velocity is better than that of the conventional method, indicating the feasibility and effectiveness of this method. This study will provide more accurate shear wave velocity data for the exploration and development of volcanic reservoirs.

Keywords: volcanic reservoir; conventional logging; shear wave velocity prediction; Xu–White model; statistical model; Bayesian inversion



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1. Introduction

Volcanic reservoirs contain a variety of resources, including subsurface thermal energy, mineral resources, groundwater, and oil and gas deposits. As the degree of exploration and development continues, the volcanic reservoir has attracted much attention as a special type of oil and gas reservoir. Since the discovery of the world's first volcanic oil and gas reservoir in California, Russia, Cuba, Argentina, Japan, and other countries have successively discovered high-yield volcanic oil and gas reservoirs [1–4]. Since the 1970s, volcanic oil and gas reservoirs have been found in the Bohai Bay Basin, the Junggar Basin in Xinjiang, the Erlian Basin in Inner Mongolia, the Jiangnan Basin, the Subei Basin, and the Tarim Basin in China [5–8]. The research methods of volcanic reservoirs mainly include geophysical methods (seismic information identification, magnetotelluric measurement, high-precision gravity and magnetic exploration, time domain field sounding, frequency domain continuous electromagnetic profile, and geophysical logging), volcanic chronology research methods, volcanic lithofacies, petrology and petrochemistry research methods, and volcanic reservoir and diagenesis research methods, as well as the conditions and mechanisms of volcanic reservoir accumulation and types of volcanic reservoirs. With the continuous discovery of volcanic reservoirs, reservoir evaluation technology has entered a new stage of development. More attention has been paid to the application of geophysical

methods such as logging and seismic. The lithology identification, reservoir description, and fracture properties of volcanic rocks have become the core content of the study.

S-wave velocity plays a key role in seismic pre-stack inversion and reservoir attribute analysis, and it is also an important parameter for petrophysical property inversion, reservoir lithology, and fracture fluid identification. However, in actual production, due to limitations in well location, logging technology, and acquisition cost, shear wave velocity information is usually scarce in logging data, making volcanic rock shear wave velocity prediction an important issue that needs to be addressed. A number of empirical formulas and petrophysical models have been proposed to establish the link between logging information and shear wave velocity. Castagna et al. [9] proposed the “mudstone line” formula and revealed the empirical relationship between P-wave and S-wave velocities. Han et al. [10] considered the influence of porosity and clay content on velocity, and then gave us the linear regression equation between P-wave velocity and S-wave velocity under different pressures. Qingzhong Li [11] proposed the nonlinear relationship between the P-wave velocity and S-wave velocity of water-saturated sandstone. Greenberg et al. [12] proposed empirical formulas for the water-saturated P-wave and S-wave velocities of various mineral components. Yan et al. [13] improved Han’s empirical formula by considering the influence of pore aspect ratio on velocity. Vernik et al. [14] proposed a mixing method based on the Greenberg–Castagna theory, and then they considered the influence of kerogen on shear wave velocity. Sohail and Hawkes [15] evaluated different empirical and petrophysical models for shear wave velocity prediction and considered the modified Xu–White model the best choice available at present.

The prediction of shear wave velocity by an empirical formula is based on the fitting relationship between the actual shear wave velocity and other logging curve data. However, because the physical mechanism of rock is not considered, the accuracy of prediction results is insufficient. The petrophysical model describes the influence of microstructure on the overall properties of rock and connects the micro characteristics of rock, such as porosity and pore structure, with macro characteristics such as elastic modulus and velocity. Berryman [16] proposed an equivalent medium self-compatible approximation (SCA) model, assuming that rocks are composed of ellipsoidal bodies with different pore aspect ratios. Xu and White [17,18] proposed an equivalent medium model for muddy sandstone reservoirs, considering the pore geometry of the reservoir. Keys and Xu [19] introduced an approximate equation for the dry rock modulus to improve the Xu–White model. Lee [20] improved the pride model to calculate the consolidation coefficient of compacted dry sandstone, followed by calculating the shear wave velocity. Ruiz and Dvorkin [21] proposed a differential equivalent medium model (DEM) to predict shear wave velocity with a fixed pore aspect ratio. Bai et al. [22] improved the Xu–White model with a variable pore aspect ratio. Bing Zhang et al. [23] established a statistical rock model suitable for anisotropic shale reservoir parameter inversion, which provides a reference for shear wave velocity prediction.

Volcanic reservoirs are characterized by irregular pore structures, diverse mineral compositions, and physical anisotropy [24–26]. Pan et al. [24] developed a petrophysical model of volcanic rocks using QAPM (Q (quartz), A (alkali feldspar), P (plagioclase), and M (magnesian iron)) and porosity based on a classification scheme, but the influence of tuffs was not considered in this model. Zhao et al. [8] developed a binary petrophysical model based on the equivalent medium self-compatible approximation (SCA) model, treating acidic volcanic rocks as matrix minerals, and medium–basic volcanic rocks as infill minerals. Wang et al. [27] applied the Greenberg–Castagna theory to volcanic reservoirs and directly used empirical formulas to predict the shear wave velocity of the reservoir. Xu et al. [28] proposed an improved model suitable for volcanic reservoirs by combining the Xu–White model and the differential equivalent medium model (DEM). In a follow-up study, Xu et al. [29] proposed an isotropic volcanic rock model and incorporated the effects of multiple minerals and pore structures.

Combining the existing research and the characteristics of volcanic rock reservoirs, this paper will propose a regional shear wave velocity prediction method based on the improved Xu–White model for the conventional logging of volcanic rock. Due to the limited data provided by conventional logging, we first analyze the key petrophysical parameters affecting velocity in the Xu–White model and fully collect the prior information to form the prior distributions for model parameter initialization. Use of the prior distribution to initialize the Xu–White model and construct a statistical petrophysical model can overcome the shortcomings of insufficient conventional logging data. Then, combined with a statistical model and Bayesian inversion method, the key petrophysical parameters are calculated and the shear wave velocity curve is predicted by matching the simulated P-wave velocity with the P-wave velocity in the actual logging. The effectiveness of the method is verified by testing the conventional logging data on actual volcanic rocks.

2. Fundamental Principles

2.1. Geological Characteristics of Volcanic Reservoir

When magma in the deep crust overflows or erupts through cracks, it solidifies rapidly in the air to form volcanic rock. Due to the special diagenetic mechanism, the volcanic reservoir has diverse mineral components, a complex pore structure, and strong heterogeneity. The mineral composition of volcanic rock is complex and diverse, mainly composed of silicates of magnesium, iron, aluminium, potassium, calcium, sodium, and oxides of magnesium and iron. The most important minerals are quartz, feldspar, pyroxene, hornblende, mica, and olivine. Volcanic reservoirs are fractured reservoirs with various types of reservoir spaces. The lithology, clastic particles, tectonics, and epigenetic diagenesis of volcanic rocks all influence the development of reservoir spaces, mainly including pores, fractures, dissolution pores, and the mutual coupling between them in the form of fractures/pores, fractures/dissolution pores, and dissolution pores/fractures. The development phase of reservoir pore space and the combination of different reservoir spaces led to great differences in the reservoir capacity of volcanic reservoirs. Accurate velocity information allows us to effectively analyze the lithology, physical properties, fluid properties, and other characteristics of volcanic rock stratum, which can be used for fracture attribute identification, fluid property analysis, and formation pressure prediction, whereafter we then judge the storage and seepage performance of the volcanic rock reservoir and detect the trapping of oil and gas in the reservoir.

2.2. Shear Wave Prediction Method Based on Han's Empirical Formula

The basis of using the empirical formula to predict shear wave velocity is the relationship among different rock attributes established by actual data. This method directly establishes the relationship between target parameters and the logging curve; it is simple to calculate and is widely used in practical shear wave velocity prediction. Han et al. [10] gave us the linear regression equation between P-wave velocity and S-wave velocity by measuring rock samples under different pressures and considering the effects of porosity and clay content in argillaceous sandstone. For water-saturated rock under 40 MPa pressure, the equations are as follows:

$$V_p(\text{km/s}) = 5.59 - 6.93\phi - 2.18C \quad (1)$$

$$V_s(\text{km/s}) = 3.52 - 4.91\phi - 1.89C \quad (2)$$

V_p is the P-wave velocity, V_s is the S-wave velocity, ϕ is the porosity, and C is the clay content.

Although the empirical formula proposed by Han is a simple method, it is widely used in practical production, and the accuracy of the predictions is acceptable.

2.3. Shear Wave Prediction Method Based on the Xu–White Model

The petrophysical model connects the microstructure and composition of rock with the overall attributes and describes the influence of the internal attributes of rock on external elastic attributes. Due to the particularity of the diagenetic mechanism of volcanic rocks, the pore types are complex and diverse, in which tuffaceous minerals are similar to clay. Considering the large proportion of tuffaceous minerals, it cannot be ignored in the construction of the volcanic rock petrophysical model. The Xu–White model can be used to build a volcanic rock petrophysical model to predict shear wave velocity.

Xu and White [17,18] proposed the Xu–White model based on the Gassmann fluid substitution equation, K-T equation, and differential equivalent medium theory (DEM), taking into account porosity, matrix properties, fluids, and pore aspect ratios. The matrix in the model consists of sand and clay, and the pores are divided into sand-related pores with a large pore aspect ratio and clay-related pores with a small pore aspect ratio. The equations are expressed as follows:

$$\phi = \phi_s + \phi_c \quad (3)$$

$$\phi_s = f_s \frac{\phi}{1 - \phi} \quad (4)$$

$$\phi_c = f_c \frac{\phi}{1 - \phi} \quad (5)$$

$$K_d - K_0 = \frac{1}{3}(K_1 - K_0) \frac{3K_d + 4\mu_0}{3K_0 + 4\mu_0} \sum_{l=s,c} \phi_l T_{ijj}(\alpha_l) \quad (6)$$

$$\mu_d - \mu_0 = \frac{(\mu_1 - \mu_0)}{5} \frac{6\mu_d(K_0 + 2\mu_0) + \mu_0(9K_0 + 8\mu_0)}{5\mu_0(3K_0 + 4\mu_0)} \sum_{l=s,c} \phi_l F(\alpha_l) \quad (7)$$

$$F(\alpha) = T_{ijj}(\alpha) - \frac{T_{ijj}(\alpha)}{3} \quad (8)$$

In order to avoid the excessive amount of iterative calculation in DEM differential calculation, Keys and Xu [19] improved the Xu–White model and determined the elastic modulus of a rock skeleton by solving the first-order linear ordinary differential equations. The formula for the elastic modulus of an argillaceous sandstone dry rock skeleton is as follows:

$$K_d = K_0(1 - \phi)^p \quad (9)$$

$$\mu_d = \mu_0(1 - \phi)^q \quad (10)$$

In Equations (1)–(8), ϕ , ϕ_s , and ϕ_c are total porosity, sand-related porosity, and clay-related porosity, respectively. f_s and f_c are the volume fraction of sand and clay, $f_s + f_c = 1 - \phi$. K_d , K_0 and K_1 are the bulk moduli of the rock skeleton, rock mineral matrix, and pore fluid, respectively. μ_d , μ_0 and μ_1 are the shear modulus of the rock skeleton, rock mineral matrix, and pore fluid, respectively. For dry rock, K_1 and μ_1 are zero, $F(\alpha)$, $T_{ijj}(\alpha)$ and $T_{ijj}(\alpha)$ are functions of the pore aspect ratio α , and p and q are geometric factors.

The equations for p and q are as follows:

$$p = \frac{1}{3} \sum_{l=s,c} f_l T_{ijj}(\alpha_l) \quad (11)$$

$$q = \frac{1}{5} \sum_{l=s,c} f_l F(\alpha_l) \quad (12)$$

f_l is the volume fraction of the relevant mineral components.

Berryman [16] proposed an n-phase mixture self-compatibility approximation model (SCA) to estimate the equivalent elastic modulus of multiphase media and revealed the scalars required to calculate p and q .

$$T_{ijj}(\alpha_l) = \frac{3F_1}{F_2} \quad (13)$$

$$F(\alpha_l) = \frac{2}{F_3} + \frac{1}{F_4} + \frac{F_4F_5 + F_6F_7 - F_8F_9}{F_2F_4} \quad (14)$$

$$F_1 = 1 + A \left[\frac{3}{2}(f + \theta) - R \left(\frac{3}{2}f + \frac{5}{2}\theta - \frac{4}{3} \right) \right] \quad (15)$$

$$F_2 = 1 + A \left[1 + \frac{3}{2}(f + \theta) - R \left(\frac{3}{2}f + \frac{5}{2}\theta \right) \right] + B(3 - 4R) + \frac{A}{2}(A + 3B)(3 - 4R) \left[f + \theta - R(f - \theta + 2\theta^2) \right] \quad (16)$$

$$F_3 = 1 + A \left[1 - \left(f + \frac{3}{2}\theta \right) - R(f + \theta) \right] \quad (17)$$

$$F_4 = 1 + \frac{A}{4} [f + 3\theta - R(f - \theta)] \quad (18)$$

$$F_5 = A \left[-f + R \left(f + \theta - \frac{4}{3} \right) \right] + B\theta(3 - 4R) \quad (19)$$

$$F_6 = 1 + A[1 + f - R(f + \theta)] + B(1 - \theta)(3 - 4R) \quad (20)$$

$$F_7 = 2 + \frac{A}{4} [3f + 9\theta - R(3f + 5\theta)] + B\theta(3 - 4R) \quad (21)$$

$$F_8 = A \left[1 - 2R + \frac{f}{2}(R - 1) + \frac{\theta}{2}(5R - 3) \right] + B(1 - \theta)(3 - 4R) \quad (22)$$

$$F_9 = A[(R - 1)f - R\theta] + B\theta(3 - 4R) \quad (23)$$

Some parameters in the above formula are calculated by the following formula:

$$A = \frac{\mu_j}{\mu_m} - 1, \quad B = \frac{1}{3} \left(\frac{K_j}{K_m} - \frac{\mu_j}{\mu_m} \right), \quad R = \frac{3\mu_m}{3K_m + 4\mu_m},$$

$$\theta = \frac{\alpha}{(1 - \alpha^2)^{\frac{3}{2}}} \left[\arccos \alpha - \alpha(1 - \alpha^2)^{\frac{1}{2}} \right], \quad f = \frac{\alpha^2}{1 - \alpha^2} (3\theta - 2)$$

In the equation, K_j and μ_j are the bulk modulus and shear modulus of each mineral component, respectively. K_m and μ_m are the bulk modulus and shear modulus of the mineral matrix, respectively.

The Gassmann fluid substitution equation [30] allows for one fluid-saturated rock velocity to predict another fluid-saturated rock velocity, and likewise the rock skeleton velocity can be used to predict fluid-saturated rock velocity. The relationship and formulas are as follows:

$$\frac{K_{\text{sat}}}{K_0 - K_{\text{sat}}} = \frac{K_d}{K_0 - K_d} + \frac{K_1}{\phi(K_0 - K_1)} \quad (24)$$

$$\mu_{\text{sat}} = \mu_d \quad (25)$$

In the equation, K_{sat} and μ_{sat} are the fluid-saturated rock bulk modulus and shear modulus.

Hill's average formula [31] can calculate the modulus of mineral matrix without considering the mineral geometry.

$$M_H = \frac{1}{2} \left(\sum_{i=1}^N f_i M_i + \frac{1}{\sum_{i=1}^N \frac{f_i}{M_i}} \right) \quad (26)$$

In the equation, i is a mineral in the matrix, M_i is the modulus corresponding to the mineral, f_i is the volume fraction corresponding to the mineral, and M_H is the Hill effective modulus.

The Wood formula [32] can calculate the equivalent modulus of the pore fluid mixture. The formula is as follows:

$$\frac{1}{K_f} = \sum_{i=1}^N \frac{f_i}{K_i} \quad (27)$$

In the formula, f_i and K_i are the volume fraction and bulk modulus of fluid components, respectively.

When seismic waves are in a homogeneous, isotropic elastic medium, the wave velocity can be expressed by the rock bulk modulus, shear modulus, and density. The formulas are as follows:

$$V_p = \sqrt{\frac{\left(K_{sat} + \frac{4}{3}\mu_{sat}\right)}{\rho}} \quad (28)$$

$$V_s = \sqrt{\frac{\mu_{sat}}{\rho}} \quad (29)$$

In the formula, ρ is the fluid-saturated rock density, and V_p and V_s are the P-wave and S-wave velocities of the fluid-saturated rock.

The equations for the density of fluid-saturated rock are as follows:

$$\rho = (1 - \phi)\rho_0 + \phi\rho_f \quad (30)$$

$$\rho_0 = f_s\rho_s + f_c\rho_c \quad (31)$$

$$\rho_f = (1 - S_w)\rho_{gas} + S_w\rho_{water} \quad (32)$$

In the equation, ρ_0 and ρ_f are the density of the mineral matrix and the density of the saturated fluid, respectively. S_w is the water saturation. ρ_s , ρ_c , ρ_{gas} and ρ_{water} are the densities of sand, clay, gas, and water, respectively.

When given the volume fraction, density, and intrinsic modulus of sand, clay, and pores, the aspect ratio of sand-related pore space and clay-related pore space, the mineral matrix modulus is first calculated using Hill's average (Equation (26)), and the skeletal modulus of the rock is calculated through the Xu–White model (Equations (9) and (10)). The pore fluid modulus is then calculated using Wood's formula (Equation (27)), and the elastic modulus of fluid-saturated rock is calculated via the Gassmann fluid substitution equation (Equations (24) and (25)). Finally, the S-wave velocity and P-wave velocity are calculated by using the velocity equation (Equations (28) and (29)).

2.4. Shear Wave Prediction Method Based on the Improved Xu–White Model

The prediction of shear wave velocity based on the empirical formula is obtained by directly establishing the relationship between logging curve and shear wave velocity. The Xu–White model is usually simplified to Shear wave velocity by assuming that the properties of the sand, clay, and pore fluids in the model do not vary in depth and that their physical parameters are constants derived from experimentation or experience. However, the properties of the mineral matrix, pore fluid, porosity, and pore aspect ratio in rocks vary greatly in depth and are variables considered to have a significant impact on the accuracy of shear wave velocity prediction. According to previous research results [33–35], the velocity of sand in the matrix, porosity, and the aspect ratio of clay-related pore space have, among the above parameters, the greatest influence on the shear wave prediction results, and the values of these parameters need to be determined after inversion calculations and then used to calculate the shear wave velocity.

When conventional methods are used to predict shear wave velocities, certain rock physical parameters are constants obtained by transforming the prior information. The prior information refers to actual data, experience, results data, etc. However, prior information

will exhibit specific statistical characteristics, and it is a variable rather than a constant. In the process of shear wave velocity prediction, the statistical characteristics of the prior information are integrated into the petrophysical model, which can improve the accuracy of prediction results. The statistical petrophysical method solves the problem of using the above prior information [36,37]. In these studies, the prior information is used to establish the distribution of key physical parameters and to apply it to the parameter inversion process. This method has been applied to the description of reservoir characteristics, which not only obtains good inversion results, but also provides some statistical information.

This paper presents a prediction method of shear wave velocity based on an improved Xu–White model. The method is based on the Xu–White model and uses parameters that severely affect rock velocity and cannot be obtained from conventional logging data as key petrophysical parameters, combining the prior information to form a distribution of key petrophysical parameters. Conventional logging data only provide some logging curves, such as density, velocity, porosity, clay content, and water saturation. However, in the Xu–White model, the P-wave velocity and S-wave velocity of sand and the aspect ratio of sand-related pores and clay-related pores are the key parameters affecting rock velocity, but they cannot be obtained from conventional logging. Pillar et al. [38] gave us the relative pore aspect ratio of sand (α_s) calculation formula:

$$\alpha_s = 0.17114 - 0.24477\phi + 0.004314f_c \quad (33)$$

Thus, the pore aspect ratio curve related to sand in the well can be estimated, but other parameters need to be determined via inversion calculation.

The P-wave and S-wave velocities of sand can be calculated by the time average formula [17,18]. The formula are as follows:

$$K_0 = \rho_0 \left[\frac{1}{(T_0^P)^2} + \frac{4}{3(T_0^S)^2} \right] \quad (34)$$

$$\mu_0 = \rho_0 \left[\frac{1}{(T_0^S)^2} \right] \quad (35)$$

$$T_0^P = (1 - f'_c)T_s^P + f'_cT_c^P \quad (36)$$

$$T_0^S = (1 - f'_c)T_s^S + f'_cT_c^S \quad (37)$$

$$f'_c = \frac{f_c}{1 - \phi} \quad (38)$$

$$\rho_0 = (1 - f'_c)\rho_s + f'_c\rho_c \quad (39)$$

In the equations, T_0^P , T_s^P and T_c^P are the P-waves propagation times for matrix, sand, and clay, respectively. T_0^S , T_s^S and T_c^S are the shear wave propagation times for matrix, sand, and clay, respectively, where the propagation time is the reciprocal of the velocity. f'_c is the normalised clay volume fraction.

The prior information is necessary to establish the distribution of key petrophysical parameters. Actual logging information is limited by well conditions, acquisition techniques, and acquisition costs, and there are only a few wells or sections of wells in the target area that have accurate and complete logging and mineral fraction information. The information from these high-quality wells or sections is used as reference information. The reference information is rich in data, including S-wave velocity and P-wave velocity, mineral information, density, and porosity. The aspect ratio of sand-related pores and the velocity of the

sand in the reference information can be found in the above Formulas (33) to (39), and the aspect ratio of clay-related pores (α_c) is calculated using the following equation:

$$\alpha_c = \arg \min \left(\left| \frac{\tilde{V}_p - V_p}{V_p} \right| + \left| \frac{\tilde{V}_s - V_s}{V_s} \right| \right) \quad (40)$$

In the formula, \tilde{V}_p and \tilde{V}_s are the P-wave and S-wave velocities simulated by the Xu–White model.

The reference information provides the numerical data of the P-wave velocity and S-wave velocity of sand, and the aspect ratio parameters of clay-related pores. These data can obtain the prior distribution of key parameters after statistics. The prior distributions and the P-wave velocities of the target wells are used to invert the S-wave velocity and P-wave velocity of the sand, as well as the aspect ratios of the clay-related pores in the target wells. The Bayesian inversion method [39] allows for the prior distribution to be incorporated into this inversion process, and its objective function is as follows:

$$m = \arg \max [P(m | V_p)] \quad (41)$$

$$P(m | V_p) \propto P(V_p | m)P(m) = N(V_p; \tilde{V}_p, V_1)N(m; E, V_2) \quad (42)$$

In these equations, m is a vector consisting of the three parameters V_{p_sand} , V_{s_sand} and α_c . $P(\cdot)$ is the probability function. $N(\cdot)$ is the probability of a normal distribution. V_1 is the variance of the noise in V_p . V_2 is the covariance matrix, and E is the vector composed of the expectation values of the three parameters.

The simulated annealing particle swarm optimization (SA–PSO) is a global optimization algorithm that solves the nonlinear objective function solution problem in inversion. This algorithm is based on the idea of simulated annealing and can improve the local optimization ability of the particle swarm optimization algorithm. It has been applied to the nonlinear inversion of reservoir characterization [40,41]. It is applied to solving the objective function to maximise the posterior distribution $P(m | V_p)$ by searching for the optimal solution of the parameter m .

Figure 1 shows the workflow of shear wave velocity prediction based on the improved Xu–White model. First, the prior information of three key petrophysical parameters is calculated by reference logging data, and the prior distribution is obtained. The Xu–White model is then initialized with prior distributions and the statistical petrophysical model is built. Finally, the Bayesian inversion method is used to match the simulated p-wave velocity with the actual p-wave velocity in order to calculate the key petrophysical parameters and predict the s-wave velocity of the target well. In this workflow, the use of prior information makes the prediction of the shear wave velocity of the target well more accurate.

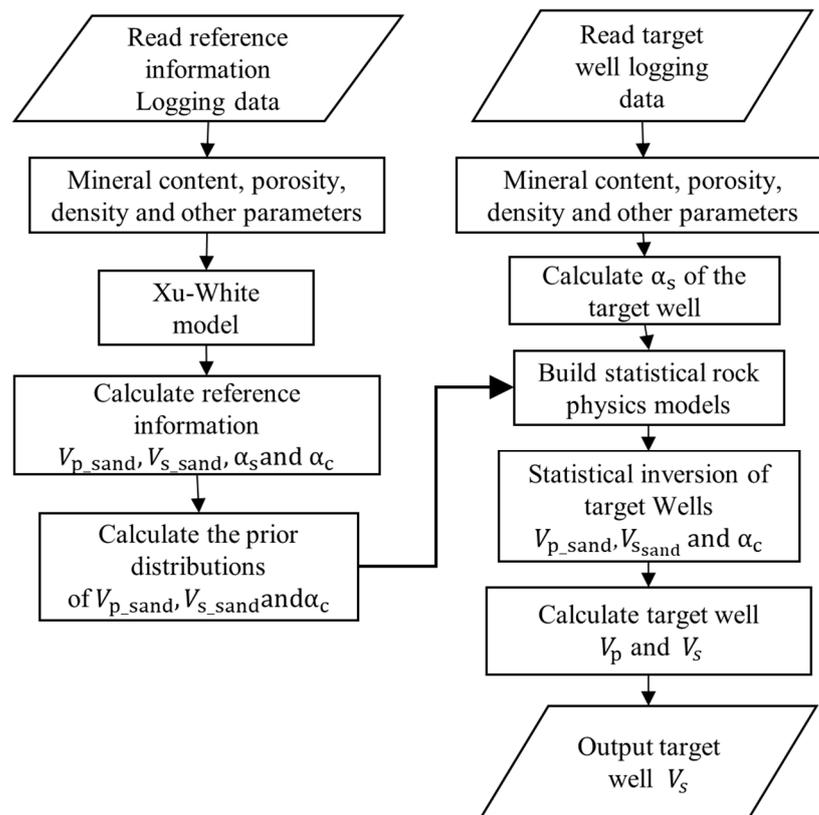


Figure 1. Work flow of the shear wave velocity prediction based on the improved Xu-White model.

3. Prediction of Shear Wave Velocities in Actual Volcanic Reservoirs

3.1. Field Well Log Data

In this paper, the actual logging data of a well location in the No. 5 structure of Nanpu Sag in eastern China are selected for research. The study area is located in the downthrow wall of the Xinanzhuang Fault in the Nanpu Sag in the northern part of the Huanghua Depression in the Bohai Bay Basin. It is in the northwest of the Nanpu No. 1 structure and is a subduction zone of dextral tectonics developed on a Middle and Palaeozoic bedrock nappe tectonic setting, controlled by the Xinanzhuang fault and its derivative faults. The main reservoir is the volcanic rocks of the Lower Tertiary Shahejie Formation, which has a wide distribution and thickness and develops a variety of stratigraphic lithological traps related to the volcanic rocks or stratigraphic overburden. The volcanic rocks are mainly volcanic lava, but some unequal thicknesses of volcanoclastic rock are also trapped in the volcanic rocks [42].

The lower half of the Nanpu well 1 is a key exploration well section with more complete logging information, and it is used as the reference well section. The mineral component information of the reference well section is shown in Figure 2, the information is derived from logging data and core data, and the modulus and density of minerals and pore fluids are shown in Table 1. The matrix minerals constitute the framework of the petrophysical model, and the mineral components directly affect the physical parameters of the petrophysical model. Silicate melt inclusions (SMI) are silicate droplets wrapped by mineral phenocrysts under a high temperature and high pressure environment, and they are widely developed in volcanic rock (especially mafic rocks) phenocrysts. Through literature research [27], it was found that the main component of silicate melt inclusions in the Nanpu 1 well is feldspar, and its modulus and density are similar to feldspar. In this paper, the silicate melt inclusions are approximated to feldspar for model construction. There are conventional logging curves such as S-wave velocity, P-wave velocity, density, clay content, water saturation, and the porosity curve in the upper half of Nanpu well 1,

as shown in Figure 3. This section can be used as a target well section to predict its shear wave velocity. The validity of the method in this paper can be verified via the actual shear wave velocity curve.

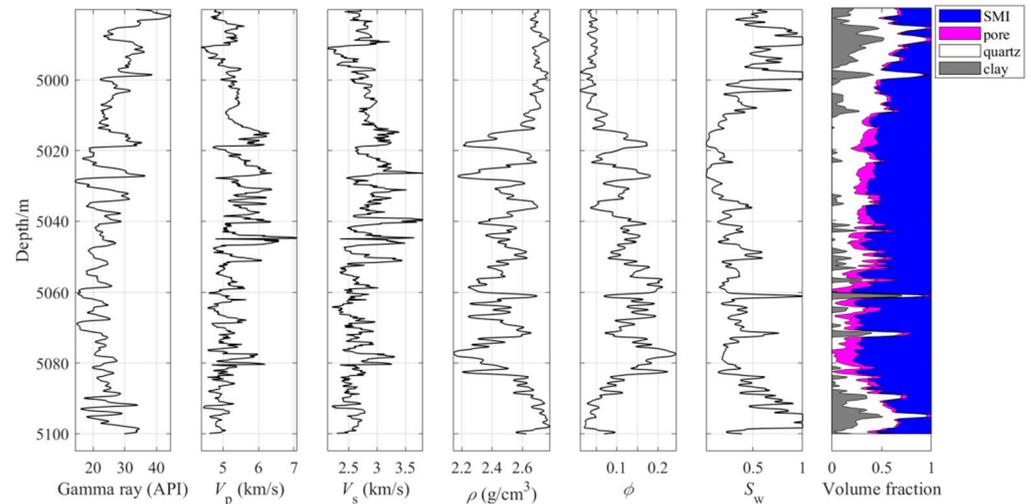


Figure 2. The logging curve and mineral composition information of the reference well section.

Table 1. The modulus and density of mineral and pore fluids [43].

	Bulk Modulus (GPa)	Shear Modulus (GPa)	Density (g/cm ³)
Quartz	37.9	44.3	2.65
SMI	37.5	15	2.62
Clay	25	9	2.55
Gas	0.336	–	0.34
Water	2.2	–	1.4

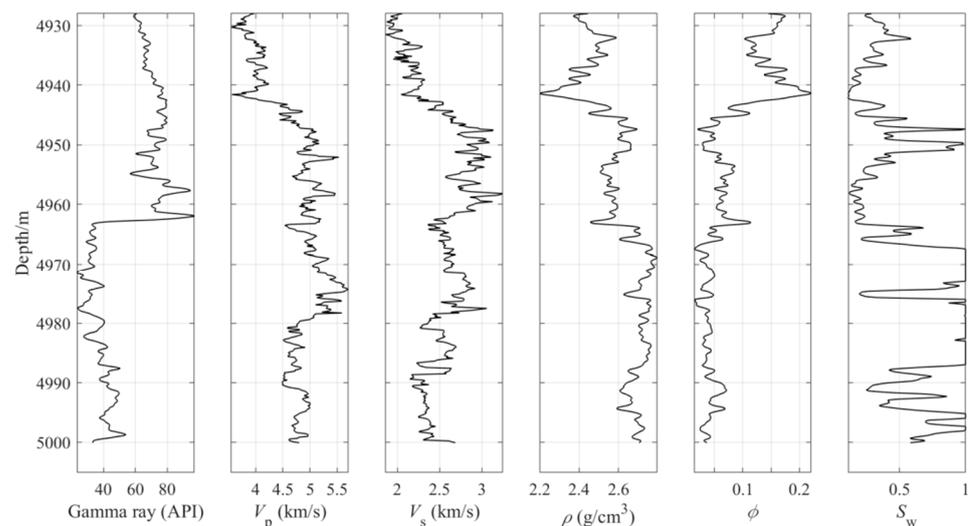


Figure 3. The logging curve of the target well section.

3.2. Shear Wave Velocity Prediction Process

Firstly, the velocity of the sand was calculated in the Xu–White model according to the mineral composition information in the logging data of the reference well section. The results are shown in Figure 4, which shows the statistics for the reference well section, and then the aspect ratio (α_c) of the clay-related pore space in the reference well section

is calculated, which is used as the key petrophysical parameter to establish the prior distribution for the next process. Then, the aspect ratio (α_s) of the sand-related pore space in the target well section is calculated. Finally, a statistical petrophysical model based on the Xu–White model is developed. This model is initialized using three prior distributions of three key parameters, and then it is combined with known logging information to calculate the shear wave velocity of the target well section.

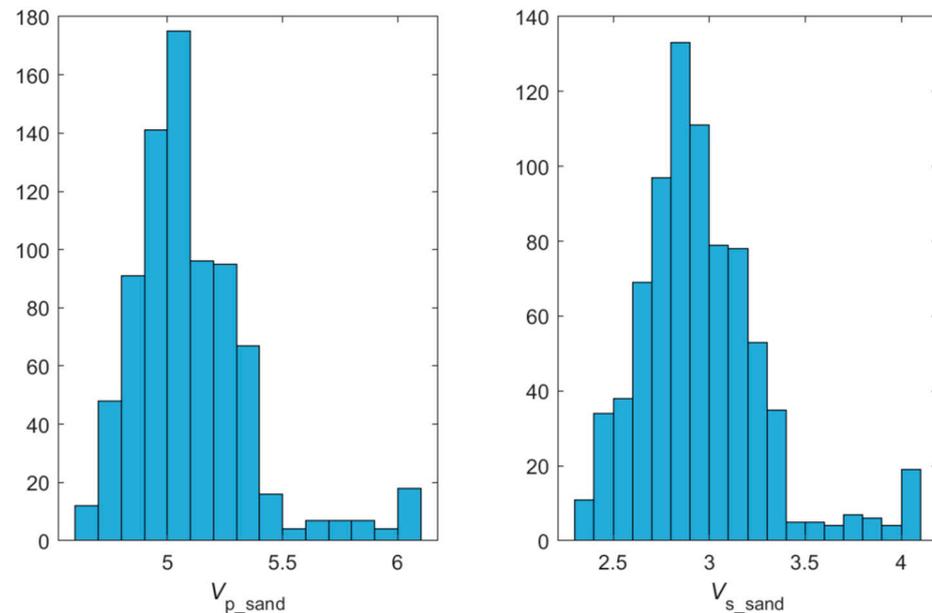


Figure 4. Prior information of statistical sand velocity in the reference section.

3.3. Results and Discussion

In order to verify the accuracy of the method, this paper uses Han’s empirical formula and the traditional Xu–White model to predict the shear wave velocity. Its results are compared to the inversion results of the statistical petrophysical model based on the Xu–White model and quantitatively evaluated by the mean square error (MSE) and correlation coefficient (r). The evaluation formulas are as follows:

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^N (\tilde{V}_i - V_i)^2 \quad (43)$$

$$r = \frac{\text{cov}(\tilde{V}, V)}{\sigma_{\tilde{V}} \sigma_V} \quad (44)$$

In the equation, i is the depth point location, \tilde{V}_i and V_i are the predicted and actual velocities, N is the total number of depth points, $\text{cov}(\cdot)$ is the covariance, and σ is the standard deviation of the velocities.

The prediction results of the three methods for the target well section are shown in Figures 5–7, and their error evaluations are shown in Table 2.

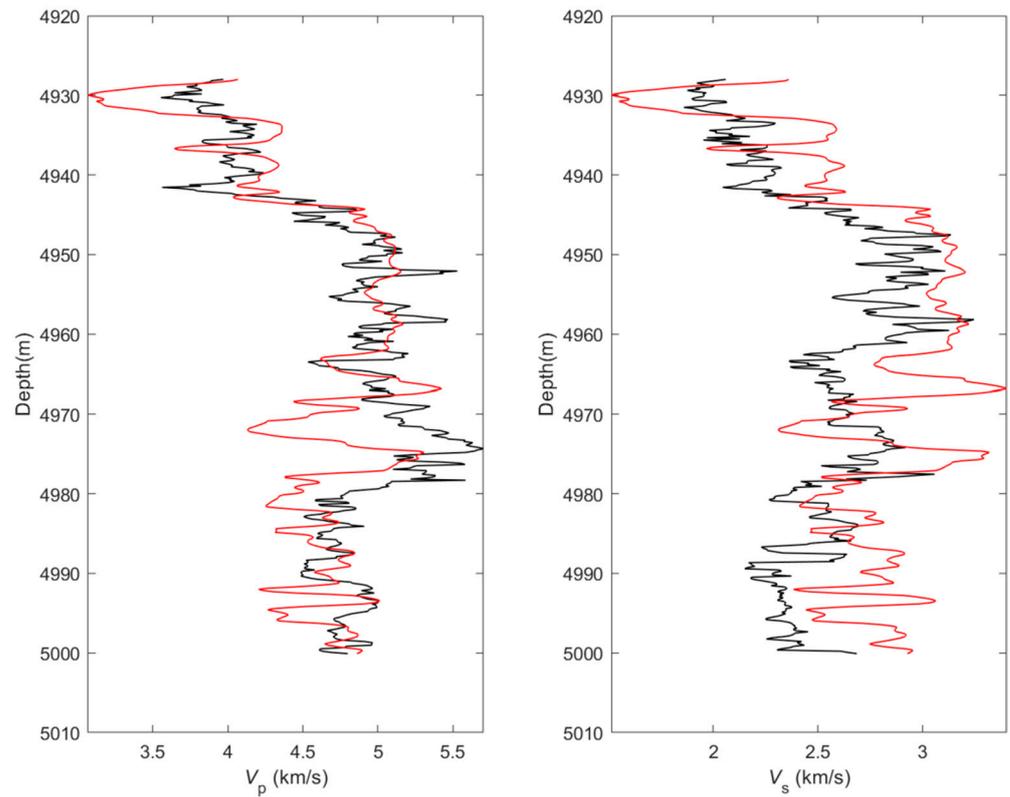


Figure 5. The prediction of P-wave and S-wave velocities based on Han's empirical formula (red is the predicted velocity, black is the actual velocity).

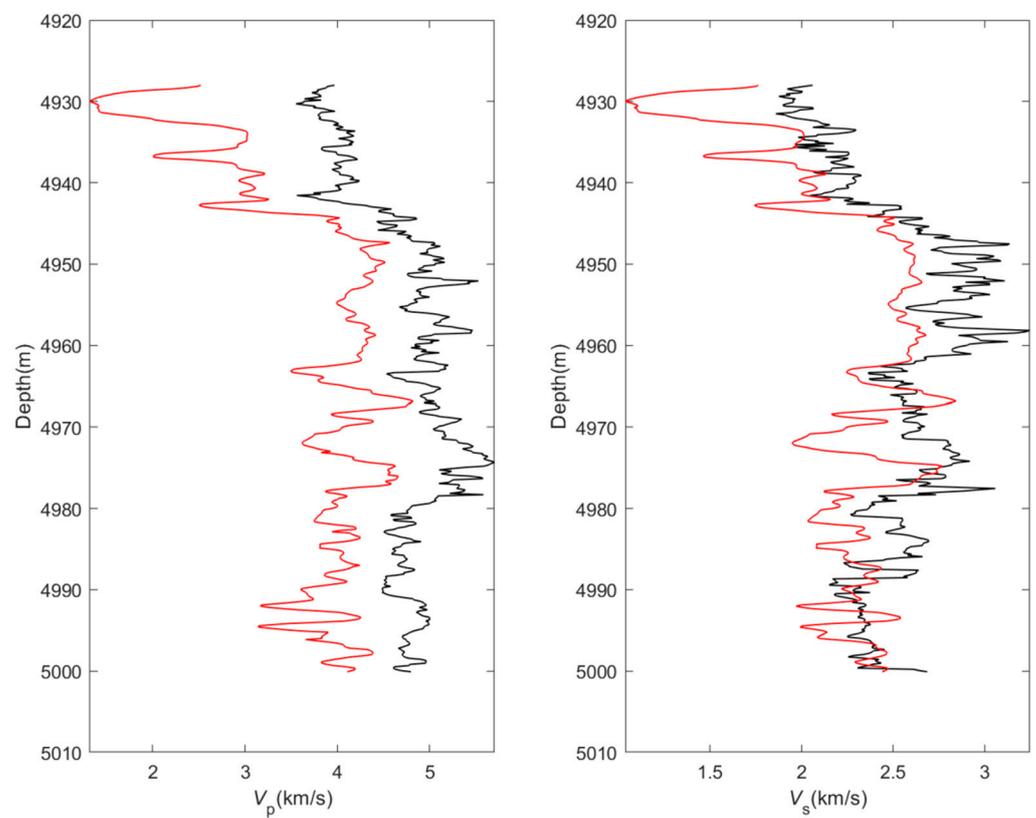


Figure 6. The prediction results of P-wave and S-wave velocities based on the Xu-White model (red is forecast speed, black is actual speed).

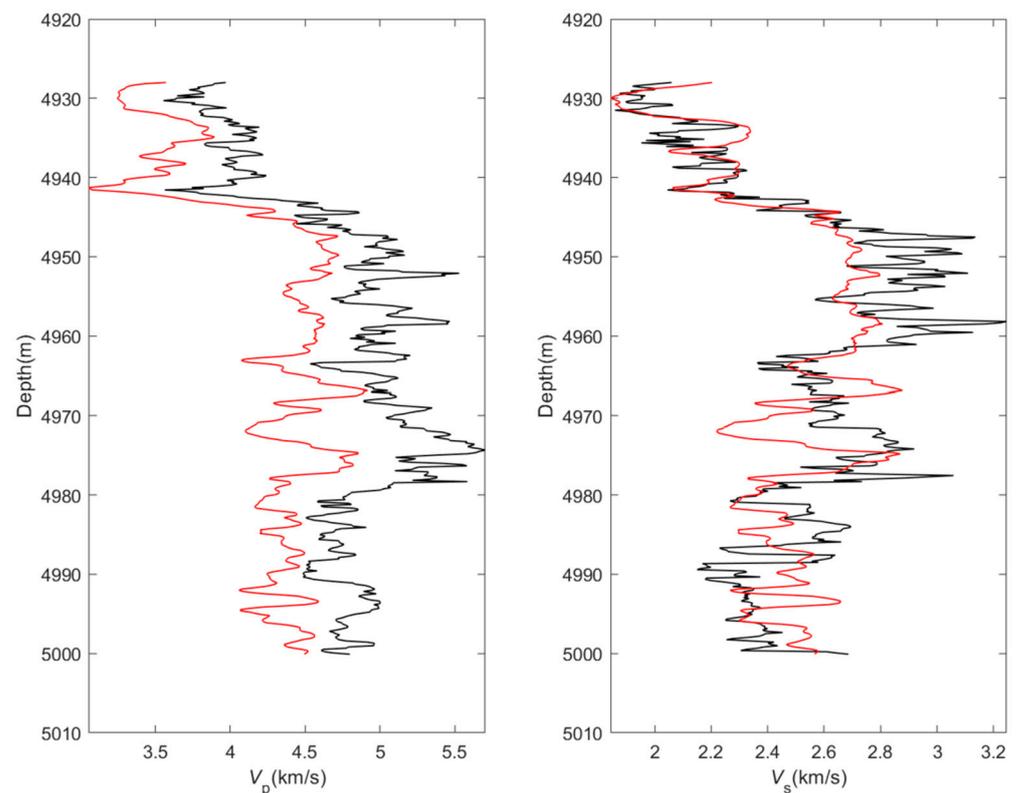


Figure 7. The prediction results of P-wave and S-wave velocities based on the improved Xu–White model (red is forecast speed, black is actual speed).

Table 2. The mean square error (MSE) and correlation coefficient (r) of V_p and V_s predicted by different models.

Model	MSE of V_p	MSE of V_s	r of V_p	r of V_s
Han Model	0.060199	0.12804	0.70823	0.70995
Xu–White Model	0.21499	0.11607	0.80821	0.73465
Improved Xu–White Model	0.05241	0.061259	0.84713	0.74737

Comparing the above results, the validity of the method in this paper applied to the prediction of shear wave velocity in the logging area of volcanic reservoirs is verified. The prediction results based on the Han empirical formula are shown in Figure 5. This method is based on the statistical fitting of a large number of S-waves and P-waves, densities information. The prediction results based on Han’s empirical formula have a good correlation and small global error. However, there are large local errors in the 4930–4940 m and 4960–4980 m sections of the well. The prediction of shear wave velocity based on Han’s empirical formula is worse than the other two methods. The mean square error of the predicting P-wave velocity is less than that of the predicting P-wave velocity based on the Xu–White model. The prediction results based on the Xu–White model are shown in Figure 6. The method uses the constant parameter initialization model. The correlation of prediction results based on the Xu–White model is greatly improved compared to the Han empirical formula method, and the prediction results of shear wave velocity are also improved. Because the influence of rock properties changing with formation pressure is not considered, and the mineral composition in silicate melt inclusion is difficult to determine, there is a deviation in the prior information of sand velocity provided in the reference well section. Because the sand velocity in the target well section is the mean value of the sand velocity in the reference well section, the global error of predicting P-wave velocity is large. The statistical petrophysical modelling method based on the improved Xu–White model

gives us the best prediction results. As shown in Figure 7, the prediction results fit the measured data well, the correlation has been further improved, the global and local errors are small, and the accuracy of the predicted S-wave velocity and P-wave velocity has been significantly improved compared to the traditional Xu–White model method. The results of actual data calculation verify the accuracy and effectiveness of the statistical petrophysical model based on the improved Xu–White model for predicting shear wave velocity.

4. Conclusions

This paper presents a prediction method of shear wave velocity in the conventional logging area of a volcanic reservoir based on an improved Xu–White model and introduces the characteristics of a volcanic reservoir, the establishment steps of the improved Xu–White model, and the workflow of shear wave velocity calculation. The method utilizes prior distributions of key petrophysical parameters to initialize the Xu–White model and constructs a statistical petrophysical model which improves the accuracy of the results when applied to conventional logging. The accuracy of prior information depends on the richness of the logging data from the reference well, as well as the core data. In addition, the similar rock properties of the target and reference sections can also ensure the accuracy of prior information. In the framework of the Bayesian inversion, combined with these key petrophysical parameters and statistical petrophysical models, the shear wave velocity information of a volcanic reservoir is predicted, improving the accuracy of prediction results and solving the problem of excessive logging data and core data being required by traditional methods. Compared to the prediction results of traditional methods, the effectiveness of the proposed method is proved. This method effectively improves the accuracy of shear wave velocity prediction in the conventional logging of volcanic rocks. The accurate S-wave velocity information provided by this method can provide important parameters for subsequent volcanic reservoir fluid attribute identification, reservoir characteristic evaluation, fracture distribution identification, pre-stack seismic inversion, and AVO analysis.

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