CSChE Systems & Control Transactions

Open Access | Original Research

Conference Proceedings
Canadian Chemical Engineering Conference 2021
October 24-27, Montréal, Québec, Canada

PSEcommunity.org/LAPSE:2021.0796

PSE PRESS

Adaptive State Feedback Stabilization of Generalized Hamiltonian Systems with Unstructured Components

Seyedabbas Alavi, Nicolas Hudon*

Queen's University at Kingston, Department of Chemical Engineering, Kingston, Ontario, Canada
* Corresponding Author: nicolas.hudon@queensu.ca.

ABSTRACT

This paper considers the problem of adaptive state feedback controller design for stabilizing the generalized Hamiltonian systems with unstructured components. This class of models enables one to exploit the dissipative-conservative structure of generalized Hamiltonian systems for feedback control design while relaxing the burden of deriving an exact structured model representation. First, an efficient adaptation law is designed such that a correct value of parameters is estimated. Assuming that the overall system is stabilizable, and under mild assumptions on the unstructured part of the dynamics, a stabilizing adaptive control law is designed to stabilize systems to the desired steady-state. The stability of the closed-loop system is demonstrated using Lyapunov stability arguments. A numerical illustration of the proposed approach is presented to demonstrate the potential of the design method.

Keywords: Adaptive stabilization, Hamiltonian system, parameter estimation, Lyapunov stability

Date Record: Original manuscript received October 20, 2021. Published October 21, 2021

INTRODUCTION

Port-Hamiltonian (pH) systems (or generalized Hamiltonian systems) have attracted the attention of many researchers in recent years as a modelling framework useful for feedback control design and analysis. The structured properties of this system facilitate the stability and tracking controller design for dynamical systems [1,2]. In applications, however, controller performance and stability properties can be lost due to model uncertainties, noise in sensors readings, input disturbance, and parameter uncertainties. As well, for a given nonlinear system, it is often difficult to compute an exact (or standard) generalized Hamiltonian representation.

Studies on feedback stabilization of approximate Hamiltonian systems were presented in [3,4] by exploiting a homotopy-based decomposition. This approach exploits the geometric structure of the anti-exact part of the dynamics. In the present note, we follow this general approach to generalized Hamiltonian systems stabilization by considering the problem of adaptive state feedback stabilization of control affine nonlinear systems where one part of the dynamics can be represented as a structured system with quadratic Hamiltonian generating function and the remaining of the dynamics is unstructured. Using this approximate representation, and under mild assumptions on the stabilizability of the overall dynamics and on the norm of the unstructured dynamics, an adaptive stabilizing state feedback law is designed based on the structured part of the dynamics.

PROBLEM FORMULATION

We consider control affine systems with parametric uncertainties, i.e.,

$$\dot{x} = f(x, \theta) + g(x)u, \tag{1}$$

where $\theta \in R^l$ is the constant parametric uncertainty.

Assumption 1: System (1) is locally stabilizable for fixed parameter θ, i.e.,

$$span\{f(x), ad_f g(x), \cdots, ad_f^k g(x) | \forall x \in \{D\}\backslash\{0\}, k \in Z_+\} = \mathbb{R}^n, \tag{2}$$

where ad_f^k is the Lie bracket operator [5].

Following [6], it is possible to express system \eqref{eq:affpar} as a pH system with dissipation

$$\dot{x} = [J(x, \theta) - R(x, \theta)]\nabla H(x, \theta) + g(x)u. \tag{3}$$

However, finding an exact transformation to express system (1) as the generalized Hamiltonian system (3) is not always possible, therefore, we consider the parameter-dependent version of a generalized Hamiltonian system with unstructured dynamics

$$\dot{x} = [J(x, \theta) - R(x, \theta)]\nabla H(x) + \Psi(x, \theta) + g(x)u, \tag{4}$$

where the generating Hamiltonian function is selected to be quadratic and not a function of the unknown parameters, i.e.,

$H(x) = \frac{1}{2}x^T x$. This can be achieved by choosing a positive definite $R(\cdot)$ and skew-symmetric $J(\cdot)$ and leaving the rest as the unstructured part of the dynamic $\Psi(x,\theta)$s.

In the sequel, we demonstrate that the proposed representation can be exploited in the derivation of a suitable state feedback adaptive stabilizing control law, under mild assumptions on the unstructured part of the dynamics Ψ. The key assumption is that the system (4) (equivalently system (1)) is stabilizable for fixed unknown parameters θ. Moreover, we make the following assumption of the unstructured component of the dynamics $\Psi(x,\theta)$.

Assumption 2: The unstructured vector $\Psi(x,\theta)$ has Lipschitz property in x and θ, and meets the inequality

$$|\Psi(x,\theta)| \le q(x,\theta)|R(x,\theta)\nabla H(x)| \quad \forall x \in \mathcal{D}\backslash\{0\}, \qquad (5)$$

where $\mathcal{D} \subset \mathbb{R}^n$ is a domain of interest centred at the origin and the state-dependent bound is bounded locally by a constant, i.e., $q(x,\theta) \le \mathrm{q}$ on $\mathcal{D}$.

In other words, the contribution of the unstructured part of the dynamics must be bounded by the natural dissipation of the system, encoded in the structured part of the dynamics.

Following [7-9], we state the following assumption on the drift vector.

Assumption 3: There is a vector function $P(x) \in R^{n\times l}$ and $Q(x) \in R^n$ such that

$$[J(x,\theta) - R(x,\theta)]\nabla H(x) = P(x)\theta + Q(x), \qquad (6)$$

where θ is a l dimensional constant parameter vector.

This assumption can be, up to a re-parameterization, met in practice, at least locally.

ADAPTIVE STABILIZING CONTROLLER DESIGN

Although we know the unstructured component, we do not use it in development of the controller. The main goal is to use the strucutured component along with the *information* about the unstructured component for stability analysis. We consider the Hamiltonian function $H(x) = \frac{1}{2}x^T x$, hence, $\nabla H(x) = x$. Using Assumption 3, we can write the system (4) as

$$\dot{x} = P(x)\theta + Q(x) + \Psi(x,\theta) + g(x)u, \qquad (7)$$

We do not have access to the true value of parameters for the controller formulation. To develop a feedback adaptive stabilizing control law, we consider the estimation $\hat{\theta}$ of the parameters. The estimated parameters is used to generate an auxiliary system as given below

$$\dot{\check{x}} = P(x)\hat{\theta} + Q(x) + \Psi(x,\hat{\theta}) + \Lambda(x - \check{x}) + g(x)u, \qquad (8)$$

where $\Lambda \in R^{n\times n}$ is a positive square diagonal matrix with entries to be assigned freely. We use the information generated by this auxiliary system to obtain a precise value for the unknown parameter. We define the estimation error by $\bar{x} = x - \check{x}$. Consequently, we obtain the following estimation error dynamic $\dot{\bar{x}} = \dot{x} - \dot{\check{x}}$ given by

$$\dot{\bar{x}} = P(x)\tilde{\theta} + \Phi - \Lambda(x - \check{x}), \qquad (9)$$

where $\Phi = \Psi(x,\theta) - \Psi(x,\hat{\theta})$, and the parameter estimation error is denoted by $\tilde{\theta} = \theta - \hat{\theta}$. One important requirement in parameter estimation is the persistent excitation in closed-loop, which is provided by making the reference signal rich enough. This requirement, however, could be *weakened* in nonlinear systems since the inherent nonlinearity of the system improves the system excitation, thus enhancing parameter convergence [10].

We are considering the case that the unknown parameter appears in the unstructured part of the dynamic. Since we do not use the unstructured component in the controller design, we need to make the reference signal rich enough to guarantee the Persistent Excitation (PE) condition. This allows recovering the true value for the unknown parameter θ.

Adaptive stabilization at the non-zero steady-state

We firstly present the results for the stabilization of the system at non-zero equilibrium points. The Hamiltonian function, centred at x^*, is expressed by.

$$H(x - x^*) = \frac{1}{2}(x - x^*)^T(x - x^*), \qquad (10)$$

We consider the controller of the form

$$u = \alpha(x, d, \hat{\theta}) + K_I(x)\xi$$
$$\dot{\xi} = -K_I^T(x)g^T(x)\nabla H(x - x^*) \qquad (11)$$
$$\dot{\hat{\theta}} = \Gamma P^T(x)(x - \check{x}) + \Gamma[P^T(x) - P^T(x^*)]\nabla H(x - x^*),$$

where $K_I(x) \in \mathbb{R}^{m\times m}$ is a positive matrix, $\xi \in \mathbb{R}^m$ is the controller dynamic, $P(x)$ is the matrix function satisfying Assumption 3 and α is going to be designed below.

The adaptation performance and parameter estimation strongly depends on the value of the $P(x)$ around the desired value x^*. As long as this value is non-zero, correct parameter estimation is guaranteed. Otherwise, we need to excite the system in order to generate sufficient information for adaptation law. This is more important for stabilization of the system at the origin since in many cases, the entries of the matrix $P(x)$ become zero around the origin. Consequently, the adaptation evolution stops before the estimated parameter settles on the true values. Following [10,11] and to achieve the desired performance, we add the dither signal $d(t)$ to the desired set-point to make the reference signal rich enough. In order to achieve the stabilization at the same time, we consider a decaying excitation signal as given below

$$d(t) = e^{-at} b \sin(\omega t), \qquad (12)$$

where a, b and c are scalar values. The following Proposition states the asymptotic stability of the system (4) at desired non-zero constant equilibrium points x^* when we add the integral action to the system. Because of the space limitation, the proofs are omitted

Proposition 1: Consider the non-exact generalized Hamiltonian system (4) with Hamiltonian function (10), interconnected with controller (11), where

$$\alpha = -[g^T(x^*)g(x)]^{-1}g^T(x^*)[\hat{A} + K_p R(x,\hat{\theta}))\nabla H(x - x^*) - g(x^*)u^* - K_d d(t)], \qquad (13)$$

where $\hat{A} = \Delta\hat{J} - \Delta\hat{R}$ and $\Delta J = J(x,\hat{\theta}) - J(x^*,\hat{\theta})$ and $\Delta R = R(x,\hat{\theta}) - R(x^*,\hat{\theta})$. $K_p, K_d \in \mathbb{R}^{n\times n}$ are positive constant gain matrix. We also assume the squared matrix $g^T(x)g(x)$ has full rank m for all $x \in \mathcal{D}$. Under local stabilizability condition (2), and assuming that the unstructured component $\Psi(x,\theta)$ meets the conditions of Assumption 2, the trajectory of system (4) is asymptotically stabilized at x^*.

Remark 1: The parameter convergence speed is manipulated by three factore: 1- nonlinearity of matrix $P(x)$ is, 2- entries values of matrix Λ, and 3- the gain Γ.

Remark 2: In this study, the auxilary system $\dot{\breve{x}}$ generates extra information to assist the adaptation law in finding the exact parameter value. As well, we can tune the gains to achieve the desired performance in closed-loop. As a result, we can *relax* the external excitation requirement for the estimation.

Adaptive stabilization at the zero steady state

The Hamiltonian function, centered at the origin, is expressed by

$$H(x) = \frac{1}{2}x^T x, \tag{14}$$

We consider the controller of the form

$$\begin{aligned} u &= \alpha(x,d,\hat{\theta}) + K_I(x)\xi \\ \dot{\xi} &= -K_I^T(x)g^T(x)\nabla H(x) \\ \dot{\hat{\theta}} &= \Gamma P^T(x)(x-\breve{x}), \end{aligned} \tag{15}$$

where $H(x) = \frac{1}{2}x^T x$, $K_I(x) \in \mathbb{R}^{m\times m}$ is a positive matrix, $\xi \in \mathbb{R}^m$ is the controller dynamic, $P(x)$ is the matrix function satisfying Assumption 3 and α is going to be designed below. The following Proposition states the asymptotic stability of the system (4) at $x^* = 0$ when we add the integral action to the system. The proof is omited to save on space.

Proposition 2: Consider the non-exact generalized Hamiltonian system (4) with Hamiltonian function (14), interconnected with controller (15), where

$$\begin{aligned} \alpha = -[g^T(x)g(x)]^{-1}g^T(x)\big[K_p R(x,\hat{\theta})\nabla H(x) \\ - K_d d(t)\big], \end{aligned} \tag{16}$$

with $K_p, K_d \in \mathbb{R}^{n\times n}$ are positive constant gain matrix. We also assume the squared matrix $g^T(x)g(x)$ has full rank m for all $x \in \mathcal{D}$. Under local stabilizability condition (2), and assuming that the unstructured component $\Psi(x,\theta)$ meets the conditions of Assumption 2, the trajectory of system (4) is asymptotically stabilized at the origin.

We illustrate the construction above for the stabilization of following nonlinear system at the origin.

ILLUSTRATIVE EXAMPLE: ISOTHERMAL VAN DE VUSSE REACTION

To illustrate the constructions on non-zero steady state stabilization, we consider the van de Vusse reaction. The main reaction involves the transformation of cyclopentadiene (component A) to the product cyclopentanol (component B). A parallel reaction takes place producing the by-product dicyclopentadiene (component D). Furthermore, cyclopentenol reacts again giving the undesired product cyclopentandiol (component C). All these reactions may be described by the reaction scheme:

$$\begin{gathered} A \rightarrow B \rightarrow C \\ 2A \rightarrow D \end{gathered}$$

The main advantage is to obtain desirable component which is component B. We focus on the two-dimesional isothermal van de Vusse system where the steady state concentration of components depend on the dilution rate Di. We first focus on the two-dimesional isothermal van de Vusse system. The goverging equations for this system are given as

$$\begin{aligned} \dot{C}_A &= -k_1 C_A - k_3 C_A^2 + Di(C_{A0} - C_A) \\ \dot{C}_B &= k_1 C_A - k_2 C_B - DiC_B \end{aligned} \tag{17}$$

where $k_i(T_{in}) = k_{0_i}\exp\left(\frac{E_i}{RT_{in}}\right)$. The numerical values of the process parameters are given in Table 1.

Table 1: van de Vusse reaction numerical values

C_{A0}	5 mol/l
T_{in}	403.15 K
Di	15 l/hr
C_p	3.01 kJ/(kg.K)
ρ	0.9434 kJ/mol
ΔH_1	4.2 kJ/mol
ΔH_2	-11 kJ/mol
ΔH_3	-41.85 kJ/mol
k_{10}	1.287 e 12 l/(mol.hr)
k_{20}	1.287 e 12 l/(mol.hr)
k_{30}	9.043 e 9 l/(mol.hr)
E_1/R	-9758.3 K
E_2/R	-9758.3 K
E_3/R	-8560 K

By setting $C_A = x_1$, $C_B = x_2$ and $Di = u$, we obtain

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-k_1x_1 - k_3x_1^2 \\ k_1x_1 - k_2x_2\end{bmatrix} + \begin{bmatrix}x_{10} - x_1 \\ -x_2\end{bmatrix}u \tag{18}$$

To check the stabilizability Assumption 1, we analyze the rank condition of the matrix $S = [f(x), ad_f g(x)]$ which is always 2 for all $x \in \mathcal{D}$ except at the origin $x_1 = x_2 = 0$; therefore, the system is stabilizable.

Application of the main results

A possible representation of the van de Vusse system is given using the Hamiltonian function

$$H(x) = \frac{1}{2}(x_1^2 + x_2^2),$$

the structured matrix

$$J(x) = \begin{bmatrix}0 & -k_1\\ k_1 & 0\end{bmatrix}, \quad R(x) = \begin{bmatrix}k_1 + k_{3x_1} & 0\\ 0 & k_2\end{bmatrix}$$

leaving the unstructured part of the dynamics

$$\Psi(x,\theta) = \begin{bmatrix}k_1x_2\\ 0\end{bmatrix}$$

We can find η and q such that the unstructured component has Lipschitz property and satisfies Assumption 2. The detailed calculations are omitted due to space limitations. We consider two cases.

k_3 is unknown

Assuming k_3 as the unknown parameter, we can find

$$P(x) = \begin{bmatrix}-x_1^2\\ 0\end{bmatrix}, \quad Q(x) = \begin{bmatrix}-k_1x_1\\ k_1x_1 - k_2x_2\end{bmatrix} \tag{19}$$

For this case, equation (9) becomes

$$\dot{\tilde{x}} = \begin{bmatrix}-x_1^2\\ 0\end{bmatrix}\tilde{k}_3 - \begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}\begin{bmatrix}x_1-\breve{x}_1\\ x_2-\breve{x}_2\end{bmatrix}, \qquad (20)$$

Initially, the reactor is operating at a steady-state of $x^* = (0.92, 0.73)$, which corresponds to $u^* = 10(1/hr)$. The reactor temperature is fixed at $403.15\,K$ and at this temperature $k_i = (39.57, 39.57, 5.47)$. We seek to reach the optimum steady-state, where C_B is maximized. As the feeding rate D increases, the concentration of A and B rises simultaneously, however, increasing the D beyond a certain range of values (roughly $20(1/hr)$) does not noticeably raise C_B up (compared to C_A). On the other side, raising D results in higher operational expenses. Hence, we select $x^* = (1.49, 0.98)$ as our optimal steady-state which is corresponding to $D = 20(1/hr)$.

Simulations are performed for initial condition for intial condition $x_0 = (2,1)$ and after selecting $\lambda_1 = 2, \lambda_2 = 7, K_p = [1 \quad 0; 0 \quad 1]$ and $k_i = 4$.

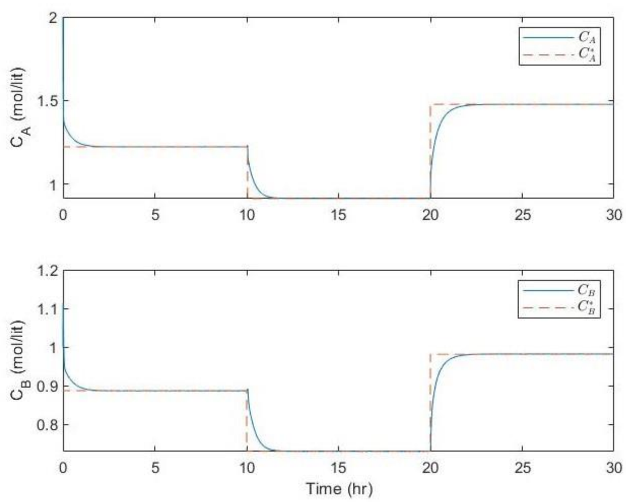

Figure 1. Closed-loop trajectory response for isothermal van de Vusse reaction when $\theta = k_3$.

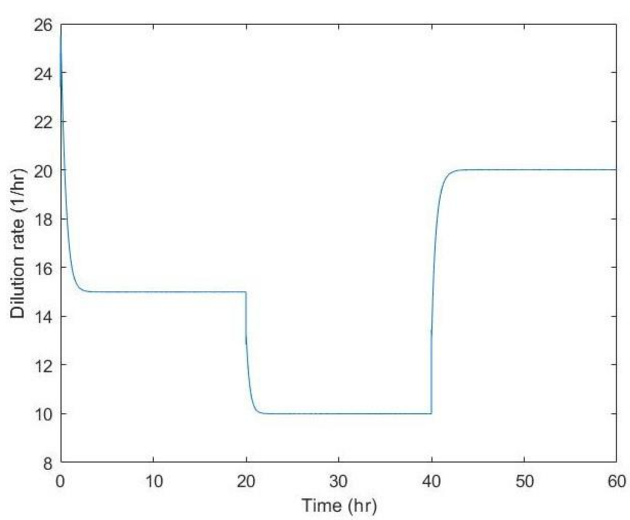

Figure 2. Adaptive controller output value for isothermal van de Vusse reaction when $\theta = k_3$.

Based on the obtained plots, the dynamic controller is able to stabilize the system and desired equilibrium points.

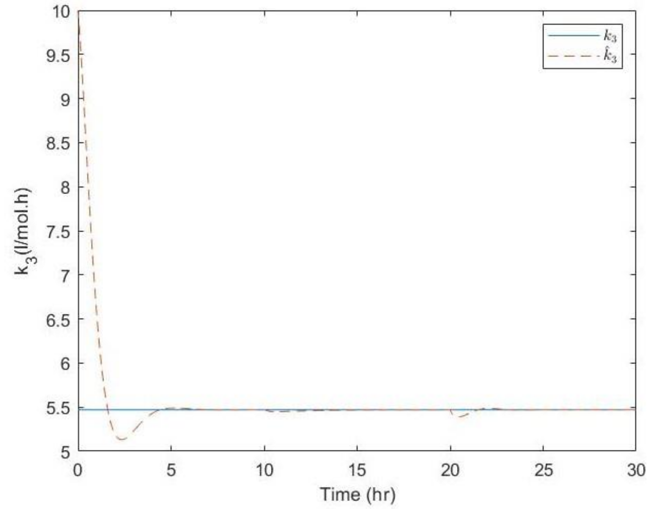

Figure 3. Closed-loop estimation response for k_3 in isothermal van de Vusse reaction when $\theta = k_3$.

Figure 1 shows that the system trajectories converge to the desired steady state. The dynamics of control input and parameter estimation responsed are given in Figure 2 and 3.

k_1, k_2 is unknown

Matrices P and Q changes to

$$P(x) = \begin{bmatrix}-x_1 - x_2 & 0\\ x_1 & -x_2\end{bmatrix}, \qquad Q(x) = \begin{bmatrix}-k_3 x_1^2\\ 0\end{bmatrix} \qquad (21)$$

Controller and trajectories responses are similar to Figures 1 and 2. Figure 4 illustrates the parameter estimation response after selecting $\lambda_1 = 2$, $\lambda_2 = 2$.

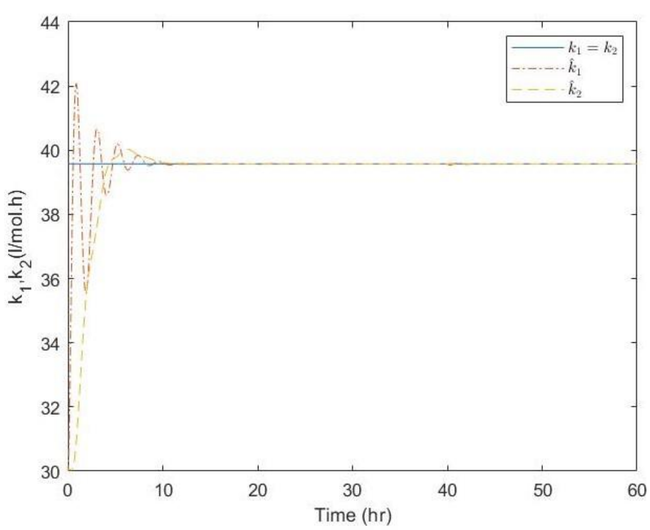

Figure 4. Closed-loop estimation response for k_1 and k_2 in isothermal van de Vusse reaction.

CONCLUSION

In this paper, we considered the problem of adaptive state feedback stabilizing controller design for a class of generalized Hamiltonian systems with unstructured component.

First, an adaptation law is designed to give an exact estimate of the parameter value. Then, a stabilizing state feedback controller is proposed using the Lyapunov stability criterion. Under a mild condition on the unstructured dynamics related to the natural dissipation of the generalized Hamiltonian system, it is shown that the overall closed-loop system is stable at the desired equilibrium point. Using this approach, a simpler controller can be designed by exploiting the generalized Hamiltonian structure without solving matching equations. Finally, the results are validated by the numerical simulation of isothermal Van de vusse reactions. Further work is underway to deal with the case that unknown parameter apears in the unstructured component as well as the adaptive output feedback desing for the proposed classes of systems.

ACKNOWLEDGEMENTS

This research is supported by the Natural Science and Engineering Research Council of Canada (NSERC) via the Discovery Grant program.

REFERENCES

1. Maschke B, Van der Schaft A. Port-Controlled Hamiltonian systems. *Modelling origins and system theoretic properties IFAC Proceedings Volumes.* 25:13:359-365 (1992)
2. Ortega R, van der Schaft A, Mareels I, Maschke B. Putting energy back in control. *IEEE Control Systems Magazine,* 21:2:18-33 (2001)
3. Hudon N, Guay M. Constructive controller design by damping feedback stabilization of control affine systems. *Systems and Control Letters* 62:11:1009-1017 (2013)
4. Guay M, Hudon N. Stabilization of nonlinear systems via potential-based realization. *IEEE Transactions on Automatic Control* 61:4:1075-1080 (2016)
5. Khalil H. Nonlinear Systems. 3rd ed. Upper Saddle River, New Jersey: prentice Hall (2002)
6. Van der Schaft A. $\mathcal{L}_2$ −Gain and Passivity Techniques in Nonlinear Control. 3rd ed, Springer (2017)
7. Praly L. Adaptive nonlinear regulation: estimaton from the Lyapunov equation. *IEEE Transactions on Automatic Control* 37:6:729-740 (1992)
8. Cao Z, Hou X, Zhao W. Adaptive robust simultaneous stabilization controller with tuning parameters design for tow dissipative Hamiltonian systems, *Archives of Control Sciences* 27:4:505-525 (2017)
9. Wei A, Wang Y. Adaptive parallel simultaneous stabilization of a set of uncertain port-controlled Hamltonian systems subject to actuator saturation, *International Journal of Adaptrive Control and Signal Processing* 28:11:1128-1144 (2014)
10. Lin J S, Kanellakopoulos I. Nonlinearities engance parameter convergence in strict feedback systems. *IEEE Transaction on Automatic Contgrol* 44:1:89-94 (1999)
11. Adelota V, Guay M. Parameter convergence in adaptive extremum-seeking control. *Automatica* 43:1:105-110 (2007)

This conference proceeding has not been peer reviewed.